Thermoelastic Damping and Frequency Shift in Kirchhoff Plate Resonators Based on Modified Couple Stress Theory With Dual-Phase-Lag Model

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ABSTRACT

The present investigation deals with study of thermoelastic damping and frequency shift of Kirchhoff plate resonators by using generalized thermoelasticity theory of dual-phase-lag model. The basic equations of motion and heat conduction equation are written with the help of Kirchhoff-Love plate theory and dual phase lag model. The analytical expressions for thermoelastic damping and frequency shift of modified couple stress dual-phase-lag thermoelastic plate have been obtained. A computer algorithm has been constructed to obtain the numerical results. Influences of modified couple stress dual-phase-lag thermoelastic plate; dual-phase-lag thermoelastic plate and Lord-Shulman (L-S, 1967) thermoelastic plate with few vibration modes on the thermoelastic damping and frequency shift are examined. The thermoelastic damping and frequency shift with varying values of length and thickness are shown graphically for clamped-clamped and simply supported boundary conditions. It is observed from the results that the damping factor and frequency shift have noticed larger value in the presence of couple stress for varying values of length but opposite effect are shown for varying values of thickness in case of both vibration modes and boundary conditions.

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Keywords : Modified couple stress theory; Kirchhoff-Love plate theory; Dual-phase-lag model; Thermoelastic damping; Frequency shift.

1 INTRODUCTION

TZOU [30-31] was the first who proposed the dual phase lag (DPL) model. This model describes the interactions between photons and electrons on the microscopic level as retarding sources causing a delayed response on the macroscopic level. The dual phase lag model was the modification of classical thermoelastic model in which the Fourier law is replaced by an approximation to a modified Fourier law with the introduction of two

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different time lags: one is the phase lag of the heat flux τ_q and is related to the thermal wave speed and other is the phase lag of the temperature gradient τ_θ that represents the time constant for electron-lattice equilibrium. A new model, known as three phase lag thermoelastic model, was introduced by Roychoudhuri [24]. The existence of couple-stress in materials was originally postulated by Voigt [32]. He assumed that the interaction between the two particles of a body through an area element is transmitted not solely the action of a force vector but also by a couple stress vector. This assumption leads to the description of stress field by means of two asymmetric tensors: the force stress tensor and the couple stress tensor. Cosserat and Cosserat [4] were the first to develop a mathematical model to analyze materials with couple stresses. Toupin [28] reviewed the basic concepts and foundations of classical theory [Cosserat and Cosserat [4]] and presented the generalization of it. He discussed some quantitative features of wave propagation in elastic materials with couple stresses and correct the equation in classical field theories for couple stresses. Mindlin and Tiersten [18] formulated a linearized theory of couple stress elasticity which contained two classical and two additional material constants for isotropic elastic materials. The effect of couple stresses on the surface concentration around a circular hole in an infinite medium under tension was studied by Mindlin [19]. Koiter [11] proposed classical couple stress elasticity theory which contains four material constants two classical and two additional for isotropic elastic materials. In these developments, the gradient of the rotation vector is used as a curvature tensor.

Yang et al. [33] developed a modified couple-stress model, in which the couple stress tensor is symmetrical and only one material length parameter is needed to capture the size effect which is caused by micro-structure. Park and Gao [21] constructed a new model for the bending of a Bernoulli-Euler beam using the minimum total potential energy principle and a modified couple stress theory. Variational formulation of a modified couple stress theory and its application to a simple shear problem was studied by Ma et al. [17]. Tsiatas [29] developed a new Kirchhoff plate model for the static analysis of isotropic micro-plates with arbitrary shape using modified couple stress theory containing only one material length scale parameter which can capture the size effect. Sun and Tohmyoh [27] derived the governing equations of coupled thermoelastic problem in axisymmetric out-of-plane vibration of circular plate and obtianted the analytical results for thermoelastic damping. Rezazadeh et al. [23] derived quality factor of thermoelastic damping of a clamped-clamped micro- beam using modified couple stress theory. Guo et al. [6] derived the formula for thermoelastic damping of a micro beam resonator in the context of dual-phase-lagging heat conduction model. Guo et al. [7] presented the thermoelastic damping effect of a circular micro-plate resonator under clamped and simply supported boundary conditions which is based on generalized thermoelasticity theory of dual-phase-lagging model. Alashti and Abolghasemi [2] developed a size- dependent Bernoulli-Euler beam formulation on the basis of new model of couple stress theory and prepared the mathematical formulation for clamped (C-C), simply supported (S-S) and cantilever (C-F) boundary conditions.

Sourki and Hoseini [26] studied free vibration of a cracked microbeam within the framework of Euler-Bernoulli beam theory using modified couple stress theory. Kakhki et al. [8] obtianted analytical solution for thermoelastic damping in a micro-beam based on modified couple stress theory in the context of one relaxation time. Kumar and Devi [10] studied thermoelastic beam in modified couple stress theory subjected to laser source and heat flux by employing the Euler-Bernoulli beam theory and Laplace transform technique. Uniqueness of solution of initial boundary value problem in thermoelasticity of bodies with voids, a porous thermoelastic body including voidage time derivative among the independent constitutive variables, effect of Thomson and initial stress in a thermo-porous elastic solid under GN electromagnetic theory can be found in (Marin [15], Marin and Florea [16], Marin et al. [1]). The vibrations of thin plate in modified couple stress thermoelastic medium with the help of Kirchhoff plate theory, thermoelastic functionally graded beam in modified couple stress theory subjected to a dual phase lag model have been discussed by Kumar and Devi [9-14].

In the present work, we investigated the vibrations of thin plate in modified couple stress thermoelastic medium by applying Kirchhoff- Love plate theory along with dual-phase-lag model. The Euler-Bernoulli theory and normal mode analysis are used to solve the generalized equations. The analytical expressions for thermoelastic damping and frequency shift of modified couple stress dual-phase-lag thermoelastic plate for clamped-clamped and simply supported boundary conditions have been obtianed for varying values of length and thickness. Special cases of interest are also given in the present problem.

2 BASIC EQUATIONS

Following Yang et al. [33], Rao [22], the constitutive equation, the equations of motion and the equation of heat

conduction in a modified couple stress generalized thermoelastic dual-phase-lag model without body forces, body couples and heat sources are

Constitutive relations

$$t_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} - \frac{1}{2} e_{kij} m_{lk,l} - \beta T \delta_{ij}, \tag{1}$$

$$m_{ij} = 2\alpha \chi_{ij}, \; \chi_{ij} = \frac{1}{2}\left(\omega_{i,j} + \omega_{j,i}\right), \; \omega_i = \frac{1}{2} e_{ipq} u_{q,p}, \tag{2}$$

Equation of motion

$$\left(\lambda + \mu + \frac{\alpha}{4}\Delta\right)\nabla(\nabla . \boldsymbol{u}) + \left(\mu - \frac{\alpha}{4}\Delta\right)\nabla^2 \boldsymbol{u} - \beta\nabla T = \rho \ddot{\boldsymbol{u}}, \tag{3}$$

Equation of heat conduction

$$\left(1 + \tau_T \frac{\partial}{\partial t}\right)K\Delta T - \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e \frac{\partial T}{\partial t} + \beta T_0 \frac{\partial e}{\partial t}\right) = 0, \tag{4}$$

where t_{ij} are the components of stress tensor, λ and μ are Lame constants, δ_{ij} is Kronecker's delta, e_{ij} are the components of strain tensor, e_{ijk} is alternate tensor, m_{ij} are the components of couple-stress, $\beta = (3\lambda + 2\mu)\alpha_t$, α_t are the coefficients of linear thermal expansion respectively, T is the temperature change, α is the couple stress parameter, χ_{ij} is symmetric curvature, ω_i is the rotational vector, u is displacement vector, ρ is the density, Δ is the Laplacian operator, ∇ is del operator. K is the coefficient of the thermal conductivity, c_e is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/T_0 \ll 1$, $E = \mu(3\lambda + 2\mu)/(\lambda + \mu)$ is Young's modulus, $\nu = \lambda/2(\lambda + \mu)$ is the Poisson ratio. Here τ_T and τ_q are the phase lags of the temperature gradient and the heat flux respectively.

3 FORMULATION OF THE PROBLEM

Let us consider a modified couple stress thermoelastic Kirchhoff plate resonators with uniform thickness h. The origin of the Cartesian coordinate system (x,y,z) is taken at the centre of the plate. In equilibrium conditions, the plate is unstrained, unstressed and continues at uniform environmental temperature T_0 everywhere. We define the displacement components $u(x,y,z,t)$, $v(x,y,z,t)$, $w(x,y,z,t)$ and temperature $T(x,y,z,t)$. According to Kirchhoff's-Love Plate theory, the displacement components are given by

$$u = -z \,\partial w / \partial x, \; v = -z \,\partial w / \partial y, \; w(x,y,z,t) = w(x,y,t). \tag{5}$$

Following Rao [22], the strain and stress components are taken as:

$$\varepsilon_{xx} = -z \frac{\partial^2 w}{\partial x^2}, \tag{6}$$

$$\varepsilon_{yy} = -z\frac{\partial^2 w}{\partial y^2}, \tag{7}$$

$$\gamma_{xy} = -2\mu z\frac{\partial^2 w}{\partial x\,\partial y} = -\frac{Ez}{(1+v)}\frac{\partial^2 w}{\partial x\,\partial y}. \tag{8}$$

$$t_{xx} = \frac{E}{(1-v^2)}\left(\varepsilon_{xx} + v\varepsilon_{yy} - (1+v)\alpha_T T\right), \tag{9}$$

$$t_{yy} = \frac{E}{(1-v^2)}\left(\varepsilon_{yy} + v\varepsilon_{xx} - (1+v)\alpha_T T\right), \tag{10}$$

$$t_{xy} = \mu\gamma_{xy}. \tag{11}$$

The bending and torsion moments are defined following (Rao [22], Chen and Li [3]) as:

$$M_x = \int_{-h/2}^{h/2} t_{xx}\,z\,dz + \int_{-h/2}^{h/2} m_{yx}\,dz, \tag{12}$$

$$M_y = \int_{-h/2}^{h/2} t_{yy}\,z\,dz - \int_{-h/2}^{h/2} m_{xy}\,dz, \tag{13}$$

$$M_{xy} = \int_{-h/2}^{h/2} t_{xy}\,z\,dz + \frac{1}{2}\int_{-h/2}^{h/2}\left(m_{yy} - m_{xx}\right)dz. \tag{14}$$

with the aid of Eqs. (5)-(11) in Eqs. (12)-(14), we obtain

$$M_x = -D^*\left(\frac{\partial^2 w}{\partial x^2} + v\frac{\partial^2 w}{\partial y^2} + \alpha_T M_T\left(1+v\right)\right) + \frac{\alpha h}{2}\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right), \tag{15}$$

$$M_y = -D^*\left(\frac{\partial^2 w}{\partial y^2} + v\frac{\partial^2 w}{\partial x^2} + \alpha_T M_T\left(1+v\right)\right) - \frac{\alpha h}{2}\left(\frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x^2}\right), \tag{16}$$

$$M_{xy} = -\frac{\partial^2 w}{\partial x\,\partial y}\left(D^*\left(1-v\right) + \alpha h\right). \tag{17}$$

The equations for shear force resultants are

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y}, \quad Q_y = \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x}. \tag{18}$$

The equation of motion (force equilibrium z in the direction) is given as:

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} - \rho h\frac{\partial^2 w}{\partial t^2} = 0. \tag{19}$$

Using Eqs. (15)-(17) in (18) and (19), then the equation of motion for micro plate with symmetry about *y*-axes is taken as:

$$\left(D^* + \frac{\alpha h}{2}\right)\frac{\partial^4 w}{\partial x^4} + \frac{E\,\alpha_T}{(1-v)\beta d}\frac{\partial^2 M_T}{\partial x^2} + \rho h\frac{\partial^2 w}{\partial t^2} = 0, \tag{20}$$

where $D^* = Eh^3/12(1-v^2)$ is the flexural rigidity of the plate, and the thermal moment is given by

$$M_T = \beta d \int_{-h/2}^{h/2} Tz \; dz \tag{21}$$

The equation of heat conduction with dual-phase-lag thermoelastic model can be written with the aid of Eq. (5) as:

$$K\left(1+\tau_T\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\rho c_e\frac{\partial T}{\partial t} - T_0\beta z\frac{\partial^3 w}{\partial x^2\partial t}\right). \tag{22}$$

We define the dimensionless quantities as:

$$\left(x', z', u', w'\right) = \frac{(x,z,u,w)}{L}, \; \left(\tau_q', \tau_T', t'\right) = \frac{(\tau_q,\tau_T,t)v}{L}, \; T' = \frac{T}{T_0}, \; M_T' = \frac{M_T}{d\,\beta T_0 h^2}, \; v^2 = \frac{E}{\rho}. \tag{23}$$

Using dimensionless quantities defined in Eq. (23) on Eqs. (20) and (22), we obtain

$$\left(1+\frac{\alpha h}{2D^*}\right)\frac{\partial^4 w}{\partial x^4} + \frac{E\,\alpha_T T_0 h^2 L}{(1-v)D^*}\left(\frac{\partial^2 M_T}{\partial x^2}\right) + \frac{\rho h v^2 L^2}{D^*}\frac{\partial^2 w}{\partial t^2} = 0, \tag{24}$$

$$\frac{K}{Lv\beta}\left(1+\tau_T\frac{\partial}{\partial t}\right)\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right) = \left(1+\tau_q\frac{\partial}{\partial t} + \frac{\tau_q^2}{2}\frac{\partial^2}{\partial t^2}\right)\left(\frac{\rho c_e}{\beta}\frac{\partial T}{\partial t} - z\frac{\partial^3 w}{\partial x^2\partial t}\right) = 0. \tag{25}$$

Following Gao et al. [7], the solution of time harmonic vibrations of the plate are taken as:

$$w(x,t) = W(x)e^{i\omega t}, \; T(x,z,t) = \Theta(x,z)e^{i\omega t}, \tag{26}$$

where ω, t denotes the frequency of the plate and time respectively. Substituting the values of T from (26) in Eq. (21), we obtain

$$M_\theta = \beta d \int_{-h/2}^{h/2} \Theta(x,z)z \; dz \tag{27}$$

Using Eqs. (26) in (24) and (25), yield

$$\left(1+\frac{\alpha h}{D^*}\right)\frac{\partial^4 W}{\partial x^4} + \left(\frac{E\,\alpha_T T_0 h^2 L}{(1-v)D^*}\right)\frac{\partial^2 M_\theta}{\partial x^2} - \frac{\rho h v^2 L^2 \omega^2}{D^*}W = 0, \tag{28}$$

$$\frac{K}{Lv\beta}\left(1+\tau_{T}i\,\omega\right)\left(\frac{\partial^{2}\Theta}{\partial x^{2}}+\frac{\partial^{2}\Theta}{\partial z^{2}}\right)=i\,\omega\left(1+\tau_{q}i\,\omega-\frac{\omega^{2}}{2}\tau_{q}^{2}\right)\left(\frac{\rho c_{e}}{\beta}\Theta-z\,\frac{\partial^{2}W}{\partial x^{2}}\right). \tag{29}$$

4   THERMAL FIELD ON THE THICKNESS DIRECTION

The thermal gradient of the plate is very small as compared to that along its thickness direction $\left(\left|\partial\Theta/\partial x\right|<<\left|\partial\Theta/\partial z\right|\right)$, then the Eq. (29) is given as:

$$\frac{\partial^{2}\Theta}{\partial z^{2}}+p^{*2}\Theta+\left(\frac{i\,\omega z\,\beta Lv\tau_{q}^{*}}{K\tau_{t}^{*}}\right)\frac{\partial^{2}W}{\partial x^{2}}=0, \tag{30}$$

where

$$p^{*2}=-\frac{\rho c_{e}i\,\omega Lv\tau_{q}^{*}}{K\tau_{t}^{*}},\ \tau_{t}^{*}=\left(1+\tau_{T}i\,\omega\right),\ \tau_{q}^{*}=\left(1+\tau_{q}i\,\omega-\frac{\omega^{2}}{2}\tau_{q}^{2}\right). \tag{31}$$

In this case, we assume that there is no heat across the upper and lower surfaces of the plate, then we get

$$\frac{\partial\Theta}{\partial z}=0\ \ \text{at}\ \ z=\pm\frac{h}{2}. \tag{32}$$

Using conditions (32), the general solution of Eq. (29) is written as:

$$\Theta(x,z)=-\frac{i\,\omega L\,\beta v\tau_{q}^{*}}{Kp^{2}\tau_{t}^{*}}\left(z-\frac{\sin\left(p^{*}z\right)}{p\cos\left(p^{*}/2\right)}\right)\frac{\partial^{2}W}{\partial x^{2}}. \tag{33}$$

The thermal moment can be written with the aid of Eq. (33) in Eq. (31) as:

$$\frac{d^{2}M_{\theta}}{dx^{2}}=-\left(\frac{i\,\omega\beta^{2}dh^{3}Lv\tau_{q}^{*}}{12Kp^{2}\tau_{t}^{*}}\right)\left(1+f\left(p^{*}\right)\right)\frac{d^{4}W}{dx^{4}}, \tag{34}$$

where

$$f\left(p^{*}\right)=\frac{24}{p^{*3}h^{3}}\left(\frac{p^{*}h}{2}-\tan\frac{p^{*}h}{2}\right) \tag{35}$$

Substituting Eq. (34) on Eq. (28), we obtain

$$\frac{d^{4}W}{dx^{4}}-\lambda^{4}W=0, \tag{36}$$

where

$$\lambda^4 = {a_\delta \omega^2}\big/{D_\omega^*}, \; a_\delta = \frac{\rho h (Lv)^2}{D^*}, D_\omega^* = \left(1 + \varepsilon\left(1 + f\left(p^*\right)\right) + \frac{\alpha h}{D^*}\right), \; \varepsilon = -\frac{i \omega E \, \alpha_T T_0 h^5 L^2 \beta^2 d v \tau_q^*}{12 K p^2 D^* (1-v) \tau_t^*}. \tag{37}$$

5 BOUNDARY CONDITIONS

We consider a micro plate whose ends are either clamped-clamped (CC) or simply supported (SS), so we have following Rao [22] for the two set of boundary conditions:
   Case (i) For Clamped-Clamped (CC)

$$W = 0, \frac{dW}{dx} = 0, \text{ at } x = 0, L \tag{38}$$

   Case (ii) For Simply supported (SS)

$$W = 0, \; \frac{d^2 W}{dx^2} = 0, \text{ at } x = 0, L \tag{39}$$

The solution of Eq. (36) is given by

$$W(x) = D_1 \sin \lambda x + D_2 \cos \lambda x + D_3 \sinh \lambda x + D_4 \cosh \lambda x . \tag{40}$$

   Substituting Eq. (40) in the boundary conditions (38) and (39), we obtain the following set of frequency equations

   Case (i) $\cos \lambda L \cosh \lambda L = 1,$ \hfill (41)

   Case (ii) $\sin \lambda L \sinh \lambda L = 0.$ \hfill (42)

The characteristic roots of the Eqs. (41) and (42) are given by

   Case (i) $\lambda = \dfrac{2n\pi}{L}, \quad n \; \varepsilon \; I$ \hfill (43)

   Case (ii) $\lambda = \dfrac{n\pi}{L}, \quad n \; \varepsilon \; I$ \hfill (44)

Substituting Eqs. (43) and (44) in (40), yield the vibration solution for thermal deflection as:

   Case (i) $w(x,t) = W(x,t) = \displaystyle\sum_{n=1}^{\infty} D_n \left(\sin\left(\frac{2n\pi}{L} x\right) - \sinh\left(\frac{2n\pi}{L} x\right) \right) e^{i \omega_n t},$ \hfill (44)

   Case (ii) $w(x,t) = W(x,t) = \displaystyle\sum_{n=1}^{\infty} D_n \sin\left(\frac{n\pi}{L} x\right) e^{i \omega_n t},$ \hfill (45)

where

Case (i) $\omega_n = \dfrac{4n^2\pi^2}{L^2}\sqrt{D_\omega^* D^* \big/ \rho h}$,   (46)

Case (ii) $\omega_n = \dfrac{n^2\pi^2}{L^2}\sqrt{\dfrac{D_\omega^* D^*}{\rho h}}$.   (47)

6   DAMPING AND FREQUENCY SHIFT

In the presence of thermoelastic coupling and thermal relaxation time, the vibration frequency of the plate can be written as:

$$\omega_n^2 = \lambda_n^4\left(\frac{D_\omega^*}{a_\delta}\right),$$ (48)

$$\omega_n = \lambda_n^2\sqrt{\frac{D_\omega^*}{a_\delta}} = \omega_0\left[1+\frac{\alpha h}{2D^*}+\varepsilon\left(1+f\left(p^*\right)\right)\right]^{\frac{1}{2}},$$ (49)

where $\omega_0 = \dfrac{\lambda_n^2}{\sqrt{a_\delta}}$ and following Sharma [25], we can replace $f\left(p^*\right)$ with $f\left(\omega_0\right)$ and expand Eq. (49) up to first order, we obtain

$$\omega_n = \omega_0\left(1+\frac{\alpha h}{4D^*}+\frac{\varepsilon}{2}\left(1+f\left(\omega_0\right)\right)\right).$$ (50)

The quantity p^{*2} in Eq. (31) are complex in nature, therefore by using Euler theorem we get

$$p^* = p_0 e^{\frac{-i\alpha_1}{2}},\; p_0 = \varepsilon_2\omega_n\left(\sqrt{\left\{\omega_n^2\tau_q^2+\left(1-\frac{\omega_n^2}{2}\tau_q^2\right)^2\right\}\left(1+\omega_n^2\tau_T^2\right)}\right),\; \alpha_1 = \tan^{-1}\left(-\frac{\left(1-\frac{\omega_n^2}{2}\tau_q^2\right)+\tau_q\tau_T\omega_n^2}{\omega_n\left\{\tau_T\left(1-\frac{\omega_n^2}{2}\tau_q^2\right)-\tau_q\right\}}\right).$$ (51)

Replacing ω_n with ω_0 in Eqs. (51), we obtain

$$p^* = \sqrt{2}p_1\left(\cos\frac{\alpha_2}{2}-i\,\sin\frac{\alpha_2}{2}\right),$$ (52)

and

$$p^* = p_1 e^{\frac{-i\alpha_1}{2}},\; p_1 = \varepsilon_2\omega_0\left(\sqrt{\left\{\omega_0^2\tau_q^2+\left(1-\frac{\omega_0^2}{2}\tau_q^2\right)^2\right\}\left(1+\omega_0^2\tau_T^2\right)}\right),\; \alpha_2 = \tan^{-1}\left(-\frac{\left(1-\frac{\omega_0^2}{2}\tau_q^2\right)+\tau_q\tau_T\omega_0^2}{\omega_0\left\{\tau_T\left(1-\frac{\omega_0^2}{2}\tau_q^2\right)-\tau_q\right\}}\right).$$ (53)

The frequency ω_n is complex in nature and hence we take

$$\omega_n = \omega_R^m + i\,\omega_I^m, \quad \omega_R^m = \text{Re}(\omega_n), \quad \omega_I^m = \text{Im}g(\omega_n), \tag{54}$$

and

$$\omega_R^n = \omega_0 \left[1 + \frac{\alpha h}{2D} + \frac{\varepsilon_1}{2} \left\{ 1 + \frac{6\cos\theta}{p_1^2 h^2} - \frac{6\sqrt{2}\cos\dfrac{3\theta}{2}}{p_1^3 h^3} \left(\frac{\sin\varsigma_1 + \tan\dfrac{3\theta}{2}\sinh(\varsigma_1\varsigma_2)}{\cos\varsigma_1 + \cosh(\varsigma_1\varsigma_2)} \right) \right\} \right], \tag{55}$$

$$\omega_I^n = \frac{\varepsilon_1}{2} \left\{ \frac{6\cos\theta}{p_1^2 h^2} - \frac{6\sqrt{2}\cos\dfrac{3\theta}{2}}{p_1^3 h^3} \left(\frac{\sin\varsigma_1 \tan\dfrac{3\theta}{2} - \sinh(\varsigma_1\varsigma_2)}{\cos\varsigma_1 + \cosh(\varsigma_1\varsigma_2)} \right) \right\}. \tag{56}$$

where $\varsigma_1 = \sqrt{2}p_1 h \cos\left(\dfrac{\theta}{2}\right)$, $\varsigma_2 = \tan\left(\dfrac{\theta}{2}\right)$.

Following Sharma [25], the thermoelastic damping and frequency shift in a thermoelastic circular plate are taken as:

$$Q^{-1} = 2 \left| \frac{\text{Im}g(\omega_n)}{\text{Re}(\omega_n)} \right|, \tag{57}$$

$$\omega_s = \left| \frac{\text{Re}(\omega_n) - \omega_0}{\omega_0} \right|. \tag{58}$$

7   PARTICULAR CASES

If couple stress parameter $(\alpha = 0)$, then Eqs. (55) and (56), yield the thermoelastic damping and frequency shift for dual-phase-lag thermoelastic plate.

Taking $\tau_T = \tau_q^2 = 0$, $\tau_q = \tau_0$, in Eqs. (55) and (56), we obtain thermoelastic damping and frequency shift for modified couple stress Lord-Shulman (LS) thermoelastic plate.

8   NUMERICAL RESULTS AND DISCUSSION

The mathematical model is prepared with magnesium material for the purpose of numerical computations. The material constants of the problem are taken from Daliwal and Singh [5], Kumar et al. [12]. The values of thermoelastic damping Q^{-1} and frequency shift ω_s of first two vibration modes have been computed from Eqs. (57) and (58) in the absence and presence of couple stress. The numerical computations have been carried out with the help of MATLAB software for magnesium material. The computed simulated results have been presented graphically in Figs. 1-8 for clamped-clamped and simply supported plates for varying values of thickness and length. In all these Figs., solid line represent to modified couple stress dual phase lag thermoelastic plate (CDPL), small dash line represent to dual phase lag thermoelastic plate (DPL), small dotted line represent to modified couple stress thermoelastic plate (LS) for few vibration modes respectively. The constants were taken as:
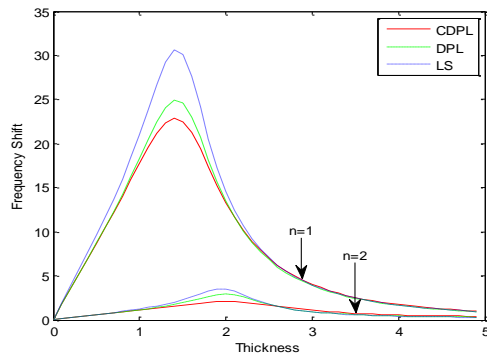
$\lambda = 2.696 \times 10^{10} Kgm^{-1}s^{-2}$, $\mu = 1.639 \times 10^{10} Kgm^{-1}s^{-2}$, $\rho = 1.74 \times 10^{3} Kgm^{-3}$, $T_0 = 298K$, $K = 1.7 \times 10^{2} Wm^{-1}K^{-1}$,

$c_e = 1.04 \times 10^{3} JKg^{-1}K^{-1}$, $\alpha_t = 1.78 \times 10^{-5}K^{-1}$, $\alpha = 25N$, $\tau_q = 0.004 Sec$, $\omega = 10 Sec^{-1}$, $\tau_T = 0.002 Sec$, $H = .01m$, $L = 100m$.
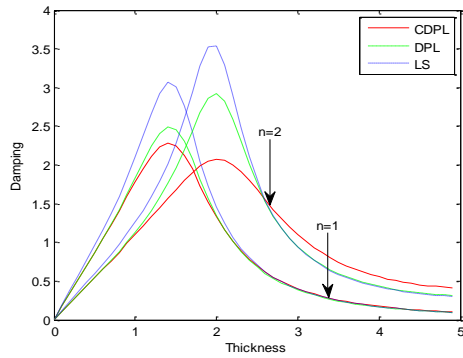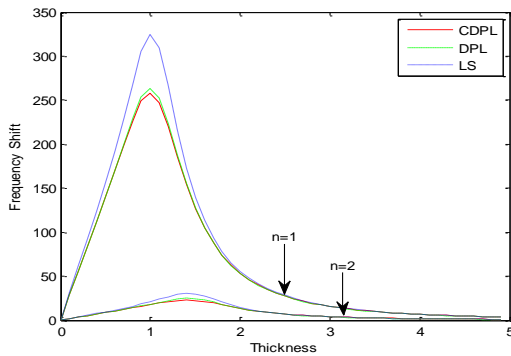
Fig.1
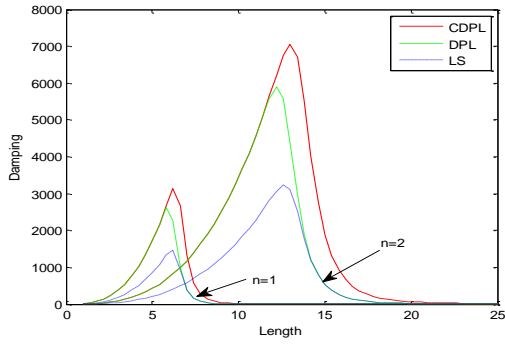Damping of few vibration modes versus thickness (h) in a clamped-clamped (CC) plate of fixed length.
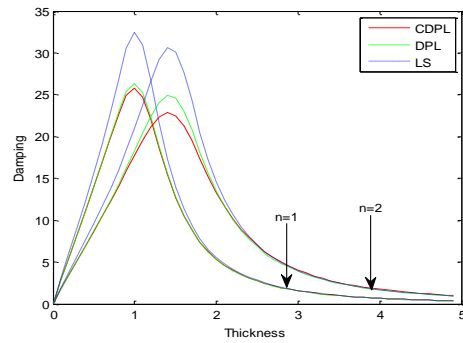
Fig.2
Damping of few vibration modes versus thickness (h) in a simply supported-simply supported (SS) plate of fixed Length.
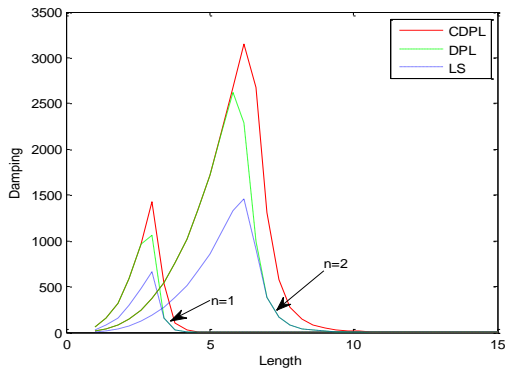
Fig.3
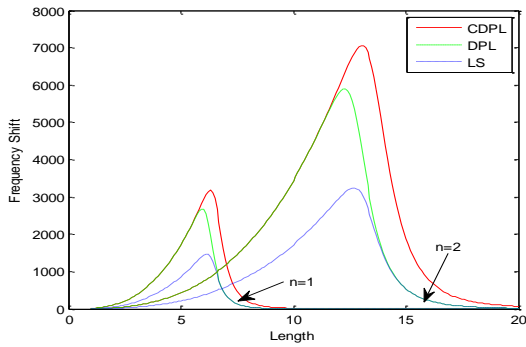Frequency shift of few vibration modes versus thickness (h) in a clamped-clamped (CC) plate of fixed length.

Fig.4
Frequency shift of few vibration modes versus thickness (h) in a simply supported-simply supported (SS) plate of fixed length.
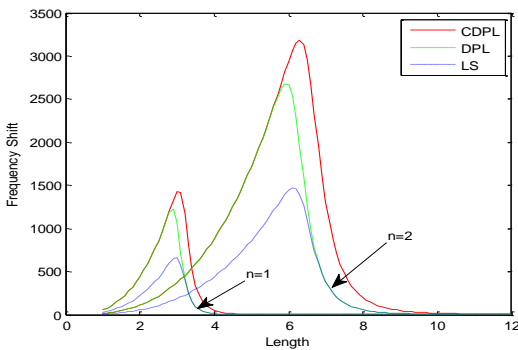
Fig.5
Damping of few vibration modes versus length (L) in a clamped-clamped (CC) plate of fixed thickness.

Fig.6
Damping of few vibration modes versus length (L) in a simply supported-simply supported (SS) plate of fixed thickness.

Fig.7
Frequency shift of few vibration modes versus length (L) in a clamped-clamped (CC) plate of fixed thickness.

Fig.8
Frequency shift of few vibration modes versus length (L) in a simply supported-simply supported (SS) plate of fixed thickness.

Fig. 1 depicts the thermoelastic damping in a clamped-clamped plate of first two vibration modes for dual-phase-lag model with varying values of thickness. It is observed that the damping factor initially increases to attain maxima and then decreases in the assumed range of thickness. The damping factor has smaller value in the presence of couple stress in comparison to absence of couple stress for dual-phase-lag model, while the damping factor

observed larger value in case of L-S theory. It is clear from the figure that the peak value of damping factor for vibration mode $(n = 2)$ is higher than that of vibration mode $(n = 1)$ for all cases of thermoelasticity. Fig. 2 represents the thermoelastic damping in a simply supported plate of first two vibration modes for dual-phase-lag model with varying values of thickness. The behaviour and variation of thermoelastic damping are similar for all cases and first two vibration modes. It is observed that the peak value of damping factor for first vibration mode is larger in comparison to second vibration mode. Moreover, the damping factor has smaller value for CDPL than that of DPL and LS theories. Fig. 3 exhibits the frequency shift in a clamped-clamped plate of first two vibration modes for dual-phase- lag model with varying values of thickness. It is observed that trend of variation of frequency shift is same for vibration modes $(n = 1, 2)$. It is also observed that the frequency shift has larger value for first vibration mode than that of second vibration mode. The frequency shift has observed higher value in case of LS theory than that of CDPL, DPL models. Fig. 4 represents the frequency shift in a simply supported plate of first two vibration modes for dual-phase-lag model with varying values of thickness. Frequency shift increases initially to attain maximum value and then decreases in the considered range of thickness for all cases and vibration modes. It is observed that the frequency shift has smaller value under the effect of couple stress dual phase lag model in comparison to dual phase lag and L-S models. Fig. 5 depicts the thermoelastic damping for dual-phase-lag model in a clamped-clamped plate of first two vibration modes with varying values of length. It is observed that the behaviour of thermoelastic damping is same but its variation is different for first two vibration modes. The damping factor initially increases smoothly with peak value and then decreases in the remaining range of thickness. Moreover, the damping factor observed larger value in case of CDPL than that of DPL and LS models. Fig. 6 exhibits the thermoelastic damping for dual-phase-lag model in a simply supported plate of first two vibration modes with varying values of length. It is observed that damping factor has observed larger value for second vibration mode in comparison with first vibration mode. The peak value of damping factor is higher in the presence of couple stress and smaller in the absence of couple stress. Fig. 7 represents the vibrations of frequency shift in case of clamped-clamped plate with varying values of length. The frequency shift increases rapidly with increasing values of thickness to attain maximum value and then decreases smoothly in the considered range of thickness. It is noticed that the frequency shift observed larger value in the presence of couple stress and smaller value for absence of couple stress. The peak value of frequency shift increases with increasing values of vibration modes. Fig. 8 depicts the vibrations of frequency shift in case of simply supported plate with varying values of length. The trend of variation is similar for all cases and both vibration modes. It is observed that frequency shift increases to attain maxima and then decreases in the assumed region for CDPL, DPL and LS models. The frequency shift observed higher value for vibration mode $(n = 2)$ than that of vibration mode $(n = 1)$ for all cases.

8 CONCLUSION

The vibrations of thin plate in modified couple stress thermoelastic medium has been discussed in the context of Kirchhoff-Love plate theory and dual-phase-lag thermoelastic model. The mathematical expressions for thermoelastic damping of vibration and frequency shift are obtained for thermoelastic plate. Damping factor and frequency shift with varying values of length and thickness are shown graphically to show the effect of couple stress and vibration modes for clamped-clamped and simply- supported boundary conditions. It is observed that the thermoelastic damping of simply supported plate has observed larger value than that of clamped plate in case of varying values of thickness for first and second modes of vibration, while thermoelastic damping observed smaller value for simply supported plate in comparison with clamped plate in case of varying values of length for few vibration modes. The frequency shift observed similar behaviour for clamped-clamped as well as simply supported plate for first two vibration modes of thermoelastic plate with dual-phase-lag model. It is also observed that the damping factor and frequency shift have noticed larger value in the presence of couple stress for varying values of length in case of both vibration modes and boundary conditions but opposite trend is observed for varying values of thickness.

REFERENCES

[1]   Abd-Elaziz E.M., Marin M., Othman M.I.A., 2019, On the effect of Thomson and initial stress in a thermo porous elastic solid under GN electromagnetic theory, *Symmetry* **11**(3): 413.

[2]   Alashti R.A., Abolghasemi A.H., 2014, A size-dependent Bernoulli-Euler beam formulation based on a new model of couple stress theory, *Transactions C: Aspects* **27**(6): 951-960.

[3]   Chen W., Li X., 2014, A new modified couple stress theory for anisotropic elasticity and microscale laminated Kirchhoff plate model, *Archive of Applied Mechanics* **84**: 323-341.

[4]   Cosserat E., Cosserat F., 1909, *Theory of Deformable Bodies*, Hermann et Fils, Paris.

[5]   Daliwal R.S., Singh A., 1980, *Dynamical Coupled Thermoelasticity*, Hindustan Publishers, Delhi, India.

[6]   Guo F.L., Wang G.Q., Rogerson G.A., 2012, Analysis of thermoelastic damping in micro-and nanomechanical resonators based on dual-phase-lagging generalized thermoelasticity theory, *International Journal of Engineering Science* **60**: 59-65.

[7]   Guo F.L., Song J., Wang G.Q., Zhou Y.F., 2014, Analysis of thermoelastic dissipation in circular micro-plate resonators using the generalized thermoelasticity theory of dual-phase-lagging model, *Journal of Sound and Vibration* **333**: 2465-2474.

[8]   Kakhki E.K., Hosseini S.M., Tahani M., 2016, An analytical solution for thermoelastic damping in a micro-beam based on generalized theory of thermoelasticity and modified couple stress theory, *Applied Mathematical Modeling* **40**(4): 3164-3174.

[9]   Kumar R., Devi S., Sharma V., 2017, A problem of thick circular plate in modified couple stress thermoelastic diffusion with phase-lags, *Multidiscipline Modeling in Materials and Structures* **12**(3): 478-494.

[10]   Kumar R., Devi S., 2017, Thermoelastic beam in modified couple stress thermoelasticity induced by laser pulse, *Computers and Concrete* **19**(6): 707-716.

[11]   Koiter W.T., 1964, Couple-stresses in the theory of elasticity, *Proceedings of the Royal Netherlands Academy of Sciences* **67**: 17-44.

[12]   Kumar R., Devi S., 2017, Response of thermoelastic functionally graded beam due to ramp type heating in modified couple stress with dual phase lag model, *Multidiscipline Modelling in Materials and Structures* **13**(3): 471-488.

[13]   Kumar R., Devi S., Sharma V., 2017, Damping in microscale modified couple stress thermoleastic circular plate resonators, *AAM* **12**(2): 924-945.

[14]   Kumar R., Devi S., 2018, Damping and frequency shift in microscale modified couple stress thermoelastic plate resonators, *Journal of Solid Mechanics* **10**(3): 621-636.

[15]   Marin M., 1997, An uniqueness result for body with voids in linear thermoelasticity, *Rendiconti di Matematica* **17**(7): 103-113.

[16]   Marin M., Florea O., 2014, On temporal behavior of solutions in thermoelasticity of porous micropolar bodies, *Analele Universitatii "Ovidius" Constanta* **22**(1): 169-188.

[17]   Ma H.M., Gao X.L., Reddy J.N., 2008, A microstructure-dependent Timoshenko beam model based on a modified couple stress theory, *Journal of Mechanics and Physics of Solids* **56**(12): 3379-3391.

[18]   Mindlin R.D., Tiersten H.F., 1962, Effects of couple-stresses in linear elasticity, *Archive for Rational Mechanics and Analysis* **11**: 415-448.

[19]   Mindlin R.D., 1963, Influence of couple stresses on stress-concentrations, *Experimental Mechanics* **3**: 1-7.

[20]   Mindlin R.D., 1964, Micro-structure in linear elasticity, *Archives for Rational Mechanics and Analysis* **15**: 51-78.

[21]   Park S.K., Gao X.L., 2006, Bernoulli–Euler beam model based on a modified couple stress theory, *Journal of Micromechanics and Microengineering* **16**: 23-55.

[22]   Rao S.S., 2007, *Vibration of Continuous Systems*, John Wiley & Sons, Inc. Hoboken, New Jersey.

[23]   Rezazadeh G., Vahdat A.S., Tayefeh-Rezaei S., Cetinkaya C., 2012, Thermoelastic damping in a micro beam resonator using modified couple stress theory, *Acta Mechanica* **223**: 1137-1152.

[24]   Roychoudhuri S.K., 2007, On a thermoelastic three-phase-lag model, *Journal of Thermal Stresses* **30**: 231-238.

[25]   Sharma J.N., 2011, Thermoelastic damping and frequency shift in micro/nanoscale anisotropic beams, *Journal of Thermal Stresses* **34**(7): 650-666.

[26]   Sourki R., Hoseini S.A.H., 2016, Free vibration analysis of size-dependent cracked microbeam based on the modified couple stress theory, *Applied Physics A* **122**: 413.

[27]   Sun Y., Tohmyoh H., 2009, Thermoelasic damping of the axisymmetric vibration of circular plate resonators, *Journal of Sound and Vibration* **319**: 392-405.

[28]   Toupin R.A., 1962, Elastic materials with couple-stresses, *Archive for Rational Mechanics and Analysis* **11**: 385-414.

[29]   Tsiatas G.C., 2009, A new Kirchhoff plate model based on a modified couple stress theory, *Journal of Solids and Structures* **46**: 2757-2764.

[30]   Tzou D.Y., 1995, A unified approach for heat conduction from macro-to-micro scales, *Journal of Heat Transfer* **117**: 8-16.

[31]   Tzou D.Y., 1997, *Macro-to-Microscale Heat Transfer: The Lagging Behaviour*, Series in Chemical and Mechanical Engineering, Taylor and Francis, Washington.

[32]   Voigt W., 1887, *Theoretische Studienuber die Elasticitatsverhaltnisse der Krystalle*, *Abhandlungen der Kŏniglichen Gesellschaften der Wissenschaften in Gŏttingen,* German.

[33]   Yang F., Chong A.C.M., Lam D.C.C., Tong P., 2002, Couple stress based strain gradient theory for elasticity, *International Journal of Solids and Structures* **39**: 2731-2743.