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Research Paper

Mathematical Study for the Rayleigh Wave Propagation in a Composite Structure with Piezoelectric Material

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ABSTRACT

The undulated characteristics of the irregular boundaries in the layered structure with piezoelectric materials generate some prominent effects on wave propagation. On the other hand, initial stress in the layered structure also play an important role in velocity characterization of the surface seismic waves. In light of the above, this paper studies the Rayleigh-type wave propagation in a composite structure with piezoelectric materials. Mathematical expressions for the mechanical displacement and electric potential function are obtained for both the piezoelectric layer and elastic substrate with the aid of coupled electromechanical field equations. Frequency equations for the waves are derived for both electrically open and short cases. The effects of the corrugation parameters, initial stress, piezoelectric constant, dielectric constant and thickness of the piezoelectric layer on the phase velocity of Rayleigh-type wave are discussed graphically for both the electrically open and short cases. Numerical examples and discussions are made to exhibit the findings graphically. The validation of the problem is made with the classical result.

Keywords : Generalized Rayleigh-type wave; Piezoelectricity; Frequency equation; Initial stress; Corrugation.

1 INTRODUCTION

NOWADAYS, most engineering and industrial instruments are designed from specialized materials with specific properties. Piezoelectric material is that material which generates electricity (identified as piezo-electricity) in response to applied mechanical stress and displays deformation under the effect of an external electric field. Piezoelectric materials are commercially constructed in ceramics as well as single-crystal form. These materials are broadly used in engineering and technology viz., nano-science, mechanical engineering, electronic and micro-system technology, mechatronics, navigation, and also in medical appliances due to their intrinsic properties.

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In addition, the application of piezoelectric materials has also been focused on vibration and noise control, position and shape control and health monitoring systems. The important characteristics of these materials are because of their internal molecular structures for which they are also known as smart structures and are used in a myriad of applications such as sensors, oscillators, actuators, etc. Moreover, piezoelectric composite materials are broadly used in acoustic wave filters, resonators, composite transducers, etc., which has encouraged researchers in the area of smart piezoelectric composite materials. The coupling nature of piezoelectric material has gained a lot of recognition due to its vast significance in modern industrial fields and engineering applications. Elastic wave propagation in layered media made up of elastic substrate in combination with a piezoelectric layer has become a topic of interest because of some possible device applications. Some useful information about piezoelectric crystal plates may be found in Mindlin [1] and Tiersten [2]. Chen and Sun [3] explained the propagation behavior in a piezoelectric plate of two-layered structure. Romos and Otero [4] introduced wave propagation in piezoelectric layer. Mesquida et al. [5] studied the wave phenomenon in layered piezoelectric structure. An attempt has been made by Liu et al. [6] to generalize the theory of Lamb waves in functionally graded material plates. Effect of a biasing electric field on the propagation of symmetric and anti-symmetric Lamb waves in piezoelectric plates was highlighted by Liu et al. [7, 8]. Later on, Liu and Wang [9] studied the propagation of Rayleigh waves in a graded half-space. Propagation of shear waves (SH waves) in a coupled plate consisting of a piezoelectric layer and an elastic layer with initial stress has been analytically investigated by Qian et al. [10]. They solved the electromechanical field equations to derive the mechanical displacement and electrical potential function for the piezoelectric coupled plates. Du et al. [11] examined the propagation of the Love wave in a functionally graded piezoelectric layered medium and presented a pioneer result. Cao et al. [12] studied the propagation of Rayleigh waves to provide theoretical guidance in nondestructive evaluation for the analysis of the reliability and durability of electronic devices made of piezoelectric wafers. Some more recent noteworthy studies regarding wave reflection and transmission in anisotropic media are done by researchers such as, Guha et al. [13], Singh et al. [14], Saha et al. [15]. The problem related to pre-stressed piezoelectric medium has been subject to continued interest due to its importance in various applications. The initial stress exists in a medium as a result of several physical factors. These factors such as overburdened layer, variation in temperature, slow process of creep, and gravitational field, have pronounced influence on the propagation of waves as these are responsible for the evolution of a large proportion of initial stress in a medium. Son and Kang [16] showed the effect of initial stress on the propagation of SH waves in piezoelectric coupled plates. Li and Jin [17] examined the excitation and propagation of shear horizontal waves in a piezoelectric layer imperfectly bonded to a metal or elastic substrate. Saroj et al. [18] discussed the propagation behavior of Love-type waves in a functionally graded piezoelectric medium embedded amidst an initially stressed layer and elastic substrate. In most of the realistic systems, the interface of the material system may be considered as undulated in nature. The undulation present in any realistic model of the material generates an impact on the waves propagating through the material. Moreover, the configuration of the system can be considered in many forms of irregularity such as rectangular, parabolic and corrugated. One of the most important irregularities is of a corrugated type which can help to get a real insight of the considered structure. Also, the non-planarity present in the material can help for appropriate modelling and interpretation of the results. Hurd [19] made an attempt to provide the electromagnetic wave propagating constantly towards infinite corrugated surfaces. Glass and Maradudin [20] draw attention to the propagation of leaky surface waves in isotropic non-dissipative media with flat and corrugated surfaces. Singh [21] made an effort to discuss the propagation of the Love wave in a layered medium enclosed by an irregular boundary surface. Recently, Singh et al. [22] investigated the effect of corrugated boundary surface and reinforcement on the propagation of torsional surface waves. Moreover, Abd-Alla et al. [23] exercised for the Rayleigh wave propagation in an initially stressed gravitating orthotropic elastic half-space under the effect of a magnetic field. Abd-Alla and Abo-Dahab [24] investigated the Rayleigh waves in magneto-thermo-viscoelastic solid with thermal relaxation times. Abo-Dahab [25] studied the propagation of Stoneley waves in magneto-thermoelastic materials with voids and two relaxation times.

In order to make surface acoustic wave (SAW) devices more effective, most scientists and engineers adopt layer or substrate models of different material properties. During the manufacturing process of piezoelectric surface acoustic wave devices, residual stress is developed due to the different materials of layer and substrate. This stress can be used to improve the performance of SAW devices which established a biasing state. Also, at the same time, initial stress is developed unavoidably in the layer and substrate during the production of devices due to a mismatch of material properties. The presence of initial stress causes changes in the speed of piezoelectric devices. Due to these reasons, we have the motivation for the current study. The final cause of this investigation is to explore the impact of piezoelectric constants, initial stress, corrugation parameter and thickness of the layer on generalized Rayleigh-type wave velocity. The obtained results in this study are not only advantageous in the making of piezoelectric structures with high functioning but are also influential for the estimation of residual stress distribution in the layered structure.

2 FORMULATION OF THE PROBLEM

We have considered a piezoelectric layer of finite thickness "*H*" lying over an elastic substrate which is presented in Fig. 1. It is assumed that there exists initial stress in the layered structure, and the upper surface of the layer is corrugated. Usually, the thickness of the elastic substrate is much greater than that of the layer, such that the structure can be treated as a piezoelectric layered structure. Also, we have taken the coordinate system in such a way that x-axis is in the direction of wave propagation along the interface between the layer and elastic substrate and z -axis is pointing vertically downwards.

Corrugation parameters $\zeta_1(x)$ is continuous functions of *x*, independent of *y*. The functions $\zeta_j(x)$ can be taken as periodic in nature and their Fourier series expansions are given by (Singh [21]) $\zeta_1(x)=\sum_{n=1}^{\infty}\left(\zeta_n e^{inpx}+\zeta_{-n}e^{-inpx}\right)$. Here $\zeta_n(x)$ and $\zeta_{-n}(x)$ represents the coefficients for Fourier series with wave number (p), wave length $\left(\dfrac{2\pi}{p}\right)$ and *n* is the order of series. Here we have assumed the wavelength very large as compared to the amplitude.

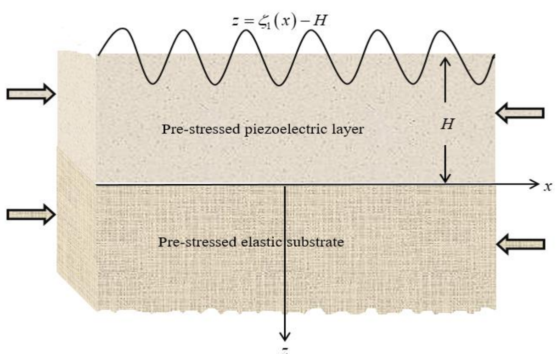


Fig.1
Schematic diagram of the problem.

3 GOVERNING EQUATIONS FOR PIEZOELECRTIC LAYER AND ELASTIC SUBSTRATE

For initially stressed piezoelectric material, the equation of motion and charge equation can be written as:

$$\left(T_{ij}+u_{j,k}\sigma_{ik}^{1}\right)_{,i}+\rho_1 f_i=\rho_1\ddot{u}_{,j}, \tag{1}$$

$$D_{i,i}=0. \tag{2}$$

where D_i and σ are the increment of electric displacement and electric charge density due to a dynamic disturbance superposed on initial state, u_i is components of displacement vector, f_i is body force per unit mass, $\sigma_{ik}{}^1$ is the stress tensor referred to initial stress, ρ_1 is the mass density of the layer. Electric displacement D_i satisfies Maxwell's equation

$$\frac{\partial D_1}{\partial x}+\frac{\partial D_2}{\partial y}+\frac{\partial D_3}{\partial z}=0. \tag{3}$$

The electric field E_1, E_2 and E_3 are given by

$$E_1 = -\frac{\partial \phi}{\partial x}, E_2 = -\frac{\partial \phi}{\partial y}, E_3 = -\frac{\partial \phi}{\partial z}. \tag{4}$$

The constitutive relations for piezoelectric material may be written as:

$$T_{11} = C_{11}\frac{\partial u}{\partial x} + C_{12}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} \tag{5}$$

$$T_{22} = C_{12}\frac{\partial u}{\partial x} + C_{11}\frac{\partial v}{\partial y} + C_{13}\frac{\partial w}{\partial z} + e_{31}\frac{\partial \phi}{\partial z} \tag{6}$$

$$T_{33} = C_{13}\frac{\partial u}{\partial x} + C_{13}\frac{\partial v}{\partial y} + C_{33}\frac{\partial w}{\partial z} + e_{33}\frac{\partial \phi}{\partial z} \tag{7}$$

$$T_{23} = C_{44}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) + e_{15}\frac{\partial \phi}{\partial y} \tag{8}$$

$$T_{13} = C_{44}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) + e_{15}\frac{\partial \phi}{\partial x} \tag{9}$$

$$T_{12} = \left(\frac{C_{11} - C_{12}}{2}\right)\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) \tag{10}$$

$$D_1 = e_{15}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial x} \tag{11}$$

$$D_2 = e_{15}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) - \varepsilon_{11}\frac{\partial \phi}{\partial y} \tag{12}$$

$$D_3 = e_{31}\frac{\partial u}{\partial x} + e_{31}\frac{\partial v}{\partial y} + e_{33}\frac{\partial w}{\partial z} - \varepsilon_{11}\frac{\partial \phi}{\partial z} \tag{13}$$

where C_{11}, C_{13}, C_{12} and C_{44} are the elastic constants, e_{15}, e_{31} and e_{33} are piezoelectric constants and ε_{11} is the dielectric constant.

Hence, the equation of motions for piezoelectric layer in the absence of body force

$$\left(C_{11} + \sigma_{11}^{1}\right)\frac{\partial^2 u_1}{\partial x^2} + \left(C_{31} + C_{44}\right)\frac{\partial^2 w_1}{\partial x\, \partial z} + \left(C_{44} + \sigma_{33}^{1}\right)\frac{\partial^2 u_1}{\partial z^2} + \left(e_{31} + e_{15}\right)\frac{\partial^2 \phi_1}{\partial x\, \partial z} = \rho_1\left(\frac{\partial^2 u_1}{\partial t^2}\right), \tag{14}$$

$$\left(C_{44} + \sigma_{11}^{1}\right)\frac{\partial^2 w_1}{\partial x^2} + \left(C_{31} + C_{44}\right)\frac{\partial^2 u_1}{\partial x\, \partial z} + \left(C_{33} + \sigma_{33}^{1}\right)\frac{\partial^2 w_1}{\partial z^2} + e_{15}\frac{\partial^2 \phi_1}{\partial x^2} + e_{33}\frac{\partial^2 \phi_1}{\partial z^2} = \rho_1\left(\frac{\partial^2 w_1}{\partial t^2}\right), \tag{15}$$

$$e_{15}\frac{\partial^2 w_1}{\partial x^2} + \left(e_{15} + e_{31}\right)\frac{\partial^2 u_1}{\partial x\, \partial z} + e_{33}\frac{\partial^2 w_1}{\partial z^2} - \varepsilon_{11}\frac{\partial^2 \phi_1}{\partial x^2} - \varepsilon_{33}\frac{\partial^2 \phi_1}{\partial z^2} = 0. \tag{16}$$

Equation of motions for the elastic substrate is given as:

$$G\left[\left(C'_{11}+\sigma_{11}^2\right)\frac{\partial^2 u_2}{\partial x^2}+C'_{13}\frac{\partial^2 w_2}{\partial x\,\partial z}+\left(C'_{44}+\sigma_{33}^2\right)\left(\frac{\partial^2 u_2}{\partial z^2}+\frac{\partial^2 w_2}{\partial x\,\partial z}\right)\right]=\rho_2\frac{\partial^2 u_2}{\partial t^2}, \tag{17}$$

$$G\left[\left(C'_{33}+\sigma_{33}^2\right)\frac{\partial^2 w_2}{\partial z^2}+C'_{13}\frac{\partial^2 u_2}{\partial x\,\partial z}+\left(C'_{44}+\sigma_{11}^2\right)\left(\frac{\partial^2 u_2}{\partial z\,\partial x}+\frac{\partial^2 w_2}{\partial x^2}\right)\right]=\rho_2\frac{\partial^2 w_2}{\partial t^2}, \tag{18}$$

$$\varepsilon'_{11}\frac{\partial^2 \phi_2}{\partial x^2}+\varepsilon'_{33}\frac{\partial^2 \phi_2}{\partial z^2}=0. \tag{19}$$

where u_1, w_1, ϕ_1 and u_2, w_2, ϕ_2 denote the mechanical displacement and the electrical potential function for the piezoelectric layer and elastic substrate, respectively. The electric potential function ϕ_0 suffices the Laplace's condition because the upper surface of the piezoelectric layer is in contact with air (vacuum) with a very low dielectric constant ε_0 .

$$\frac{\partial^2 \phi_0}{\partial x^2}+\frac{\partial^2 \phi_0}{\partial y^2}=0. \tag{20}$$

4 BOUNDARY CONDITIONS

For generalized Rayleigh-type wave propagation in the piezoelectric layer lying over an elastic substrate, the following boundary conditions are satisfied:

1. Traction will be zero at the free surface $\left(z=\zeta_1-H\right)$ i.e.

$$\left(\tau_{zx}-\zeta_1'\tau_{xx}\right)=0 \tag{21}$$

$$\left(\tau_{zz}-\zeta_1'\tau_{zx}\right)=0 \tag{22}$$

2. Electrically short condition at the free surface $\left(z=\zeta_1-H\right)$ may be expressed as:

$$\phi_1\left(-H,y\right)=0 \tag{23}$$

3. Electrically open conditions at the free surface $\left(z=\zeta_1-H\right)$ can be written as:

$$\phi_1\left(-H,y\right)=\phi_0\left(-H,y\right) \tag{24}$$

$$D_1\left(-H,y\right)=D_0\left(-H,y\right) \tag{25}$$

4. Continuity of stresses, mechanical displacement, electrical potential and electrical displacement at the interfaces $\left(z=0\right)$

$$u_1=u_2, \tag{26}$$

$$w_1 = w_2, \tag{27}$$

$$\phi_1 = \phi_2. \tag{28}$$

$$\left(\tau_{zx}\right)_1 = \left(\tau_{zx}\right)_2 \tag{29}$$

$$\left(\tau_{zz}\right)_1 = \left(\tau_{zz}\right)_2 \tag{30}$$

$$\left(D_x\right)_1 = \left(D_x\right)_2 \tag{31}$$

5 DYNAMIC OF THE PROBLEM

5.1 Solution for upper piezoelectric medium

Solution for Eqs. (14), (15) and (16) can be written as:

$$u_1 = U_1 e^{ik(x-ct)},\ w_1 = W_1 e^{ik(x-ct)} \text{ and } \phi_1 = \overline{\phi}_1 e^{ik(x-ct)} \tag{32}$$

Using Eq. (32) in Eqs. (14), (15) and (16), we obtain

$$\left(C_{44}+\sigma_{33}^{1}\right)U_1'' + \left(k^2\rho_1 c^2 - k^2\left(C_{11}+\sigma_{11}^{1}\right)\right)U_1 + ik\left(C_{13}+C_{44}\right)W_1' + ik\left(e_{31}+e_{15}\right)\overline{\phi}_1' = 0, \tag{33}$$

$$\left(C_{33}+\sigma_{33}^{1}\right)W_1'' + \left(k^2\rho_1 c^2 - k^2\left(C_{44}+\sigma_{11}^{1}\right)\right)W_1 + ik\left(C_{13}+C_{44}\right)U_1' + e_{33}\overline{\phi}_1'' - e_{15}k^2\overline{\phi}_1 = 0, \tag{34}$$

$$e_{33}W_1'' - e_{15}k^2 W_1 + ik\left(e_{15}+e_{31}\right)U_1' - \varepsilon_{33}\overline{\phi}_1'' + \varepsilon_{11}k^2\overline{\phi}_1 = 0. \tag{35}$$

On solving Eqs. (33), (34) and (35), we get

$$\left(D^6 P_1 + P_2 D^4 + P_3 D^2 + P_4\right)\left(U_1, W_1, \overline{\phi}_1\right) = 0 \tag{36}$$

where $P_1, P_2, P_3, P_4, \upsilon_1$ and υ_2 are defined in Appendix A.

Therefore, for the piezoelectric layer, mechanical displacement and electric potential function can be written as:

$$u_1 = \left(A e^{-ik\lambda_1 z} + B e^{-ik\lambda_2 z} + C e^{-ik\lambda_3 z} + A' e^{ik\lambda_1 z} + B' e^{ik\lambda_2 z} + C' e^{ik\lambda_3 z}\right) e^{ik(x-ct)}, \tag{37}$$

$$w_1 = \left(\overline{A} e^{-ik\lambda_1 z} + \overline{B} e^{-ik\lambda_2 z} + \overline{C} e^{-ik\lambda_3 z} + \overline{A}' e^{ik\lambda_1 z} + \overline{B}' e^{ik\lambda_2 z} + \overline{C}' e^{ik\lambda_3 z}\right) e^{ik(x-ct)}, \tag{38}$$

$$\overline{\phi}_1 = \left(\overline{\overline{A}} e^{-ik\lambda_1 z} + \overline{\overline{B}} e^{-ik\lambda_2 z} + \overline{\overline{C}} e^{-ik\lambda_3 z} + \overline{\overline{A}}' e^{ik\lambda_1 z} + \overline{\overline{B}}' e^{ik\lambda_2 z} + \overline{\overline{C}}' e^{ik\lambda_3 z}\right) e^{ik(x-ct)}. \tag{39}$$

where

$\lambda_1^2 + \lambda_2^2 + \lambda_3^2 = \dfrac{-P_2}{P_1}$, $\lambda_1^2\lambda_2^2 + \lambda_2^2\lambda_3^2 + \lambda_3^2\lambda_1^2 = \dfrac{P_3}{P_1}$, $\lambda_1^2\lambda_2^2\lambda_3^2 = \dfrac{-P_4}{P_1}$, $A, B, C, A', B', C', \overline{A}, \overline{B}, \overline{C}, \overline{A}', \overline{B}', \overline{C}'$,

$\overline{\overline{A}}, \overline{\overline{B}}, \overline{\overline{C}}, \overline{\overline{A}}', \overline{\overline{B}}', \overline{\overline{C}}'$ $\overline{A} = P'A$, $\overline{B} = Q'B$, are constants defined as $\overline{C} = R'C$, $\overline{A}' = SA'$, $\overline{B}' = TB'$, $\overline{C}' = VC'$, $\overline{\overline{A}} = P''A$, $\overline{\overline{B}} = Q''B$,

$\overline{\overline{C}} = R''C$, $\overline{\overline{A}}' = S'A'$, $\overline{\overline{B}}' = T'B'$, $\overline{\overline{C}}' = V'C'$ and $P', Q', R', P'', Q'', R'', S, T, V$ are defined in Appendix A.

5.2 *Solution for elastic substrate*

We consider the solution of Eqs. (17), (18) and (19) as:

$$u_2 = U_2 e^{ik(x-ct)},\ w_2 = W_2 e^{ik(x-ct)} \text{ and } \phi_2 = \overline{\phi_2} e^{ik(x-ct)} \tag{40}$$

Using Eq. (40) in Eqs. (17), (18) and (19), we obtain

$$\left(C'_{44}+\sigma_{33}^2\right)U_2'' - k^2\left(\left(C'_{11}+\sigma_{11}^2\right)-\frac{\rho c^2}{G}\right)U_2 + ik\left(C'_{13}+\left(C'_{44}+\sigma_{33}^2\right)\right)W_2' = 0, \tag{41}$$

$$C'_{33}W_2'' - k^2\left(\left(C'_{11}+\sigma_{11}^2\right)-\frac{\rho c^2}{G}\right)W_2 + ik\left(C'_{13}+\left(C'_{44}+\sigma_{33}^2\right)\right)U_2' = 0, \tag{42}$$

$$\varepsilon'_{11}k^2\overline{\phi_2} - \varepsilon'_{33}\overline{\phi_2''} = 0. \tag{43}$$

Eqs. (41) and (42) may be written as:

$$L_1 D^4 + L_2 D^2 + L_3 = 0 \tag{44}$$

where L_1, L_2, L_3 are provided in Appendix A.

Solutions of Eqs. (44) and (43) may be written as:

$$u_2 = A_1 e^{-ik\lambda_4 z} + B_1 e^{-ik\lambda_5 z}, \tag{45}$$

$$w_2 = A_1' e^{-ik\lambda_4 z} + B_1' e^{-ik\lambda_5 z}, \tag{46}$$

$$\overline{\phi}_2 = J e^{-ik\sqrt{\frac{\varepsilon_{11}}{\varepsilon_{33}}}z}. \tag{47}$$

with $\lambda_4^2 + \lambda_5^2 = \dfrac{-L_2}{L_1}$, $\lambda_4^2\lambda_5^2 = \dfrac{L_3}{L_1}$.

Hence, mechanical displacement and electric potential for the elastic substrate are

$$u_2 = \left(A_1 e^{-ik\lambda_4 z} + B_1 e^{-ik\lambda_5 z}\right)e^{ik(x-ct)}, \tag{48}$$

$$w_2 = \left(A_1' e^{-ik\lambda_4 z} + B_1' e^{-ik\lambda_5 z}\right)e^{ik(x-ct)}, \tag{49}$$

$$\overline{\phi}_2 = J e^{-ik\sqrt{\frac{\varepsilon_{11}}{\varepsilon_{33}}}z} e^{ik(x-ct)}. \tag{50}$$

where J, A_1, B_1, A_1', B_1' are constant.

$A_1' = K_1 A_1$, $B_1' = K_2 B_1$ and K_1, K_2 are defined in Appendix A.

6 FREQUENCY EQUATIONS

From the boundary conditions, we obtained the following equations

$$\begin{aligned}
&A\left[\left(-C_{44}ik\,\lambda_1+aikC_{44}+a^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}-ik\,\lambda_1 aC_{13}+a^{'}e_{31}\right)\right]e^{-ik\,\lambda_1(\zeta_1-H)}+\\
&B\left[\left(-C_{44}ik\,\lambda_2+bikC_{44}+b^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}-ik\,\lambda_2 bC_{13}+b^{'}e_{31}\right)\right]e^{-ik\,\lambda_2(\zeta_1-H)}+\\
&C\left[\left(-C_{44}ik\,\lambda_3+cikC_{44}+c^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}-ik\,\lambda_3 cC_{13}+c^{'}e_{31}\right)\right]e^{-ik\,\lambda_3(\zeta_1-H)}+\\
&A^{'}\left[\left(C_{44}ik\,\lambda_1+dikC_{44}+d^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}+ik\,\lambda_1 dC_{13}+d^{'}e_{31}\right)\right]e^{ik\,\lambda_1(\zeta_1-H)}+\\
&B^{'}\left[\left(C_{44}ik\,\lambda_2+eikC_{44}+e^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}+ik\,\lambda_2 eC_{13}+e^{'}e_{31}\right)\right]e^{ik\,\lambda_2(\zeta_1-H)}+\\
&C^{'}\left[\left(C_{44}ik\,\lambda_3+fikC_{44}+f^{'}ike_{15}\right)-\zeta_1^{'}\left(ikC_{11}+ik\,\lambda_3 fC_{13}+f^{'}e_{31}\right)\right]e^{ik\,\lambda_3(\zeta_1-H)}=0
\end{aligned}\tag{51}$$

$$\begin{aligned}
&A\left[\left(C_{13}ik-\lambda_1 aikC_{33}-a^{'}ik\,\lambda_1 e_{33}\right)-\zeta_1^{'}\left(aikC_{44}-ik\,\lambda_1 C_{44}+a^{'}ike_{15}\right)\right]e^{-ik\,\lambda_1(\zeta_1-H)}+\\
&B\left[\left(C_{13}ik-\lambda_2 bikC_{33}-b^{'}ik\,\lambda_2 e_{33}\right)-\zeta_1^{'}\left(bikC_{44}-ik\,\lambda_2 C_{44}+b^{'}ike_{15}\right)\right]e^{-ik\,\lambda_2(\zeta_1-H)}+\\
&C\left[\left(C_{13}ik-\lambda_3 cikC_{33}-c^{'}ik\,\lambda_3 e_{33}\right)-\zeta_1^{'}\left(cikC_{44}-ik\,\lambda_3 C_{44}+c^{'}ike_{15}\right)\right]e^{-ik\,\lambda_3(\zeta_1-H)}+\\
&A^{'}\left[\left(C_{13}ik+dik\,\lambda_1 C_{33}+d^{'}ik\,\lambda_1 e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_1+ikdC_{44}+d^{'}ike_{15}\right)\right]e^{ik\,\lambda_1(\zeta_1-H)}+\\
&B^{'}\left[\left(C_{13}ik+eik\,\lambda_2 C_{33}+e^{'}ik\,\lambda_2 e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_2+ikeC_{44}+e^{'}ike_{15}\right)\right]e^{ik\,\lambda_2(\zeta_1-H)}+\\
&C^{'}\left[\left(C_{13}ik+fik\,\lambda_3 C_{33}+f^{'}ik\,\lambda_3 e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_3+ikfC_{44}+f^{'}ike_{15}\right)\right]e^{ik\,\lambda_3(\zeta_1-H)}=0
\end{aligned}\tag{52}$$

$$\begin{aligned}
&P^{"}Ae^{-ik\,\lambda_1(\zeta_1-H)}+Q^{"}Be^{-ik\,\lambda_2(\zeta_1-H)}+R^{"}Ce^{-ik\,\lambda_3(\zeta_1-H)}+S^{'}A^{'}e^{ik\,\lambda_1(\zeta_1-H)}+\\
&T^{'}B^{'}e^{ik\,\lambda_2(\zeta_1-H)}+V^{'}C^{'}e^{ik\,\lambda_3(\zeta_1-H)}=0
\end{aligned}\tag{53}$$

$$\begin{aligned}
&AP^{"}e^{-ik\,\lambda_1(\zeta_1-H)}+BQ^{"}e^{-ik\,\lambda_2(\zeta_1-H)}+CR^{"}e^{-ik\,\lambda_3(\zeta_1-H)}+A^{'}S^{'}e^{ik\,\lambda_1(\zeta_1-H)}+\\
&B^{'}T^{'}e^{ik\,\lambda_2(\zeta_1-H)}+C^{'}T^{'}e^{ik\,\lambda_3(\zeta_1-H)}-A_0e^{-k(\zeta_1-H)}=0
\end{aligned}\tag{54}$$

$$\begin{aligned}
&A\left(P^{'}-\lambda_1\right)e^{-ik\,\lambda_1(\zeta_1-H)}+B\left(Q^{'}-\lambda_2\right)e^{-ik\,\lambda_2(\zeta_1-H)}+C\left(R^{'}-\lambda_3\right)e^{-ik\,\lambda_3(\zeta_1-H)}+\\
&A^{'}\left(S+\lambda_1\right)e^{ik\,\lambda_1(\zeta_1-H)}+B^{'}\left(T+\lambda_2\right)e^{ik\,\lambda_2(\zeta_1-H)}+C^{'}\left(V+\lambda_3\right)e^{ik\,\lambda_3(\zeta_1-H)}=0
\end{aligned}\tag{55}$$

$$A+B+C+A^{'}+B^{'}+C^{'}-A_1-B_1=0\tag{56}$$

$$P^{'}A+Q^{'}B+R^{'}C+SA^{'}+TB^{'}+VC^{'}-K_1A_1-K_2B_1^{'}=0\tag{57}$$

$$P^{"}A+Q^{"}B+R^{"}C+S^{'}A^{'}+T^{'}B^{'}+V^{'}C^{'}-J=0\tag{58}$$

$$\begin{aligned}
&A\left(C_{44}P^{'}-C_{44}\lambda_1+P^{"}e_{15}\right)+B\left(C_{44}Q^{'}-C_{44}\lambda_2+Q^{"}e_{15}\right)+C\left(C_{44}R^{'}-C_{44}\lambda_3+R^{"}e_{15}\right)+\\
&A^{'}\left(C_{44}S+C_{44}\lambda_1+S^{'}e_{15}\right)+B^{'}\left(C_{44}T+C_{44}\lambda_2+T^{'}e_{15}\right)+C^{'}\left(C_{44}V+C_{44}\lambda_1+V^{'}e_{15}\right)\\
&-A_1\left(K_1\left(C^{'}_{44}+\sigma_{33}^2\right)-\lambda_1^1\left(C^{'}_{44}+\sigma_{33}^2\right)\right)-B_1\left(K_2\left(C^{'}_{44}+\sigma_{33}^2\right)-\lambda_2^1\left(C^{'}_{44}+\sigma_{33}^2\right)\right)-e_{15}^{'}J=0
\end{aligned}\tag{59}$$

$$
\begin{aligned}
&A\left(C_{13}-C_{33}P^{'}\lambda_1-e_{31}P^{"}\lambda_1\right)+B\left(C_{13}-C_{33}Q^{'}\lambda_2-e_{31}Q^{"}\lambda_2\right)+C\left(C_{13}-C_{33}R^{'}\lambda_3-e_{31}R^{"}\lambda_2\right)+\\
&A^{'}\left(C_{13}+C_{33}S\lambda_1+e_{31}S^{'}\lambda_1\right)+B^{'}\left(C_{13}+C_{33}T\lambda_2+e_{31}T^{'}\lambda_2\right)+C^{'}\left(C_{13}+C_{33}V\lambda_3+e_{31}V^{'}\lambda_3\right)-\\
&A_1\left(C_{13}^{'}-C_{33}^{'}\lambda_1^{'}K_1\right)-A_2\left(C_{13}^{'}-C_{33}^{'}\lambda_2^{'}K_2\right)=0
\end{aligned}
\tag{60}
$$

$$
\begin{aligned}
&A\left(P^{'}-\lambda_1\right)+B\left(Q^{'}-\lambda_2\right)+C\left(R^{'}-\lambda_3\right)+A^{'}\left(S+\lambda_1\right)+B^{'}\left(T+\lambda_2\right)+C^{'}\left(V+\lambda_3\right)-\\
&A_1\left(C_{13}^{'}-\lambda_1^{'}K_1C_{33}^{'}\right)-B_1\left(C_{13}^{'}-\lambda_2^{'}K_2C_{33}^{'}\right)+Je_{13}^{'}\sqrt{\frac{\varepsilon_{11}}{\varepsilon_{33}}}=0
\end{aligned}
\tag{61}
$$

6.1 Frequency equation for electrically open case

Eliminate constants in Eqs. (51)-(52) and (54)-(61) and the necessary and sufficient condition for the existence of non-trivial solutions are that the determinant of the coefficient matrix of A_i 's has to vanish. For the electrically open case, we have

$$\left|A_{ij}\right|_{10\times 10}=0. \tag{62}$$

where $A_{11}, A_{12}, A_{13}, ..., A_{1010}$ are provided in Appendix A.

The above Eq. (62) is the required dispersion relation for generalized Rayleigh-type wave propagation in a piezoelectric layer overlying an elastic substrate.

6.2 Frequency equation for electrically short case

For the electrically shorted case, the number of boundary conditions is less by one than that of the electrically open case (Eq. (20) and A_0 are superfluous) as indicated in condition (24). Thus, the dispersion relation for generalized Rayleigh-type wave for the electrically shorted case is determined as follows:

$$\left|A_{ij}\right|_{9\times 9}=0. \tag{63}$$

Thus Eqs. (62)-(63) provide the relationship between phase velocity and wave number for both the cases i.e., for electrically open and short cases, where $A_{31}, A_{32}, A_{33}, A_{34}, A_{35}, A_{36}$ are defined in Appendix A and all the remaining notations will be the same as that of Eq. (62).

6.3 Special case (validation of the problem)

If we consider only isotropic half-space without initial stress, then Eq. (62) transforms to

$$\left(2-\frac{c^2}{c_2^{\,2}}\right)^2=4\sqrt{\left[\left(\frac{c^2}{c_1^{\,2}}-1\right)\left(\frac{c^2}{c_2^{\,2}}-1\right)\right]}, \tag{64}$$

where $c_1^{\,2}=\dfrac{\lambda+2\mu}{\rho}$ and $c_2^{\,2}=\dfrac{\mu}{\rho}$.

Eq. (64) is in well agreement with the classical result obtained by Rayleigh [26].

7 NUMERICAL EXAMPLES AND DISCUSSIONS

Up to now, we have obtained the expression for phase velocity, stress field and mechanical displacement for piezoelectric material. In order to demonstrate the behavior of generalized Rayleigh-type wave in the considered geometry, we have taken the following parameters from Weis and Gaylord [27] to show the effect of various parameters on wave propagation. The computational material parameters and all the material constants used in the computation are summarized in Table 1.

Table 1
The materials properties constants.

Parameters	Piezoelectric layer (PZT-5H)	Elastic substrate
$C_{11}\left(\text{Nm}^{-2}\right)$	12.1×10^{10}	2.03×10^{11}
$C_{33}\left(\text{Nm}^{-2}\right)$	11.7×10^{10}	2.424×10^{11}
$C_{44}\left(\text{Nm}^{-2}\right)$	2.34×10^{10}	0.595×10^{11}
$C_{13}\left(\text{Nm}^{-2}\right)$	8.41×10^{10}	0.752×10^{11}
$e_{33}\left(\text{cm}^{-2}\right)$	23.3	0
$e_{13}\left(\text{cm}^{-2}\right)$	-6.5	0
$e_{15}\left(\text{cm}^{-2}\right)$	17	0
ε_{33}	3400	28.7
ε_{11}	3100	85.2
$\rho\left(10^3\text{kgm}^{-3}\right)$	7.7	4.647

7.1 Effect of piezoelectric constant on frequency equation of generalized Rayleigh-type wave

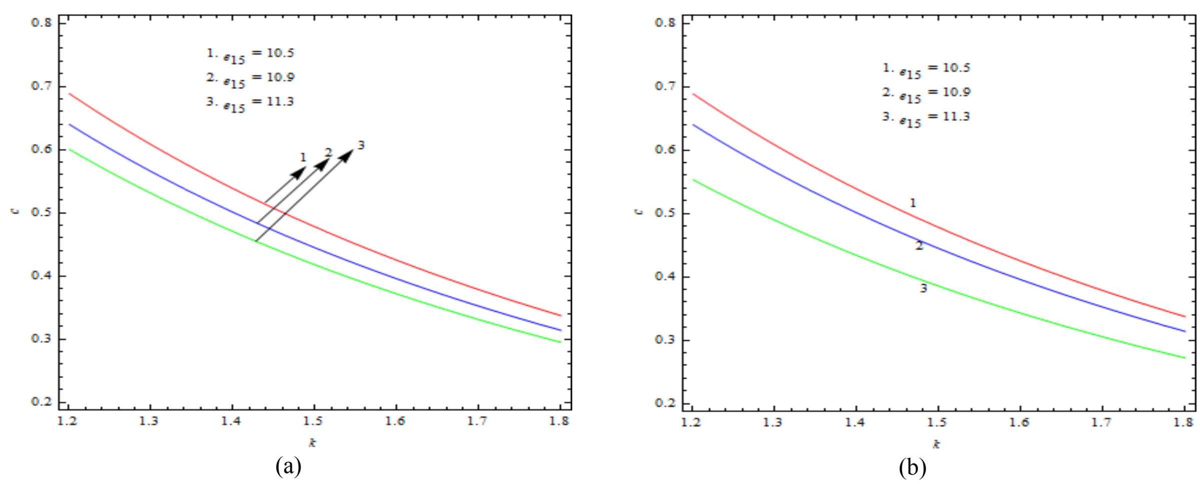


Fig.2
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of piezoelectric coefficient e_{15} of piezoelectric layer: (a) for electrically open case (b) for an electrically short case.

The effect of the piezoelectric coefficient on generalized Rayleigh-type wave velocity is taken into account. Figs. 2(a) and 2(b) represent the dispersion curves of generalized Rayleigh-type waves for different values of piezoelectric coefficient for electrically open and short cases, respectively. From both the figures, we observed that generalized Rayleigh-type wave velocity decreases with increasing values of piezoelectric coefficient for both electrically open and short cases.

7.2 Effect of dielectric coefficient on frequency equation of generalized Rayleigh-type wave

To show the effect of the dielectric coefficient on generalized Rayleigh-type wave velocity in electrically open and short cases, Figs. 3(a) and 3(b) are plotted. Curves on the graph indicate that Rayleigh-type wave velocity decrease as we increase the values of dielectric coefficient for both electrically open and short cases.

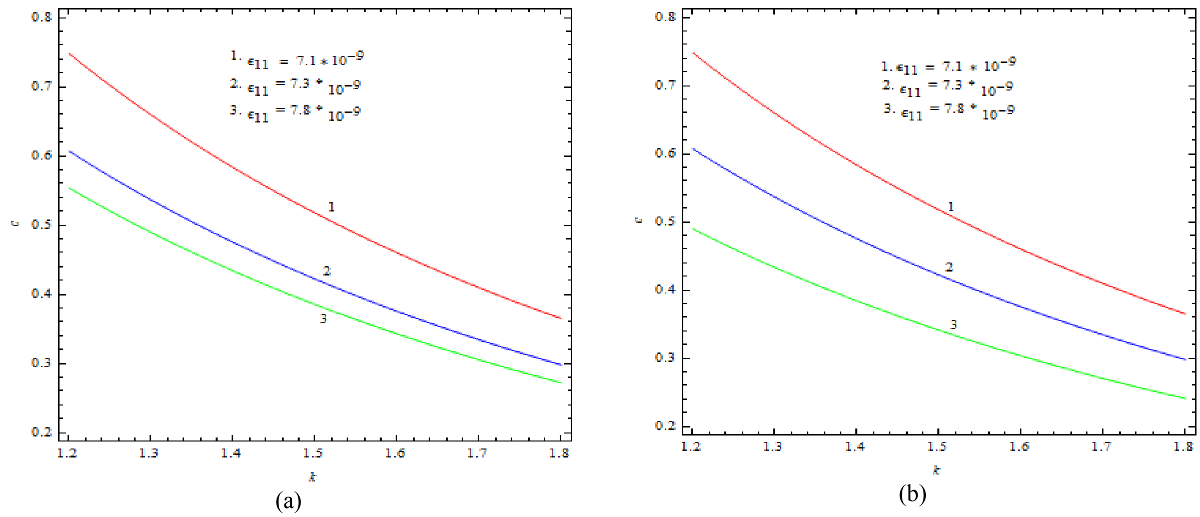


Fig.3
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of dielectric coefficient ε_{11} of piezoelectric layer: (a) for electrically open case (b) for electrically short case.

7.3 Effect of corrugation parameter on frequency equation of generalized Rayleigh-type wave

The substantial impact of the corrugation parameter of the piezoelectric layer on the frequency curves has been delineated through Figs. 4(a) and 4(b). From both the figures (Fig. 4(a) and Fig. 4(b)) it is observed that generalized Rayleigh-type wave velocity decreases with increment in the corrugation parameter of the piezoelectric layer.

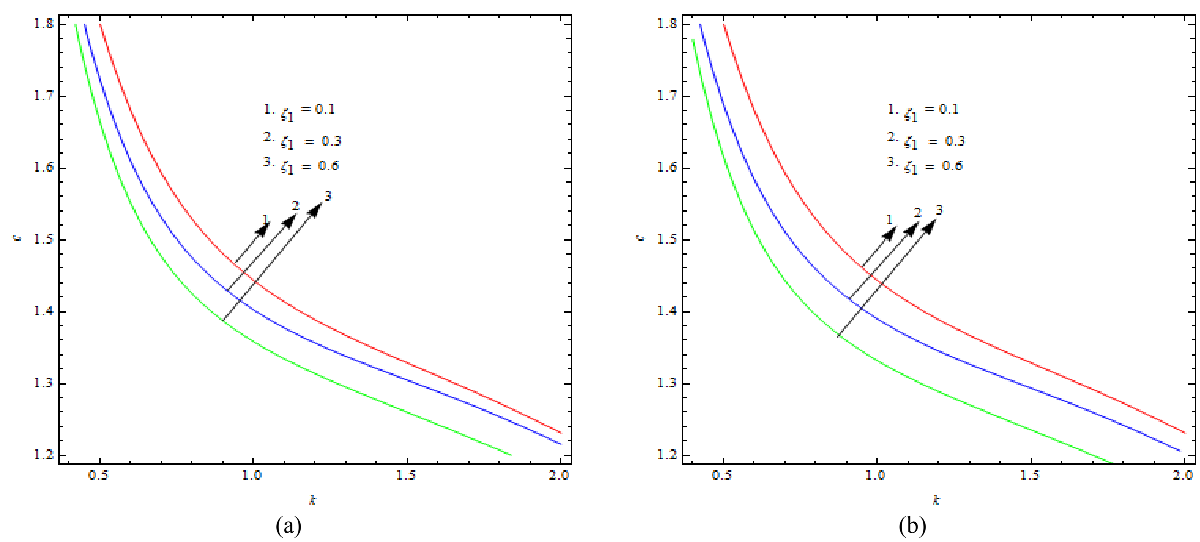
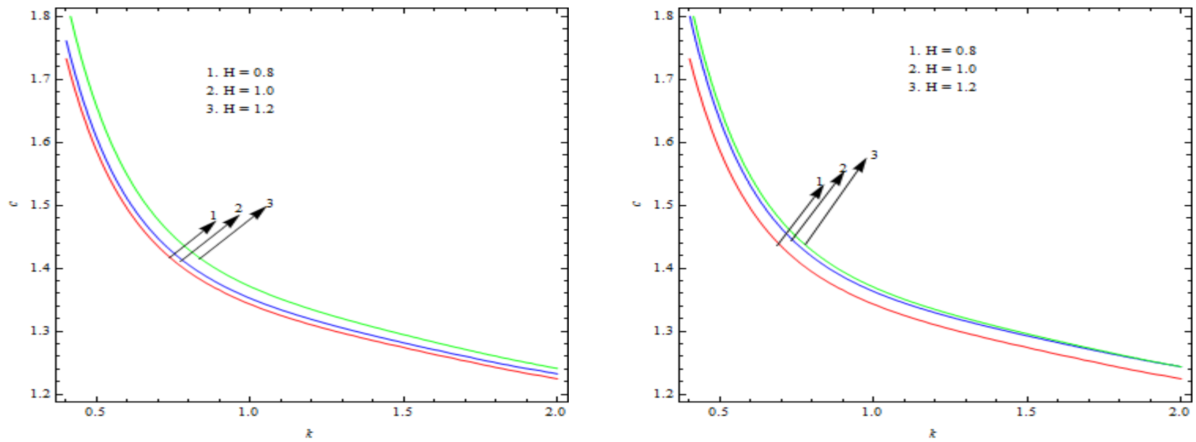


Fig.4
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of corrugation parameter ζ_1 of the upper boundary surface of piezoelectric layer: (a) for electrically open case (b) for electrically short case.

7.4 Effect of thickness of the layer on frequency equation of generalized Rayleigh-type wave

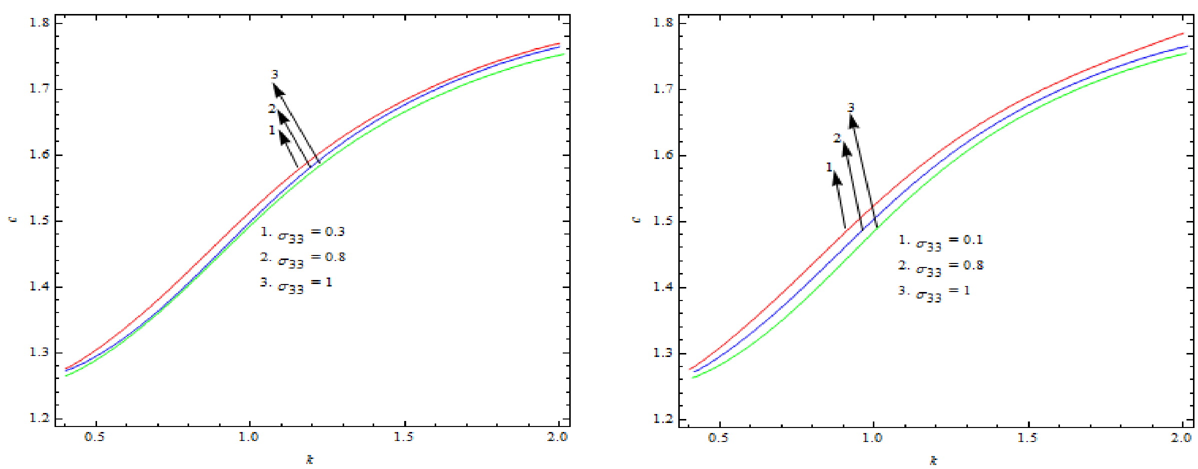


(a) (b)

Fig.5
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of H for piezoelectric layer: (a) for electrically open case (b) for electrically short case.

Figs. 5(a) and 5(b) manifest the profound effect of the height of the piezoelectric layer on the frequency curves. It seems that for electrically open and short cases, generalized Rayleigh-type wave velocity increases with increasing value of the height of the layer.

7.5 Effect of initial stress on frequency equation of generalized Rayleigh-type wave



(a) (b)

Fig.6
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of σ_{33} for piezoelectric layer: (a) for electrically open case (b) for electrically short case.

Figs. (6) to (7) manifest the profound effect of initial stress of both, the piezoelectric layer and elastic substrate on the frequency curves. The effect of initial stress on the piezoelectric layer has been represented in Fig. 6(a) and Fig. 6(b) for electrically open and short cases respectively. Both the figures (Fig. 6(a) and Fig. 6(b)) reveal that generalized Rayleigh-type wave velocity decreases with increasing the value of initial stress. Fig. 7(a) and Fig. 7(b) distinctly study the effect of initial stress of elastic substrate on the frequency curves for electrically open and short cases respectively. It is established that Rayleigh-type wave velocity increases with increasing value of initial stress for electrically open and short cases.

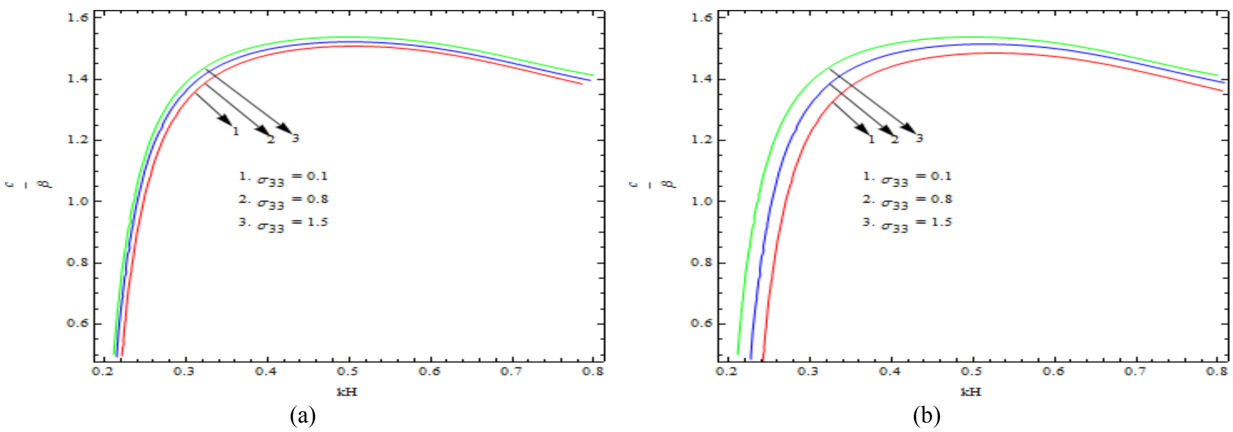


Fig.7
Variation of Rayleigh-type wave velocity (c) against wave number (k) for different values of σ_{11} for elastic substrate: (a) for electrically open case (b) for electrically short case.

8 CONCLUSIONS

The following outcomes can be drawn from the present study:

- Generalized Rayleigh-type wave velocity decreases with increment in piezoelectric constant, dielectric constant and initial stress of the piezoelectric layer in electrically open as well as short cases.
- The size of the corrugated layer significantly affects the Rayleigh-type wave velocity. Size of the corrugation parameter disfavors the phase velocity for electrically open and short cases both.
- For electrically open and short cases, it is observed that generalized Rayleigh-type wave velocity increases with increasing value of initial stress of elastic substrate and depth of the piezoelectric layer.

The results obtained in this paper are fundamental and can promote a better understanding of the behavior of generalized Rayleigh-type wave propagating in elastic non-homogeneous media (e.g., in piezoelectric materials and composites). The present study has its practical importance in material science and other engineering branches. Moreover, this study may be useful in signal processing, sound system, wireless communication, and defense equipment.

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APPENDIX A

$$P_1=\frac{-\left(\varepsilon_{33}\left(C_{33}+\sigma_{33}^1\right)+e_{33}{}^2\right)}{e_{33}\left(C_{33}+\sigma_{33}^1\right)},\upsilon_1=\frac{\left(C_{11}+\sigma_{11}{}^1\right)}{\left(C_{44}+\sigma_{33}{}^1\right)},\upsilon_2=\frac{\left(C_{44}+\sigma_{33}{}^1\right)}{\left(C_{33}+\sigma_{33}{}^1\right)},$$

$$P_2=\frac{2e_{15}}{\left(C_{33}+\sigma_{33}{}^1\right)}+\frac{e_{33}}{\left(C_{33}+\sigma_{33}{}^1\right)}+\frac{\varepsilon_{33}}{e_{33}}\left(\upsilon_1+\upsilon_2\right)+\frac{\varepsilon_{11}}{e_{33}}-\frac{\varepsilon_{33}\left(C_{13}+C_{44}\right)^2}{e_{33}\left(C_{33}+\sigma_{33}{}^1\right)\left(C_{44}+\sigma_{33}{}^1\right)}$$
$$-\frac{\left(e_{15}+e_{31}\right)\left(C_{13}+C_{44}\right)}{e_{33}\left(C_{44}+\sigma_{33}{}^1\right)}+\frac{\left(e_{15}+e_{31}\right)^2}{e_{33}\left(C_{44}+\sigma_{33}{}^1\right)},$$

$$P_3=\frac{\varepsilon_{11}}{e_{33}}(\upsilon_1+\upsilon_2)-\frac{(e_{15})^2}{e_{33}(C_{13}+C_{44})}-\frac{2e_{15}a_1}{(C_{33}+\sigma_{33}{}^1)}-\frac{e_{33}a_1a_2}{(C_{33}+\sigma_{33}{}^1)}+\frac{\varepsilon_{11}(C_{13}+C_{44})^2}{e_{33}(C_{33}+\sigma_{33}{}^1)(C_{44}+\sigma_{33}{}^1)}$$

$$+\frac{2(e_{15}+e_{31})(C_{13}+C_{44})e_{15}}{e_{33}(C_{44}+\sigma_{33}{}^1)(C_{33}+\sigma_{33}{}^1)}-\frac{(e_{15}+e_{31})^2\upsilon_2}{e_{33}(C_{44}+\sigma_{33}{}^1)},\ P_4=\frac{\varepsilon_{11}\upsilon_1\upsilon_2}{e_{33}}+\frac{(e_{15})^2\upsilon_1}{e_{33}(C_{33}+\sigma_{33}{}^1)},$$

$$P'=\frac{-P''}{(s_{11}+S)},Q'=\frac{-Q''}{(m_{11}+T)},R'=\frac{-R''}{(n_{11}+V)},$$

$$P''=\frac{(a_{12}-a_{11}k^2\lambda_1^2)(a_{22}-a_{21}k^2\lambda_1^2)+(a_{24}-ik\lambda_1a_{23})(a_{14}+k^2\lambda_1^2a_{13})}{i\lambda_1ka_{15}(a_{22}-a_{21}k^2\lambda_1^2)+(e_{33}k^2+k^2\lambda_1^2e_{33})(a_{14}+k^2\lambda_1^2a_{13})},$$

$$Q''=\frac{(a_{12}-a_{11}k^2\lambda_2^2)(a_{22}-a_{21}k^2\lambda_2^2)+(a_{24}-ik\lambda_2a_{23})(a_{14}+k^2\lambda_2^2a_{13})}{i\lambda_2ka_{15}(a_{22}-a_{21}k^2\lambda_2^2)+(e_{33}k^2+k^2\lambda_2^2e_{33})(a_{14}+k^2\lambda_2^2a_{13})},$$

$$R''=\frac{(a_{12}-a_{11}k^2\lambda_3^2)(a_{22}-a_{21}k^2\lambda_3^2)+(a_{24}-ik\lambda_3a_{23})(a_{14}+k^2\lambda_3^2a_{13})}{i\lambda_3ka_{15}(a_{22}-a_{21}k^2\lambda_3^2)+(e_{33}k^2+k^2\lambda_3^2e_{33})(a_{14}+k^2\lambda_3^2a_{13})},$$

$$S=\frac{-k^2\lambda_1(e_{15}+e_{31})(e_{15}k^2+k^2\lambda_1^2e_{33})+(\varepsilon_{11}{}'k^2+k^2\lambda_1^2\varepsilon_{11})(a_{24}+ik\lambda_1a_{23})}{(a_{22}-k^2\lambda_1{}^2a_{22})(\varepsilon_{11}{}'k^2+k^2\lambda_1^2\varepsilon_{11})-(e_{15}k^2+k^2\lambda_1^2e_{33})(e_{15}k^2+k^2\lambda_1^2e_{33})},$$

$$T=\frac{-k^2\lambda_2(e_{15}+e_{31})(e_{15}k^2+k^2\lambda_2^2e_{33})+(\varepsilon_{11}{}'k^2+k^2\lambda_2^2\varepsilon_{11})(a_{24}+ik\lambda_2a_{23})}{(a_{22}-k^2\lambda_2{}^2a_{22})(\varepsilon_{11}{}'k^2+k^2\lambda_2^2\varepsilon_{11})-(e_{15}k^2+k^2\lambda_2^2e_{33})(e_{15}k^2+k^2\lambda_2^2e_{33})},$$

$$V=\frac{-k^2\lambda_3(e_{15}+e_{31})(e_{15}k^2+k^2\lambda_3^2e_{33})+(\varepsilon_{11}{}'k^2+k^2\lambda_3^2\varepsilon_{11})(a_{24}+ik\lambda_3a_{23})}{(a_{22}-k^2\lambda_3{}^2a_{22})(\varepsilon_{11}{}'k^2+k^2\lambda_3^2\varepsilon_{11})-(e_{15}k^2+k^2\lambda_3^2e_{33})(e_{15}k^2+k^2\lambda_3^2e_{33})},$$

$$L_1=C_{33}{}'(C_{44}{}'+\sigma_{33}^2),L_3=\left[k^2(C_{11}{}'+\sigma_{11}{}^2)-\frac{\rho c^2}{G}\right]^2,$$

$$L_2=k^2(C_{44}{}'+\sigma_{33}^2)\left[(C_{11}{}'+\sigma_{11}^2)-\frac{\rho c^2}{G}\right]+k^2C_{33}{}'\left[(C_{11}{}'+\sigma_{11}^2)-\frac{\rho c^2}{G}\right]+\left[ik\left(C_{13}{}'+(C_{44}{}'+\sigma_{33}^2)\right)\right]^2,$$

$$K_1=\frac{G(C'_{44}+\sigma_{33}^2)\lambda_4{}^2+G(C'_{11}+\sigma_{11}^2)-\rho c^2}{GC'_{13}\lambda_4+G(C'_{44}+\sigma_{33}^2)\lambda_4},K_2=\frac{G(C'_{44}+\sigma_{33}^2)\lambda_5{}^2+G(C'_{11}+\sigma_{11}^2)-\rho c^2}{GC'_{13}\lambda_5+G(C'_{44}+\sigma_{33}^2)\lambda_5},$$

$$A_{11}=\left[(-C_{44}ik\lambda_1+aikC_{44}+a'ike_{15})-\zeta_1'(ikC_{11}-ik\lambda_1aC_{13}+a'e_{31})\right]e^{-ik\lambda_1(\zeta_1-H)},$$

$$A_{12}=\left[(-C_{44}ik\lambda_2+bikC_{44}+b'ike_{15})-\zeta_1'(ikC_{11}-ik\lambda_2bC_{13}+b'e_{31})\right]e^{-ik\lambda_2(\zeta_1-H)},$$

$$A_{13}=\left[(-C_{44}ik\lambda_3+cikC_{44}+c'ike_{15})-\zeta_1'(ikC_{11}-ik\lambda_3cC_{13}+c'e_{31})\right]e^{-ik\lambda_3(\zeta_1-H)},$$

$$A_{14}=\left[(C_{44}ik\lambda_1+dikC_{44}+d'ike_{15})-\zeta_1'(ikC_{11}+ik\lambda_1dC_{13}+d'e_{31})\right]e^{ik\lambda_1(\zeta_1-H)},$$

$$A_{15}=\left[(C_{44}ik\lambda_2+eikC_{44}+e'ike_{15})-\zeta_1'(ikC_{11}+ik\lambda_2eC_{13}+e'e_{31})\right]e^{ik\lambda_2(\zeta_1-H)},$$

$$A_{16}=\left[(C_{44}ik\lambda_3+fikC_{44}+f'ike_{15})-\zeta_1'(ikC_{11}+ik\lambda_3fC_{13}+f'e_{31})\right]e^{ik\lambda_3(\zeta_1-H)},$$

$$A_{17}=0=A_{18}=A_{19}=A_{110},$$

$$A_{21}=\left[(C_{13}ik-\lambda_1aikC_{33}-a'ik\lambda_1e_{33})-\zeta_1'(aikC_{44}-ik\lambda_1C_{44}+a'ike_{15})\right]e^{-ik\lambda_1(\zeta_1-H)},$$

$$A_{22}=\left[(C_{13}ik-\lambda_2bikC_{33}-b'ik\lambda_2e_{33})-\zeta_1'(bikC_{44}-ik\lambda_2C_{44}+b'ike_{15})\right]e^{-ik\lambda_2(\zeta_1-H)},$$

$$A_{23}=\left[(C_{13}ik-\lambda_3cikC_{33}-c'ik\lambda_3e_{33})-\zeta_1'(cikC_{44}-ik\lambda_3C_{44}+c'ike_{15})\right]e^{-ik\lambda_3(\zeta_1-H)},$$

$$A_{24}=\left[\left(C_{13}ik+dik\lambda_1C_{33}+d^{'}ik\lambda_1e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_1+ikdC_{44}+d^{'}ike_{15}\right)\right]e^{ik\lambda_1(\zeta_1-H)},$$
$$A_{25}=\left[\left(C_{13}ik+eik\lambda_2C_{33}+e^{'}ik\lambda_2e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_2+ikeC_{44}+e^{'}ike_{15}\right)\right]e^{ik\lambda_2(\zeta_1-H)},$$
$$A_{26}=\left[\left(C_{13}ik+fik\lambda_3C_{33}+f^{'}ik\lambda_3e_{33}\right)-\zeta_1^{'}\left(ikC_{44}\lambda_3+ikfC_{44}+f^{'}ike_{15}\right)\right]e^{ik\lambda_3(\zeta_1-H)},$$
$$A_{27}=0=A_{28}=A_{29}=A_{210},A_{31}=P^{''}e^{ik\lambda_1(\zeta_1-H)},A_{32}=Q^{''}e^{ik\lambda_2(\zeta_1-H)},A_{33}=R^{''}e^{ik\lambda_3(\zeta_1-H)},$$
$$A_{34}=S^{'}e^{-ik\lambda_1(\zeta_1-H)},A_{35}=T^{'}e^{-ik\lambda_2(\zeta_1-H)},A_{36}=V^{'}e^{-ik\lambda_3(\zeta_1-H)},A_{310}=-e^{-k(\zeta_1-H)},A_{37}=0=A_{38}=A_{39},$$
$$A_{41}=\left(P^{'}-\lambda_1\right)e^{ik\lambda_1(\zeta_1-H)},A_{42}=\left(P^{'}-\lambda_2\right)e^{ik\lambda_2(\zeta_1-H)},A_{43}=\left(P^{'}-\lambda_3\right)e^{ik\lambda_3(\zeta_1-H)},$$
$$A_{44}=\left(S+\lambda_1\right)e^{-ik\lambda_1(\zeta_1-H)},A_{45}=\left(T+\lambda_2\right)e^{-ik\lambda_2(\zeta_1-H)},A_{46}=\left(V+\lambda_3\right)e^{-ik\lambda_3(\zeta_1-H)},$$
$$A_{47}=0=A_{48}=A_{49}=A_{410},A_{51}=A_{52}=A_{53}=A_{54}=A_{55}=A_{56}=1,A_{57}=-1=A_{58},A_{59}=0=A_{510},$$
$$A_{61}=P^{'},A_{62}=Q^{'},A_{63}=R^{'},A_{64}=S,A_{65}=T,A_{66}=V,A_{67}=K_1,A_{68}=K_2,A_{69}=0=A_{610},$$
$$A_{71}=P^{''},A_{72}=Q^{''},A_{73}=R^{''},A_{74}=S^{'},A_{75}=T^{'},A_{76}=V^{'},A_{77}=0,A_{78}=0,A_{79}=-1,A_{710}=0,$$
$$A_{81}=\left(C_{44}P^{'}-C_{44}\lambda_1+P^{''}e_{15}\right),A_{82}=\left(C_{44}Q^{'}-C_{44}\lambda_2+Q^{''}e_{15}\right),A_{83}=\left(C_{44}R^{'}-C_{44}\lambda_3+R^{''}e_{15}\right),$$
$$A_{84}=\left(C_{44}S+C_{44}\lambda_1+S^{'}e_{15}\right),A_{85}=\left(C_{44}T+C_{44}\lambda_2+T^{'}e_{15}\right),A_{86}=\left(C_{44}V+C_{44}\lambda_3+V^{'}e_{15}\right),$$
$$A_{87}=-\left(\left(C^{'}_{44}+\sigma_{33}^2\right)K_1-\left(C^{'}_{44}+\sigma_{33}^2\right)\lambda_1^{'}\right),A_{88}=-\left(\left(C^{'}_{44}+\sigma_{33}^2\right)K_2-\left(C^{'}_{44}+\sigma_{33}^2\right)\lambda_2^{'}\right),$$
$$A_{89}=-e_{15}^{'},A_{810}=0,A_{91}=\left(C_{13}-C_{33}\lambda_1P^{'}-P^{''}e_{31}\lambda_1\right),A_{92}=\left(C_{13}-C_{33}\lambda_2Q^{'}-Q^{''}e_{31}\lambda_2\right),$$
$$A_{93}=\left(C_{13}-C_{33}\lambda_3R^{'}-R^{''}e_{31}\lambda_3\right),A_{94}=\left(C_{13}+C_{33}\lambda_1S+S^{'}e_{31}\lambda_1\right),A_{95}=\left(C_{13}+C_{33}\lambda_2T+T^{'}e_{31}\lambda_2\right),$$
$$A_{96}=\left(C_{13}+C_{33}\lambda_3V+V^{'}e_{31}\lambda_3\right),A_{97}=-\left(C_{13}^{'}-\left(C^{'}_{33}+\sigma_{11}^2\right)\lambda_1^{'}K_1\right),A_{98}=-\left(C_{13}^{'}-\left(C^{'}_{33}+\sigma_{11}^2\right)\lambda_2^{'}K_2\right),$$
$$A_{99}=e_{31}^{'}\sqrt{\frac{\varepsilon_{11}}{\varepsilon_{33}}},A_{910}=0,A_{101}=\left(P^{'}-\lambda_1\right),A_{102}=\left(Q^{'}-\lambda_2\right),A_{103}=\left(R^{'}-\lambda_3\right),A_{104}=\left(S+\lambda_1\right),$$
$$A_{105}=\left(T+\lambda_2\right),A_{106}=\left(V+\lambda_3\right),A_{107}=-\left(\left(C^{'}_{33}+\sigma_{11}^2\right)-\lambda_1^{'}K_1\left(C^{'}_{33}+\sigma_{11}^2\right)\right),$$
$$A_{108}=-\left(C_{13}^{'}-\lambda_2^{'}K_2\left(C^{'}_{33}+\sigma_{11}^2\right)\right),A_{109}=e_{31}^{'}\sqrt{\frac{\varepsilon_{11}}{\varepsilon_{33}}},A_{1010}=0,$$
$$A_{31}=P^{''}e^{-ik\lambda_1(\zeta_1-H)},A_{32}=Q^{''}e^{-ik\lambda_2(\zeta_1-H)},A_{33}=R^{''}e^{-ik\lambda_3(\zeta_1-H)},A_{34}=S^{'}e^{ik\lambda_1(\zeta_1-H)}$$
$$A_{35}=T^{'}e^{ik\lambda_2(\zeta_1-H)},A_{36}=V^{'}e^{ik\lambda_3(\zeta_1-H)}.$$

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