Journal of Solid Mechanics Vol. 12, No. 1 (2020) pp. 189-203
DOI: 10.22034/jsm.2019.1867399.1431

Nonlocal Dispersion Analysis of a Fluid – Conveying Thermo Elastic Armchair Single Walled Carbon Nanotube Under Moving Harmonic Excitation

M. Mahaveersree Jayan[1], R. Kumar [2], R. Selvamani [1,*], J. Rexy [1]

[1]*Department of Mathematics, Karunya Institute of Technology and Sciences, Coimbatore, Tamil Nadu, India*
[2]*Department of Mathematises, Kurukshetra University, Kurukshetra, Haryana, India*

Received 15 December 2019; accepted 18 February 2020

ABSTRACT
In this work, the nonlocal elastic waves in a fluid conveying armchair thermo elastic single walled carbon nanotube under moving harmonic load is studied using Eringen nonlocal elasticity theory via Euler Bernoulli beam equation. The governing equations that contains partial differential equations for single walled carbon nanotube is derived by considering thermal and Lorenz magnetic force. The small scale interactions induced by the nano tubes are simulated by the non-local effects. The time domain responses are obtained by using both modal super position method and Newmarks's direct integration method. The effect of nonlocal parameter, thermal load, magnetic field of the moving harmonic load on the dynamic displacement of SWCNT is discussed. The results obtained show that the dynamic displacement of fluid conveying SWCNT ratio is significantly affected by the load velocity and the excitation frequency. This type of results presented here, will provide useful information for researchers in structural nano science to understand the small scale response of elastic waves coupled with thermo elasticity and some field forces.

Keywords: Nonlocal model; Thermo elastic nanotube; Harmonic load; Newmarks's direct integration method; Armchair.

1 INTRODUCTION

SINCE the scientific advancement of nano and micro electro mechanical systems in medicine, biology, and mechanical engineering, microscale wave propagation has become an active research topic in the recent years. Whenever the dynamical responses of mechanical structure of nano-size elements are required, the size dependent continuum theories shall be utilized. The analyses of small-scale structures require more sophisticated theories than commonly used classical continuum mechanics that ignores interatomic forces. Currently, there is an enormous deal of theoretical and experimental works on mechanical behaviour of carbon nanotubes (CNTs). Experimentally

*Corresponding author.
E-mail address: selvam1729@gmail.com (R. Selvamani).

observed data are highly required; however, the technical challenge and expense of testing at the nano-scale restricts the availability of these data. Such a fact has motivated researches to exploit theoretical models to characterize mechanical behaviours of CNTs. The structural members belonging to fluid conveying magneto thermo elastic SWCNT have received wide attention by the various fields research communities like hydro elasticity of nano tube and nano electro mechanical system. Because of the coupling of thermo mechanical and magnetic fields, extensive study has been made for thermo magneto- elastic (TME) nano materials. Therefore, with increasing usage of TME structures in various engineering fields, wave propagation in TME nano material with mechanical load has also attracted many researchers. Considerable motivation has been increased to investigate the mechanical, thermal, chemical, electrical and electromagnetic properties of CNTs after the advent of CNT's, Lijima [1]. Since the theory of classical or local continuum cannot predict the characteristics of the nano-scales structures, the small-size scale and nano-scale surface effect in nanotechnology become more significant. As per the local continuum theory, the stress at a point depends only on the strain at the same point. But, by the nonlocal elasticity theory of Eringen [2], the stress at a point is a function of strains at all points in the continuum. The idea in the Erignen's non-local elasticity is based on the atomic theory of lattice dynamics. In the Erignen's context, there are many studies related to the static, buckling and vibration analysis of CNTs by using the local and nonlocal beam theories in literature. For example, Peddieson, Buchanan and Mcnitt [3] was first applied the nonlocal elasticity theory to static deformation analysis of Euler-Bernoulli beams. Lu, Lee, Lu, and Zhang [4] established a nonlocal Euler beam model to obtain frequency equations and model shape functions of simply supported, clamped and cantilever beams. The torsional vibration analysis of double walled carbon nanotubes was investigated by Aydogdu and Arda [5] using nonlocal elasticity. Civalck, Demir and Akgoz [6] used differential quadrature method to study the static deflection of SWCNTs based on nonlocal theory of elasticity. Yambae Tokyo [7] investigated the recent development of carbon nanotube and their properties. Wu, Zhang, Leung and Zhong [8] developed the energy- equivalent model on studying the mechanical properties of single walled carbon nano tubes. The thermal effect on vibration characteristics of armchair and zigzag single walled carbon nanotubes using nonlocal parabolic beam theory is reads from Baghdadi, Tournsi, Zidour and Benzairi [9]. Naceri, Zidour, Semmah, Houari, Benzair and Tounsi [10] studied the sound wave propagation in armchair single walled carbon nanotubes under thermal environment. Bedia, Benzair, Semmah, Tounsi and Mahmoud [11] studied the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity. Murmu and Pradhan [12] applied the nonlocal Euler-Bernoulli beam model to study the buckling analysis of a simply supported SWCNTs subjected to an axial compressive load and with the effect of temperature change and surrounding elastic medium. The thermal effect on ultrasonic wave propagation characteristics of a nano plate are studied by Narender and Gopalakrishnan [13] based on nonlocal continuum theory. In their study, the axial stress caused by the thermal effect was considered. Farzed Ebrahimi and Mahmoodi [14] analysed the thermal loading effect on free vibration characteristics of carbon nanotubes (CNT) with multiple cracks by taking various boundary conditions for nanotube. Farzed Ebrahimi, Boreiry and Shaghaghi [15] demonstrated the nonlinear vibration analysis of electro hygro-thermally actuated embedded nano beams with various boundary conditions. They concluded that, the two types of thermal loading, namely uniform and linear temperature rises through thickness direction. Because of excellent mechanical properties, chemical and thermal stability, and hollow geometry, the carbon nanotube CNT promises many new applications in Nano biological devices and Nano mechanical systems such as fluid storage, fluid transport, and targeted drug delivery. The influence of internal moving fluid on free vibration and stability of CNT's using classical beams model for both supported and cantilevered systems is reads from Yoon, Ru and Miodochowski [16, 17]. They found that the internal moving fluid could substantially affect resonant frequencies especially for larger radius at higher flow velocity, and the critical flow velocity for structural instability of flutter could fall within the range of practical significance. The surrounding elastic medium can significantly reduce the effect of internal moving fluid on resonant frequencies. Zhang, Liu and Liu [18] studied the thermal effect on transverse vibrations of DWCNTs using classical continuum model. Chang [19] developed a model for the thermal- mechanical vibration and instability of a fluid conveying single walled carbon nanotube embedded in an elastic medium based on nonlocal elasticity theory. They concluded that the effect of temperature change, nonlocal parameter and elastic medium constant on the vibration frequency and buckling instability of SWCNT conveying fluid. Wang [20] studied the reliability of various theoretical beam modes via elasticity theories for wave propagation analysis of fluid conveying SWCNTs with either Euler-Bernoulli beam theory or Timoshenko beam theory and either stress or strain gradients. It is found that the combined strain /inertia gradient Timoshenko beam theory is more suitable for analysing the dynamical behaviours of fluid –conveying nanotubes. Mohammad and Goughari [21] proposed a method to analyse the effect of a longitudinal magnetic field on the transverse vibration of a magnetically sensitive single walled carbon nanotube conveying fluid. The problem of predicting the transverse vibrations of continuous media obtained from the passage of harmonic moving loads is of considerable practical interest in the structural dynamics area. Kiani and Mehri [22] assessed the nanotubes structures under a moving nano particle using nonlocal beam theories. Simsek, Simsek and Kocaturk [23-25] investigated the vibration analysis of a SWCNTs subjected to a moving harmonic load based on Eringen's nonlocal elasticity theory.

In the present study, the nonlocal elastic waves in a fluid conveying armchair thermo elastic single walled carbon nanotube under moving harmonic load is studied using Eringen nonlocal elasticity theory in the context of Euler Bernoulli beam equation. The governing equations that contains partial differential equations for single walled carbon nanotube is derived by considering thermal and Lorenz magnetic force along with the nonlocal parameters. The computed dynamic displacement is presented as dispersion curves.

2 ATOMIC STRUCTURE OF THE ARMCHAIR SINGLE WALLED CARBON NANOTUBE

The theoretical assumption of single walled carbon nanotube (SWCNT) is the rolled form of a graphene sheet. The fundamental structure of carbon nanotubes can be classified into three categories as armchair, zigzag and chiral in terms of the chiral vector $\vec{C}_h$ shown in Fig.1. The chiral vector can be expressed in terms of base vectors $\vec{a}_1$ and $\vec{a}_2$ as in Fig.1

$$\vec{C}_h = m\,\vec{a}_1 + n\,\vec{a}_2$$

where the integer pair (*m, n*) represents the indices of translation, which decide the structure around the circumference.

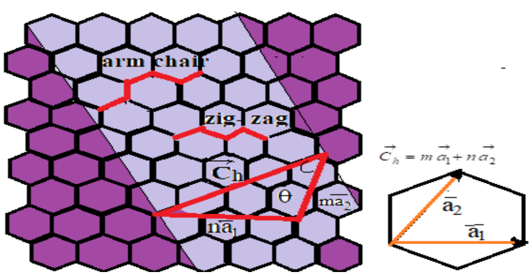

Fig.1
Hexagonal lattice of graphene sheet with base vectors.

The diameter of armchair single walled carbon nanotube for $(n = m)$ is given by Yamabe [9] as:

$$d = \frac{3ns}{\pi}$$

In which s is the length of the carbon band which is $0.142\times10^{9}m$. Wu et al. [8] developed energy –equivalent model for studying the mechanical properties of single walled carbon nanotube based on the relation between molecular mechanics and solid mechanics. Using the same method, the equivalent Young's modulus of armchair nanotubes are expressed as:

$$P_a = \frac{4\sqrt{3}}{3}\frac{VW}{3Wt + 4Vs^2t(\gamma^2_{a1} + \gamma^2_{a2})} \qquad \text{Radius} = \frac{\left|\vec{C}_h\right|}{2\pi} = b\sqrt{3(m^2 + mn + n^2 / 2\pi}$$

where V and W are the force constants, t is the thickness of the nanotube, and the parameters γ_{a1} and γ_{a2} are given by

$$\gamma_{a1} = \frac{4 - \cos^2(\pi/2n)}{16 + 2\cos^2(\pi/2n)} \qquad \gamma_{a2} = \frac{\sqrt{12 - 3\cos^2(\pi/2n)}\cos(\pi/2n)}{32 + 4\cos^2(\pi/2n)}$$

The expressions of Young's modulus of a graphene sheet is obtained by letting $n \to \infty$ as follows:

$$E_g = \frac{8\sqrt{3}VW}{18Wt + Vs^2 t}$$

3 ERINGEN NONLOCAL THEORY OF ELASTICITY

Under Eringen's non local theory, the stress state is considered as a function of strain in the entire nearby region of the body. The general form of the constitutive equations in the non-local form of elasticity contains an integral over the entire region of interest. The integral contains a non-local kernel function, which describes the relative influences of the strains at various locations on the stress at a given location. The constitutive equations of linear, homogeneous, isotropic, non-local elastic solid with zero body forces are given by Eringen [2] as follows:

$$\Pi_{ij,i} + \rho(f_j - \ddot{u}_j) = 0 \tag{1}$$

$$\Pi_{ij}(x) = \int_V \alpha(|x - x'|), \tau)\Pi_{ij}^{c}(x')dV(x') \tag{2}$$

$$\Pi_{ij}^{c} = C_{ijkl}\varepsilon_{kl} \tag{3}$$

$$\varepsilon_{ij}(x') = \frac{1}{2}\left(\frac{\partial u_i(x')}{\partial x_{j'}} + \frac{\partial u_i(x')}{\partial x_{i'}}\right) \tag{4}$$

where $\Pi_{ij,i}, \rho, f_j, \ddot{u}_j$ are the stress tensor, mass density, body force density and displacement vector at a reference point *x* in the body, respectively, at the time *t*, Eq. (3) is the classical constitutive relation where $\Pi^{c}_{ij}(x')$ is the classical stress tensor at any point x' in the body, which is related to the linear strain tensor $\varepsilon_{ij}(x')$ at the same point. Eq. (4) is the classical strain displacement relationship. The kernel function $\alpha(|x - x'|, \tau)$ is the attenuation function which incorporated the nonlocal effect in the constitutive equations. It is clear that, the only difference between Eqs. (1)-(4) and the corresponding equations of classical elasticity in Eq. (2) replaces the Hooke's law in Eq. (3) by Eq. (2). Eq. (2) consists the parameters which correspond to the non-local modulus has dimensions of $(lengh)^{-3}$ and so it depends on a characteristic length (lattice parameter, size of grain, granular distance, etc.) and "*l*" is an external characteristic length of the system (wavelength, crack length, size or dimensions of sample, etc.). Therefore the non-local modulus can be written in the following form;

$$\alpha = \alpha(|x - x'|, \tau), \tau = \frac{e_o a}{l} \tag{5}$$

where $e_0 a$ is a constant corresponding to the material's and has to be determined for each materials independently and "$|x - x'|$" is the Euclidian distance. Then, the integro-partial differential Eq. (2) of non-local elasticity can be derived to partial differential equation as follows:

$$(1-\tau^2 l^2 \nabla^2)\Pi_{ij}(x) = \Pi_{ij}^{c}(x) = C_{ijkl}e_{kl}(x) \tag{6}$$

where C_{ijkl} is the elastic modulus tensor of classical isotropic elasticity and e_{ij} is the strain tensor. Where ∇^2 denotes the second-order spatial gradient applied on the stress tensor $\Pi_{ij,i}$. Eringen proposed $e_o = 0.39$ by the matching of the dispersion curves via non-local theory for place wave and Born-Karman model of lattice dynamics at the end of the Brillouin zone $(ka = \pi)$, where a is the distance between atoms and k is the wave number in the phonon analysis.

4 PROBLEM FORMULATION

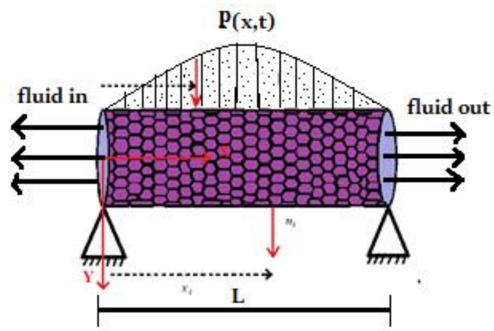

Fig.2
Geometry of the problem.

The simply supported fluid conveying armchair thermo elastic SWCNT under the harmonic load is shown in Fig.2. In the analytical model, we assumed that, the internal fluid is steady and incompressible with zero gravity. The partial differential equation which governs the free vibration of the fluid- conveying armchair thermo elastic SWCNT under the influence of moving harmonic load can be represented as:

$$\frac{\partial Q}{\partial x} + N_t \frac{\partial^2 y}{\partial x^2} - m_c \frac{\partial^2 y}{\partial x^2} + F_p = p(x,t) + \rho A \frac{\partial^2 y}{\partial t^2} \tag{7}$$

where $P(x,t)$ is the distributed load in the transverse direction of x axis, ρ mass density, y is the transverse bending of the beam, m_c is a mass of SWCNT per unit length, A is the cross section of SCNT and t denotes the time. The resultant shear force Q on the cross section of the nanotube is defined in the following equilibrium equation

$$Q = \frac{\partial M}{\partial x} \tag{8}$$

The thermal term N_t represents the additional axial thermal load in terms of the temperature T and α_x is the thermal expansion of coefficient in x direction. This thermo elastic force can be given as:

$$N_t = -EA\alpha_x T \tag{9}$$

Due to the plug flow fluid, the fluid force for unit length is taken as:

$$F_p = m_f \left(2v \frac{\partial^2 y}{\partial t \partial x} + v^2 \frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial t^2} \right) \tag{10}$$

where v is the uniform mean flow velocity of the fluid, m_f is the mass of fluid per unit length. y is the bending in the transverse directions. The resultant bending moment M in Eq. (8) is represented in the following form

$$M = \int_A z \prod_{xx} dA, \tag{11}$$

where Π_{xx} is the nonlocal axial stress defined by nonlocal continuum theory. The constitutive equation of a homogeneous isotropic elastic solid for one-dimensional nanotube is considered from Eringen [2] as:

$$\Pi_{xx} - (e_0 a)^2 \frac{\partial^2 \Pi_{xx}}{\partial x^2} = E\,\varepsilon_{xx} \tag{12}$$

where x is the axial coordinate , ε_{xx} is the axial strain, $(e_0 a)$ is a nonlocal parameter which represents the impact of nonlocal scale effect on the structure. a is an internal characteristic length and E is young modulus. The nonlocal relations in Eq. (12) can be written with temperature environment as follows:

$$\Pi_{xx} - (e_0 a)^2 \frac{\partial^2 \Pi_{xx}}{\partial x^2} = E\,\varepsilon_{xx} - E\,\alpha_x T \tag{13}$$

In the context of Bernoulli –Euler model, the axial strain ε_{xx} for small deflection is defined as:

$$\varepsilon_{xx} = -z \frac{\partial^2 y}{\partial x^2} \tag{14}$$

where z is the transverse co-ordinate in the positive direction of deflection. By using Eqs. (13)-(14), in Eq. (11), the bending moment M can be expressed as:

$$M - (e_0 a)^2 \left[\frac{\partial^2 M}{\partial x^2}\right] = EI \frac{\partial^2 y}{\partial x^2} \tag{15}$$

where $I = \int_A z^2 dA$ is the moment of inertia. By substituting Eqs. (7)-(9) into Eq. (15), the nonlocal bending moment M and shear force Q can be expressed as follows:

$$M - (e_0 a)^2 \left[(m_c + \rho A)\frac{\partial^2 y}{\partial t^2} + F_p + EA\,\alpha_x T + p(t)\right] = EI \frac{\partial^2 y}{\partial x^2} \tag{16}$$

and

$$Q - (e_0 a)^2 \left[(m_c + \rho A)\frac{\partial^3 y}{\partial x^2 \partial t^2} + \frac{\partial F_p}{\partial x} + EA\,\alpha_x T\right] = EI \frac{\partial^3 y}{\partial x^3} \tag{17}$$

For the transverse displacement, the equation of motion Eq. (17) can be expressed under the harmonic excitation, distributed pressure and thermal interaction as follows:

$$\begin{aligned} p(x,t) = {} & EI \frac{\partial^4 y(x,t)}{\partial x^4} + EA\,\alpha_x T \frac{\partial^2 y(x,t)}{\partial x^2} + m_f v^2 \frac{\partial^2 y(x,t)}{\partial x^2} + 2m_f v \frac{\partial^2 y(x,t)}{\partial t\,\partial x} + (\rho A + m_c + m_f)\frac{\partial^2 y(x,t)}{\partial t^2} \\ & - \left((e_0 a)^2 \left(\begin{aligned} & EA\,\alpha_x T \frac{\partial^4 y(x,t)}{\partial x^4} + m_c \frac{\partial^2 y(x,t)}{\partial x^2} + m_f v^2 \frac{\partial^4 y(x,t)}{\partial x^4} \\ & + 2m_f v \frac{\partial^4 y(x,t)}{\partial t^2 \partial x^2} + m_f \frac{\partial^4 y(x,t)}{\partial t^4} \end{aligned} \right) \right) \end{aligned} \tag{18}$$

The SWCNT is subjected to a moving harmonic load $p(x,t)$ which moves in the axial direction of the nanotube with constant velocity v_p as:

$$p(x,t)=p(t)\delta(x-x_p) \tag{19}$$

where $p(t)=p_o\sin(\Omega t)$ and $\delta(x-x_p)$ is the Dirac delta function, x_p is the coordinate of the moving harmonic load, P_0 is the amplitude of the harmonic load, Ω is the excitation frequency of the moving harmonic load. Introduction of Eq. (19) into Eq. (18) gives the following particle differential equation with constant coefficients in the nonlocal form

$$EI\frac{\partial^4 y}{\partial x^4}-EA\alpha_x T\frac{\partial^2 y}{\partial x^2}+m_f v^2\frac{\partial^2 y}{\partial x^2}+2m_f v\frac{\partial^2 y}{\partial t\,\partial x}-(e_0a)^2\left(\begin{array}{l}(m_c+m_f+\rho A)\dfrac{\partial^4 y}{\partial X\,\partial t^2}+EA\alpha_x T\dfrac{\partial^4 y}{\partial x^4}+\rho A\dfrac{\partial^2 y}{\partial x^2}\\ +m_f v^2\dfrac{\partial^4 y}{\partial x^4}+2m_f v\dfrac{\partial^4 y}{\partial t^2\partial x^2}+m_f\dfrac{\partial^4 y}{\partial t^4}\end{array}\right)$$
$$=P_o\sin\Omega t\,\delta(x-x_p)-(e_oa)\frac{\partial^2 P_o\sin\Omega t}{\partial x^2} \tag{20}$$

The dynamic displacement of the SWCNT in the transverse direction is represented as follows:

$$w(x,t)=\sum_{i=1}^{\infty}\Phi_i(x)q_i(t) \tag{21}$$

$q_i(t)$ are the unknown time-dependent generalized coordinates and $\Phi_i(x)$ are the Eigen-modes of an undamped simply supported beam which can be chosen as follows:

$$\Phi_i(x)=\sin\left(\frac{i\pi x}{l}\right),\quad i=1,2,3,4......... \tag{22}$$

Employing the Eqs. (21)-(22), into Eq. (20), will yields

$$P_o\sin\Omega t\,\delta(x-x_p)-(e_oa)\frac{\partial^2 P_o\sin\Omega t}{\partial x^2}=EI\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^4 q_i(t)\right)-EA\alpha_x T\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^2 q_i(t)\right)+m_f v^2\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^2 q_i(t)\right)$$
$$+2m_f v\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)\dot{q}_i(t)\right)-(e_0a)^2\left(\begin{array}{l}(m_c+m_f+\rho A)\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^2\ddot{q}_i(t)\right)+EA\alpha_x T\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^4 q_i(t)\right)+\rho A\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^2 q_i(t)\right)\\ +m_f v^2\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^4 q_i(t)\right)+2m_f v\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\left(\frac{\pi}{l}\right)^2\ddot{q}_i(t)\right)+m_f\left(\sum_{i=1}^{\infty}\sin\left(\frac{i\pi x}{l}\right)\ddot{q}_i(t)\right)\end{array}\right) \tag{23}$$

where prime denotes the derivative with respect to *x*, by taking the integrating on both sides $\Phi_i(x)$ into above equation can be written as:

$$\int_0^l P(x,t)-(e_0a)^2\frac{\partial^2 P(x,t)}{\partial X^2}\phi_i(x)dx=\sum_{i=1}^{\infty}qi(t)\int_0^l EI\,\ddot{\phi}_i\,\phi_j(x)dx-\sum_{i=1}^{\infty}qi(t)\int_0^l EA\alpha_x T\,\dot{\phi}_i\,\phi_j(x)dx$$
$$+\sum_{i=1}^{\infty}qi(t)\int_0^l M_f V^2\,\dot{\phi}_i\,\phi_j(x)dx+\sum_{i=1}^{\infty}qi(t)\int_0^l 2m_f v\,\dot{\phi}_i\,\phi_j(x)dx$$
$$-(e_0a)^2\left[\begin{array}{l}\sum_{i=1}^{\infty}\dot{q}_i(t)\int_0^l(mc+mf+\rho A)\dot{\phi}_i\,\phi_j(x)dx+\sum_{i=1}^{\infty}q_i(t)\int_0^l EA\alpha_x T\,\ddot{\phi}_i\,\phi_j(x)dx\\ +\sum_{i=1}^{\infty}q_i(t)\int_0^l\rho A\,\dot{\phi}_i\,\phi_j(x)dx+\sum_{i=1}^{\infty}q_i(t)\int_0^l M_f V^2\,\dot{\phi}_i\,\phi_j(x)dx+\sum_{i=1}^{\infty}\dot{q}_i(t)\int_0^l 2m_f v\,\dot{\phi}_i\,\phi_j(x)dx\\ +\sum_{i=1}^{\infty}\ddot{q}_i(t)\int_0^l m_f\,\phi_i\phi_j(x)dx\end{array}\right] \tag{24}$$

In the present Equation by considering the following orthogonally conditions

$$\int_0^l \Phi_i \Phi_j(x)dx = \begin{cases} \dfrac{l}{2} & i=j \\ 0 & i \neq j \end{cases} \tag{25}$$

and using the following general property of Dirac delta function (Fryba [26]) for the load terms on the right- hand side

$$\int_{x_1}^{x_2} \Gamma(x)\delta^{(n)}(x-x_0)dx = \begin{cases} (-1)^n g^{(n)}(x_0) & if \quad x_1 < x_0 < x_2 = j \\ 0 & otherwise \end{cases} \tag{26}$$

The n^{th} derivative of Direc-Delta function is represented as $\delta^n(x-x_0)$ and also the following relations are considered

$$k_j = \int_0^l EI\left(\frac{i\pi}{l}\right)^4 \Phi_i\Phi_j(x)dx \tag{27a}$$

$$\alpha_j = \int_0^l \left(mfv^2 + EA\alpha_x T\right)\left(\frac{i\pi}{l}\right)^2 \Phi_i\Phi_j(x)dx \tag{27b}$$

$$M_j = \int_0^l \rho A((e_o a)^2 - 1 + m_f + m_c)\left(\frac{i\pi}{l}\right)^2 \Phi_i\Phi_j(x)dx \tag{27c}$$

$$\beta_j = \int_0^l (e_o a)^2\left(N_t + m_f v^2\right)\left(\frac{i\pi}{l}\right)^4 \Phi_i\Phi_j(x)dx - (e_o a)\int_0^l (\rho A)\left(\frac{i\pi}{l}\right)^2 \Phi_i\Phi_j(x)dx \tag{27d}$$

$$D_j = (e_o a)^2\int_0^l (2m_f v + m_f)\left(\frac{i\pi}{l}\right)^4 \Phi_i\Phi_j(x)dx - (e_o a)\int_0^l (\rho A)\left(\frac{i\pi}{l}\right)^2 \Phi_i\Phi_j(x)dx \tag{27e}$$

$$f_i(t) = p(x,t)\int_0^l \left[\delta(x-x_p) - (e_o a)^2 \frac{\partial^2(x-x_p)}{\partial x^2} - Kw\right]\phi_i d(x) \tag{27f}$$

which will develop the following differential equation of the i^{th} mode of the generalized deflection

$$\ddot{q}_j(t) + \left[\omega_j^2 + \zeta_j^2 + \psi_j^2\right]\dot{q}_j(t) + Z_j^2 q_j(t) = \frac{1}{M_j} f_i(t) \quad i,\ j = 1,2,3,4...... \tag{28}$$

where $\varsigma_i = \sqrt{\dfrac{D_i}{M_j}}$, $\omega_j = \sqrt{\dfrac{k_i}{M_j}}$, $\psi_j = \sqrt{\dfrac{\alpha_i}{M_j}}$, $Z_i = \sqrt{\dfrac{\beta_i}{M_j}}$ and $f_i(t) = 0$ for $t > 0$ where $x_p(0 \le x_P = v_p)$ are the coordinate of the following moving harmonic load. Then by considering the following equation.

$$\frac{1}{M_j} f_i(t) = S_i(t)$$

where also by employing the following equation:

$$S_i(t)=\frac{2p(t)}{\rho AL}\sin\left(\frac{i\pi x_p t}{L}\right) \tag{29}$$

For the homogeneous initial conditions, Eq. (28) may be solved as follows according to Simsek [27]:

$$q_i(t)=\frac{1}{M_j}\int_0^t S_i(\lambda)\sin m_j(t-\lambda)d\lambda \tag{30}$$

Substituting Eq. (30) into Eq. (29) and applying integration leads to the following results:

$$q_j(t)=\frac{\left((\rho A+m_f+m_c)-\rho A(e_0a)^2\right)p_o}{\rho AL}\sum_{i=1}^{\infty}\left[\frac{\cos\left(\Omega t-\dfrac{i\pi x_p(t)}{L}\right)-\cos(M_j t)}{M_j^2-\left(\Omega t-\dfrac{i\pi x_p(t)}{L}\right)^2}-\frac{\cos\left(\Omega t+\dfrac{i\pi x_p(t)}{L}\right)-\cos(M_j t)}{M_j^2-\left(\Omega t+\dfrac{i\pi x_p(t)}{L}\right)^2}\right] \tag{31}$$

Substituting Eq. (31) into Eq. (21), finally, the total dynamic deflection can be achieved as follows:

$$w(x,t)=\frac{\left((\rho A+m_f+m_c)-\rho A(e_0a)^2\right)p_o}{\rho AL}\sum_{i=1}^{\infty}\left[\frac{\cos\left(\Omega t-\dfrac{i\pi x_p(t)}{L}\right)-\cos(Mt)}{M^2-\left(\Omega t-\dfrac{i\pi x_p(t)}{L}\right)^2}-\frac{\cos\left(\Omega t+\dfrac{i\pi x_p(t)}{L}\right)-\cos(Mt)}{M^2-\left(\Omega t+\dfrac{i\pi x_p(t)}{L}\right)^2}\right]\times\sin\left(\frac{i\pi x}{L}\right) \tag{32}$$

5 NUMERICAL RESULT AND DISCUSSION

In this section, the forced nonlocal wave propagation in an armchair thermo fluid conveying simply supported SWCNT under the impact of the moving harmonic load is investigated. The choice of effective wall thickness and the elastic modulus E is a series structural issue in the elastic waves of SWCNT. However, this issue has been recently addressed and resolved in Wang and Liew [28]. The Numerical values of the involved parameters is taken as $\rho=2300\,kg\,m^{-3}$, $t_p=0.35nm$. According to the calculation, the mass of fluid per unit length in the SWCNT is $1.52\times10^{-16}kg/m^2$ and the mass per unit length of the SWCNT is $2.75\times10^{-15}kg/m^2$.The thermal expansion coefficient in room temperature $\alpha_x=-1.5\times10^{-6}C^{-1}$ and $l=10nm$, $d=1nm$ .The non-dimensional velocity parameter of the moving harmonic load is represented as $\xi=\dfrac{v_p}{v_{cr}}$ the critical velocity of the SWCNT is $v_{cr}=\dfrac{\psi_1 L}{\pi}$ where ψ_1 is the fundamental frequency of the SWCNT. The influence of the excitation frequency of the moving harmonic load Ω is represented by the frequency ratio θ where $\theta=\Omega/\psi_1$ the dimensionless time parameter is defined by $t^*=x_p/L$.

Figs. 3-4 shows the variation dynamic displacement of the elastic SWCNT with harmonic load velocity for the values $N_t=0.2,0.5$ with different non-local constants $e_oa=0,0.5,1.0,1.5$ and armchair value (3,3). From these figures, it is observed that the dynamic deflection of the SWCNT is highly influence by the values of non-local parameters and has higher magnitude at the lower values of the velocity of the harmonic loads. Fig.4 reveals dispersion trend in the wave propagation due to the increasing of the thermal parameter values. A comparative illustration is made between the dynamic displacements of the elastic SWCNT with harmonic load velocity for the values $N_t = 0.2, 0.5$ with different non local constants $e_o a = 0, 0.5, 1.0, 1.5$ .respectively and shown in the Figs.5-6. From the Figs.5 and 6, it is clear that, at the lower range of harmonic velocity the dynamic displacement of SWCNT attain maximum value in both cases of $N_t = 0.2$ and $N_t = 0.5$, but there is deviation in elastic wave behaviour when armchair (10,10) in Figs.5 and 6. This may happen due to the effect of increase in thermal effect and chirality value.

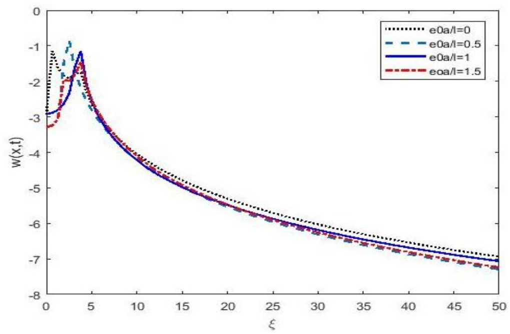

Fig.3
Variation dynamic displacement versus harmonic load velocity with $N_T = 0.2$ and armchair (3,3).

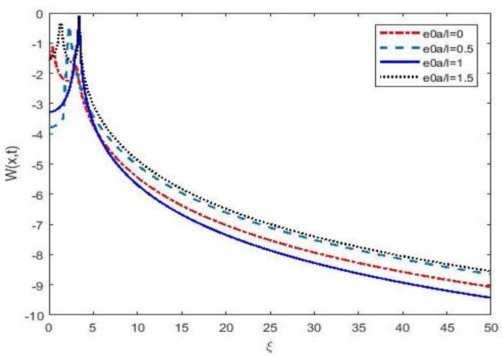

Fig.4
Variation dynamic displacement versus harmonic load velocity with $N_T = 0.5$ and armchair (3,3).

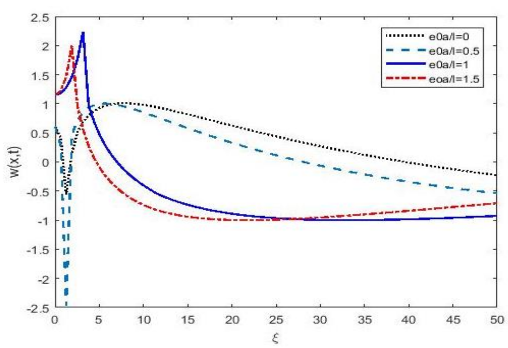

Fig.5
Variation dynamic displacement versus harmonic load velocity with $N_T = 0.2$ and armchair (10,10).

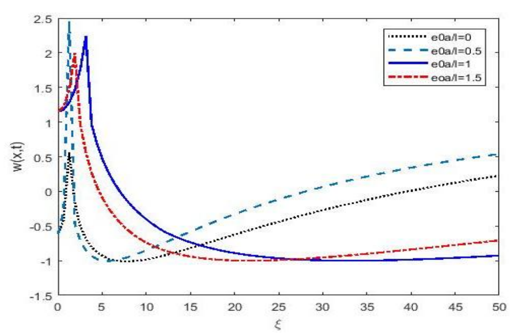

Fig.6
Variation dynamic displacement versus harmonic load velocity with $N_T = 0.5$ and armchair (10,10).

Figs. 7-8 investigate the variation dynamic displacement of the elastic SWCNT with the frequency ratio θ for the values $e_0a = 0.2, 0.5$ and armchair (3,3) with different thermal parameter values $N_t = 0.5, 1.0, 1.5, 2.0$. From these figures, it seen that the dynamic deflection of the SWCNT is varying significantly by the varying values of thermal parameter. Also the dynamic displacement increases as the frequency ratio increases and starts to decrease at $\theta = 35$ .Fig.8 presents the wave propagation nature in the trend line due to the increasing values of the non local parameter. Figs. 9 and 10 illustrate the comparison between the dynamic displacement of the elastic SWCNT versus the frequency ratio for the values $e_0a = 0.2, 0.5$ and armchair (10,10) with the varying thermal parameter values $N_t = 0.5, 1.0, 1.5, 2.0$ .respectively. From the Fig.9, it is clear that, the dynamic displacement is getting peak values in the frequency ratio range $5 \le \theta \le 15$,but in Fig.10 it attains peak values at $\theta = 35$ due to the increase in non-local effect and magnetic field value of fluid conveying SWCNT.

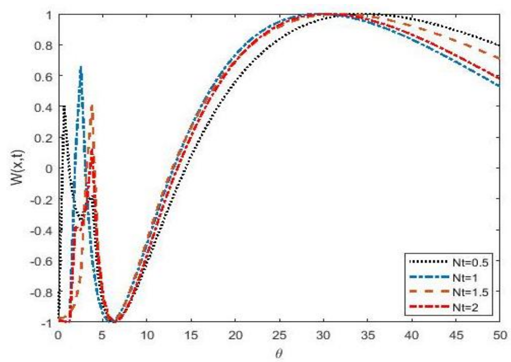

Fig.7
Variation dynamic displacement versus frequency ratio with $e_0a = 0.2$ and armchair (3,3).

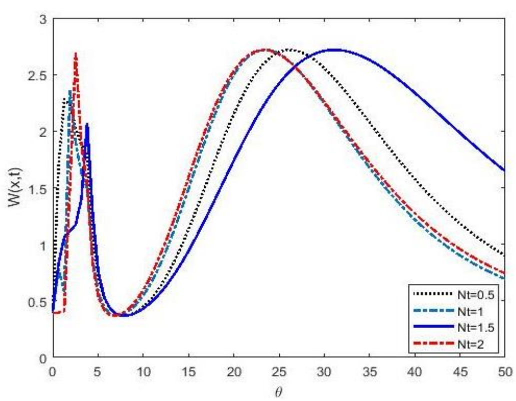

Fig.8
Variation dynamic displacement versus frequency ratio with with $e_0a = 0.5$ and armchair (3,3).

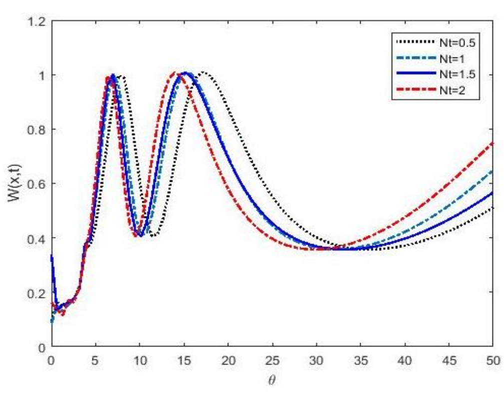

Fig.9
Variation dynamic displacement versus frequency ratio with $e_0a = 0.2$ and armchair (10,10).

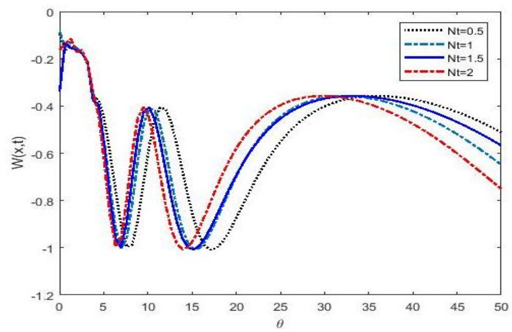

Fig.10
Variation dynamic displacement versus frequency ratio with $e_0a = 0.5$ and armchair (10,10).

Figs.11-12 discusses the dispersion curves for the dynamic displacement versus mode *n* with $e_0a = 0.2, 0.5$ of fluid conveying elastic SWCNT for the armchair values (3,3) and (10,10). From the Figs.11 and 12, it is observed that the dynamic displacement is propagating from the starting values of the mode number and varying steadily in the higher values as the mode values increases. The armchair with higher temperature is rendering lower displacement due to the variation of the diameter of the tube. Figs.13-14 explains the variation of the dynamic displacement over the aspect ratio (*L/d*) with $e_0a = 0.2, 0.5$ of fluid conveying elastic SWCNT for the armchair values (3,3) and (10,10). According to this figures, at the lower level of aspect ratio the effect of displacement gets maximum and decreases in the higher aspect ratio as the non-local parameter and the armchair value increases.

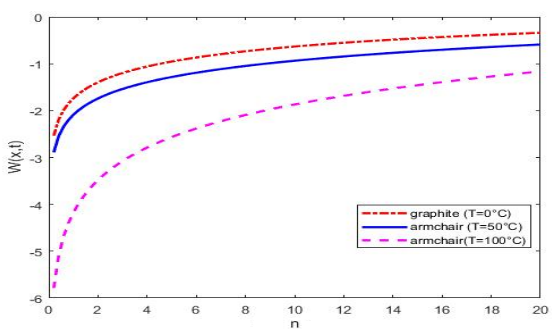

Fig.11
Variation dynamic displacement versus mode with $e_0a = 0.2$ and armchair (3,3).

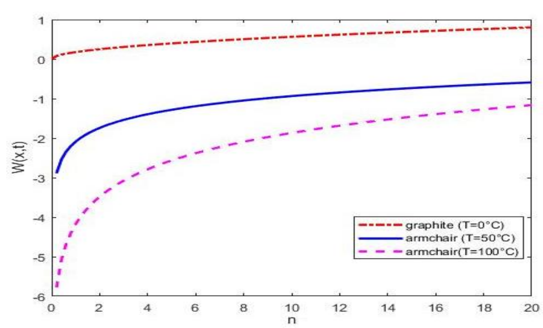

Fig.12
Variation dynamic displacement versus mode with $e_0a = 0.5$ and armchair (10,10).

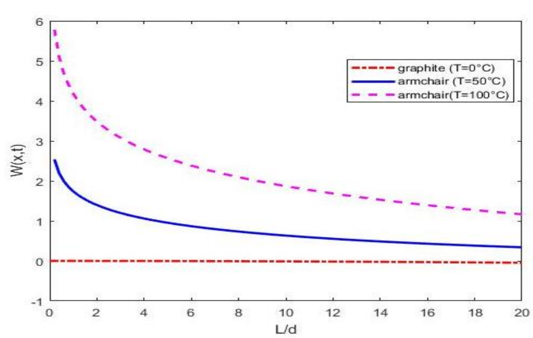

Fig.13
Variation dynamic displacement versus aspect ratio with $e_0a = 0.2$ and armchair (3,3).

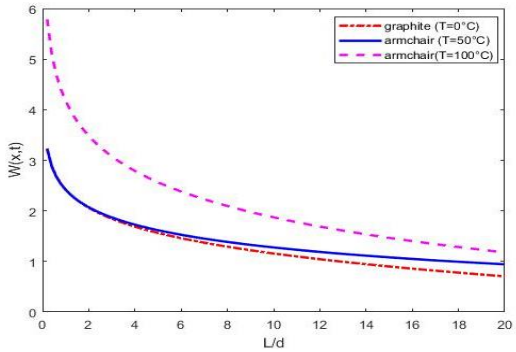

Fig.14
Variation dynamic displacement versus aspect ratio with $e_0a = 0.5$ and armchair (10,10).

The 3D curves in Figs 15-18, clarifies the variation the dynamic displacement against the time and velocity ratio with and without fluid force for the values of thermal constant and non-local parameter. These curves explain the dependence of dynamic displacement on the different physical parameter t^*, ξ, N_t, F_p with increasing non-local parameter and armchair values.

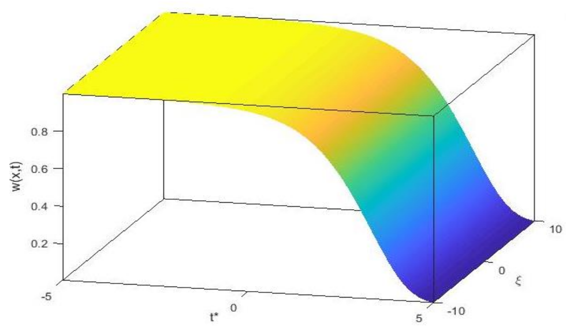

Fig.15
3D Distribution of dynamic displacement with t^* and ξ for $e_0a = 0.5$, $N_t = 0, F_p = 0$ with armchair (3,3).

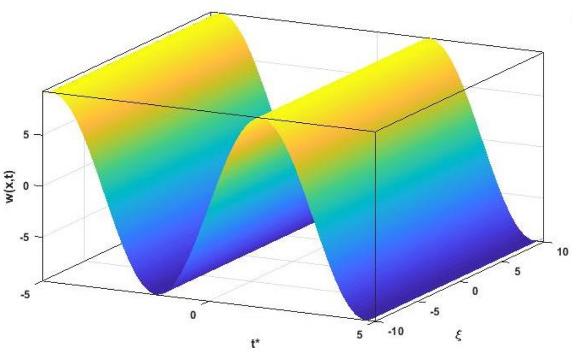

Fig.16
3D Distribution of dynamic displacement with t^* and ξ for $e_0a = 0.5$, $N_t = 0.5, F_p = 0$ with armchair (10,10).

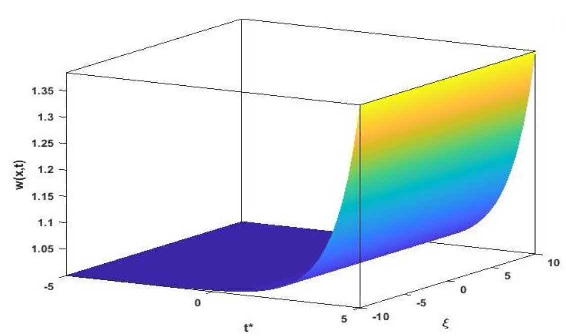

Fig.17
3D Distribution of dynamic displacement with t^* and ξ for $e_0a = 0.5$, $N_t = 0.5, F_p = 0.5$ with armchair (3,3).

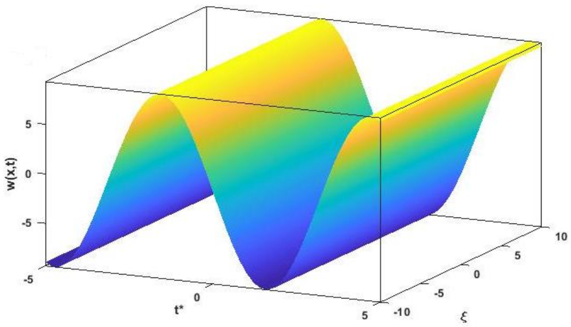

Fig.18
3D Distribution of dynamic displacement with t^* and ξ for $e_0a = 0.5$, $N_t = 0.5, F_p = 0.5$ with armchair (10,10).

6 CONCLUSIONS

This study provides an analytical model to obtain the dynamic displacement of SWCNT due to the harmonic load velocity, excitation frequency, non-dimensional time parameter, temperature vector and nonlocal constants using Eringen's nonlocal elasticity theory in the context of Euler Bernoulli beam equation. The time domain responses are obtained by using both modal super position method and Newmarks's direct integration method. From the results indicated, we can conclude that:

- The values of mentioned physical quantities are coverages to zero for lower values of mechanical loads and other parameter, and all functions are continuous.
- The amplitude of all dynamic displacement increases and decreases when the value of harmonic load velocity and excitation frequency increases.
- The values of dynamic displacement is increases and in wave propagation trend when the non-dimensional time parameter increases.
- The non-local scale effect, chirality and temperature field vectors have an important role on the distribution of dynamic displacement.
- The effect of fluid environment also influences the dynamic displacement as the load velocity and non-dimensional time increases. Deformation of a nano material depends on the type of armchair as well as the chirality vectors.

REFERENCES

[1] Lijima S., 1991, Synthesis of carbon nanotubes, *Nature* **354**: 56-58.
[2] Eringen A.C., 1983, On differential equation of nonlocal elasticity and solutions of screw dislocation and surface waves, *Journal of Applied Physics* **54**: 4703-4710.
[3] Peddieson T., Buchanan G.R., McNitt R.P., 2003, Application of nonlocal continuum model of technology, *International Journal of Engineering Sciences* **41**: 305-312.
[4] Lu P., Lee L.P., Lu C., Zhang P.Q., 2006, Dynamic properties of flexural beams using nonlocal elasticity model , *Journal of Applied Physics* **99**: 073510-9.
[5] Aydogdu M., Arda M., 2016, Torsional vibration analysis of double walled carbon nanotubes using nonlocal elasticity, *International Journal of Mechanical Designs* **12**(1): 71-84.
[6] Civalek O., Demir C., Akgoz B., 2009, Static analysis of single walled carbon nanotubes based on eringen's nonlocal elasticity theory, *Journal of Sound Vibrations* **2**: 47-50.
[7] Yamabe T., 1995, Recent development of carbon nanotube, *Synthetic Metals* **70**: 1511-1518.
[8] Wu Y., Zhang Y., Leung A.Y., Zhong W., 2006, En energy- equivalent model on studying the mechanical properties of single walled carbon nanotubes, *Thin- Walled Structures* **44**: 667-676.
[9] Baghdadi H., Tounsi A., Zidour A., Benzairi A., 2014, Thermal effect on vibration charecterstics of armchair and zigzag single walled carbon nanotubes using nonlocal parabolic beam theory, *Nanotubes and Carbon Nanostructures* **23**: 266-272.
[10] Naceri M., Zidour M., Semmah A., Houari M.S.A., Benzair A., Tounsi A., 2011, Sound wave propagation in armchair single walled carbon nanotubes under thermal environment, *Journal of Applied Physics* **110**: 124322-7.
[11] Bedia W.A., Benzair A., Semmah A., Tounsi A., Mahmoud S.R., 2015, On the thermal buckling characteristics of armchair single-walled carbon nanotube embedded in an elastic medium based on nonlocal continuum elasticity, *Journal of Physics* **45**(2): 225-233.

[12] Murmu T., Pradhan S.C., 2010, Thermal effect on stability of embedded carbon nanotubes, *Computational Materials Science* **47**: 721-726.

[13] Narender S., Gopalakrishnan S., 2012, Temperature effects on wave propagation in nanoplates, *Composites Part -B Engineering* **43**: 1275-1281.

[14] Ebrahimi F., Mahmoodi F., 2018, Vibration analysis of carbon nanotubes with multiple cracks in thermal environment, *Advances in Nano Research* **6**(1): 2287-2388.

[15] Ebrahimi F., Boreiry M., Shaghaghi G.R., 2018, Nonlinear vibration analysis of electro – hygro - thermally actuated embedded nanobeams with various with various boundary conditions, *Microsystem Technologies* **24**(12): 5037-5054.

[16] Yoon J., Ru C.Q., Mioduchowski A., 2005, Vibration and instability of carbon nanotubes conveying fluid, *Composites Science and Technologies* **65**: 1326-1336.

[17] Yoon J., Ru C.Q., Mioduchowski A., 2006, Flow –induced flutter instability of cantilever carbon nanotubes, *International Journal of Solid Structures* **43**: 3337-3349.

[18] Zhang Y.Q., Liu X., Liu G.R., 2007, Thermal effect on transverse vibration of double walled carbon nanotube, *Nanotechnology* **18**: 445701-7.

[19] Chang T.P., 2012, Thermal- mechanical vibration and instability of a fluid conveying single walled carbon nanotube embedded in a n elastic medium based on nonlocal elasticity theory, *Applied of Mathematical Modelling* **36**: 1964-1973.

[20] Wang L., 2010, Wave propagation of fluid conveying single walled carbon nanotubes via gradient elasticity theory, *Computational Materials Sciences* **49**: 761-766.

[21] Mohammad H., Goughari S., 2016, Vibration and instability analysis of nanotubes conveying fluid subjected to a longitudinal magnetic field, *Applied Mathematical Modelling* **40**: 2560-2576.

[22] Kiani K., Mehri B., 2010, Assessment of nanotubes structures under a moving nanoparticle using nonlocal beam theories, *Journal of Sound Vibrations* **329**(11): 2241 - 2264.

[23] Simsek M., Kocaturk T., 2009, Nonlinear dynamic analysis of an eccentrically prestressed damped beam under a concentrated moving harmonic load, *Journal of Sound Vibrations* **320**: 235-253.

[24] Simsek M., Kocaturk T., 2009, Free and forced vibration of functionally graded beam subjected to concentrated moving harmonic load, *Composites Structures* **90**: 465-473.

[25] Simsek M., 2010, Vibration of a single walled carbon nanotube under action of a moving harmonic load based on nonlocal elasticity theory, *Physica E: Low -Dimensional Systems and Nanostructures* **43**(1): 182-191.

[26] Fryba L., 1972, *Vibration of Solids and Structures under Moving Loads*, Noordhoff International, Groningen, The Netherlands.

[27] Simsek M., 2011, Nonlocal effect in the forced vibration of an elastically connected double- carbon nanotube system under a moving nanoparticle, *Composites Materials Sciences* **50**: 2112-2123.

[28] Wang Q., Liew K.M., 2007, Application of nonlocal continuum mechanics to static analysis of micro and nano structure, *Physics Letters A* **363**(3): 236-242.