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Dispersion of Torsional Surface Wave in a Pre-Stressed Heterogeneous Layer Sandwiched Between Anisotropic Porous Half-Spaces Under Gravity

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ABSTRACT
The study of surface waves in a layered media has their viable application in geophysical prospecting. This paper presents an analytical study on the dispersion of torsional surface wave in a pre-stressed heterogeneous layer sandwiched between a pre-stressed anisotropic porous semi-infinite medium and gravitating anisotropic porous half-space. The non-homogeneity within the intermediate layer and upper semi-infinite medium is assumed to rise up, because of quadratic variation and exponential variation in directional rigidity, pre-stress, and density respectively. The displacement dispersion equation for the torsional wave velocity has been expressed in the term of Whitaker function and their derivatives. Dispersion relation and the closed-form solutions have been obtained analytically for the displacement in the layer and the half-spaces. It is determined that the existing geometry allows torsional surface waves to propagate and the observe exhibits that the layer width, layer inhomogeneity, frequency of heterogeneity in the heterogeneous medium has a great impact on the propagation of the torsional surface wave. The influence of inhomogeneities on torsional wave velocity is also mentioned graphically by means plotting the dimensionless phase velocity against non-dimensional wave number for distinct values of inhomogeneity parameters.

Keywords: Torsional surface wave; Anisotropic; Pre-stress; Porosity; Inhomogeneity; Gravity.

1 INTRODUCTION

THE Earth crust and mantle are not homogeneous and initial stress-free throughout and it contains some hard and soft rock or mantle and non-homogeneity, initially stressed property are the trivial characteristic of the Earth. In recent years, dispersion of torsional wave in the heterogeneous medium has considerable application in applied mathematics and geophysics engineering because of Earth's inhomogeneity properties. Due to significant

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applications, the variation in rigidity, density, and initial stress has approximated linearly (Meissner [1]). Additionally, quadratic variation in rigidity, density, and pre-stress for the Earth crust is found in many places. As a consequence, the present paper aims to study the propagation of the torsional surface wave in a heterogeneous layer sandwiched between two anisotropic porous half-spaces under the influence of the gravity field. The study of the seismic wave gives significant information about the layered earth structure and has been used to appropriately determine the earthquake epicenter correctly.

The work has been carried out on propagation of torsional waves in an inhomogeneous layer over an inhomogeneous half-space by Chattopadhyay et al. [2]. Additionally, propagation of love waves in non-homogeneous substratum over initially stressed heterogeneous half-space was analyzed by Gupta et al. [3] and Gupta et al. [4] pointed out propagation of S-waves in a non-homogeneous anisotropic and initially stressed medium. A torsional surface wave is a seismic wave which requires only circumferential displacement that needs to be independent of the azimuthal angle. These are the waves where the particles of the medium twist or spiral clockwise and anticlockwise in regards to the motion of the waves. These forms of earthquake waves are torsional. Among the different surface wave, the torsional wave plays an important role in its own torsional field.. Propagation of torsional surface waves in a heterogeneous half-space underneath a rigid layer was analyzed by Dey et al. [5]. Kumari and Sharma [6] gave a thought regarding propagation of torsional waves in a viscoelastic layer over an inhomogeneous half space. An analysis of the effect of inhomogeneity on the propagation of surface waves giving an intriguing field to the utilization of mathematical technique and moreover, is of reasonable significance to seismologists due to the fact in any sensible model of the earth there may be a continuous change in the elastic properties of the material in the vertical path giving upward thrust to inhomogeneity.

Likewise, Earth's interior and surface geographical structures might be assumed to be along with may be assumed to be along with liquid-filled porous layer at which density and elastic moduli change intermittently. The layers of the Earth are fluid-saturated poroelastic and absolutely anisotropic in nature. The interaction between fluid motion deformation in the porous medium is depicted by using poroelastic models. Inside the Earth crust liquid soaked permeable rocks are available as limestone and others pervaded by groundwater or oil or gasoline. Most of these hydrocarbons are actual abundant like a harder sponge, abounding holes however not compressible. The holes or pores can incorporate water or oil or fuel and rock will probably be saturated with any such three. The holes are abundant tinier than spong holes but they are still holes and they are alleged porosity and the layer are referred to as the porous layer. The phenomenal of wave propagation in fluid-saturated porous media having one-of-a-kind sort of fabric houses have been a subject of a couple of investigators as a result of its fundamental application in geophysics and seismology.

In view that Biot established the poroelasticity concept of the fluid-saturated porous media [7, 8]. Additionally, Wang and Tian [9] pointed out the acoustoelastic principle for fluid-saturated porous media. Free vibration analysis of sandwich nano-plate with functionally graded porous core and piezoelectric face sheets was analyzed by Arani and Zamani [10]. Dynamic analysis of a rectangular plate made of porous materials lying over an elastic foundation was done by Arani et al. [11]. Chattaraj and Samal [12] introduced an idea about dispersion of Love type surface wave in an anisotropic porous layer with periodic non-uniform boundary surface. The propagation of Love waves in a fluid-saturated porous anisotropic layer was examined by Konczak [13]. Saroj and Sahu [14] discussed reflection of plane wave at traction-free surface of a pre-stressed functionally graded piezoelectric material half-space. Love waves in an inhomogeneous fluid saturated porous layered half-space with linearly varying properties was distinctived by Ke et al. [15] . Prasad and Kundu [16] investigated the propagation of the torsional surface wave in a pre-stressed dry sandy layer over a gravitating anisotropic porous half-space. Ghorai et al. [17] contemplated Love waves in a fluid-saturated porous layer under a rigid boundary and lying over an elastic half-space under gravity. Possibility of Love wave propagation in a porous layer under the effect of linearly varying directional rigidities was detailed by Gupta et al [18]. Free vibration of a rectangular plate made of porous materials having Y-foam, G-foam, and coustone and it was compared with each other by Arani et al. [19].

This present paper endeavors to study the dispersion of torsional surface wave in an anisotropic pre-stressed heterogeneous layer sandwiched between an anisotropic porous semi-infinite layer and gravitating anisotropic porous half-space. The rigidities, initial stress, and density of the porous layer are assumed to vary exponentially whereas rigidities, initial stress, and density of the heterogeneous layer are supposed to vary quadratically. The z-axis is vertical positive downward co-ordinate whose origin at the interface of anisotropic porous half-space underneath gravity. The mathematical statements of the velocity of the torsional wave are obtained which coincides with the classical result.

2 BASIC PRELIMINARIES AND PROBLEM FORMULATION

We consider a pre-stress heterogeneous layer (M_2) of finite thickness H lying over a gravitating anisotropic porous semi-infinite medium (M_3) and underlying a pre-stress anisotropic porous layer (M_1). To examine torsional surface waves, a cylindrical co-ordinate system (r,θ,z) is introduced with z-axis directed to downward positive. Where r and θ be the radial and circumferential co-ordinate respectively. The wave is assumed to be propagated along radial (r) direction and particle displacement along an azimuthal (θ) direction. The geometry of the problem is depicted in Fig.1.

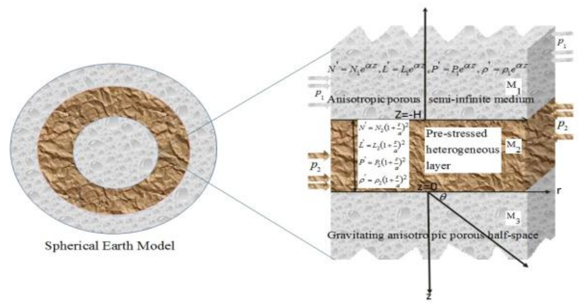

Fig.1
Geometry of problem.

The dynamical mathematical statement for porous layer under compressive initial stress p' in the absence of the viscosity of liquid and body force are given by Biot [20]

$$\left.\begin{aligned}
&\frac{\partial s'_{rr}}{\partial r}+\frac{1}{r}\frac{\partial s'_{r\theta}}{\partial r}+\frac{\partial s'_{rz}}{\partial z}+\frac{s'_{rr}-s'_{\theta\theta}}{r}-p'\left(\frac{\partial \omega'_z}{\partial \theta}+\frac{\partial \omega'_\theta}{\partial z}\right)=\frac{\partial^2}{\partial t^2}\left(\rho'_{rr}v'_r+\rho'_{r\theta}V'_r\right)\\
&\frac{\partial s'_{r\theta}}{\partial r}+\frac{2}{r}s'_{r\theta}+\frac{1}{r}\frac{\partial s'_{\theta\theta}}{\partial \theta}+\frac{\partial s'_{\theta z}}{\partial z}-p'\frac{\partial \omega'_z}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho'_{rr}v'_\theta+\rho'_{r\theta}V'_\theta\right)\\
&\frac{\partial s'_{rz}}{\partial r}+\frac{s'_{rz}}{r}+\frac{1}{r}\frac{\partial s'_{z\theta}}{\partial \theta}+\frac{\partial s'_{zz}}{\partial z}+p'\frac{\partial \omega'_z}{\partial \theta}=\frac{\partial^2}{\partial t^2}\left(\rho'_{rr}v'_z+\rho'_{r\theta}V'_z\right),
\end{aligned}\right\} \quad (1)$$

$$\left.\begin{aligned}
&\frac{\partial s'}{\partial r}=\frac{\partial^2}{\partial t^2}\left(\rho'_{r\theta}v'_r+\rho'_{\theta\theta}V'_r\right)\\
&\frac{\partial s'}{\partial \theta}=\frac{\partial^2}{\partial t^2}\left(\rho'_{r\theta}v'_\theta+\rho'_{\theta\theta}V'_\theta\right)\\
&\frac{\partial s'}{\partial z}=\frac{\partial^2}{\partial t^2}\left(\rho'_{r\theta}v'_z+\rho'_{\theta\theta}V'_z\right),
\end{aligned}\right\} \quad (2)$$

where $s'_{ij}\ (i,j=r,\theta,z)$ and s' are the incremental stress components of solid and stress in liquid respectively. $v'_r,\ v'_\theta,\ v'_z$ and $V'_r,\ V'_\theta,\ V'_z$ are the components of displacement vector of solid and liquid respectively. The rotational components are given by

$$\left.\begin{aligned}
&\omega'_r=\frac{1}{2r}\left(\frac{\partial v'_z}{\partial \theta}-\frac{\partial v'_\theta}{\partial z}\right)\\
&\omega'_\theta=\frac{1}{2}\left(\frac{\partial v'_r}{\partial z}-\frac{\partial v'_z}{\partial r}\right)\\
&\omega'_z=\frac{1}{2r}\left(\frac{\partial}{\partial r}\left(rv'_\theta\right)-\frac{\partial v'_r}{\partial \theta}\right).
\end{aligned}\right\} \quad (3)$$

The strain components are given by

$$[E]=\begin{bmatrix} e'_{rr}=\dfrac{\partial v'_r}{\partial r} & e'_{r\theta}=\dfrac{1}{2}\left(\dfrac{1}{r}\dfrac{\partial v'_r}{\partial\theta}-\dfrac{v'_\theta}{r}+\dfrac{\partial v'_\theta}{\partial r}\right) & e'_{rz}=\dfrac{1}{2}\left(\dfrac{\partial v'_r}{\partial z}+\dfrac{\partial v'_z}{\partial r}\right)\\ e'_{\theta r}=\dfrac{1}{2}\left(\dfrac{1}{r}\dfrac{\partial v'_r}{\partial\theta}-\dfrac{v'_\theta}{r}+\dfrac{\partial v'_\theta}{\partial r}\right) & e'_{\theta\theta}=\dfrac{1}{r}\left(\dfrac{\partial v'_\theta}{\partial\theta}+v'_r\right) & e'_{\theta z}=\dfrac{1}{2}\left(\dfrac{\partial v'_\theta}{\partial z}+\dfrac{1}{r}\dfrac{\partial v'_z}{\partial\theta}\right)\\ e'_{zr}=\dfrac{1}{2}\left(\dfrac{\partial v'_r}{\partial z}+\dfrac{\partial v'_z}{\partial r}\right) & e'_{z\theta}=\dfrac{1}{2}\left(\dfrac{\partial v'_\theta}{\partial z}+\dfrac{1}{r}\dfrac{\partial v'_z}{\partial\theta}\right) & e'_{zz}=\dfrac{1}{2}\dfrac{\partial v'_z}{\partial z} \end{bmatrix} \tag{4}$$

The stress vector s' due to liquid is related to the fluid pressure p_f by the relation $-s'=\psi p_f$.
where ψ is the porosity of layer. The mass coefficients ρ'_{rr}, $\rho'_{r\theta}$ and $\rho'_{\theta\theta}$ are related to the densities ρ',ρ'_s and ρ'_l of the layer, solid and liquid respectively by Biot [8]

$$\rho'_{rr}+\rho'_{r\theta}=(1-\psi)\rho'_s$$
$$\rho'_{r\theta}+\rho'_{\theta\theta}=\psi\rho'_l.$$

Therefore, aggregate mass density of solid-liquid is

$$\begin{aligned}\rho' &= \rho'_{rr}+2\rho'_{r\theta}+\rho'_{\theta\theta}\\ &=\left(\rho'_{rr}+\rho'_{r\theta}\right)+\left(\rho'_{r\theta}+\rho'_{\theta\theta}\right)\\ &=\rho'_s+\psi\left(\rho'_l-\rho'_s\right).\end{aligned}$$

These mass coefficients also obey the following inequalities,

$$\rho'_{rr}>0,\;\; \rho'_{\theta\theta}>0,\;\; \rho'_{r\theta}<0,\;\; \rho'_{rr}\rho'_{\theta\theta}-\rho'^{2}_{r\theta}>0.$$

3 DYNAMICS OF POROUS LAYER UNDER INITIAL STRESS

For the torsional surface waves propagating along the radial direction *r*, the stress and displacement components are independent of θ and we have

$$v'_r=v'_z=0;\; v'=v'(r,z,t) \qquad V'_r=V'_z=0;\; V'=V'(r,z,t),$$

where, v'_r, v'_θ, v'_z and V'_r, V'_θ, V'_z are the components of displacement vector of solid and liquid respectively. Therefore,

$$\left.\begin{aligned} & s'_{rr}=s'_{\theta\theta}=s'_{zz}=s'_{zr}=0\\ & e'_{r\theta}=\frac{1}{2}\left(\frac{\partial v'_\theta}{\partial r}+\frac{v'_\theta}{r}\right);\; e'_{\theta z}=\frac{1}{2}\frac{\partial v'_\theta}{\partial z}\\ & \omega'_r=-\frac{1}{2}\frac{\partial v'_\theta}{\partial z};\; \omega'_z=\frac{1}{2r}\frac{\partial}{\partial r}\left(r v'_\theta\right);\; \omega'_\theta=0 \end{aligned}\right\} \tag{5}$$

The strain-stress relations are

$$\left.\begin{aligned} s'_{r\theta} &= 2N' e'_{r\theta} = N'\left(\frac{\partial v'_\theta}{\partial r} - \frac{v'_\theta}{r}\right) \\ s'_{z\theta} &= 2L' e'_{r\theta} = L'\frac{\partial v'_\theta}{\partial z}, \end{aligned}\right\} \tag{6}$$

where N' and L' are the rigidity of the porous medium along r and z-direction respectively. Using the relation (5) in Eqs. (1) and (2), the equation of motion can be written as:

$$\frac{\partial s'_{r\theta}}{\partial r} + \frac{2}{r}s'_{r\theta} + \frac{\partial s'_{\theta z}}{\partial z} - p'\frac{\partial \omega'_z}{\partial r} = \frac{\partial^2}{\partial t^2}\left(\rho'_{rr}v'_\theta + \rho'_{r\theta}V'_\theta\right), \tag{7}$$

$$\frac{\partial^2}{\partial t^2}\left(\rho'_{r\theta}v'_\theta + \rho'_{\theta\theta}V'_\theta\right) = 0. \tag{8}$$

Using stress-strain relation (6) and strain displacement (4), Eq. (7) may be written as:

$$\left(N' - \frac{p'}{2}\right)\left(\frac{\partial^2 v'_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v'_\theta}{\partial r} - \frac{v'_\theta}{r^2}\right) + \frac{\partial}{\partial z}\left(L'\frac{\partial v'_\theta}{\partial z}\right) = \frac{\partial^2}{\partial t^2}\left(\rho'_{rr}v'_\theta + \rho'_{r\theta}V'_\theta\right). \tag{9}$$

Eliminating V'_θ from Eqs. (9) and (8), one gets

$$\left(N' - \frac{p'}{2}\right)\left(\frac{\partial^2 v'_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v'_\theta}{\partial r} - \frac{v'_\theta}{r^2}\right) + \frac{\partial}{\partial z}\left(L'\frac{\partial v'_\theta}{\partial z}\right) = d'\frac{\partial^2 v'_\theta}{\partial t^2}, \tag{10}$$

The solution of Eq. (10), when wave propagating along radial direction with amplitude of displacement as function of dept may be written as:

$$v'_\theta = v(z)J_1(kr)e^{i\omega t}, \tag{11}$$

ω is circular frequency of the wave, J_1 is the Bessel's function of first kind of order one and $v(z)$ is solution of

$$\frac{d^2 v(z)}{dz^2} + \frac{1}{L'}\frac{dL'}{dz}\frac{dv(z)}{dz} - \frac{k^2 N'}{L'}\left(\left(1-\xi'\right) - \frac{d\rho' c^2}{N'}\right)v(z) = 0, \tag{12}$$

where, $c = \dfrac{\omega}{k}$ is the velocity of propagation of torsional wave, $\xi' = \dfrac{p'}{2N'}$ is the dimensionless parameter due to initial stress p', $\gamma_{11} = \dfrac{\rho'_{rr}}{\rho'}$, $\gamma_{12} = \dfrac{\rho'_{r\theta}}{\rho'}$, $\gamma_{22} = \dfrac{\rho'_{\theta\theta}}{\rho'}$ are the dimensioless parameter for the material of porous medium, $d = \dfrac{d'}{\rho'} = \dfrac{1}{\rho'}\left(\rho'_{rr} - \dfrac{\rho'^2_{r\theta}}{\rho'_{\theta\theta}}\right) = \gamma_{11} - \dfrac{\gamma_{12}^2}{\gamma_{22}}$.

On substituting, $v(z) = \dfrac{v_1(z)}{\sqrt{L'}}$ in Eq. (12) we have

$$\frac{d^2v_1(z)}{dz^2}-\frac{1}{2L'}\left\{\frac{d^2L'}{dz^2}-\frac{1}{2L'}\left(\frac{dL'}{dz}\right)^2\right\}v_1(z)-\frac{k^2N'}{L'}\left(\left(1-\xi'\right)-\frac{d\rho'c^2}{N'}\right)v_1(z)=0, \tag{13}$$

Now, the variation of directional rigidity (elastic moduli), initial stress and density has been taken as:

$$\left.\begin{aligned} N' &= N_1e^{\alpha z}\\ L' &= L_1e^{\alpha z}\\ p' &= p_1e^{\alpha z}\\ \rho' &= \rho_1e^{\alpha z}\end{aligned}\right\} \tag{14}$$

where α is a constants having dimension equal to inverse of the length. Using Eq. (14), Eq. (13) becomes

$$\frac{d^2v_1(z)}{dz^2}-m_1^2v_1(z)=0, \tag{15}$$

where, $m_1^2=k^2\left\{\frac{\alpha^2}{4k^2}+\frac{N_1}{L_1}\left(\left(1-\xi_1\right)-\frac{dc^2}{c_1^2}\right)\right\}$, $\xi_1=\frac{p_1}{2N_1}$ and $c_1=\sqrt{\frac{N_1}{\rho_1}}$, is the shear wave velocity in the porous layer. Solution of Eq. (15) is obtained by

$$v_1(z)=D_1e^{-m_1z}+D_2e^{m_1z}, \tag{16}$$

where D_1 and D_2 are arbitrary constants. In this way, the displacement in the anisotropic porous layer is

$$v'_\theta=u'_1(say)=\frac{D_1e^{-\left(m_1+\frac{\alpha}{2}z\right)}+D_2e^{\left(m_1-\frac{\alpha}{2}z\right)}}{\sqrt{L_1}}J_1(kr)e^{i\omega t}. \tag{17}$$

4 DYNAMIC OF PRE-STRESSED HETEROGENEOUS ANISOTROPIC MEDIUM

The dynamic equation of motion for the propagation of torsional wave in a prestressed anisotropic elastic medium is given by Biot [20] as:

$$\frac{\partial\sigma'_{r\theta}}{\partial r}+\frac{\partial\sigma'_{z\theta}}{\partial z}+\frac{2\sigma'_{r\theta}}{r}-\frac{\partial}{\partial z}\left(\frac{p'}{2}\frac{\partial v'_\theta}{\partial z}\right)=\rho'\frac{\partial^2v_\theta}{\partial t^2}, \tag{18}$$

where p' is the compressive initial stress along radial direction r, $v'_\theta=v'_\theta\left(r,z,t\right)$ the displacement along circumferential direction θ and p' the density of the medium $\sigma'_{r\theta}$ and $\sigma'_{z\theta}$ are the incremental stress component in the heterogeneous anisotropic elastic medium and given as, $\sigma'_{r\theta}=2N'e'_{r\theta}$, $\sigma'_{z\theta}=2L'e'_{z\theta}$.

where $e'_{r\theta}=\frac{1}{2}\left(\frac{\partial v'_\theta}{\partial r}-\frac{v'_\theta}{r}\right)$ and $e'_{z\theta}=\frac{1}{2}\left(\frac{\partial v'_\theta}{\partial z}\right)$, N' and L' are the directional rigidities of the medium along r and z directions respectively.

Using the above relations, Eq. (18) can be written as:

$$N^{'}\left\{\frac{\partial^2 v_\theta^{'}}{\partial r^2}+\frac{1}{r}\frac{\partial v_\theta^{'}}{\partial r}-\frac{v_\theta^{'}}{r^2}\right\}+\frac{\partial}{\partial z}\left(\Phi^{'}\frac{\partial v_\theta^{'}}{\partial z}\right)=\rho^{'}\frac{\partial^2 v_\theta^{'}}{\partial t^2}, \tag{19}$$

where $\Phi^{'}=L^{'}-\frac{p^{'}}{2}$.

The solution of Eq. (19), when wave propagating along radial direction with amplitude of displacement as fuction of dept may be written as:

$$v_\theta^{'}=v(z)J_1(kr)e^{i\omega t}, \tag{20}$$

where ω is the circular frequency of the wave, where $v(z)$ is solution of

$$\frac{d^2v(z)}{dz^2}+\frac{1}{\Phi^{'}}\frac{dv}{dz}\frac{d\Phi^{'}}{dz}-\frac{k^2N^{'}}{\Phi^{'}}\left(1-\frac{\rho c^2}{N^{'}}\right)v(z)=0, \tag{21}$$

where $c=\frac{\omega}{k}$ is the velocity of the dispersion of torsional surface wave and J_1 is the Bessel's function of first kind and of order one.

By letting $v(z)=\frac{v_2(z)}{\sqrt{\Phi^{'}}}$, Eq. (21) can be expressed as:

$$\frac{d^2v_2(z)}{dz^2}-\frac{1}{2\Phi^{'}}\left\{\frac{d^2\Phi^{'}}{dz^2}-\frac{1}{2\Phi^{'}}\left(\frac{d\Phi^{'}}{dz}\right)^2\right\}v_2(z)=\frac{k^2N^{'}}{\Phi^{'}}\left(1-\frac{\rho c^2}{N^{'}}\right)v_2(z), \tag{22}$$

Now, the variation of directional rigidity(elastic moduli), initial stress and density in the upper layer has been taken as:

$$\left.\begin{aligned} N^{'}&=N_2\left(1+\frac{z}{a}\right)^2\\ L^{'}&=L_2\left(1+\frac{z}{a}\right)^2\\ p^{'}&=p_2\left(1+\frac{z}{a}\right)^2\\ \rho^{'}&=\rho_2\left(1+\frac{z}{a}\right)^2 \end{aligned}\right\}, \tag{23}$$

where a is constant having dimension equal to the length. Using relation (23), Eq. (22) takes the form

$$\frac{d^2v_2(z)}{dz^2}-m_2^2v_2(z)=0, \tag{24}$$

where, $m_2^2=\frac{k^2N_2}{L_2(1-\xi_2)}\left(1-\frac{c^2}{c_2^2}\right)$, $\xi_2=\frac{p_2}{2L_2}$ is the dimensionless initial stress parameter, $c_2=\sqrt{\frac{N_2}{\rho_2}}$ is the shear wave velocity in the upper layer along radial direction.

The solution of Eq. (24) is obtained as:

$$v_2(z) = D_3 e^{-m_2 z} + D_4 e^{m_2 z}, \tag{25}$$

where D_3 and D_4 are arbitrary constants. Thus, the displacement in pre-stress asisotropic heterogeneous medium is

$$v'_\theta = u'_2(say) = \frac{D_3 e^{-m_2 z} + D_4 e^{m_2 z}}{\sqrt{L_2}\sqrt{1-\xi_2}\left(1+\frac{z}{a}\right)} J_1(kr) e^{i\omega t}. \tag{26}$$

5 DYNAMIC OF POROUS LAYER UNDER INFLUENCE OF GRAVITY

The dynamic equation of motion for anisotropic porous medium under the effect of gravity and in the absence of body forces for the dispersion of torsional surface is given by Biot [20] as:

$$\frac{\partial s'_{r\theta}}{\partial r} + \frac{\partial s'_{z\theta}}{\partial z} + 2\frac{s'_{r\theta}}{r} - \frac{\partial}{\partial z}\left(d' gz e'_{z\theta}\right) - d' gz \frac{\partial}{\partial z}\left\{\frac{1}{2}\left(\frac{\partial v'_\theta}{\partial r} + \frac{v'_\theta}{r}\right)\right\} = \frac{\partial^2}{\partial t^2}\left(\rho'_{rr} v'_\theta + \rho'_{r\theta} V'_\theta\right), \tag{27}$$

$$\frac{\partial s'}{\partial \theta} = \frac{\partial^2}{\partial t^2}\left(\rho'_{r\theta} v'_\theta + \rho'_{\theta\theta} V'_\theta\right). \tag{28}$$

where, $v'_\theta(r,\theta,z)$ and $V'_\theta(r,\theta,z)$ are the displacement along circumferential direction of solid and liquid respectively and g is the acceleration due to gravity, s'_{ij} and s' are the stress component of solid and liquid respectively.

For gravitating anisotropic porous medium the stress are related to strain by

$$s'_{r\theta} = 2N' e'_{r\theta},\; s'_{z\theta} = 2L' e'_{z\theta},\; e'_{r\theta} = \frac{1}{2}\left(\frac{\partial v'}{\partial r} - \frac{v'}{r}\right),\; e'_{z\theta} = \frac{1}{2}\frac{\partial v'}{\partial z}, \tag{29}$$

where N' and L' be the directional rigidity of the porous medium along r and z direction respectively. Under the assumption that there is no relative motion between solid and liquid of porous layer, the mass coefficients ρ'_{rr}, $\rho'_{r\theta}$ and $\rho'_{\theta\theta}$ are related to the density ρ', ρ'_s and ρ'_l of the layer, solid and liquid respectively by Biot [8]

$$\rho'_{rr} + \rho'_{r\theta} = (1-\psi)\rho'_s$$
$$\rho'_{r\theta} + \rho'_{\theta\theta} = \psi\rho'_l.$$

Therefore, mass density of the bulk material is

$$\begin{aligned}\rho' &= \rho'_{rr} + 2\rho'_{r\theta} + \rho'_{\theta\theta} \\ &= \left(\rho'_{rr} + \rho'_{r\theta}\right) + \left(\rho'_{r\theta} + \rho'_{\theta\theta}\right) \\ &= \rho'_s + \psi\left(\rho'_l - \rho'_s\right),\end{aligned}$$

where, ψ being porosity of porous layer and

$$\rho'_{rr} > 0,\; \rho'_{\theta\theta} > 0,\; \rho'_{r\theta} < 0,\; \rho'_{rr}\rho'_{\theta\theta} - {\rho'_{r\theta}}^2 > 0.$$

Introducing the Eq. (29), dynamic equation of motion obtained by Eqs. (27)-(28) reduces to

$$\left(N' - \frac{1}{2}d'gz\right)\left\{\frac{\partial^2 v'_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v'_\theta}{\partial r} - \frac{v'_\theta}{r^2}\right\} + \left(N' - \frac{1}{2}d'gz\right)\frac{\partial^2 v'_\theta}{\partial z^2} - \frac{d'g\partial v'}{2\partial z} = d'\frac{\partial^2 v'_\theta}{\partial t^2}, \tag{30}$$

For the wave propagating along *r* direction we may assume the harmonic solution of Eq. (30) as:

$$v'_\theta = v'_3(z)J_1(kr)e^{i\omega t}. \tag{31}$$

Here $v'_3(z)$ is the solution of

$$\frac{d^2 v'_3(z)}{dz^2} - \frac{Gdk}{2\left(1-\frac{Gdkz}{2}\right)}\frac{dv'_3(z)}{dz} - k^2\left[\frac{N'\left(1-\frac{GdkzL'}{2N'}\right)}{L'\left(1-\frac{Gdkz}{2}\right)} - \frac{c^2 d}{c_3^2\left(1-\frac{Gdkz}{2}\right)}\right]v'_3(z) = 0, \tag{32}$$

where, $c_3 = \sqrt{\frac{L'}{\rho'}}$ is the velocity of the shear wave in anisotropic elastic medium along *r* direction, $G = \frac{g\rho'}{L'k}$, Biot's gravity parameter, $\gamma_{11} = \frac{\rho'_{rr}}{\rho'}$, $\gamma_{12} = \frac{\rho'_{r\theta}}{\rho'}$, $\gamma_{22} = \frac{\rho'_{\theta\theta}}{\rho'}$ are the dimensionless parameter for the material of porous medium, $d = \frac{d'}{\rho'} = \frac{1}{\rho'}\left(\rho'_{rr} - \frac{\rho'^2_{r\theta}}{\rho'_{\theta\theta}}\right) = \gamma_{11} - \frac{\gamma_{12}^2}{\gamma_{22}}$.

By letting $v'_3(z) = \frac{\psi(z)}{\sqrt{\left(1-\frac{Gdkz}{2}\right)}}$ in Eq. (32), we obtain

$$\frac{d^2\psi(z)}{dz^2} + \left\{\frac{G^2 d^2 k^2}{16}\left(1-\frac{Gdkz}{2}\right)^{-2}\right\}\psi(z) - k^2\left[\left\{\frac{N'\left(1-\frac{GdkzL'}{2N'}\right)}{L'\left(1-\frac{Gdkz}{2}\right)} - \frac{c^2 d}{c_3^2\left(1-\frac{Gdkz}{2}\right)}\right\}\right]\psi(z) = 0, \tag{33}$$

Again letting $\delta = \frac{4}{Gd}\left(1-\frac{Gdkz}{2}\right)$, the Eq. (33) can be expressed as:

$$\psi''(\delta) + \left[\frac{1}{4\delta^2} + \frac{m_3}{\delta} - \frac{1}{4}\right]\psi(\delta) = 0, \tag{34}$$

where, $m_3 = \frac{c^2}{c_3^2 G} + \frac{1}{Gd}\left(1-\frac{N'}{L'}\right)$, Eq. (34) is well known Whittaker equation, whose solution is obtained by

$$\psi(\delta) = D_5 W_{m_3,0}(\delta) + D_6 W_{-m_3,0}(-\delta).$$

As the solution of the Eq. (34) must be bounded and vanishes as $z\to\infty$ for the surface wave. i.e, $\delta\to\infty$, we make take the solution as:

$$\psi(\delta) = D_6 W_{-m_3,0}(-\delta).$$

Therefore, the displacement in porous layer half-space is given by

$$v'_\theta = u'_3(say) = \frac{D_6 W_{-m_3,0}\left[-\frac{4}{Gd}\left(1-\frac{Gdkz}{2}\right)\right]}{\sqrt{\left(1-\frac{Gdkz}{2}\right)}} J_1(kr)e^{i\omega t}. \tag{35}$$

6 BOUNDARY CONDITION AND FREQUENCY EQUATION

Since the thickness of the intermediary layer (M_2) is H and at $z=-H$, there is another semi-infinite medium that is (M_1). It is assumed that the dispersion of wave is continuous at the interface of the media, that is, at $z=-H$, the velocity of the (M_1) is equal to the velocity of (M_2). Also, stress at the interface $z=-H$ of the media are contained like any other internal surface that means stress of the intermediate layer is the same as that of the upper semi-infinite medium.

So, mathematically, boundary conditions at $z=-H$ are:

i. $u'_1 = u'_2$ i.e, displacement components are continuous.

ii. $L_1 \frac{\partial u'_1}{\partial z} = L_2 \frac{\partial u'_2}{\partial z}$ i.e, stress components are continuous.

Again at the interface of the layer (M_2) and the lower half-space (M_3), the displacement component is continuous with any other internal surface in the media at $z=0$, which behaves as the stress on one side of it is same as that on the other side.

Mathematically, boundary conditions at $z=0$ are:

i. $u'_2 = u'_3$ i.e, displacement components are continuous.

ii. $L_2 \frac{\partial u'_2}{\partial z} = L' \frac{\partial u'_3}{\partial z}$ i.e, stress components are continuous.

Apply boundary condition **(1)** in Eq. (17) and in Eq. (26) it becomes

$$D_2\sqrt{L_2}\sqrt{1-\xi_2}\left(1-\frac{H}{a}\right)e^{\left(m_1+\frac{\alpha}{2}H\right)} - D_3\sqrt{L_1}e^{m_2 H} - D_4\sqrt{L_1}e^{-m_2 H} = 0, \tag{36}$$

$$\begin{aligned} &-D_2\sqrt{L_1}\sqrt{1-\xi_2}\left(1-\frac{H}{a}\right)^2\frac{\alpha}{2}e^{\left(m_1+\frac{\alpha}{2}H\right)} + D_3\left\{\sqrt{L_2}m_2\left(1-\frac{H}{a}\right)e^{m_2 H} + \sqrt{L_2}\frac{1}{a}e^{m_2 H}\right\} \\ &-D_4\left\{\sqrt{L_2}m_2\left(1-\frac{H}{a}\right)e^{-m_2 H} - \sqrt{L_2}\frac{1}{a}e^{-m_2 H}\right\} = 0. \end{aligned} \tag{37}$$

Apply boundary condition **(2)** in Eq. (26) and in Eq. (35) it becomes

$$D_3 + D_4 - D_6 Q_1\sqrt{L_2}\sqrt{1-\xi_2} = 0, \tag{38}$$

where, $Q_1 = \left[\frac{D_6 W_{-m_3,0}\left[-\frac{4}{Gd}\left(1-\frac{Gdkz}{2}\right)\right]}{\sqrt{\left(1-\frac{Gdkz}{2}\right)}}\right]_{z=0}$

$$D_3\left(m_2+\frac{1}{a}\right)\sqrt{L_2}+D_4\left(m_2-\frac{1}{a}\right)\sqrt{L_2}-D_6L^{'}Q_2\sqrt{1-\xi_2}=0, \tag{39}$$

where, $Q_2=\frac{\partial}{\partial z}\left[\frac{D_6W_{-m_3,0}\left[-\frac{4}{Gd}\left(1-\frac{Gdkz}{2}\right)\right]}{\sqrt{\left(1-\frac{Gdkz}{2}\right)}}\right]_{z=0}$.

Now eliminating D_2, D_3, D_4 and D_6 from Eqs. (36), (37), (38) and (39) we have

$$\begin{vmatrix} \sqrt{L_2}\sqrt{1-\xi_2}\left(1-\frac{H}{a}\right)e^{\left(m_1+\frac{\alpha}{2}H\right)} & -\sqrt{L_1}e^{m_2H} & -\sqrt{L_1}e^{-m_2H} & 0 \\ -\sqrt{L_1}\sqrt{1-\xi_2}\left(1-\frac{H}{a}\right)^2\frac{\alpha}{2}e^{\left(m_1+\frac{\alpha}{2}H\right)} & R_1 & -R_2 & 0 \\ 0 & 1 & 1 & -\sqrt{L_2}\sqrt{1-\xi_2} \\ 0 & \left(m_2+\frac{1}{a}\right)\sqrt{L_2} & \left(m_2-\frac{1}{a}\right)\sqrt{L_2} & -L^{'}Q_2\sqrt{1-\xi_2} \end{vmatrix}=0,$$

where,

$$\left.\begin{aligned} R_1&=\left\{\sqrt{L_2}m_2\left(1-\frac{H}{a}\right)e^{m_2H}+\sqrt{L_2}\frac{1}{a}e^{m_2H}\right\} \\ R_2&=\left\{\sqrt{L_2}m_2\left(1-\frac{H}{a}\right)e^{-m_2H}-\sqrt{L_2}\frac{1}{a}e^{-m_2H}\right\} \end{aligned}\right\} \tag{40}$$

Expanding the determinant, we have

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2(1-\xi_2)}}\right]kH= \\ \frac{L_2\left(1-\frac{kH}{ak}\right)^2m_2^{'}L^{'}Q_2+L_2^2\left(1-\frac{kH}{ak}\right)^2m_2^{'}\frac{1}{ak}Q_1+L_1\left(1-\frac{kH}{ak}\right)^2\frac{\alpha}{2k}m_2^{'}L_2Q_1-L_2^2\left(1-\frac{kH}{ak}\right)m_2^{'}\frac{1}{ak}Q_1}{L_2^2\left(1-\frac{kH}{ak}\right)^2m_2^{'2}Q_1-L_2\left(1-\frac{kH}{ak}\right)\frac{1}{ak}L^{'}Q_2-L_2^2\left(1-\frac{kH}{ak}\right)\frac{1}{a^2k^2}Q_1+L_1\left(1-\frac{kH}{ak}\right)^2\frac{\alpha}{2k}\left(L^{'}Q_2+\frac{1}{ak}L_2Q_1\right)}, \tag{41}$$

where, $m_2^{'}=\sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2(1-\xi_2)}}$.

Eq. (41) gives the required dispersion equation of torsional surface wave in a pre-stress anisotropic inhomogeneous layer lying over an anisotropic porous layer half-space under gravity and underlying a pre-stress anisotropic porous layer . It should be noted that if the upper layer is non-porous solid than $\psi\to 0$ and $\rho_s^{'}\to\rho^{'}$, which leads to $\gamma_{11}+\gamma_{12}\to 1$ and $\gamma_{12}+\gamma_{22}\to 0$, which gives to $\gamma_{11}-\frac{\gamma_{12}^2}{\gamma_{22}}\to 1$ or $d\to 1$. Again if $\psi\to 1$ then $\rho_l^{'}\to\rho^{'}$ and liquid becomes fluid $\gamma_{11}-\frac{\gamma_{12}^2}{\gamma_{22}}\to 0$ or $d\to 0$. Hence for porous layer $0<d<1$.

7 PATICULAR CASES

Case-i

When the upper porous layer is homogeneous i.e. $\alpha = 0$ then Eq. (41) reduces to

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2(1-\xi_2)}}\right]kH = \frac{L_2\left(1-\frac{kH}{ak}\right)^2 m_2' L' Q_2 + L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2' \frac{1}{ak} Q_1 - L_2^2\left(1-\frac{kH}{ak}\right) m_2' \frac{1}{ak} Q_1}{L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2'^2 Q_1 - L_2\left(1-\frac{kH}{ak}\right)\frac{1}{ak} L' Q_2 - L_2^2\left(1-\frac{kH}{ak}\right)\frac{1}{a^2k^2} Q_1}$$

which is the dispersion equation of torsional surface wave in a homogeneous porous layer under the gravity.

Case-ii

When the intermediate inhomogeneous layer is free from pre-stress i.e. $p_2 = 0$ then Eq. (41) becomes

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2}}\right]kH =$$

$$\frac{L_2\left(1-\frac{kH}{ak}\right)^2 m_2'' L' Q_2 + L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2'' \frac{1}{ak} Q_1 + L_1\left(1-\frac{kH}{ak}\right)^2 \frac{\alpha}{2k} m_2'' L_2 Q_1 - L_2^2\left(1-\frac{kH}{ak}\right) m_2'' \frac{1}{ak} Q_1}{L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2''^2 Q_1 - L_2\left(1-\frac{kH}{ak}\right)\frac{1}{ak} L' Q_2 - L_2^2\left(1-\frac{kH}{ak}\right)\frac{1}{a^2k^2} Q_1 + L_1\left(1-\frac{kH}{ak}\right)^2 \frac{\alpha}{2k}\left(L' Q_2 + \frac{1}{ak} L_2 Q_1\right)}$$

where, $m_2'' = \sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2}}$, which is the dispersion equation of torsional surface wave in an initially stress free inhomogeneous layer underlying a inhomogeneous porous layer and lying over an anisotropic porous half-space under gravity.

Case-iii

When the lower half-space is elastic isotropic solid and free from gravity i.e. $N' = L'$, $d \to 1$ and $G = 0$ then Eq. (41) reduce to

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)\frac{N_2}{L_2(1-\xi_2)}}\right]kH =$$

$$\frac{L_2\left(1-\frac{kH}{ak}\right)^2 m_2' L'\left(1-\frac{c^2}{2c_3^2}\right) + L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2' \frac{1}{ak} + L_1\left(1-\frac{kH}{ak}\right)^2 \frac{\alpha}{2k} m_2' L_2 - L_2^2\left(1-\frac{kH}{ak}\right) m_2' \frac{1}{ak}}{L_2^2\left(1-\frac{kH}{ak}\right)^2 m_2'^2 - L_2\left(1-\frac{kH}{ak}\right)\frac{1}{ak} L'\left(1-\frac{c^2}{2c_3^2}\right) - L_2^2\left(1-\frac{kH}{ak}\right)\frac{1}{a^2k^2} + L_1\left(1-\frac{kH}{ak}\right)^2 \frac{\alpha}{2k}\left(L'\left(1-\frac{c^2}{2c_3^2}\right) + \frac{1}{ak} L_2\right)}$$

Case-iv

If the inhomogeneous layer is homogeneous, isotropic and pre-stress free i.e. $\frac{1}{a} = 0$, $N_2 = L_2$, and $p_2 = 0$ then Eq. (41) takes the form

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}\right]kH=\frac{L_2\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}L'Q_2+L_1\frac{\alpha}{2k}\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}L_2Q_1}{L_2^2\left(\frac{c^2}{c_2^2}-1\right)Q_1+L_1\frac{\alpha}{2k}L'Q_2}$$

which is the dispersion equation of torsional surface wave in a homogeneous, isotropic and initial stress free layer lying under an anisotropic porous medium and lying over a gravitating anisotropic porous half-space.

Case-v

If the upper half-space is omitted, case (4) reduces to

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}\right]kH=\frac{\mu_3}{\mu_2}\frac{\left(1-\frac{c^2}{2c_3^2}\right)}{\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}},$$

which on approximation gives

$$\tan\left[\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}\right]kH=\frac{\mu_3}{\mu_2}\frac{\sqrt{\left(1-\frac{c^2}{c_3^2}\right)}}{\sqrt{\left(\frac{c^2}{c_2^2}-1\right)}}$$

which is the well known classical dispersion equation of torsional surface wave.

8 NUMERICAL CALCULATION AND DISCUSSION

Mathematical statement (41) is the frequency equation giving the velocity of torsional surface wave in an initially stressed anisotropic heterogeneous layer sandwiched between a pre-stressed anisotropic porous semi-infinite medium and an anisotropic porous half-space under gravity. Numerical computation of dispersion Eq. (41) has been performed to exhibit the consequence of initial stress, porosity, gravity and inhomogeneity parameters related to the accepted medium at the dispersion of torsional surface wave. For numerical computation, we take a few numerical data for porous medium from Samal and Chattaraj [22] and Gubins [23] for heterogeneous.

For upper and lower anisotropic porous half-space

$$L_1=L'=0.1387\times10^{10}N/m^2 \qquad N_1=N'=0.2774\times10^{10}N/m^2 \qquad \rho_{rr}=1.926137\times10^3Kg/m^3$$
$$\rho_{r\theta}=-0.002137\times10^3Kg/m^3 \qquad \rho_{\theta\theta}=0.215337\times10^3Kg/m^3 \qquad f=0.26$$

For heterogeneous layer

$$N_2=6.34\times10^{10}N/m^2 \qquad L_2=3.99\times10^{10}N/m^2 \qquad \rho_2=3364Kg/m^3$$

All the figures have been plotted on the vertical axis as dimensionless phase velocity c/c_2 towards horizontal axis as non-dimensional wave number *kH*. For graphical illustration, numerical values of all the figures had been taken from Table 1.

Table 1
Values of various dimensionless parameter.

	Fig.2	Fig.3	Fig.4	Fig.5	Fig.6	Fig.7
α/k	-	0.1	0.2	0.1	0.1	0.2
$1/ak$	0.1	-	0.1	0.3	-	0.1
d	0.9	0.9	-	0.9	0.9	-
G	0.3	0.5	0.3	-	0.5	0.3
p_2	0.4	0.2	0.5	0.6	0.2	0.5

Using above numerical data, results are depicted in Fig. 2-7. In Fig. 2, phase velocity c/c_2 has been plotted against wave number kH for the different value of inhomogeneity parameter α/k associated with directional rigidity, density and initial stress of the upper semi-infinite porous medium. It is inferred that as rigidity, density and porosity of the upper half-space increases, the phase velocity c/c_2 increases for a fixed value of wave number kH. This leads to fact that the phase velocity of torsional wave is directly proportional to the wave number for different value of α/k .

Fig.3 represents the dispersion curve for the torsional surface wave when inhomogeneity parameter $1/ak$ related to directional rigidity, density and pre-stress of the heterogeneous layer. It has been observed that as the inhomogeneity parameter $1/ak$ of the heterogeneous layer increases, the phase velocity decreases at a particular wave number thereby reflects the reality that phase velocity of the torsional surface wave is inversely proportional to the inhomogeneity of the heterogeneous layer.

Fig.4 signifies the effect of porosity of the half-space under the influence of the gravity. The dispersion curves are plotted for different value of d with fixed value of gravity, rigidity, pre-stress and inhomogeneity parameters. It has been seen that as the porosity of the half-space decreases, the phase velocity of the torsional wave increases for a fixed value of wave number. It reflects the facts that porosity and velocity of torsional wave are inversely proportional to each other.

In Fig.5 study has been made to obtain the effect of gravity G in the anisotropic porous half-space. In this figure c/c_2 has been plotted against kH for different value of gravity G. These dispersion curves show that the velocity of the torsional surface wave increases as the value of G decreases.

Fig.6 has been plotted to depict the influence of inhomogeneity $1/ak$ if the upper half-space space not noted beneath have an effect on of gravity. It gives the identical impact as compared to Fig.3.

Fig.7 manifests the effect of porosity d of the half-spaces in the absence of gravity in the lower half-space. As compared to Fig.4 it has been seen that the phase velocity of the torsional wave raises with decreases of d for fixed wave number. Which well-known shows that absence of gravity, plays an important role in torsional wave propagation.

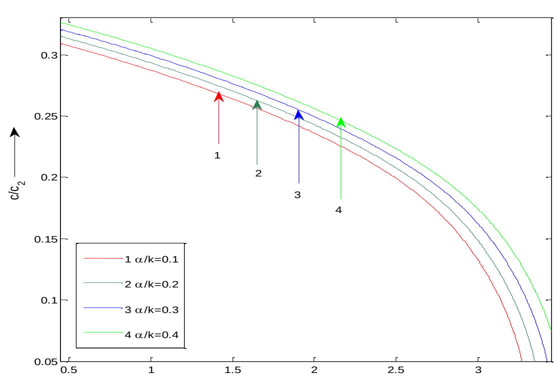

Fig.2
Variation of dimensionless phase velocity c/c_2 with respect to non-dimensional wave number kH for different values of inhomogeneity a/k in upper porous layer.

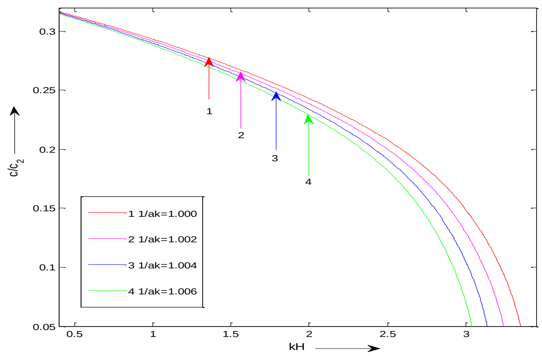

Fig.3
Comparison of dimensionless phase velocity c/c_2 against dimensionless wave number kH for various values of inhomogeneity $1/ak$ for non-homogeneous layer between porous half-space.

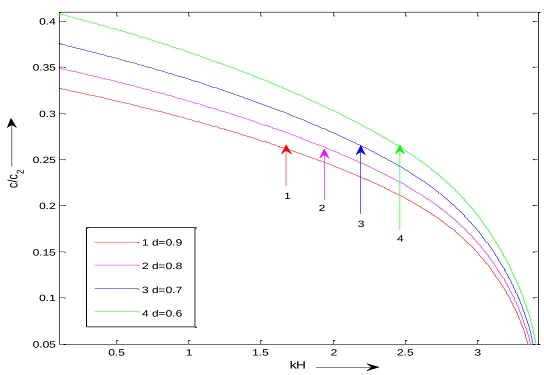

Fig.4
Correlation of dimensionless phase velocity c/c_2 against non-dimensional wave number kH for different values of porosity d in porous half-spaces.

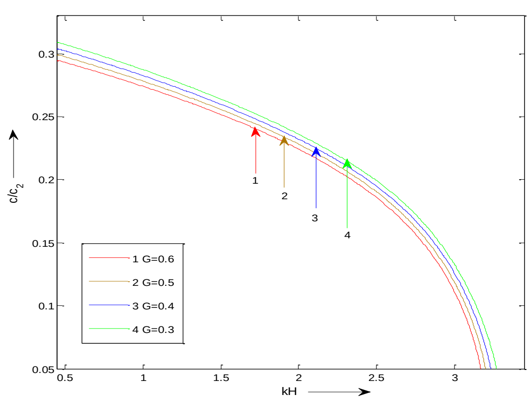

Fig.5
Assortment of dimensionless phase velocity c/c_2 versus a function dimensionless wave number kH for various values of gravity G in gravitating anisotropic porous half-space.

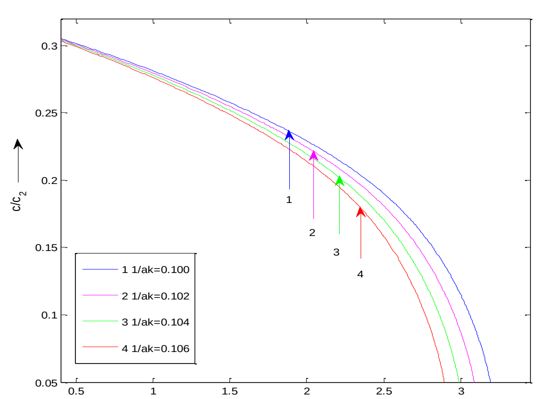

Fig.6
Variation of dimensionless phase velocity c/c_2 with respect to dimensionless wave number kH for different values of inhomogeneity $1/ak$ in the absence of initial stress p_2 in upper porous half-space.

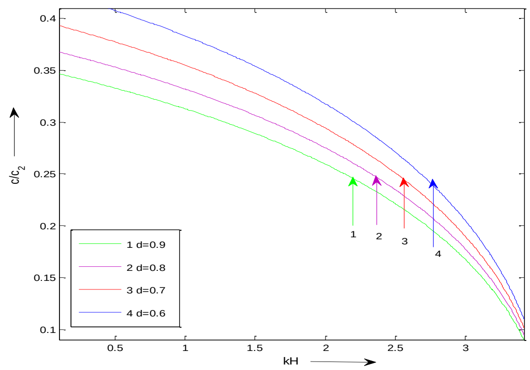

Fig.7
Correlation of dimensionless phase velocity c/c_2 against non-dimensional wave number *kH* for various values of porosity *d* in porous half-spaces under gravity.

9 CONCLUSIONS

In this paper, we have got studied the dispersion of the torsional surface wave in a heterogeneous finite thickness layer with quardratic variation in rigidity, density and initial stress, underlying an anisotropic porous semi-infinite medium with exponential variation in rigidity, density, pre-stress and lying over an anisotropic porous half-space underneath the affect of gravity. We determined that the existing geometry of this problem lets in the torsional wave to propagate. Phase velocity is computed numerically and the consequence of applicable parameters are studied and shown graphically the use of MATLAB software program. The closed-form solutions have been obtained for dispersion equations pertinent to numerous surface boundary condition. From the aforementioned figures, the outcome can be summarized as comply with:

(i) All the figures exhibit that phase velocity of the torsional surface wave decreases with the raise of dimensionless wave number, which is well nature of seismic wave, i.e., as depth raises, the rate of surface wave decreases.

(ii) When the upper semi-infinite porous medium is omitted, the dispersion Eq. (41) coincide with the classical dispersion relation of Love wave. It has been seen that as heterogeneity increments in an intermediate layer, the rate of phase velocity decreases. It prompts actuality that phase velocity and wave number *kH* for various estimation of $1/ak$ is contrarily corresponding to each other.

(iii) The speed of torsional surface wave increments as abatement of porosity *d*, for a specific estimation of directional rigidity, initial stress and gravity. It is likewise seen in the absence of gravity field, phase velocity increments as abatement of porosity.

(iv) Decreases of inhomogeneity $1/ak$ in heterogeneous layer, speed of phase velocity increases and velocity of wave increases with increases of inhomogeneity α/k in the upper porous layer in the presence of gravity, pre-stress, rigidity and density.

(v) Without pre-stress p_2 of the heterogeneous layer, the torsion surface wave increments for reductions of wave number *kH*. From the dispersion relation, it is far proven that the upper anisotropic porous semi-infinite medium is exempted from the impact of pre-stress p_1 i.e, the torsional surface wave is not getting effected by initial stress p_1.

(vi) As gravity in lower half-space diminishes the velocity of torsional surface wave increments.

Since the real Earth is not homogeneous and can be considered as made out of various inhomogeneous layers, in this manner, it is more sensible to consider the inhomogeneity talked about in the present issue to contemplate the engendering of torsional surface waves in heterogeneous Earth medium. Likewise, the earth is gravity medium, the speeding up because of gravity *G* has an awesome significance in investigating the dynamic and static problems of the earth. The present study might be valuable for geophysical utilizations of propagation of torsional waves in various layered Earth's media.

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