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An Interval Parametric Approach for the Solution of One Dimensional Generalized Thermoelastic Problem

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ABSTRACT

This paper is presenting the solutions of the one dimension generalized thermo-elastic coupled equations by considering some thermo-elastic constants as interval numbers. As most of the elastic constants are obtained using the experimental methods. Thus, there might be some deficiency of exactness to obtain such constants. This kind of deficiency might cause the results on a micro-scale. L-S model has been considered to study the effect of such an interval parametric approach to generalized Thermoelasticity. Laplace transform method applied to obtain a system of coupled ordinary differential equations. Then the vector-matrix differential form is used to solve these equations by the eigenvalue approach in Laplace transformed domain. The solution in the space-time domain obtained numerically. The numerical solutions obtained by using some suitable inverse transformation method. The solutions are graphically represented for different values of the parameter of interval parametric form and the significance of obtained results are described along with the behavior of the solutions.

Keywords : Eigen value; Generalized Thermoelasticity; Interval number; Laplace transformation; Vector matrix differential equation.

1 INTRODUCTION

TO describe the physical phenomena mathematics leads an important role. One of such physical phenomena is the elasticity and particularly the Thermoelasticity. The theory of elasticity was first developed by R. Hooke (1678) and Mariotee (1680), Navier in 1821 and many more. There are so many thermo-elastic material constants such as thermal expansion coefficients, Young's modulus, Lame's constants etc. arise during the development of the theory of Thermoelasticity [1-3]. The values of material constants are generally obtained by different kind of experiments such as Searle's method etc. Hence the value of such material constants may differ [4], i.e. uncertainties arises for these kinds of error described by Muhanna and Mullen [5]. Interval analysis method used by S. S. Rao and L. Berke [6] to describe uncertainties in structural systems. Therefore, a tiny difference of such constants

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can affect the system depending on these constants for a particular metal. Hence, we consider the concept of interval number proposed by J.J. Dinkle and M.J. Tretter in 1987 [7] to sensitivity analysis of geometric programming problems. The same concept is used by D.Pal et al. in the paper [8] to investigate the analysis of the predator-prey model. Lord and Shuman(L-S) [9] presented the generalized dynamical theory of elasticity with one relaxation time parameters in 1967 whereas Green and Lindsay (G-L) proposed thermoelastic theory with two relaxation time parameter [10]. The several problems has been solved based on L-S theory and Green and Lindsay (G-L). 'In both the theories, the conventional Fourier law of heat conduction has been modified to a hyperbolic type of equation. As such both the theories ensure finite speed of propagation of waves and eliminate the paradox of infinite speed of propagation inherent in both the uncoupled and coupled theories of Thermoelasticity automatically. Lord and Shulman developed their L-S theory for isotropic media in the absence of heat source [11]. In the presence of heat source, an extension for anisotropic body L-S theory made by Dhaliwal and Sherief [12]. The combined generalized Thermoelasticity for both L-S and G-L model derived by Noda et al. [13]. the solution procedure of such coupled or generalized thermoelastic problem is based on to select a compatible function. The limitations of such approach discussed in Bahar and Hetnarski [14].

In this paper, the one-dimensional problem of coupled partial differential equations of generalized Thermoelasticity for homogeneous isotropic body occupying the half-space [15] has been considered. The solutions of such coupled equations proposed by A.Lahiri et al. using the matrix method [16,17] Since all the constants in the displacement equation and heat conduction equation are obtained by the experimentally, values of this constants can be assumed as interval number.

2 PRELIMINARIES

Definition 1: Interval number. An interval number $\hat{A}$ is closed interval $\left[A_L, A_U\right]$where A_L and A_U are the lower limit and upper limit of the interval $\hat{A}$ respectively i.e. $\hat{A}=\{x \in R : A_L \le x \le A_U, A_L < A_U\}$.

Definition 2: Let $\hat{A}=\left[A_L, A_U\right]$ and $\hat{B}=\left[B_L, B_U\right]$ are two interval number then $\hat{A} \le \hat{B}$ iff $A_L \le B_L$ and $A_U \le B_U$.

Definition 3: Interval valued function. Let $\hat{A}=\left[A_L, A_U\right]$ is an interval number with $A_L > 0$. Then interval valued function $g:[0,1] \to \hat{A}=\left[A_L, A_U\right]$ is defined as $g_{\hat{A}}(r)=A_L^{1-r}A_U^r, r \in [0,1]$. Here for the different value of the parameter (r) we get the different value within the interval $[A_L, A_U]$.

Properties 4: Let $\hat{A}=\left[A_L, A_U\right]$and $\hat{B}=\left[B_L, B_U\right]$ are two interval numbers with $A_L > 0$.

a) Addition: $\hat{A}+\hat{B}=[A_L, A_U]+[B_L, B_U]=[A_L+B_L, A_U+B_U]$ and the corresponding interval valued function is given by $g_{\hat{A}+\hat{B}}(r)=\left(A_L+B_L\right)^{1-r}\left(A_U+B_U\right)^r$.

b) Subtraction: $\hat{A}-\hat{B}=[A_L, A_U]-[B_L, B_U]=[A_L-B_L, A_U-B_U]$ and the corresponding interval valued function is given by $g_{\hat{A}-\hat{B}}(r)=\left(A_L-B_L\right)^{1-r}\left(A_U-B_U\right)^r$

c) Scalar multiplication: The multiplication of interval number A by the scalar k is defined as $k\hat{A}=k\left[A_L, A_U\right]=\begin{cases}\left[kA_L, kA_U\right] \text{ if } k \ge 0\\ \left[kA_U, kA_L\right] \text{ if } k < 0\end{cases}$.

and the corresponding interval valued function $g:[0,1]\to\hat{A}=\left[A_L, A_U\right]$ is given by $g_{k\hat{A}}(r)=\begin{cases}(kA_L)^{1-r}(kA_U)^r \text{ if } k \ge 0\\ (|k|A_U)^{1-r}(|k|A_L)^r \text{ if } k \le 0\end{cases}$.

Lemma 5: Let $\hat{A}=\left[A_L, A_U\right]$is an interval number with $A_L > 0$ then $g_{\hat{A}}(r)=A_L^{1-r}A_U^r$ is strictly monotone increasing continuous function for $0 \le r \le 1$.

Proof: It is obvious that A_L^{1-r}, A_U^r are continuous function of *r* for $0 < A_L \le A_U$. Therefore $g_{\hat{A}}(r)$ is a continuous function of *r* then $\frac{d[g_{\hat{A}}(r)]}{dr} = A_L\left(\frac{A_L}{A_U}\right)^r \ln\left(\frac{A_L}{A_U}\right) \ge 0$ as $\left(\frac{A_L}{A_U}\right)^r \ge 0$ and $\ln\left(\frac{A_L}{A_U}\right) \ge 0$ for $A_L \ge 0$. Therefore $g_{\hat{A}}(r)$ is monotonically increasing functions of *r*.

Lemma 6: Let $\hat{A} = [A_L, A_U]$ and $\hat{B} = [B_L, B_U]$ are two closed intervals if $\hat{A} \le \hat{B}$ then $A_L^{1-r}A_U^r \le B_L^{1-r}B_U^r$. The proof is visible.

3 PROBLEMS

According to Wilms and Cohen in 1985 [18] the displacement equation of motion and heat conduction equation for the one dimension generalized Thermoelasticity are given by

$$\begin{aligned}
\frac{\partial^2 u}{\partial x^2} &= \frac{1}{v^2}\frac{\partial^2 u}{\partial t^2} + \frac{\beta}{\rho v^2}\frac{\delta}{\delta x}\left(1+\alpha\frac{\delta}{\delta t}\right)T \\
\frac{\partial^2 T}{\partial x^2} &= \frac{C}{v^2}\frac{\partial}{\partial t}\left(1+\alpha_0\frac{\partial}{\partial t}\right)T + \frac{\rho\varepsilon C}{\beta}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x\,\partial t}
\end{aligned} \tag{1}$$

or,

$$\frac{\partial^2 T}{\partial x^2} = \frac{C}{v^2}\frac{\partial}{\partial t}\left(1+\alpha_0\frac{\delta}{\delta t}\right)T + \frac{\rho\beta C T_0}{c_\varepsilon(\lambda+2\mu)}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x\,\partial t} \tag{2}$$

Stress,

$$\sigma = \rho v^2\frac{\partial u}{\partial x} - \beta(1+\alpha\frac{\partial}{\partial t})T \tag{3}$$

where the displacement *u* and temperature *T* are the functions of position *x* and time *t*. i.e, $u = u(x,t)$, $T = T(x,t)$.

$\rho =$ Mass density of the metal, $v = \sqrt{\frac{\lambda+2\mu}{\rho}}$ = Dilation wave speed (λ, μ are Lame's constant), $\beta = \alpha_t(3\lambda+2\mu)$,

α_t = Linear coefficient of thermal expansion, $C = \frac{v^2}{k}$ = Thermoelastic frequency, $k = \frac{\delta}{\rho c_\varepsilon}$ = Thermal diffusivity,

$\delta =$ Thermal conductivity, $c_\varepsilon =$ Specific heat of metal, $\varepsilon = \frac{\beta^2 T_0}{c_\varepsilon(\lambda+2\mu)}$ = Coupling constant. $T_0 =$ Absolute temperature. α, α_0 and τ are the thermal relaxation parameters.

For the different values of these relaxation parameters, we get the different types of problems of Thermoelasticity as:

(i) if $\alpha = 0, \alpha_0 = 0$ and $\tau = 0$ then the equations reduced to the problem of classical Thermoelasticity (CTE).

(ii) if $\alpha = 0, \alpha_0 = 0$ and $\tau \ne 0$ then the problem reduced to the problem of extended Thermoelasticity i.e. L-S theory (ETE).

(iii) if $\alpha \ne 0, \alpha_0 \ne 0$ and $\tau = 0$ then the problem becomes temperature rate dependent Thermoelasticity (TRDTE) problem. It is commonly known as GL theory.

Since here all these constants are determined by different kind of experimental methods for a fixed metal, therefore the experimental values of these physical constants are not exact for that metal. So we can set the values of these constants as interval numbers. Now we will study the effect of change of such physical constants in the Eqs. (1),(2) and (3).

Let $\hat{\lambda},\hat{\mu}$ are the interval counterpart of $\lambda(>0),\mu(>0)$ respectively, also the interval counterpart of the constants *v, C* and β are: Eq. (1) and (2) become (3), Hence, $\hat{v}$,$\hat{C}$ and $\hat{\beta}$ respectively.

$$\frac{\partial^2 u}{\partial x^2}=\frac{1}{\hat{v}^2}\frac{\partial^2 u}{\partial t^2}+\frac{\hat{\beta}}{\rho\hat{v}^2}\frac{\delta}{\delta x}\left(1+\alpha\frac{\delta}{\delta t}\right)T \tag{4}$$

$$\frac{\partial^2 T}{\partial x^2}=\frac{\hat{C}}{\hat{v}^2}\frac{\partial}{\partial t}\left(1+\alpha_0\frac{\delta}{\delta t}\right)T+\frac{\rho\hat{\beta}\hat{C}T_0}{c_\varepsilon(\hat{\lambda}+2\hat{\mu})}\left(1+\tau\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x\,\partial t} \tag{5}$$

$$\sigma=\rho\hat{v}^2\frac{\partial u}{\partial x}-\hat{\beta}(1+\alpha\frac{\partial}{\partial t})T \tag{6}$$

where $\hat{\lambda}\in[\lambda_L,\lambda_U]$, $\hat{\mu}\in[\mu_L,\mu_U]$,$\hat{v}\in[v_L,v_U]$, $\hat{C}\in[C_L,C_U]$ and $\hat{\beta}\in[\beta_L,\beta_U]$.

$$\left.\begin{aligned}\hat{\lambda}&=\lambda_L^{\,1-r}\lambda_U^{\,r}\\ \hat{\mu}&=\mu_L^{\,1-r}\mu_U^{\,r}\\ \hat{v}&=v_L^{\,1-r}v_U^{\,r}\\ \hat{C}&=C_L^{\,1-r}C_U^{\,r}\\ \hat{\beta}&=\beta_L^{\,1-r}\beta_U^{\,r}\end{aligned}\right\}\text{ for } 0\le r\le 1 \tag{7}$$

$$\left.\begin{aligned}v_i&=\sqrt{\frac{\lambda_i+2\mu_i}{\rho}}\\ C_i&=\frac{v_i^{\,2}}{\kappa}\\ \beta_i&=\alpha_t\left(3\lambda_i+2\mu_i\right)\end{aligned}\right\}\text{ (i=L,U)} \tag{8}$$

Now we will solve the above system of Eqs.(4-5) using (7), (8) along with the boundary condition

$$\left.\begin{aligned}\sigma&=H(t)\\ T&=0\end{aligned}\right\}\text{ at } x=0 \tag{9}$$

where $H(t)$ is Heaviside function.

4 SOLUTIONS OF THE PROBLEM

Denote Laplace transform of $f(x,t)$ as $\bar{f}(x,p)=\bar{f}$ defined by $\bar{f}=\bar{f}(x,p)=\int_0^\infty e^{-pt}f(x,t)dt$ Re(p)>0 .Taking Laplace Transformation of Eqs. (4), (5) and (6) by using (7) and (8) we obtain

$$\frac{d^2\bar{u}}{dx^2}=\frac{p^2}{{v_L}^{2(1-r)}{v_U}^{2r}}\bar{u}+\frac{{\beta_L}^{1-r}{\beta_U}^{r}}{\rho {v_L}^{2(1-r)}{v_U}^{2r}}(1+\alpha p)\frac{d\bar{T}}{dx} \tag{10}$$

$$\frac{d^2\bar{T}}{dx^2}=\frac{{C_L}^{1-r}{C_U}^{r}p(1+\alpha_0 p)}{{v_L}^{2(1-r)}{v_U}^{2r}}\bar{T}+\frac{\rho T_0(C_L{\beta_L}^{1-r})(C_U\beta_U)^r}{c_\varepsilon({\lambda_L}^{1-r}{\lambda_U}^{r}+2{\mu_L}^{1-r}{\mu_U}^{r})}p\left(1+\tau p\right)\left(\frac{d\bar{u}}{dx}\right) \tag{11}$$

$$\bar{\sigma}=\rho {v_L}^{2-2r}{v_U}^{2r}\frac{d\bar{u}}{dx}-{\beta_L}^{1-r}{\beta_U}^{r}\left(1+\alpha p\right)\bar{T} \tag{12}$$

Here, initially (i.e. at $t = 0$) u, T and their partial derivatives with respect to t are considered as zero. The corresponding boundary condition (9) becomes:

$$\left.\begin{aligned}\bar{\sigma}&=\frac{1}{p}\\ \bar{T}&=0\end{aligned}\right\}\text{ at } x=0 \tag{13}$$

Put $T'=\dfrac{d\bar{T}}{dx}$, $u'=\dfrac{d\bar{u}}{dx}$ and set $a_{31}=\left(\dfrac{c_L}{{v_L}^2}\right)^{(1-r)}\left(\dfrac{c_U}{{v_U}^2}\right)^{r}p\left(1+\alpha_0 p\right)$; $a_{34}=\rho\left(c_L\beta_L\right)^{(1-r)}\left(c_U\beta_U\right)^{r}\dfrac{pT_0\left(1+\tau p\right)}{c_\varepsilon\left({\lambda_L}^{1-r}{\lambda_U}^{r}+2{\mu_L}^{1-r}{\mu_U}^{r}\right)}$; $a_{42}=\dfrac{p^2}{{v_L}^{2(1-r)}{v_U}^{2r}}$; $a_{43}=\left(\dfrac{\beta_L}{{v_L}^2}\right)^{(1-r)}\left(\dfrac{\beta_U}{{v_U}^2}\right)^{r}\dfrac{\left(1+\alpha p\right)}{\rho}$. Then Eqs. (10), (11) can be written as vector matrix differential form as $\dfrac{d}{dx}\begin{pmatrix}\bar{T}\\ \bar{u}\\ \bar{T}'\\ \bar{u}'\end{pmatrix}=\begin{pmatrix}0&0&1&0\\0&0&0&1\\a_{31}&0&0&a_{34}\\0&a_{42}&a_{43}&0\end{pmatrix}\begin{pmatrix}\bar{T}\\ \bar{u}\\ \bar{T}'\\ \bar{u}'\end{pmatrix}$ or, $\dfrac{d\chi}{dx}=A\chi$ where $A=\begin{pmatrix}0&0&1&0\\0&0&0&1\\a_{31}&0&0&a_{34}\\0&a_{42}&a_{43}&0\end{pmatrix}$, $\chi=\begin{pmatrix}\bar{T}\\ \bar{u}\\ \bar{T}'\\ \bar{u}'\end{pmatrix}$.

The characteristic equation of $\boldsymbol{A}$ is given by $|A-\Gamma I_4|=0$ or, $\Gamma^4-(a_{42}+a_{34}a_{43}+a_{31})\Gamma^2+a_{31}a_{42}=0$.

Hence the eigen values of A obtained as $\Gamma=\pm\Gamma_1,\pm\Gamma_2$ where $\Gamma_1=\dfrac{1}{\sqrt{2}}\sqrt{a_{31}+a_{42}+a_{34}a_{43}-\sqrt{-4a_{31}a_{42}+(a_{31}+a_{42}+a_{34}a_{43})^2}}$, $\Gamma_2=\dfrac{1}{\sqrt{2}}\sqrt{a_{31}+a_{42}+a_{34}a_{43}+\sqrt{-4a_{31}a_{42}+(a_{31}+a_{42}+a_{34}a_{43})^2}}$.

The eigenvector corresponding to the Eigen-value Γ is $\begin{pmatrix}a_{42}-\Gamma^2\\ -\Gamma a_{43}\\ \Gamma(a_{43}-\Gamma^2)\\ -\Gamma^2 a_{43}\end{pmatrix}$.

Therefore the solution of above matrix differential equation becomes:

$$\begin{pmatrix}\bar{T}\\ \bar{u}\\ \bar{T}'\\ \bar{u}'\end{pmatrix}=d_1\begin{pmatrix}a_{42}-{\Gamma_1}^2\\ -\Gamma_1 a_{43}\\ \Gamma_1(a_{43}-{\Gamma_1}^2)\\ -{\Gamma_1}^2 a_{43}\end{pmatrix}e^{\Gamma_1 x}+d_2\begin{pmatrix}a_{42}-{\Gamma_1}^2\\ \Gamma_1 a_{43}\\ -\Gamma_1(a_{43}-{\Gamma_1}^2)\\ -{\Gamma_1}^2 a_{43}\end{pmatrix}e^{-\Gamma_1 x}+d_3\begin{pmatrix}a_{42}-{\Gamma_2}^2\\ -\Gamma_2 a_{43}\\ \Gamma_2(a_{43}-{\Gamma_2}^2)\\ -{\Gamma_2}^2 a_{43}\end{pmatrix}e^{\Gamma_2 x}+d_4\begin{pmatrix}a_{42}-{\Gamma_2}^2\\ \Gamma_2 a_{43}\\ -\Gamma_2(a_{43}-{\Gamma_2}^2)\\ -{\Gamma_2}^2 a_{43}\end{pmatrix}e^{-\Gamma_2 x}$$

Hence the solution of the system of Eqs. (10)-(11) obtained by considering the regularity condition of solution taking integrating constants d_1, d_3 associated with the corresponding terms $e^{\Gamma_1 x}, e^{\Gamma_2 x}$ as zero. Therefore solution for temperature and displacement becomes as follows:

$$\bar{T} = d_2\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_1^2\right)e^{-\Gamma_1 x} + d_4\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_2^2\right)e^{-\Gamma_2 x}$$

$$\bar{u} = \left(\frac{\beta_L}{v_L^2}\right)^{(1-r)}\left(\frac{\beta_U}{v_U^2}\right)^r \frac{(1+\alpha p)}{\rho}(\Gamma_1 d_2 e^{-\Gamma_1 x} + \Gamma_2 d_4 e^{-\Gamma_2 x})$$

where d_2, d_4 the constants are determined by using the boundary conditions (13) as follows:

$$d_2 = -\left(\frac{v_L^2}{\beta_L}\right)^{(1-r)}\left(\frac{v_U^2}{\beta_U}\right)^r \frac{1}{(1+p\alpha)(\Gamma_1^2 - \Gamma_2^2)p^3}\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_2^2\right)$$

$$d_4 = \left(\frac{v_L^2}{\beta_L}\right)^{(1-r)}\left(\frac{v_U^2}{\beta_U}\right)^r \frac{1}{(1+p\alpha)(\Gamma_1^2 - \Gamma_2^2)p^3}\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_1^2\right)$$

Therefore, we obtain the solution of the Eqs. (10)-(11) as:

$$\bar{T} = -\frac{\left(\frac{v_L^2}{\beta_L}\right)^{(1-r)}\left(\frac{v_U^2}{\beta_U}\right)^r}{(1+p\alpha)(\Gamma_1^2 - \Gamma_2^2)p^3}\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_2^2\right)\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_1^2\right)\left(e^{-\Gamma_1 x} - e^{-\Gamma_2 x}\right)$$

$$\bar{u} = \left(\frac{\beta_L}{v_L^2}\right)^{(1-r)}\left(\frac{\beta_U}{v_U^2}\right)^r \frac{(1+\alpha p)}{\rho}\left(\frac{v_L^2}{\beta_L}\right)^{(1-r)}\left(\frac{v_U^2}{\beta_U}\right)^r \frac{1}{(1+p\alpha)(\Gamma_1^2 - \Gamma_2^2)p^3}\left(-\Gamma_1\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_2^2\right)e^{-\Gamma_1 x} + \Gamma_2\left(\frac{p^2}{v_L^{2(1-r)} v_U^{2r}} - \Gamma_1^2\right)e^{-\Gamma_2 x}\right) \tag{14}$$

or,

$$\bar{u} = \frac{1}{\rho(\Gamma_1^2 - \Gamma_2^2)p^3}\left\{-\Gamma_1\left(\frac{p^2}{v_1^{2(1-r)} v_2^{2r}} - \Gamma_2^2\right)e^{-\Gamma_1 x} + \Gamma_2\left(\frac{p^2}{v_1^{2(1-r)} v_2^{2r}} - \Gamma_1^2\right)e^{-\Gamma_2 x}\right\} \tag{15}$$

Rewrite the Eq.(12) as:

$$\bar{\sigma} = \rho v_L^{2-2r} v_U^{2r}\frac{\partial \bar{u}}{\partial x} - \beta_L^{1-r}\beta_U^r(1+\alpha p)\bar{T} \tag{16}$$

Analytically it is very difficult to find *u, T* and σ from (14),(15) and (16). Now to find *u, T* and σ a suitable numerical method of inverse Laplace Transformation has been applied.

5 NUMERICAL RESULTS AND DISCUSSION

To illustrate our solution numerically the copper metal has been chosen with the following material property [19]:

$\rho = 8954 kg/m^2$, $k = 386 N/Ks$, $c_\epsilon = 383.1 m^2/K$, $\alpha_t = 1.78\times10^{-5}K^{-1}$, $T_0 = 293K$, $\alpha = 0.1$, $\alpha_0 = 0.05$, $\tau = 0$, $\lambda = 7.76\times10^{10} N/m^2$, $\mu = 3.86\times10^{10} N/m^2$.

Now investigate the effect for taking λ, μ as interval number and taking the following hypothetical values of parameter λ, μ for three cases:

(a) Only μ as interval number. In this case we take: $\mu_L = 3.85 \times 10^{10} N/m^2$, $\mu_U = 3.87 \times 10^{10} N/m^2$.

(b) Only λ as interval number. In this case we take: $\lambda_L = 7.75 \times 10^{10} N/m^2$, $\lambda_U = 7.77 \times 10^{10} N/m^2$.

(c) Both λ and μ as interval number. In this case we take $\mu_L = 3.85 \times 10^{10} N/m^2$, $\mu_U = 3.87 \times 10^{10} N/m^2$, $\lambda_L = 7.75 \times 10^{10} N/m^2$, $\lambda_U = 7.77 \times 10^{10} N/m^2$.

The temperature, displacement and stress distribution are represented graphically for the above cases at immediate after the impact, i.e. at $t = 0.01$. Wolfram Mathematica 9 has been used to find the numerical values of inverse Laplace transformations of $\bar{u}, \bar{T}$ and $\bar{\sigma}$ with the help of Zakian method [20-22] and MATLAB R2016b to perform numerical simulation experimentation.

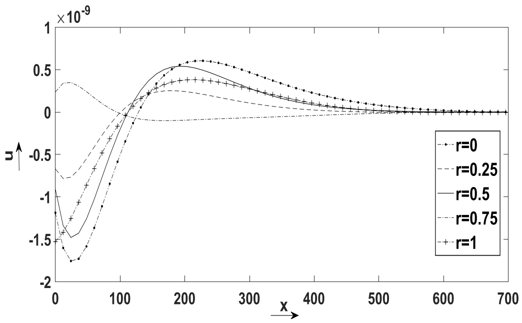

Fig.1
Distribution of displacement (u) assuming only μ as interval number case (a).

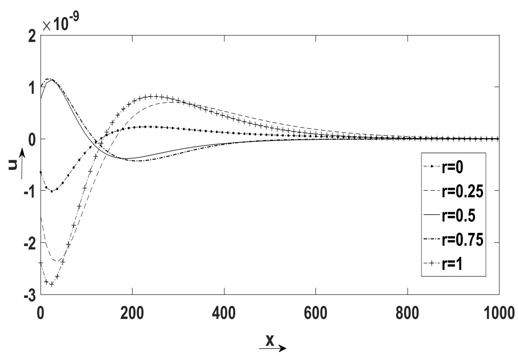

Fig.2
Displacement (u) distribution assuming only λ as interval number case (b).

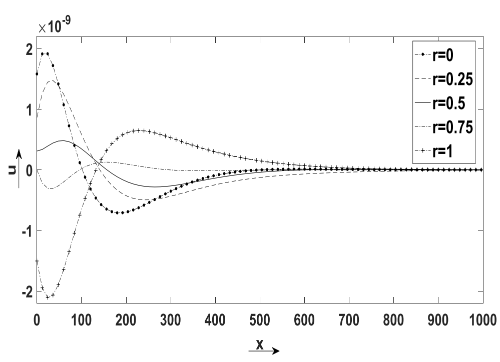

Fig.3
Displacement (u) distribution assuming λ and μ both as interval number case (c).

Figs. 1-3 shows the distribution for the displacement at time $t = 0.01$ for the three cases (a),(b) and (c) respectively. In Fig. 1, the behaviour of the distribution of u is slightly differed near the $x = 0$ for the different values of r, for considering only μ as interval number. But for λ as interval number the distributions of u are closer for the values of $r = 0.5$ and 0.75, for $r = 0$, 0.25, 1 there are remarkable differences at $x = 25$ (approx.). From the Fig. 3, it is clear that the distribution of u for $r = 0.5$ lies between the distribution of u for $r = 0$. The distribution of u for

r=1(also for r = 0.75) i.e. we can roughly say the distribution of u for r = 0.5 is like mean distribution of u for other values of r.

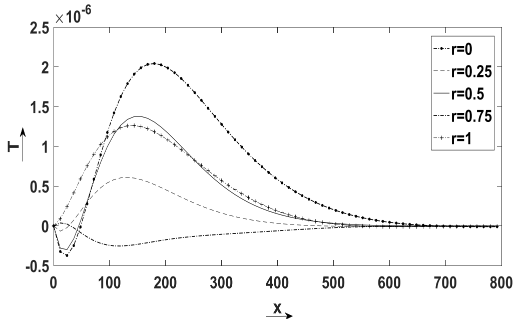

Fig.4
Temperature (T) distribution assuming only μ as interval number case (a).

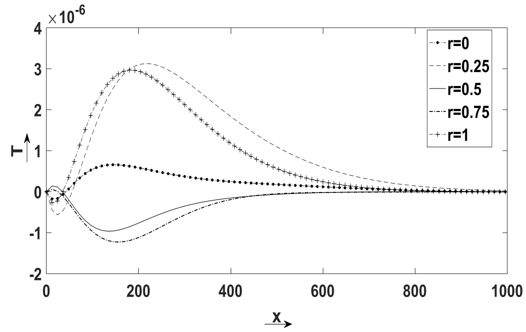

Fig.5
Temperature (T) distribution assuming λ as interval number case(b).

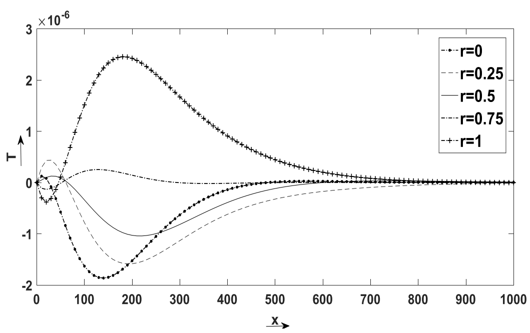

Fig.6
Temperature (T) distribution assuming λ and μ both as interval number case (c).

Figs. 4-6 show, the distribution for the temperature T at time t = 0.01 for the three cases (a), (b) and (c) respectively. The temperature distribution, T for only μ as interval number the Fig. 4 shows that for r = 0.5, the curve is closed to the curve for r = 0(x less than 100) as x increase the curve for r = 0, become closer to the curve for r = 1. Fig. 5 represents the distribution of temperature for only λ as interval number. Here for r = 0.5 and r = 0.75, the curves are similar type and closer to each other and also the curves for r = 0.25 and r = 0.75 are the same type, whereas curve for r = 0 has remarkably differed from other curves. Fig. 6 represents the distribution for both μ and λ as interval number, in this case, distribution of T for different values of r lies between the distributions for r = 0 to r = 1.

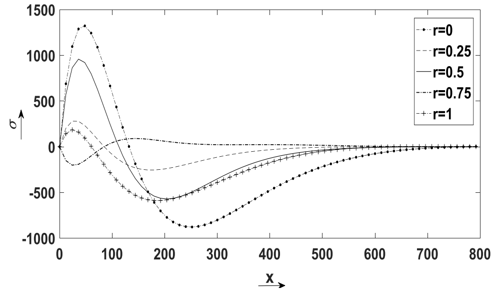

Fig.7
Stress (σ) distribution assuming μ as interval number case (a).

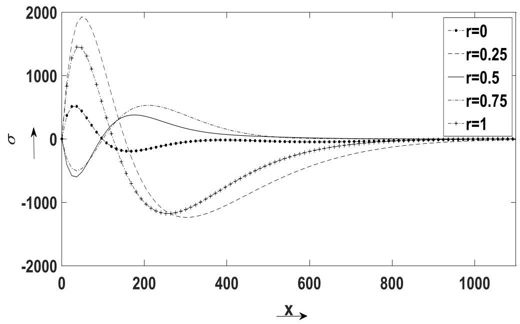

Fig.8
Stress (σ) distribution assuming λ as interval number case(b).

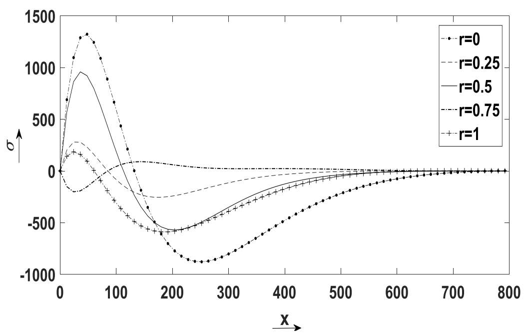

Fig.9
Stress (σ) distribution assuming λ and μ both as interval number case (c).

Stress distributions at time $t = 0.01$ for the cases (a),(b) and (c) are shown in Figs.7-9. In Fig. 7 stress distribution has different curves for different values of r for the only μ as interval number. In this case, the stress distributions for $r = 0.5$ and $r = 1$ are closer for x greater than 175 (approx). The stress distribution for only λ as interval number shown in Fig. 8 where the stress distribution for $r = 0.5$ and $r = 0.75$ are much closer than other values of r. Lastly, the Fig. 9 of stress distribution for both μ and λ as interval number shows the curves for $r = 0.5$ and $r = 1$ are closer for x greater than 200 (approx), and all the other curves very much different for different values of r.

6 CONCLUSION

In the previous section, the effects of the uncertainty of some thermo-elastic constants have been noted. A minimal change of thermo-elastic constants such as Lames' constants etc., gives effective transition to the displacement, temperature and stress. This might play an effective role in future engineering fields that deals with thermoelastic effect in micro scale. It can be applied to constructional projects.

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