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Research Paper

Fatigue Failure Analysis of Trailing Arm Using Numerical Methods

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ABSTRACT

The suspension system, one of the essential parts of any vehicle which has a significant role in vehicle steering, accelerating, and braking. One of the suspension system's main components is the trailing arm, which is exposed to frequent loadings and is made by welding method. Due to the use and nature of this piece, its fatigue analysis is crucial. Since this part is made by welding process, its fatigue analysis is much more complicated than other parts. In this paper, fatigue life of the trailing arm is investigated by using different numerical method. At first, safety factor of the component is calculated using dang van criteria. Dang van is one of the most famous and appropriated method for multi axial non proportional loads. However it is not a good criterion in order to calculate the damage of the weld line. So Volvo method that developed base on the weld process and properties is consider for fatigue analysis of the weld line. The obtained results improve the necessity of using this kind of method for welding process. Finally, it could be concluded that for fatigue analysis of a welded component such as trailing arm, using both method are necessary. Considering two different criteria for a component and comparing the obtained results of the trailing arm under non proportional applied load is one of the achievement of this paper. Of course by using this method, the calculated fatigue life of the trailing arm is accurate. At the end, it should be noted that the both applied methods, Dang Van and Volvo, are completely verified by the available experimental result in the reliable references.

Keywords : Fatigue analysis; Dang van theory; Non-proportional loads; Seam weld analysis; Volvo method; Trailing arm.

1 INTRODUCTION

THE vehicle's suspension system has two important responsibilities: absorbing the vibrations of the wheels caused by road roughness and forming an effective contact between wheel tires and road surface. In this system,

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two issues are always discussed, riding quality and vehicle steering. These two always oppose each other. In other words, one causes difficulties in the other [1]. In addition to bearing the weight of the car, the suspension system must absorb the impact of bumps on the vehicle and prevent it from being transmitted to the car body and damped its oscillation and must be designed in such a way that maintain tire contact with the road surface in different conditions [2]. The general responsibilities of the suspension system consist of [3]:

1- Bearing the weight of the vehicle
2- Absorbing the road shocks and turning them into vibrations
3- Maintaining the geometry of the steering system and the position of the wheels
4- Increasing the car's maneuverability
5- Increasing vehicle stability
6- Keeping the wheels in contact with the road surface

The trailing arm suspension is mainly used on the rear axle of cars, although in some vehicles, such as the Volkswagen Beetle (1938 - 2003), it was used simultaneously on the front and rear axles [4]. This suspension is independent and was mainly used in older cars, but it is still used in some cars today. This system has an almost triangular arm, as one end is the point of support for the wheel axel, and its vertex is perpendicular to the chassis. A set of springs is also connected to this arm near the wheels [5]. When the vehicle is on rough roads, the trailing arm moves up and down around the axis connected to the chassis, absorbing the roughness through the shock absorber. In this system, only the wheels are allowed to move up and down, and any lateral and transverse movement and change of camber angle through the suspension set is impossible. The camber angle in this system only depends on the condition of the car body. As the car enters a turn, the camber angle changes equal to the rotation of the car body, and both wheels incline toward the outside of the turn. This situation causes low steering in the car. The main advantages of this type of suspension are simplicity of design and construction, stability and good steering, and the independence of single-axle wheels in suspension [6]. A schematic illustration of this suspension is shown in Fig.1. One of the main and load-bearing components of this suspension system is the trailing arm, which plays a key role in transferring forces from the wheel to the entire suspension system. Therefore, the fatigue analysis of it is so important [7]. But manufacturing this part by welding process makes the subject of fatigue analysis of trailing arm more complicated than normal parts. Because the use of normal methods of fatigue analysis to estimate the fatigue life of welded component has not provided accurate results. Therefore, to calculate the weld line of the structure, more accurate theories that are specific to the calculation and evaluation of welds should be used. In various studies [8-10], the cause of failure of a suspension arm, made of high-strength steel, has been investigated under the influence of fatigue loading. The analysis of this research indicates cracks formation in the weld depth due to the concentration of stress. Variables affecting the fatigue life of this part are shielding gas flow rate, welding speed, the voltage used, and welding quality [11-12]. Vertical force F_Z is mainly responsible for the failure of the rear suspension because when the left and right wheels' vertical forces are not equal while driving, a generated torque is applied to the whole suspension system [13]. Emre et al. [14] improved the behavior of the suspension system statically and dynamically by increasing the stiffness of the used material in the suspension. They stated that simulation could guide designers to achieve optimal suspension parameters for vehicles. Various studies have been done on the breakdown of car components such as suspension arms [15]. In this study, Nadota and Denier used experimental and numerical methods to evaluate the failure of this part. Kashyzadeh et al. [16] investigates the effect of changes in wheel primary angles such as Camber and Toe angles on the fatigue life of vehicle steering knuckle under multi-input random non-proportional 3D stress components. The obtined results showed that the highest and lowest fatigue life of steering knuckle are related to the values of 2 positive and negative degrees of camber angle, respectively. The stress level is reduced in the various equivalent load histories by changing the toe angle to 0.2 negative, resulting in an increase in the fatigue life of steering knuckle. Also kashyzadeh and amiri [17] investigated the Effect of Vehicle Velocity on the Ride Comfort of a Car on a Road with Different Types of Roughness by experimental method. Finally they determined the maximum speed for not feeling the discomfort of occupants and the maximum human tolerance at continuous travel. Farrahi et al [18] simulated the spot weld failures of the vehicle body structure due to fatigue damage induced on the body during standardized maneuvers. This research was accomplished by using a combination of multi-body dynamics and finite element analyses. The effect of nugget diameter on the fatigue life of the spot welds was also investigated in this research. Kashyzadeh [19] investigated the effects of different loading conditions including axial and multiaxial variable amplitude loading on the fatigue life assessment of automotive components under various maneuvers. One of the major achievements of this study is that for the components with complex geometry and under multi-input loading like the steering knuckle, it is essential to perform fatigue analysis by considering all real conditions and cannot be only focused to the destructive loading. Also Kashyzadeh [20] presented a new algorithm for fatigue life assessment of automotive safety components based on the probabilistic approach. He compared the obtained results by numerical method with the experimental ones. Shariyat [21-22] has presented a new energy-based equivalent stress criteria for fatigue life determination in components with complicated geometry under random nonproportional 3D stress fields. Also, he has developed random fatigue model by modification of Gough's theory [23]. Zoroufi and Fatemi have studied the effect of various parameters of the production process on the fatigue life of steering knuckle made of forged steel by using experimental [24] and analytical [25] methods. Zoroufi et al. have studied the experimental durability assessment and life prediction of steering knuckles made of different materials including forged steel, cast iron, and aluminum alloy [26]. They also considered a number of complexities such as material property variability, variable-amplitude and multiaxial loadings, manufacturing parameters, and environmental effects on durability assessment.

In this paper, the fatigue life of the suspension arm is evaluated from two different approaches; Dang van theory for the whole of the component, and Volvo Method for the weld line of the component. First, the development histories of these methods are reviewed. Subsequently, the underlying fundamental theories are described and compared in details. The similarity and difference between these two methods in the basic theories are highlighted. All the applied theories verified by the available experimental results. In the following, whole of the structure is evaluated using Dang Van theory, one of the most famous and appropriate theories for estimating the fatigue life of parts even in non-proportionate loads [27-29]. A multiaxial fatigue damage theory was developed by Dang Van in 1999 [30] whereby the theory of shakedown is used to describe the behavior of plastic flow on a granular level. The theorem defines three different stages of shakedown: elastic shakedown, plastic shakedown and ratcheting [31-32]. Elastic shakedown occurs when a particular material volume reverts into behaving elastically again after an initial plastic deformation. Plastic shakedown or ratcheting, on the other hand, cause an accumulation of plastic strain which will eventually lead to the exceedance of ductility and thus crack initiation. It should be emphasized that the Dang Van criterion identifies whether crack initiation is likely to occur but cannot be used to obtain any information on fatigue lifetime [33]. Dang Van criterion is used for the whole structure and is not suitable for estimating the fatigue life of the weld line, because welding effects and its influence on the fatigue life are not included in this criterion. So at the same time with using dang Van criterion an appropriate criterion that considered welding process is used to estimate the weld line fatigue life. Volvo proposed and developed one of the best criterion for evaluating the fatigue life and fatigue damage of the welding line [34]. Generally, the fatigue strength of welded joints is significantly less than that of the parts which are welded together, or of the parent plate, so the weld joints are usually the weakest part of vehicle systems. And so, a reliable and validated analytical method to assess fatigue life is needed to guide vehicle designs. Many fatigue life assessment methods, such as nominal stress method, structural stress method, hot spot structural stress method, notch stress method, and local strain method, are available for analyzing welded structures [35] and some of these methods have been implemented into engineering codes [36, 37]. However, one of the major disadvantages of these methods is that these methods usually do not predict consistent results because of mesh sensitivity. This issue is especially critical for virtual life assessment based on finite element analysis (FEA). To reduce the mesh sensitivity and to obtain a consistent prediction, during the last decade, based on the existing finite element frameworks, several nodal force based structural (traction) stress concepts have been proposed and developed for welded structures. Two structural stress methods have gained prominent positions in engineering applications: the Volvo method and the Verity method. Both the concepts, the theories, and the stress and life evaluation procedures are strongly desired [38]. Finally, the results show that although Dang Van's theory is very suitable for analyzing the whole structure, it does not have the accuracy to evaluate the weld line. In performing both analyses, nCode Design software is used, one of the most common fatigue analysis software based on the finite element method.

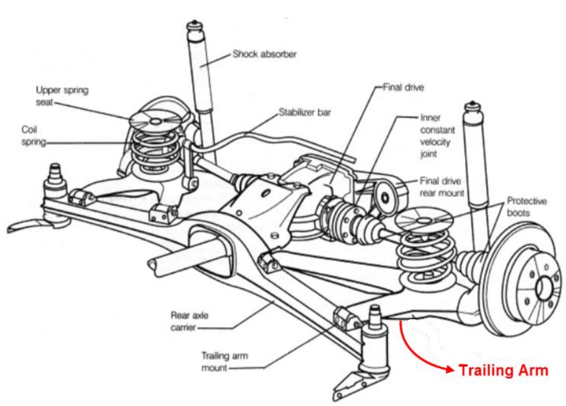

Fig.1
Schematic representation of the trailing arm.

2 NUMERICAL THEORIES USED FOR ESTIMATATION THE FATIGUE LIFE OF THE TRAILING ARM

Structures that are subject to repetitive stresses even less than the yield strength of the material will be damaged and cracked over time. If the cracks spread on the surface, they will lead to the total failure of structure that called fatigue failure. Since fatigue failure is not visible and occurs suddenly, it is essential to monitor and predict when it will occur. Fatigue damage usually occur where the geometry changes abruptly and stress is concentrated. In the welding structures, the weld line is more likely to fail due to the stress concentration. Therefore, in welding structure, analyzing of the both weld line and the analysis of the whole structure (parent part) are crucial and strongly recommended. Fatigue failure consists of three stages [39]: In the first stage, called the onset of cracking or germination, fatigue damage causes microscopic cracks on the structure's surface that are not visible to the naked eye. In the next stage, called crack growth, the cracks grow perpendicular to the stress and enter the macroscopic scale. Finally, due to the growth of cracks and stress concentration, the structure undergoes plastic deformation, and fatigue failure occurs.

2.1 Dang Van theory

Environmental and operational loads on automobile structures may introduce a multiaxial stress state in certain components and structural details. Such multiaxial stress states can have a detrimental effect on the fatigue resistance and may result in premature fatigue damage. For components and structural details experiencing predominantly uniaxial stress variations, generally accepted approaches have been developed which enable to make a fatigue lifetime estimation. However, when experiencing multiaxial stress variations additional complexity is introduced to the phenomenon of fatigue. Over the last decades a wide variety of methods and approaches have been developed specifically for multiaxial fatigue problems. However, validation often requires more investigation and experimental work. The simple uniaxial safety factor approach is fine for many cases, but where the loading is multiaxial, and especially non-proportional, a more sophisticated method, such as the Dang Van model, is required. The Dang Van criterion is the most popular solution not only in research papers, but also in commercial implementations [40].

In high cycle fatigue, Dang Van's approach lead to the definition of a widely used multiscale criterion corresponding to the unlimited endurance condition. The basic framework is the following [41]:

- ✓ the fatigue damage is controlled by mechanisms at the grain scale and therefore a description at this mesoscopic scale is necessary;
- ✓ at this scale, most of the metallic materials are aggregates of crystals with a random distributed crystallographic orientations, that can be considered isotropic and homogeneous at the macroscopic scale;
- ✓ among all grains and possible slip planes, only some well-oriented slip planes, maximizing the shear stress for a given loading path, will develop plasticity and create localized slip bands inducing crack initiation;
- ✓ below the fatigue limit, microscopic plastic strains homogenize to negligible macroscopic plastic strains, which matches the fact that macroscopic stresses are small with respect to the yield limit.

The Dang Van criterion is then based on the Lin–Taylor homogenization assumption in order to relate meso- and macroscopic mechanical fields and, based on shakedown concepts, is defined in terms of a linear combination between the hydrostatic pressure, *Ph*, and the mesoscopic resolved shear stress amplitude τ [41]. In other words, Shear stress is the cause of crack initiation, and hydrostatic stress is the cause of crack growth. The relationship between shear stress and hydrostatic stress is shown in Eq. (1) [42]:

$$\tau + a.Ph = b \tag{1}$$

which τ is the microscopic shear stress, and *Ph* is the hydrostatic stress. "*a*" and "*b*" are the parameters related to the material. "*a*" is related to the hydrostatic sensitivity factor, and "*b*" is related to the shear stress range to the extent of fatigue under shear (torsional) load. In the case of pure shear loading, $Ph = 0$ and thus $b=\tau$. Material parameters *a* and *b* can be determined by determining fatigue stress amplitude under two different uniaxial bending-pressure loads or pure bending or shear, both of which have a stress ratio of $R = -1$. In general, in this theory, crack initiation is predicted to occur if:

$$\tau(t) + a.P(t) \geq b \tag{2}$$

There is no definite relationship to determine the parameters "*a"* and "*b"* of the Dang Van criterion in principle. However, in many references, extensive research (mainly on steels) has been done to provide a relationship between the fatigue properties of materials and these parameters with their static properties [43]. Using the theory of elasticity, it is clear that the ratio between static resistance to torsion and tension is 0.577. However, in fatigue loading, this ratio varies in a range depending on the nature of the material and the dimensions of the samples tested.

After compiling approximately 500 fatigue tests on several steels with a UTS between 350 *MPa* and 2000 *MPa*. Moore, Jasper and Mac Adam observed a range of 0.44-0.71 for the same limits ratio. Finally, Föppl found a range 0.48-0.75 for steels and 0.54-0.65 for aluminum alloys [43-44]. nCode Design Life software uses the Föppl relationship to calculate the Dang Van parameters because it correlates quite well with the data available [39]. For steels, the mean ratio between torsional (t) and rotating bending (f) limits is (0.48+0.75)/2=0.615. The slope of the Dang Van criterion line (Fig. 2) is calculated from Eq. (3).

$$a = \left[3\times\left(t - f\ /2\right)\right]/2 = \left[3\times\left(0.615-0.5\right)\times f\ \right]/f\ = 0.345 \tag{3}$$

In this equation, f is usually considered approximately equal to half the ultimate tensile strength of the material (UTS) for materials with a tensile strength below 1400 *MPa*. In nCode Design Life software, this coefficient is assumed to be 0.45UTS for more certainty. Therefore, parameter "*b*" will be:

$$b = 0.615\times 0.45UTS = 0.28UTS \tag{4}$$

The relation between the macroscopic stresses $\sum_{ij}(t)$ and the microscopic ones $\sigma_{ij}(t)$ is given by [40]:

$$\sigma_{ij} = \sum_{ij}(t) + \rho_{ij}^{*} \tag{5}$$

ρ_{ij}^{*} being the stabilized local residual stress tensor. Dang Van demonstrated that this tensor is dependent on the microscopic plastic strains, so it is a deviatoric. The microscopic hydrostatic pressure is then equal to the macroscopic one:

$$p(t) = 1/3\times trace\left[\sum_{ij}(t)\right] \tag{6}$$

Due to the widespread application of this theory in various structures, which is because of its features (multi-axis and three-dimensional), this theory is one of the most common nCode Design Life software theories. One of the most important outputs of this theory in this software is structure's safety factor under applied load, calculated from Eq. (4).

$$SF = \frac{b}{\max\left(\tau(t) + a\times P(t)\right)} \tag{7}$$

Another output of Dang Van's theory is a parameter called Danger Factor, which is calculated by two methods, normal (Eq. (6)) and oblique (Eq. (7)), at the critical point of the loading path [39].

$$DF_{Normal} = \frac{\tau_{Critical}}{b - a.P_{Critical}} - 1 = \frac{\tau_{Critical}}{A} - 1 \tag{8}$$

$$DF_{Oblique} = \frac{\tau_{Critical} + a.P_{Critical}}{b} - 1 = \frac{\tau_{Critical}}{B} - 1 = \frac{1}{SF} - 1 \tag{9}$$

which $\tau_{Critical}$ and $P_{Critical}$ are critical microscopic shear stress and critical hydrostatic stress, respectively. These values are microscopic shear stress and hydrostatic stress at the critical point of the loading path, determined by the reliability coefficient. These are shown in Fig. 2.

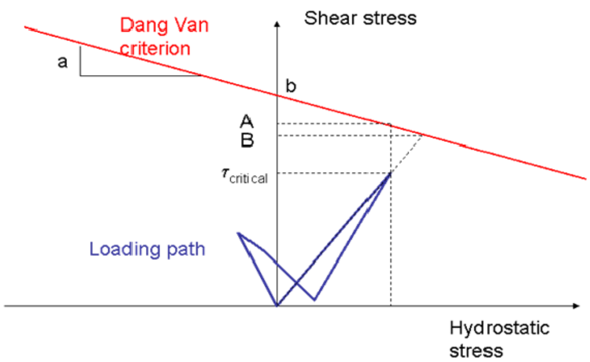

Fig.2
Normal and Oblique definitions of the Dang Van Danger Factor.

2.2 Validation of Dang Van's theory

Due to the method's numerical nature, validation of it is of great importance. Therefore, in this section, Dang Van's theory and the nCode Design Life software are evaluated with the experimental results of reputable references. For this purpose, the experimental results of Chmelko and Margetin have been used [45]. They investigated fatigue life of some samples made from ST52 by the experimental method. The modeling was performed exactly like the experimental test conditions, and the results were compared with each other. The geometric dimensions of the sample are shown in Fig. 3. As shown, the same dimensions have been used to simulate the sample in nCode Design Life and ANSYS software.

The experimental test results' graph and the lifespan for the sample under the tensile test are shown in Fig. 4. Considering that the output of ANSYS ncode Design software is the safety factor, to validate the numerical results with experimental results, the loading range and number of loading cycles are extracted from Fig. 4. The sample reliability coefficient under the mentioned fatigue is obtained from the diagram. Safety factor for different samples are shown in Table 1. It can be assumed that the reliability of the experimental test in the applied load and the amplitude of the oscillation extracted from Fig. 4 is the same, and therefore the results of the experimental and numerical tests can be easily compared. As shown in this table, the obtained numerical results have an excellent agreement with the experimental test. The safety factor contour of a sample that is obtained by the numerical simulation is shown in Fig. 5.

As shown in the pictures, the weakest point of the samples is their middle part, which finally failed in experimental tests, too. The appropriate agreement and accurate verification of the numerical results obtained with the experimental tests of the samples in the range of different oscillations and with different numbers of cycles is shown.

Table 1
Comparison of numerical analysis and experimental results.

sample	2Nf	Sa (*MPa*)	Numerical analysis safety factor
1	4×10^4	348	1.270
2	9×10^4	328	1.192
3	8.5×10^5	308	1.159
4	3×10^6	278	1.106

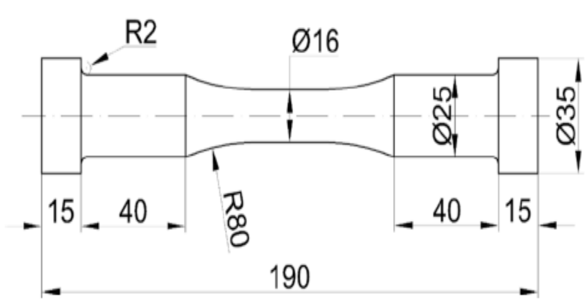

Fig.3
Dimensions of the sample used in the experimental test and numerical simulation.

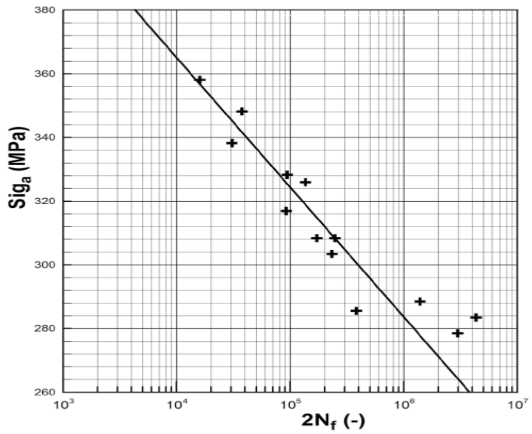

Fig.4
Experimental test results of the sample under uniaxial tensile loading.

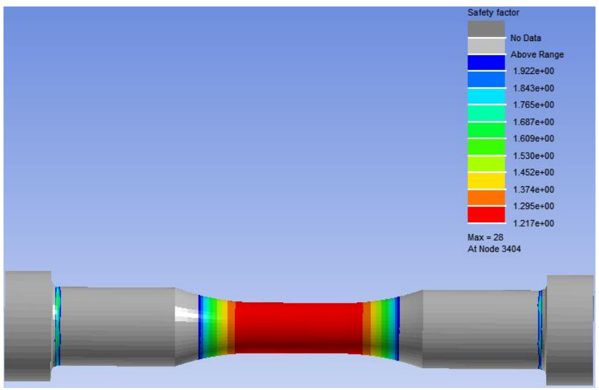

Fig.5
Safety factor contour for the sample with 4×10^4 cycle and 348 *MPa* stress range.

2.3 Fatigue theory used for seam weld

Welding is used in many industries as an effective and economical way to make structural joints between metal parts. However, the nature of the welding process means that welded joints generally have a level of fatigue resistance that is lower than that of joined parts [46]. At the same time, welds often tend to be made at geometric features or changes in section in the structure. The result of these facts is that even in a well-designed structure, it is typically the welded joints that are most likely to fail by fatigue. Any evaluation of the durability of a welded structure must therefore place a high priority on a fatigue assessment of the welded joints.

The general reasons for low weld resistance can be divided into the following general categories.

1. The geometry of the weld usually causes stress concentration (except in butt welds). Typically, the stress at the root or the toe of the weld is the highest.
2. The welding process often causes defects in the structure that can act as crack initiation sites – slag inclusions, incomplete fusion, porosity etc. These are some sample evidence that why the weld line fatigue life is often affected by crack growth.
3. There is a heat-affected zone (HAZ) in which the parent material is heated to a high temperature and allowed to cool relatively quickly around the fusion zone. This phenomenon may cause fundamental changes in the microstructure and properties in this area.
4. Often the welding process leads to the production of residual stresses in the structure, which overshadows the yield strength of the structure.

These factors make the fatigue strength of a welded joint quite different from that of joint parts. Therefore, estimating the weld line's fatigue life from the life of the welded parts can lead to significant and terrible errors. Over the years, several different standards have been proposed to evaluate the fatigue of welded joints. Examples include Euro code 3 [47], the American Society of Engineers Code for Boilers and Pressure Equipment [48], the Swedish Welded Structures Code [49], and the English standard BS7608 [36]. A method similar to the above standards is provided for aluminum parts in code BS8118 [50]. All the cases mentioned have the same principles, the main principles of which are briefly mentioned below:

- ✓ These standards are not highly dependent on material specifications. However, the main range of their application is in construction steels with yield strength less than 700 *MPa*. Many researchers have pointed out that although the fatigue strength of steel varies greatly with composition and heat treatment, after welding, the strength of the resulting joints falls into a scattered zone. This is very convenient as it allows the same design curves to be used for a range of materials.
- ✓ Welds are classified according to the type of connection, geometry, load direction, and type of possible damage.
- ✓ Each welding class is associated with an S-N curve that the designer uses in the welding life estimation step.
- ✓ The location and nature of the stress used to calculate fatigue are defined in the standard for each case. Still, the calculations are typically based on the principal stress with the highest amplitude. This maximum amplitude often occurs in a high-temperature area (Hot Spot) or area containing the concentration coefficient of geometric stress in the structure and the vicinity of the weld joint (Weld Toe). For practical calculations of fatigue based on FE, determining this stress is one of the main challenges that the mentioned standards have provided guidelines in this regard. In other words, one of the most decisive topics in different methods of estimating the fatigue life of welds is to calculate the mentioned stress level from analyzes such as finite element.
- ✓ Residual stresses in welding that are practically unavoidable are among those considered in all standards by modifying S-N diagrams.
- ✓ These cases are considered in the standards by implementing corrections. For example, in the BS7608 standard, Eq. (8) is used [36]:

$$S = S_B \left(\frac{t_B}{t} \right)^{0.25} \tag{10}$$

In this equation, t_B indicates the reference thickness, which is considered 16 *mm* in this standard. The ncode design software can use various standards to calculate the welding fatigue life, and the details of these standards can be considered in different parts of the software. However, most of these standards, such as the BS7608 standard, are not very popular in industries such as the automobile industry for several reasons:

- ✓ These standards are mainly suitable for structures with thick sheets, while in most cars, the joint sheets are equal to or less than 3 *mm* thick.
- ✓ Due to the complex geometry and load combination in the automotive industry, using classified systems for structures such as bridges or pressure vessels is not easily possible.

These and other uncertainties were the primary motivation for providing a practical method in the automotive industry for calculating fatigue life by Volvo in collaboration with the ncode software. The Volvo method was originated from a joint project by Volvo Car Corporation and Chalmers University of Technology and first appeared as a thesis for master degree by Andrėasson and Frodin with the purpose to reduce the mesh-sensitivity and use coarse mesh [51] in FEA, and later, the method was formally published as a SAE technical paper by Fermėr, Andrėasson and Frodin [52]. The key concept of the method is to construct structural stress over the whole cross section of the plate/shell of interest by utilizing the nodal force and moment information from FEA output results with the help of a linear interpolation method. Since the structural stress is obtained by solving global force/moment equilibrium on a part section with certain characteristic length scale, such as the thickness of a shell, this method is deemed to be less mesh-sensitive as compared to the traditional finite element method, which can result in localized stress as mesh size decreases if a characteristic length scale is not introduced. Even though it is essentially empirical, the structural stress obtained in this way is believed to correlate the fatigue life well and provide a simple approach for engineering applications [38].

The primary purpose of this method is to provide a suitable method for the structures of the automotive industry, based on finite element analysis and considering all the complexities of parts loadings in this industry. The general and basic principles of this method are briefly described below:

- ➢ Modeling strategy: In this method, a straightforward solution is considered for modeling seam welds. The weld line is represented by one or two rows of shell elements, and in this case no classification is required for the type of weld to be analyzed.
- ➢ Bending ratio: As shown in Fig. 6, the stress used in the fatigue calculations is the sum of the bending stress and the membrane stress of the structure. However, various experiments show that the fatigue life of flexible joints, in which bending stresses play a dominant role, is significantly longer than rigid joints in

which membrane stresses play a more prominent role. The basis of this method is based on a coefficient called the bending ratio, which is the ratio of bending stress to total stress as described in Eq. (11):

$$r = \frac{|\sigma_b|}{|\sigma_b| + |\sigma_m|} \tag{11}$$

- Material properties: Weld fatigue is determined using two pairs of S-N curves, one for determining the weld fatigue resistance under pure membrane stresses (rigid joints), and the other for determining weld fatigue under pure bending (flexible joints). Interpolation is performed between the two diagrams if the bending ratio is more than a specific limit at any computational point. Otherwise, the problem is analyzed with a more rigorous assumption, which is essentially for rigid connections. This concept is shown schematically in Fig. 7.
- Size effect: Like other standards, in this method, the thickness of the sheet also affects the results, and the resistance of the joint fatigue decreases as the thickness of the sheet exceeds the reference thickness by Eq. (12).

$$factor = \left(\frac{T_{ref}}{r}\right)^n \tag{12}$$

In the Eq. (12), the reference thickness is 1 *mm*, and the power “n” is 1/6.

- Effect of average thickness: Studies have shown that the effect of average stress on the fatigue calculation of thin sheets is greater than thick sheets.

In the following, structural stress formulation for the Volvo method is described briefly. Based on calculated line moment $m_y\ (y)$ [Nm/m] and line force, $n_x\ (y)$ [*N/m*], the structural stress can be calculated as [38]:

$$\sigma(y) = \sigma_b(y) + \sigma_n(y) = \frac{12 m_y(y) z}{t^3} - \frac{n_x(y)}{t} \tag{13}$$

where σ_b is bending stress, σ_n is normal stress, *z* is the distance from the mean surface in the local *z*-direction and *t* is the sheet thickness. The structural stress expressed in Eq. (13) includes the most important information about the stress state at the weld toe, which is believed to correlate fatigue life, but it does not include very local effects. It was suggested that the linear moment, $m_y\ (y)$, and force, $n_x\ (y)$ at any location *y* can be obtained from the nodal forces and moments using the following formulae:

$$m_y^{(i)}(y) = \frac{2}{l_y^{(i)}}\left[M_{y1}^{(i)}\left(1 - \frac{y}{l_y^{(i)}}\right) + M_{y2}^{(i)}\frac{y}{l_y^{(i)}}\right] \quad \left[\frac{Nm}{m}\right] \tag{14}$$

$$n_x^{(i)}(y) = \frac{2}{l_y^{(i)}}\left[N_{x1}^{(i)}\left(1 - \frac{y}{l_y^{(i)}}\right) + N_{x2}^{(i)}\frac{y}{l_y^{(i)}}\right] \quad \left[\frac{N}{m}\right] \tag{15}$$

where $l_y^{(i)}$ is the element edge length between the grid points. The superscript represents the element; the first and the second subscripts represent the direction and node number, respectively. The extreme values can be found for $y = 0$ and $y = l_y^{(i)}$ as following [51-52]:

$$m_y^{(i)}(0) = \frac{2}{l_y^{(i)}} M_{y1}^{(i)}; \qquad m_y^{(i)}\left(l_y^{(i)}\right) = \frac{2}{l_y^{(i)}} M_{y2}^{(i)} \tag{16}$$

$$n_x^{(i)}(0)=\frac{2}{l_y^{(i)}}N_{x1}^{(i)}; \qquad n_x^{(i)}\left(l_y^{(i)}\right)=\frac{2}{l_y^{(i)}}N_{x2}^{(i)} \tag{17}$$

Clearly, the linear force/moment at a node is only a function of the nodal force/moment of that node, and is independent of other nodes. For a shell/plate weld there are two values at each node, and it was proposed to use the maximum magnitude, with maintained sign, for each node.

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$$\sigma(0)=-\frac{12}{l_y^{(i)}t^2}M_{y1}^{(i)}-\frac{2}{l_y^{(i)}t}N_{x1}^{(i)} \tag{18}$$

$$\sigma\left(l_y^{(i)}\right)=-\frac{12}{l_y^{(i)}t^2}M_{y2}^{(i)}-\frac{2}{l_y^{(i)}t}N_{x2}^{(i)} \tag{19}$$

As an alternative to using stresses directly from the FE results file, nCode DesignLife also includes an approach in which the stresses required for a fatigue analysis are derived from nodal forces and moments. For each element for which a calculation will be made (weld toe, weld root, and possibly weld throat), knowing the grid point forces would be required. As it was mentioned, before calculation the structural stresses, it need to determine the line forces and moments (force/moment per unit length) along the weld toe. The nCode version Volvo stress evaluation procedure for elements showed in Fig. 8 is listed below [39] :

[1] Identify weld toe elements and surfaces.

[2] Define a local coordinate system:

A local coordinate system (x',y',z') is defined where the x' axis is normal to the element edge, and the z' axis is the average of the normal to element 6 at nodes 7 and 8, in an upward direction (relative to the surface at the weld toe), as illustrated in Fig. 8.

[3] Calculate the total grid point forces:

At node 7 the total grid point forces and moments are:

$$F^7=F_5^7+F_6^7 \qquad M^7=M_5^7+M_6^7 \tag{20}$$

At node 8 we also need to include the triangle element.

$$F^8=F_6^8+F_7^8+F_8^8 \qquad M^8=M_6^8+M_7^8+M_8^8 \tag{21}$$

[4] At each grid point, partition the forces and moments in proportion to edge length.

At node 7,

$$F_{RIGHT}^7=F^7.\frac{l_2}{l_1+l_2} \qquad F_{RIGHT}^7=M^7.\frac{l_2}{l_1+l_2} \tag{22}$$

And at node 8,

$$F_{LEFT}^8=F^8.\frac{l_2}{l_2+l_3} \qquad M_{LEFT}^8=M^8.\frac{l_2}{l_2+l_3} \tag{23}$$

[5] Calculate the line forces and moments:

The line forces and moments *f*, *m*, are the forces and moments per unit length along the weld toe. The line forces and moments are distributed along the element edge, giving rise to the grid point forces.at first, the value of the line forces at the node in each element, corresponding to the shear of the grid point forces calculated in the last step:

$$f_{RIGHT}^{7} = \frac{2}{l_2}\left(2.F_{RIGHT}^{7} - F_{LEFT}^{8}\right) \qquad m_{LEFT}^{7} = \frac{2}{l_2}\left(2.M_{RIGHT}^{7} - M_{LEFT}^{8}\right) \tag{24}$$

$$f_{LEFT}^{8} = \frac{2}{l_2}\left(2.F_{RIGHT}^{8} - F_{LEFT}^{7}\right) \qquad m_{LEFT}^{8} = \frac{2}{l_2}\left(2.M_{RIGHT}^{8} - M_{LEFT}^{7}\right) \tag{25}$$

Then, the line forces and moments would be averaged to give the value at the middle of the weld toe edge of element 6.

$$f_6 = \frac{f_{RIGHT}^{7} + f_{LEFT}^{8}}{2} \qquad m_6 = \frac{m_{RIGHT}^{7} + m_{LEFT}^{8}}{2} \tag{26}$$

[6] Resolve to local coordinates:
The line forces are then resolved into the local coordinate system.
[7] Calculate the stress normal to the weld toe.
The stress normal to the weld toe has a membrane and bending component. It can be calculated as follows:

$$\sigma_{top,normal}^{6} = \sigma_{membrane}^{6} + \sigma_{bending}^{6} = \frac{f_{x'}^{6}}{t} + 6\frac{m_{y'}^{6}}{t^2} \tag{27}$$

$$\sigma_{bottom,normal}^{6} = \sigma_{membrane}^{6} - \sigma_{bending}^{6} = \frac{f_{x'}^{6}}{t} - 6\frac{m_{y'}^{6}}{t^2} \tag{28}$$

where *t* is the thickness of the weld toe elements.

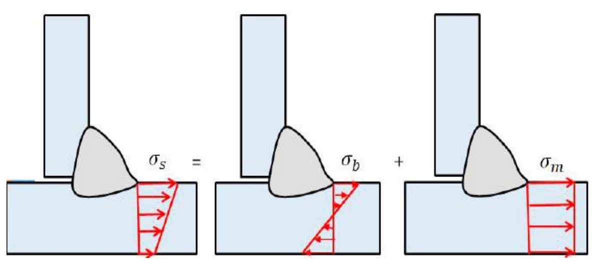


Fig.6
Upper surface tension is a combination of membrane and bending stress.

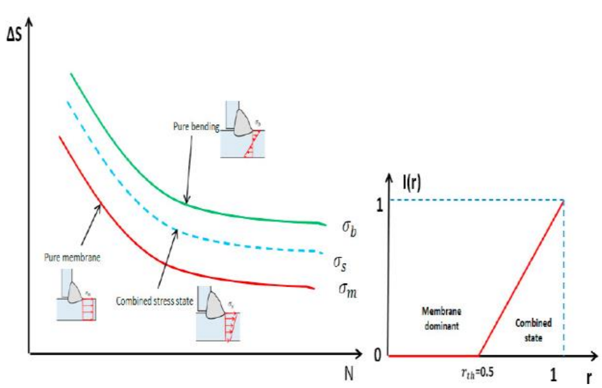


Fig.7
Interpolation of welding properties in two modes of pure bending and pure membrane.

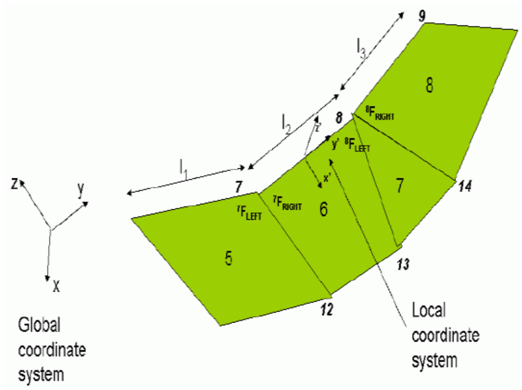


Fig.8
Local coordinate system for weld toe element.

2.3 Validation of weld line fatigue analysis

Tests performed by Lindqvist [53] were used to validate the numerical solution of weld line fatigue. In this research, an experimental test method has evaluated the connection of different parameters on the fatigue life of welded joints. High strength rolled steel sheet was used for the tests. The minimum yield strength of the sheet is 550 *MPa*, and the minimum tensile strength is 600, and the maximum is 760 *MPa*. The thickness of the sheets used is 12 *mm*, the width of the sample is 84 *mm*, and the length of the second piece is 36 *mm*. The welding angle is 45 degrees, and the welding leg length is 9.2 *mm*. The image of the sample made for the experimental tests [53] and the numerical model meshed in the software are shown in Fig. 9.

The experimental test results [53] and the numerical solution obtained in ncode Design Life software are compared in Fig. 10. As mentioned, to validate the selected method, the Lindqvist experimental bending test results for a sample with a thickness of 12 *mm* have been used. The blue dots in the diagram show the results of the experimental test for different stress ranges. The blue line is drawn by Lindqvist, which estimates the welded joint life at different stress ranges based on experimental data from different points. As is evident, the numerical results are in good agreement with the experimental results, which confirms the method's accuracy for numerical analysis of the weld joint. Numerical analysis was performed for five samples with different stress ranges and the same as the experimental test. The slight difference between the numerical and experimental results can be due to the defects in the construction of experimental parts to the ideal numerical sample, which is evident in the welding process.

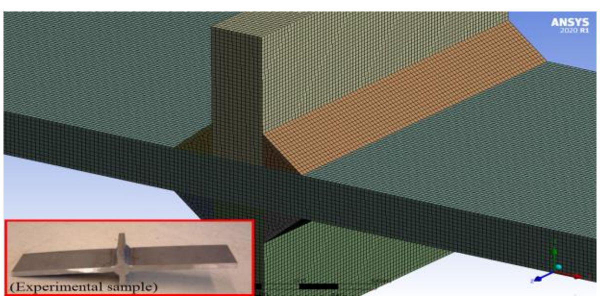


Fig.9
Experimental sample [35] and meshed model for welding fatigue test.

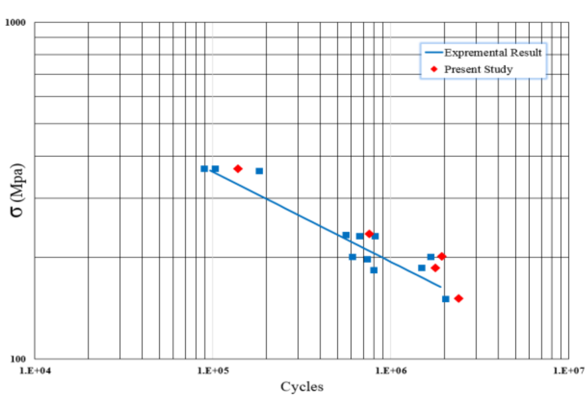


Fig.10
Experimental [35] and numerical results obtained for connection with 12 *mm* thick sheet under bending loading.

3 NUMERICAL ANALYSIS OF THE TRAILING ARM

In this study, a trailing arm was used as an example of a welded metal equipment for analysis. This equipment is usually used in the suspension and independent suspension systems of cars. In an independent suspension, one end of the suspension arm is attached to one wheel and the other end to the vehicle axle to prevent unwanted wheel rotation. This connection is mainly used in the rear wheels of cars [54]. In this mechanism, a thin sheet with a thickness of 2.8 *mm* is welded to a thick circular arm. A rectangular piece has been used as the connector of this thick circular rod and thin plate to reduce the weight and production costs. The geometry and meshed model of this mechanism is shown in Fig. 11. A total of 5,565 elements have been used to mesh the structure for static and fatigue analyses. Fig. 12 shows the mesh independency of the obtained result that is so crucial for the numerical analyses. In this chart, the obtained results of static analysis in two direction (x and y) compare with the number of elements. As can be seen, the change of the obtained result is less than 5 %, and so the obtained result is completely independent from the mesh number.

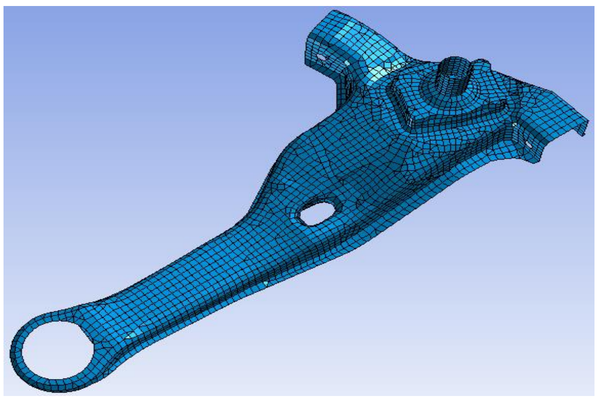


Fig.11
The geometry and meshed model of the trailing arm.

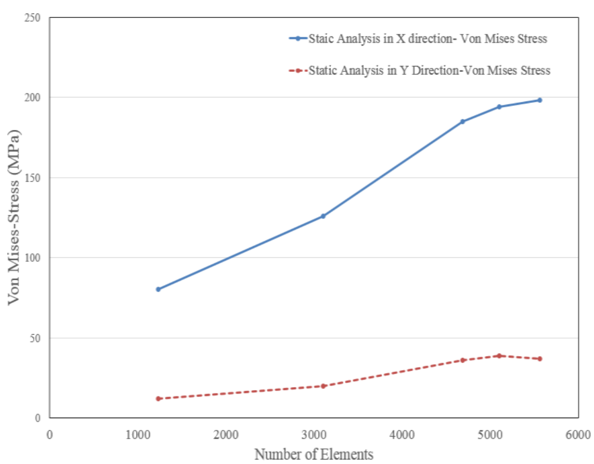


Fig.12
Mesh independency of the obtained result.

As mentioned in the previous section, the S-N diagram is presented in two modes of pure bending and zero bending to define the welding material. Then, based on the condition of each weld line point (bending or membrane or a combination of the two), the property of the material is extrapolated. The specifications of the sheet used in the suspension arm analysis are shown in Table 2.

Table 2
Material properties of the trailing arm.

Parameter	Value
Ultimate tensile strength	18000 *MPa*
elastic modulus	210000 *MPa*
Fatigue limit	430 *MPa*
Sensitivity to hydrostatic stress	0.35

All components of the fatigue forces applied to the structure by the wheel are shown in Fig. 13. As shown in this picture, the input force is in the form of three force components and three-moment components in the *x, y*, and *z* directions (6 components in total). It should be noted that these components do not have the same changes, and their changes vary over time, which is quite evident given the nature of the structure. Hence, as can be seen, it is practically multi-axial non-proportional loading. Non-proportional loading occurs when, during loading, the position of the main axes relative to the loading axis does not remain constant and changes. During a periodic load with a constant amplitude, when the magnitude of the applied stresses changes with time, the size of Mohr's circle (the stress circle) also changes with time. In some cases, due to the fact that the size of Mohr's circle changes during loading, the position of the main axes relative to the loading axis remains constant.

This loading is called non proportional loading. The importance of this issue is that when the loading is non-proportional, usual method such as S-N methods cannot be used to check the fatigue life of the structure, and in principle, using a critical plate approach become necessary [55]. Critical plane analysis refers to the analysis of stresses or strains as they are experienced by a particular plane in a material, as well as the identification of which plane is likely to experience the most extreme damage. Significance of this approach has increased during last years, because of its effectiveness and broad application range. The applied method, Dang van, is one of the best method by this approach that its use has widespread in recent years.

In this paper, the fatigue life of the trailing arm is analyzed by two theories. First, the whole structure, including the weld line, was analyzed by Dang Van's theory. Then the welding line is evaluated with a unique and relevant theory of welding. The solution method in both analyses is to obtain the stress field by analyzing the structure with a unit load on each component of force and moment. Next, fatigue cycles of different components of loads and moments is attributed to the obtained stress field from the static analyzing the structure by the unit loads and moments. Therefore, in both fatigue theories (Dang Van and Volvo method for welding), the structure is initially statically analyzed.

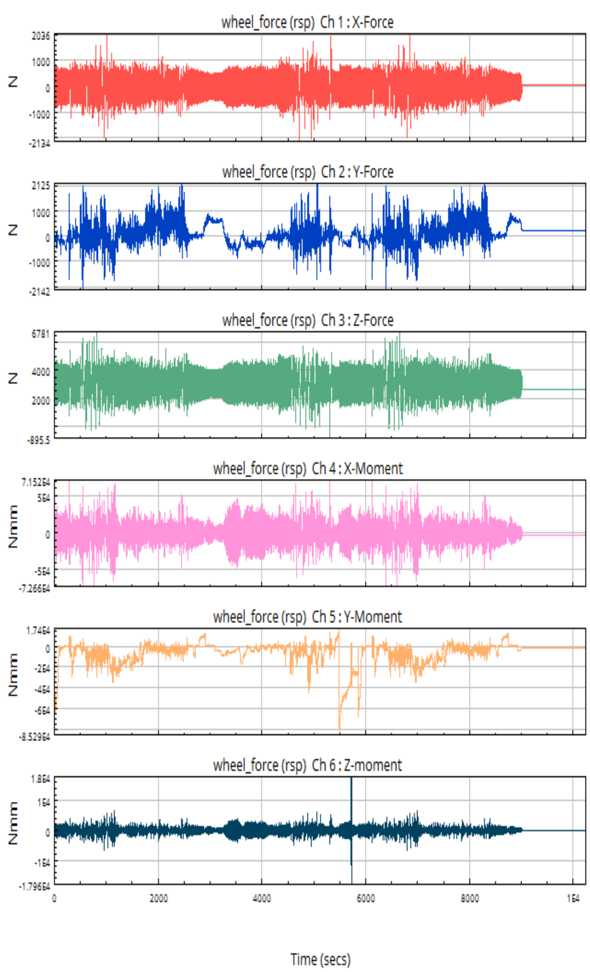
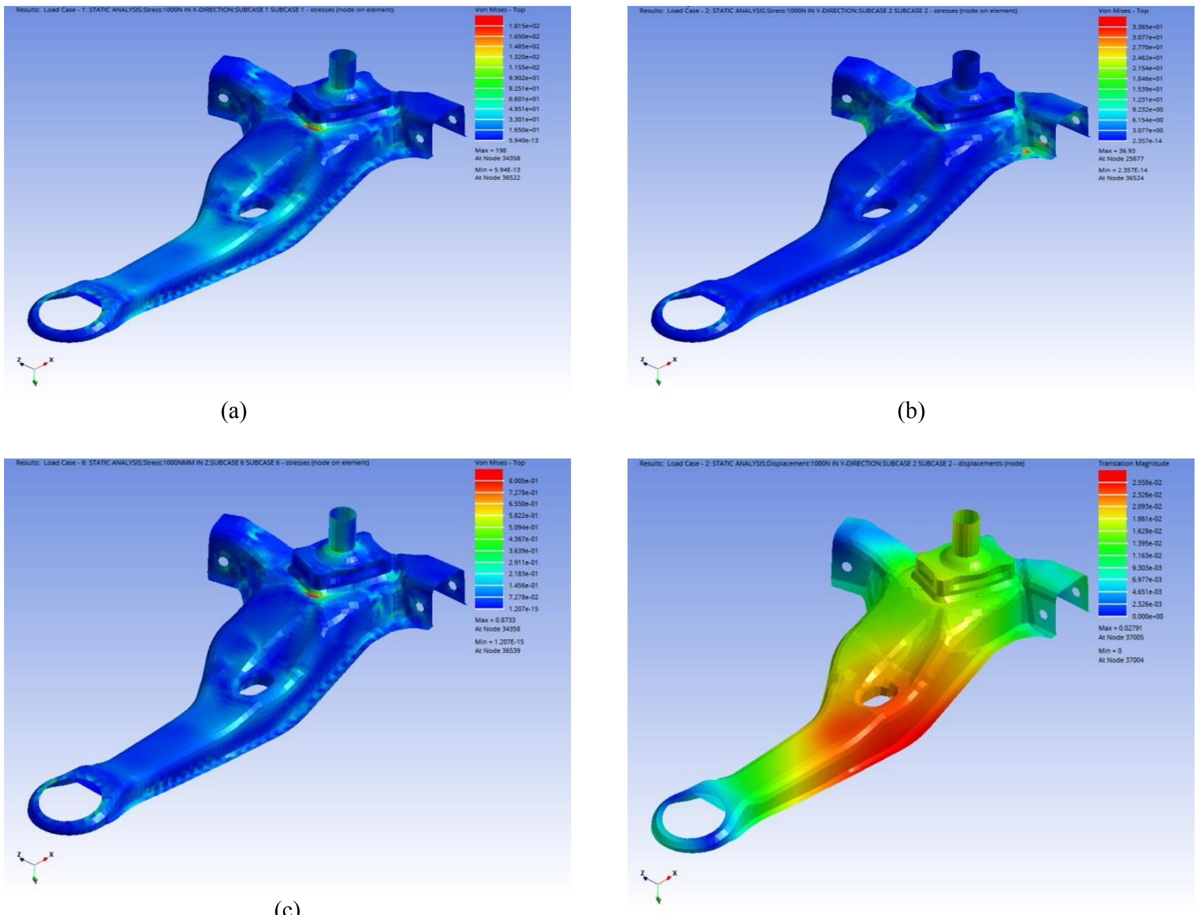


Fig.13
Changes in the force and moment components applied to the trailing arm over time.

The equivalent stress and total displacement of the suspension arm under the unit load and the unit moment in different directions are shown in Fig. 14. As shown in this figure, some loading modes, such as force in the *x*-direction or moment in the *z*-direction, cause high stress on the weld line; in a way that the highest stress in the whole structure belongs to the welding line. As mentioned earlier, the obtained stress fields are the basis for estimating the fatigue life of the part with different criteria. Next, each load and moment component's oscillations applied to the part in different directions correspond to the stress field resulting from the unit load in that direction. This method is valid in both Dang Van criteria and calculation of welding life by Volvo method, and in general, it is a kind of basis for all numerical fatigue theories.



(a) (b) (c) (d)

Fig.14
Equivalent stress field and displacement for the suspension arm due to force and unit moment in different directions: a) stress field for a unit load of 1 *kN* along the *x*-axis, b) stress field for a unit load of 1 *kN* along the *y*-axis, c) The stress field for a unit moment of 1 *kN / mm* along the *z*-axis, d) Displacement field for a unit load of 1 *kN* along the *y*-axis.

Initially, total parts of the components was analyzed using the Dang Van criterion. As mentioned in the previous sections, the output of this criterion is a safety factor. A safety factor of less than one means that the loaded structure will not withstand the number of cycles and would be failed during the time. On the contrary, a safety factor higher than one means that the structure's design is suitable for the applied load. The contour of the safety factor obtained from the Dang Van method is shown in Fig. 15. As shown in this figure, the weakest point of the structure has a safety factor of 1.259, which means that this criterion predicts that all parts of the structure will withstand the applied loads and will not be damaged under the desired working conditions. In this figure, the elements attached to the welding line are shown separately. According to the figure, this criterion predicts the welding line will not be damaged in this loading, and the minimum safety factor of the welding line elements is 2.63.

At the next step, fatigue life of the weld line is calculated using Volvo method by ncode Design Life. Contrary to Dang Van's theory, which provides the safety factor as a result of the analysis, damage of the structure would be obtained by using the Volvo method. A damage value of more than one indicates that the structure would be failed under the applied loads. Since this theory is specific to welding, it also calculates damage to weld root elements in addition to the weld throat. The root and throat elements of the welding line of the suspension arm both are considered in simulation. The damage of welding line elements is shown in Fig. 16.

As shown in Fig. 16, the damage to the weld line occurred mainly in its root and throat area. The important point is that according to the theory specific to welding, the maximum damage is 2.78, which practically indicates that the weld would be failed. While according to Dang Van's theory, the welding line, like other parts of the structure, has a safety factor higher than one, which is the wrong estimation. So, using the weld specific theory, for the weld line is necessary to estimate the fatigue life of the structure correctly. Of course using other appropriate theories such as Dang van is necessary, but it is not enough for welded structure.

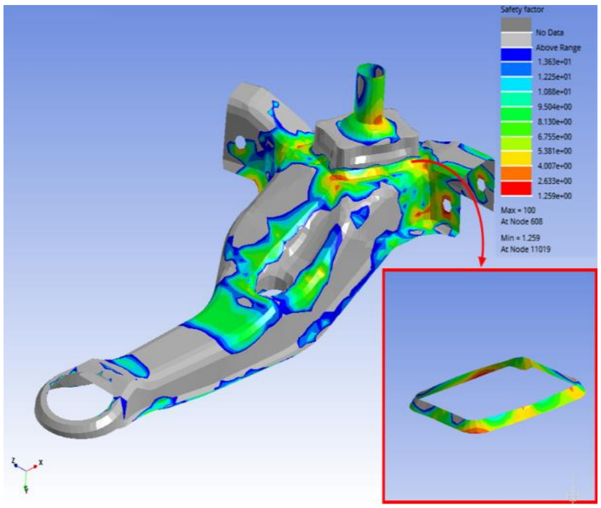


Fig.15
The safety factor contour of the trailing arm obtained by Dang Van method.

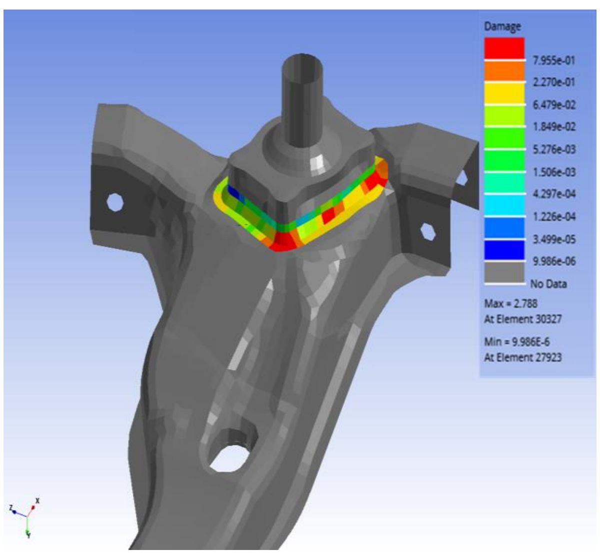


Fig.16
Welding line damage in the trailing arm structure.

4 CONCLUSIONS

In this paper, fatigue analysis is performed on the trailing arm, a welded component under multi axial non proportional loading, with the Dang Van criterion and the specialized analysis of the welding line (Volvo method). Using Dang Van criterion, which is an excellent theory for structures with multi axial non proportional loads, the fatigue life of the parent parts has been evaluated correctly. Using the critical plan method is one of the most important feature of Dang Van criterion that makes it suitable for this type of loading. Before analyzing the trailing arm, the applied theory is compared with the available experimental test results. The obtained numerical results verified very well with the experimental ones. Finally, based on the Dang Van criterion, safety factor of the component is above 1 that means the trailing arm would tolerated the applied fatigue load. Of course the Dang Van criterion is an appropriate method for a components such as trailing arm, but it does not considered welding properties. So it in the welding area, the obtained results are not accurate. In other words, specialized welding theories should be used to assess the damage or life of weld line.

As Dang Van's theory showed, the weld line will not be damaged under the load, and its safety factor is more than one, while under the same load, Volvo's method determines a damage value of more than one for the seam weld, which means connection would fail under the applied loads. Finally, the following results can be summarized:

- All loads components and their repetitions are considered exactly in the fatigue analysis of the nCode Design software.
- Dang van criterion is a very suitable for the analysis of engineering structures especially the structures under multi-axial and non-proportional loads.
- However, Dang van criterion cannot estimate correctly the life and performance of the weld.
- For simulation the seam weld and estimation the life time or damage of the weld line a special criteria developed based on the welding process must be used.
- Volvo method is developed based on the weld process parameters, tries to present a practical method in the automotive industry for calculating fatigue life by considering welding parameters and their influence on the fatigue life and fatigue damage.
- Volvo method in the nCode Design software is considered the effect of all important parameters on the fatigue life of weld line, such as: Effect of average thickness, Size effect, Material properties, Bending ratio and etc.
- Based on the Dang van method, safety factor of the seam weld is 1.25, but Volvo method shows that the damage of weld line is higher than 1, in other words, based on this theory, the weld line would be failed under the fatigue load during load cycle.
- Totally, for estimation of the life time of a welded structure such as trailing arm, using two method is necessary and crucial: a method for calculating the fatigue life and fatigue damage of the weld line, and on other method for assessment the parent sheets and whole of the structure.

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