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Effects of Viscosity on a Thick Circular Plate in Thermoelastic Diffusion Medium

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ABSTRACT

The problem treated here is to determine the viscosity effect on stresses, temperature change and chemical potential in a circular plate. The mathematical formulation is applied to two theories of thermoelastic diffusion developed by Sherief et al. [27] with one relaxation time and Kumar and Kansal [9] with two relaxation times. Laplace and Hankel transform techniques are used to obtain the expression for the displacement components, stresses, temperature change and chemical potential. The resulting quantities are computed numerically and depicted graphically by using numerical inversion technique for a particular model. Effect of viscosity is shown in the normal stress, tangential stress, temperature change and chemical potential. Some particular cases of interest are also deduced. Viscoelastic materials play an important role in many branches of engineering, technology and, in recent years, biomechanics. Viscoelastic materials, such as amorphous polymers, semi crystalline polymers, and biopolymers, can be modelled in order to determine their stress or strain interactions as well as their temporal dependencies.

Keywords : Visco thermoelastic; Thick circular plate; Laplace and Hankel transforms; Viscosity.

1 INTRODUCTION

THE coupling between the thermal and strain fields gives rise to the coupled theory of thermoelasticity. For static problems, this coupling vanishes and the thermal field becomes independent of the strain field. The coupling between the strain and temperature fields firstly studied by Duhamel [3] who derived equations for the distribution of strains in an elastic medium subjected to temperature gradients. Neumann [18] and several authors worked on Duhamel's theory and solved a number of interesting problems based on this theory. Boit [1] derived the coupled theory of thermoelasticity, which includes the dilatational term based on the thermodynamics of irreversible processes. The extended form of thermoelasticity with one relaxation time was introduced by Lord and Shulman [13] and with two relaxation times, was developed by Green and Lindsay [8]. Viscoelasticity is the property of materials that exhibit both viscous and elastic characteristics when undergoing deformation. Viscoelastic materials play an important role in many branches of civil engineering, geotechnical engineering, technology and biomechanics. Viscoelastic materials, such as amorphous polymers, semi crystalline polymers and biopolymers.

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Kumar and Chawla [10] showed the effect of viscosity on wave propagation in anisotropic thermoelastic medium with three-phase-lag model. Sharma and Kumar [25] investigated the problem of propagation of plane waves and fundamental solution with voids by using thermos-visco-elastic medium. Kumar et al. [11] studied reflection of plane waves in transversely isotropic micro polar visco-thermoelastic solid in the context of Lord and Shulman (L-S), Green and Lindsay (G-L) and Coupled Thermoelastic (C-T) models. In addition, fundamental solution to a system of differential equations in micro polar visco-thermo-elastic solids with voids was studied by Kumar et al. [12]. Svanadze [29] studied the problem of plane waves and steady vibrations in the theory of viscoelasticity for Kelvin-Voigt materials with double porosity. Sharma et al. [26] investigated the effect of magnetic field on transient wave in visco-thermo-elastic half space by using Laplace and Fourier transform techniques.

Thermo diffusion in an elastic solid is due to the coupling of the fields of temperature, mass diffusion and strain. Heat and mass exchange with the environment during the process of the thermos diffusion in an elastic solid. The concept of thermos diffusion is used to describe the processes of thermomechanical treatment of metals (carboning, nitriding steel, etc.) and these processes are thermally activated, and their diffusing substances being, e.g. nitrogen, carbon etc. They are accompanied by deformations of the solid. Podstrigach[24], Nowacki [19-20-21-22] developed the theory of thermoelastic with mass diffusion. In this theory, the coupled thermoelastic model is used. This implies infinite speeds of propagation of thermoelastic waves. Sherief and Saleh [27] developed the theory of generalized thermoelastic diffusion that predicts finite speeds of propagation for thermoelastic and diffusive waves. Ezzat et al. [5-6] studied the problem of generalized thermos-visco-elasticity with one and two relaxation times by using state space approach and Laplace transform technique. Othman et al. [23] discussed the two-dimensional problem of thermos-visco-elasticity with two relaxation times. They obtained the expressions for temperature distribution, thermal stresses and displacement components. Comparisons are made within the coupled theory and generalized G-L theory. Ezzat [7] constructed the equations of generalized thermos-visco-elasticity for a conducting isotropic media in the presence of a constant magnetic field within the context of one relaxation time [Lord-Shulman (L-S)] and two relaxation times [Green-Lindsay (G-L)]. He applied state space approach for the solution of one–dimensional problems in the absence or presence of heat sources and used Laplace transform technique. Sherief and Saleh [28] worked on a problem of a thermoelastic half space with a permeating substance in contact with the bounding plane in the context of the theory of generalized thermoelastic diffusion with one relaxation time. Kumar and Kansal [9] studied generalized thermoelastic diffusion for Green Lindsay (GL-model) theory and discussed the Lamb waves. Marin [14] constructed the existence and uniqueness theorems of the generalized solutions for the boundary value problems in elasticity of initially stressed bodies with porous materials. Some basic theorems for micro-stretch thermoelastic materials using the Lagrange identity was studied by Marin [15]. Maghraby and Halim [4] used Laplace and Hankel transforms technique to solve the problem of generalized thermoelasticity in Lord-Shulman theory [13] a half space subjected to a known axisymmetric temperature distributions. Marin and Stan [16] studied the weak solutions in elasticity of dipolar bodies with stretch. Tripathi et al. [30] investigated the temperature distribution and thermal stresses in a semi-infinite cylinder with heat sources in thermoelastic theory with one relaxation time. Thick circular plate with axisymmetric heat supply in a generalized thermoelastic diffusion by using integral transform technique was discussed by Tripathi et al. [31].

The purpose of this paper is to study the problem of thick circular plate in a viscothermoelastic diffusion medium by using Laplace and Hankel transform techniques. The generalized theories of thermoelastic diffusion developed by Sherief and Saleh [27] and Kumar and Kansal [9] are used to investigate the problem.The normal stress, tangential stress, temperature change and chemical potential are computed and presented graphically for different values of radial distance. Some particular cases are also derived from the present investigation. The results presented here will be useful in engineering problems related to visco-thermo-elastic diffusion in isotropic elastic solids.

2 BASIC EQUATIONS

Following [Sherief et al. [27], Kumar and Kansal [9], Kumar et al. [11]], the constitutive relations and the equations of motion in a generalized thermoelastic with mass diffusion in the absence of body forces, body couples, heat and mass diffusion sources are given by

(i) Constitutive relations

$$t_{ij,j} = \rho \ddot{u}_i , \tag{1}$$

$$t_{ij} = \lambda_I e_{kk}\delta_{ij} + 2\mu_I e_{ij} - \beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)T\delta_{ij} - \beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)C\delta_{ij}, \tag{2}$$

$$P = -\beta_2 e_{kk} - a\left(1+\tau_1\frac{\partial}{\partial t}\right)T + b\left(1+\tau^1\frac{\partial}{\partial t}\right)C, \quad i, j, k = 1, 2, 3. \tag{3}$$

where t_{ij} are the components of stress tensor, δ_{ij} is Kronecker's delta, e_{ij} are the components of strain tensor. The parameters λ_I and μ_I are defined as:

$$\lambda_I = \lambda\left(1+\frac{\lambda^*}{\lambda}\frac{\partial}{\partial t}\right), \quad \mu_I = \mu\left(1+\frac{\mu^*}{\mu}\frac{\partial}{\partial t}\right), \tag{4}$$

where $\beta_1 = (3\lambda_I + 2\mu_I)\alpha_t$, $\beta_2 = (3\lambda_I + 2\mu_I)\alpha_c$; λ and μ are material constants, $Q_1 = \frac{\lambda^*}{\lambda}, Q_2 = \frac{\mu^*}{\mu}$ are visco-thermo-elastic relaxation times, Here α_t , α_c are the coefficients of linear thermal expansion and diffusion expansion respectively, T is the temperature change, C is the mass concentration, P is the chemical potential of the material per unit mass, a is the coefficient describing the measure of thermoelastic diffusion, b is the coefficient describing the measure of mass diffusion effects.

(ii) Equations of motion

$$(\lambda_I + \mu_I)\nabla(\nabla.\boldsymbol{u}) + \mu_I\nabla^2\boldsymbol{u} - \beta_1\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla T - \beta_2\left(1+\tau^1\frac{\partial}{\partial t}\right)\nabla C = \rho\ddot{\boldsymbol{u}}, \tag{5}$$

where $\boldsymbol{u} = (u_1, u_2, u_3)$ is the components of displacement vector, ρ is the density, Δ is the Laplacian operator, ∇ is del operator.

(iii) Equation of heat conduction

$$K\Delta T - \rho c_e\left(\frac{\partial}{\partial t} + \tau_0\frac{\partial^2}{\partial t^2}\right)T - aT_0\left(\frac{\partial}{\partial t} + \gamma\frac{\partial^2}{\partial t^2}\right)C = T_0\beta_1\left(\frac{\partial}{\partial t} + \tau_0\eta_0\frac{\partial^2}{\partial t^2}\right)(\nabla.\boldsymbol{u}), \tag{6}$$

where K is the coefficient of thermal conductivity, c_e is the specific heat at constant strain, T_0 is the reference temperature assumed to be such that $T/T_0 << 1$.

(iv) Equation of mass diffusion

$$D\beta_2\Delta(\nabla\boldsymbol{u}) + Da\left(1+\tau_1\frac{\partial}{\partial t}\right)\Delta T + \left(\frac{\partial}{\partial t} + \tau^0\eta_0\frac{\partial^2}{\partial t^2}\right)C - Db\Delta\left(1+\tau^1\frac{\partial}{\partial t}\right)C = 0, \tag{7}$$

where D is the thermoelastic diffusion constant. Here τ^0, τ^1 are the diffusion relaxation times with $\tau^1 \geq \tau^0 \geq 0$ and τ_0, τ_1 are thermal relaxation times with $\tau_1 \geq \tau_0 \geq 0$ and here $\tau_1 = \tau^1 = 0$, $\eta_0 = 1, \gamma = \tau_0$, for Lord-Shulman (L-S) model and $\eta_0 = 0, \gamma = \tau^0$, for Green Lindsay (G-L) model.

3 FORMULATION OF THE PROBLEM

We consider a homogeneous, isotropic, generalized viscothermoelastic diffusion thick plate of thickness $2d$ defined by $0 \leq r \leq \infty$, $-d \leq z \leq d$. The initial temperature in the thick plate is given by a constant temperature T_0 and the heat flux $g_0 F(r,z)$ is prescribed on the upper and lower boundary surfaces. The cylindrical polar coordinates (r,ϕ,z) having origin on the surface $z=0$, between the lower and upper surfaces of the plate and the *z*-axis is assumed the axis of symmetry. Due to symmetry about *z*-axis, all the field quantities are independent of the coordinate θ.

For the two-dimensional problem, we take the displacement components, temperature change and mass concentration as:

$$\boldsymbol{u} = \big(u_r(r,z,t), 0, u_z(r,z,t)\big), T(r,z,t) \ \ and \ \ C(r,z,t) \tag{8}$$

We introduce the dimensionless quantities

$$\begin{gathered}(r',z') = \frac{\omega^*}{c_1}(r,z),\ (u_r',u_z') = \frac{\omega^*}{c_1}(u_r,u_z),\ t_{ij}' = \frac{t_{ij}}{\beta_1 T_0}, m_{ij}' = \frac{\omega^* m_{ij}}{c_1\beta_1 T_0},\ (T',C') = \frac{(\beta_1 T,\beta_2 C)}{\rho c_1^2}, P' = \frac{P}{\beta_2},\\ (t',\gamma',\tau_1',\tau_0',\tau^{0'},\tau^{1'},Q_i') = \omega^*(t,\gamma,\tau_1,\tau_0,\tau^0,\tau^1,Q_i),\ c_1^2 = \frac{\lambda+2\mu}{\rho},\ \omega^{*2} = \frac{\lambda}{(\mu t^2+\rho\alpha)}.\end{gathered} \tag{9}$$

where ω^* and c_1 are characteristic frequency and longitudinal wave velocity in the medium.

Upon introducing (9) in Eqs. (5)- (7), with the aid of (4), (8) and after suppressing the primes, we obtain

$$b_1\frac{\partial e}{\partial r} + b_2\left(\nabla^2 - \frac{1}{r^2}\right)u_r - \tau_{tt}\frac{\partial T}{\partial r} - \tau_{tt}^1\frac{\partial C}{\partial r} = \frac{\partial^2 u_r}{\partial t^2}, \tag{10}$$

$$b_1\frac{\partial e}{\partial z} + b_2\nabla^2 u_z - \tau_{tt}\frac{\partial T}{\partial z} - \tau_{tt}^1\frac{\partial C}{\partial z} = \frac{\partial^2 u_z}{\partial t^2}, \tag{11}$$

$$\nabla^2 T - b_3\tau_{tt}^0 T - b_4\tau_{t\gamma}^0 C = b_5\tau_\eta^0 e, \tag{12}$$

$$b_6\nabla^2 e + b_7\tau_{tt}\nabla^2 T + \left(\tau_{tt}^{20} - b_8\tau_{tt}^1\nabla^2\right)C = 0, \tag{13}$$

where

$$\begin{gathered}b_1 = \frac{(\lambda_I+\mu_I)}{\rho c_1^2}, b_2 = \frac{\mu_I}{\rho c_1^2}, b_3 = \frac{\rho c_e c_1^2}{K\omega^*}, b_4 = \frac{aT_0\beta_1 c_1^2}{\beta_2 K\omega^*}, b_5 = \frac{T_0\beta_1^2}{\rho K\omega^*}, b_6 = \frac{\beta_2^2 D\omega^*}{\rho c_1^4}, b_7 = \frac{a\beta_2 D\omega^*}{\beta_1 c_1^2},\\ b_8 = \frac{bD\omega^*}{c_1^2}, \tau_{tt} = \left(1+\tau_1\frac{\partial}{\partial t}\right), \tau_{tt}^1 = \left(1+\tau^1\frac{\partial}{\partial t}\right), \tau_{tt}^0 = \left(\frac{\partial}{\partial t}+\tau_0\frac{\partial^2}{\partial t^2}\right), \tau_\eta^0 = \left(\frac{\partial}{\partial t}+\eta_0\tau_0\frac{\partial^2}{\partial t^2}\right),\\ \tau_{t\gamma}^0 = \left(\frac{\partial}{\partial t}+\gamma\frac{\partial^2}{\partial t^2}\right),\ \tau_{tt}^{20} = \left(\frac{\partial}{\partial t}+\eta_0\tau^0\frac{\partial^2}{\partial t^2}\right), \nabla^2 = \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{\partial^2}{\partial z^2}, e = \frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{\partial u_z}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}(ru_r)+\frac{\partial u_z}{\partial z}.\end{gathered}$$

Following Mukhopadhyay and Kumar [17], the displacement components u_r and u_z in terms of potential functions ϕ and ψ in the dimensionless form are given by

$$u_r = \frac{\partial\phi}{\partial r} + \frac{\partial^2\psi}{\partial r\partial z}, \quad u_z = \frac{\partial\phi}{\partial z} - \left(\frac{\partial^2\psi}{\partial r^2} + \frac{1}{r}\frac{\partial\psi}{\partial r}\right). \tag{14}$$

We define Laplace and Hankel transforms as:

$$\bar{f}\left(r,z,p\right)=\int_0^\infty f\left(r,z,t\right)\,e^{-pt}dt$$
$$\hat{f}\left(\varsigma,z,p\right)=H\left[\bar{f}\left(r,z,p\right)\right]=\int_0^\infty \bar{f}\left(r,z,p\right)\,rJ_n\left(\eta r\right)dr, \tag{15}$$

where p is the Laplace transform parameter, ς is the Hankel transform parameter and $J_n(\,)$ is the Bessel function of the first kind of order n.

Making use of (14) in (10)-(13) and applying the Laplace and Hankel transforms defined by (15) on the resulting equation (after simplification), we obtain

$$\left(G_1D^6+G_2D^4+G_3D^2+G_4\right)\left(\hat{\phi},\hat{T},\hat{C}\right)=0, \tag{16}$$

$$\left(D^4-B_1D^2+B_2\right)\hat{\psi}=0, \tag{17}$$

where G_1, G_2, G_3, G_4, B_1 and B_2 are given in Appendix A.

The general solution of Eq. (16) can be written as:

$$\hat{\phi}=\hat{\phi}_1+\hat{\phi}_2+\hat{\phi}_3, \tag{18}$$

where $\hat{\phi}_i$ $(i=1,2,3)$ is a solution of the homogeneous differential equation given by

$$\left(D^2-\lambda_i^2\right)\hat{\phi}_i=0, i=1,2,3 \tag{19}$$

The solution of the Eq. (19) can be written as:

$$\hat{\phi}_i=\sum_{i=1}^{3}A_i\cosh\left(\lambda_i z\right), \tag{20}$$

where λ_1, λ_2 and λ_3 are the roots of the characteristic equation given by

$$G_1D^6+G_2D^4+G_3D^2+G_4=0.$$

The solution of the Eq. (16), with the aid of (20) in (18) can be written as:

$$\left(\hat{\phi},\hat{T},\hat{C}\right)\left(\varsigma,z,p\right)=\sum_{i=1}^{3}\left(1,a_i,d_i\right)\left(A_i\cosh(\lambda_i z)\right), \tag{21}$$

where

$$a_i=\sum_{i=1}^{3}\frac{b_5\tau_{tt}^{55}\left(\lambda_i^2-\varsigma^2\right)\left(\tau_t^{66}-b_8\left(\lambda_i^2-\varsigma^2\right)\tau_{tt}^{22}\right)-b_4b_6\tau_{tt}^{44}\left(\lambda_i^2-\varsigma^2\right)^2}{\left[\left(-\left(\lambda_i^2-\varsigma^2\right)+b_3\tau_{tt}^{33}\right)\left(\tau_{tt}^{66}-\left(\lambda_i^2-\varsigma^2\right)\left(b_8\tau_{tt}^{22}+b_4b_7\tau_{tt}^{11}\tau_{tt}^{44}\right)\right)\right]},$$

$$d_i = \sum_{i=1}^{3} \frac{\left(\lambda_i^2-\varsigma^2\right)^2\left[b_5 b_7 \tau_{tt}^{11}\tau_{tt}^{55}+b_6\left(\left(\lambda_i^2-\varsigma^2\right)-b_3\tau_{tt}^{33}\right)\right]}{\left[\left(-\left(\lambda_i^2-\varsigma^2\right)+b_3\tau_{tt}^{33}\right)\left(\tau_{tt}^{66}-\left(\lambda_i^2-\varsigma^2\right)\left(b_8\tau_{tt}^{22}+b_4 b_7\tau_{tt}^{11}\tau_{tt}^{44}\right)\right)\right]}, \quad i=1,2,3$$

Following the same procedure, we take the solution of (17) as:

$$\hat{\psi} = \sum_{i=4}^{5} A_i \sinh(\lambda_i z), \tag{22}$$

where λ_4 and λ_5 are the roots of the characteristic Eq. (17).

4 BOUNDARY CONDITIONS

$$\frac{\partial T}{\partial z} = \pm g_1 F\left(r,z\right) \quad \text{at} \quad z = \pm d, \tag{23}$$

$$t_{zz} = t_{zr} = 0 \ \text{at} \ \ z = \pm d, \tag{24}$$

$$P = \delta\left(t\right) f\left(r\right) \text{at} \ \ z = \pm d, \tag{25}$$

where

$$F\left(r,z\right) = z^2 e^{-\omega r}, \quad \omega > 0. \tag{26}$$

$$f\left(r\right) = H\left(a_0 - r\right) \tag{27}$$

In addition, g_1 is the constant temperature applied on the boundary.

The non-dimensional values of t_{zz}, t_{zr} and P are given by

$$t_{zz} = b_9 e + 2b_{10}\left(\frac{\partial u_z}{\partial z}\right) - b_{11}\left\{\left(1+\tau_1\frac{\partial}{\partial t}\right)T + \left(1+\tau^1\frac{\partial}{\partial t}\right)C\right\}, \tag{28}$$

$$t_{zr} = b_{10}\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right), \tag{29}$$

$$P = -e + b_{12}\tau_{tt}^{1} C - b_{13}\tau_{tt} T, \tag{30}$$

where

$$b_9 = \frac{\lambda_I}{\beta_1 T_0},\ b_{10} = \frac{\mu_I}{\beta_1 T_0},\ b_{11} = \frac{\rho c_1^2}{\beta_1 T_0},\ b_{12} = \frac{b\rho c_1^2}{\beta_2^2},\ b_{13} = \frac{a\rho c_1^2}{\beta_1\beta_2}.$$

Applying Laplace and Hankel transforms defined by (15) on Eqs. (26) and (27), we obtain

$$\hat{F}(\varsigma,z)=\frac{z^{2}\omega}{p\left(\omega^{2}+\varsigma^{2}\right)^{3/2}}, \tag{31}$$

$$\hat{f}(\varsigma)=\frac{a_0 J_1(\varsigma a_0)}{\varsigma}, \tag{32}$$

Applying Eq. (15) on the Eqs. (23)-(25) and substituting the values of $\hat{\phi}$, $\hat{T}$, $\hat{C}$ and $\hat{\psi}$ from Eqs. (21) and (22) in the resulting equations and with the aid of Eqs. (14), (28)-(32), we obtain the expressions for displacements, normal stress, tangential stress, temperature change and chemical potential as:

$$\hat{u}_r=-\varsigma\left[\sum_{i=1}^{3}A_i\cosh(\lambda_i d)+\sum_{i=4}^{5}A_i\sinh(\lambda_i d)\right], \tag{33}$$

$$\hat{u}_z=\left[\sum_{i=1}^{3}\lambda_i A_i\sinh(\lambda_i d)+\varsigma^{2}\sum_{i=4}^{5}A_i\cosh(\lambda_i d)\right], \tag{34}$$

$$\hat{T}=\sum_{i=1}^{3}b_i A_i\cosh(\lambda_i d), \tag{35}$$

$$\hat{t}_{zz}=\left[\sum_{i=1}^{3}M_i A_i\cosh(\lambda_i d)+\sum_{i=4}^{5}M_i A_i\cosh(\lambda_i d)\right], \tag{36}$$

$$\hat{t}_{zr}=-\varsigma\left[\sum_{i=1}^{3}N_i A_i\sinh(\lambda_i d)+\sum_{i=4}^{5}N_i A_i\sinh(\lambda_i d)\right], \tag{37}$$

$$\hat{P}=\left[\sum_{i=1}^{3}K_i A_i\cosh(\lambda_i d)+\sum_{i=4}^{5}K_i A_i\cosh(\lambda_i d)\right], \tag{38}$$

$$\hat{C}=\sum_{i=1}^{3}d_i A_i\cosh(\lambda_i d), \tag{39}$$

where A_1, A_2, A_3, A_4 and A_5, $\sum_{i=1}^{5}M_i$, $\sum_{i=1}^{5}N_i$, $\sum_{i=1}^{5}K_i$ are given in Appendix B.

5 PARTICULAR CASES

In the absence of diffusion $\left(a=D=\tau_{tt}^{1}=0\right)$, in Eqs. (33)-(39), we obtain the components of displacement, stresses and temperature change in a visco-thermo-elastic medium.

In the absence of viscosity $\left(Q_1=Q_2=0\right)$, in Eqs. (33)-(39), we obtain the components of displacement, stresses, temperature change and chemical potential in a thermoelastic diffusion medium. Our result in a special case are similar with those obtained by Tripathi et al. [31].

If $\tau_1 = \tau^1 = 0,\ \eta_0 = 1,\ \gamma = \tau_0$, in Eqs. (33)- (39), we obtain the corresponding results for viscothermoelastic diffusion for Lord Shulman (L-S) model.

If $\eta_0 = 0,\ \gamma = \tau^0$, in Eqs. (33)- (39), we obtain the corresponding results for visco-thermo-elastic diffusion for Green Lindsay (G-L) model.

6 NUMERICAL INVERSION OF THE TRANSFORMS

The solution of the problem is obtained in physical domain; we must invert the transforms in (33)-(39), for all the theories. Here the displacement components, normal and tangential stresses, temperature change, chemical potential and mass concentration are functions of *z*, the parameters of Laplace and Hankel transforms *p* and η respectively and hence are of the form $\hat{f}(\eta, z, p)$. We first invert the Hankel transform, which gives the Laplace transform expression $\bar{f}(r, z, p)$ of the function $f(r, z, t)$ as:

$$\bar{f}(r, z, p) = \int_0^\infty \eta \hat{f}(\eta, z, p) J_n(\eta r) d\eta \tag{40}$$

Now for fixed values of η, r and *z*, the function $\bar{f}(r, z, p)$ in (39) can be considered as the Laplace transform $\bar{g}(s)$ of the same function $g(t)$.

7 NUMERICAL RESULTS AND DISCUSSION

For numerical computations, following Daliwal and Singh [3], we take the magnesium material (thermoelastic diffusion solid) as:

$$\lambda = 2.696\times10^{10}\text{Kgm}^{-1}\text{s}^{-2},\ \mu = 1.639\times10^{10}\text{Kgm}^{-1}\text{s}^{-2},\ \rho = 1.74\times10^{3}\text{Kg m}^{-3},\ T_0 = 0.293\times10^{3}\text{K},$$
$$a = 1.02\times10^{4}\text{m}^2\text{s}^{-2}\text{K}^{-1},\ c_e = 1.04\times10^{3}\text{JKg}^{-1}\text{K}^{-1},\ \alpha_t = 1.78\times10^{-5}\text{K}^{-1},\ \alpha_c = 1.98\times10^{-4}\text{m}^3\text{Kg}^{-1},$$
$$D = 0.85\times10^{-8}\text{Kgm}^{-3}\text{sec},\ b = 9\times10^{5}\text{Kg}^{-1}\text{m}^5\text{s}^{-2},\ K = 1.7\times10^{2}\ \text{Wm}^{-1}\text{K}^{-1},\ \omega = 10s^{-1}, t = 1s,$$
$$d = 1,\ \tau_0 = 0.01s,\ \tau^0 = 0.03s,\ \tau_1 = 0.02s,\ \tau^1 = 0.04s.$$

MATLAB software has been used to determine the normal stress, tangential stress, temperature change and mass concentration for both L-S and G-L theories are computed numerically and shown graphically in Figs. 1-4 respectively. From Figs. 1-4, solid line $(-)$, corresponds to thermoelastic L-S model (TE) $(Q_1{=}Q_2{=}0)$, solid line with centre symbol $(-*-)$, corresponds to visco-thermo-elastic L-S model (VTE) $(Q_1{=}0.5, Q_2{=}1)$. Small dash line $(--)$, corresponds to thermoelastic G-L model (TE) $(Q_1{=}Q_2{=}0)$, small dash line with centre symbol $(--*--)$, corresponds to visco-thermo-elastic G-L model (VTE) $(Q_1{=}0.5, Q_2{=}1)$.

Fig.1 depicts the variations of normal stress t_{zz} with *r* . The behavior of normal stress with respect to *r* is same i.e. oscillatory for both L-S and G-L theories of thermoelastic and visco-thermo-elastic diffusion medium. On the other hand, the values of t_{zz} for G-L theory is higher in comparison to L-S theory for thermoelastic medium (TE) and opposite behavior is observed for visco-thermo-elastic medium (VTE). Fig. 2 represents that the variation of tangential stress t_{zr} with radial distance *r*. The values of tangential stress remain oscillatory in the whole range for both cases (TE and VTE) and for both theories of thermoelasticity (L-S and G-L). Similarly, the values of tangential stress for L-S (TE) is higher in comparison to L-S (VTE) theory and similar behavior is noticed for both cases (TE and VTE) for G-L theory. Fig. 3 shows the variation of temperature change T with radial distance r . It is noticed that the values of temperature change increases and decreases alternately w.r.t radial distance for both LS(TE,VTE) and GL(TE,VTE) theories of thermoelasticity. Due to the presence and absence of viscosity, the values of T for L-S theory is higher in comparison to G-L theory. Fig. 4 depicts the variations of chemical potential P with radial distance r . Similar behavior is noticed for both theories of thermoelastic and visco-thermo-elastic diffusion medium. The values of chemical potential P increases due to the presence of viscosity and decreases in the absence of viscosity for G-L theory in comparison to L-S theory.

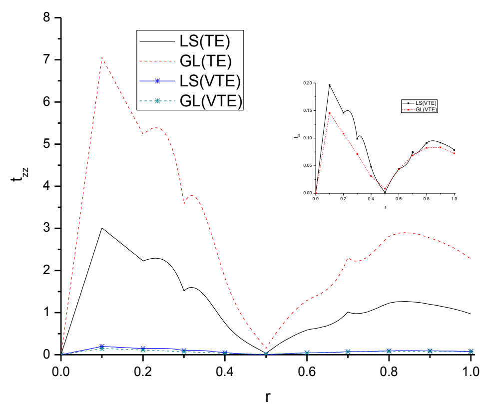

Fig.1
Variation of normal stress with radial distance.

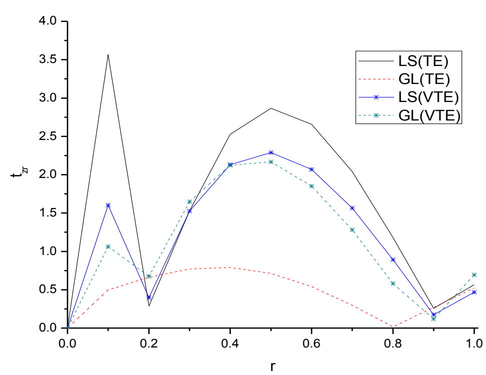

Fig.2
Variation of tangential stress with radial distance.

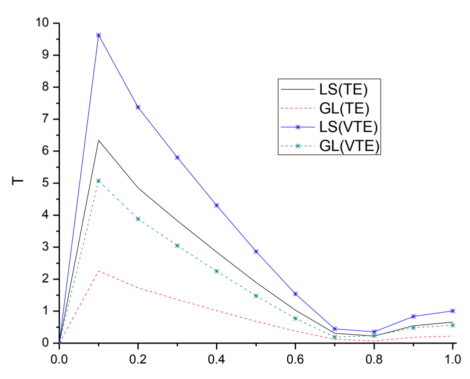

Fig.3
Variation of temperature change with radial distance.

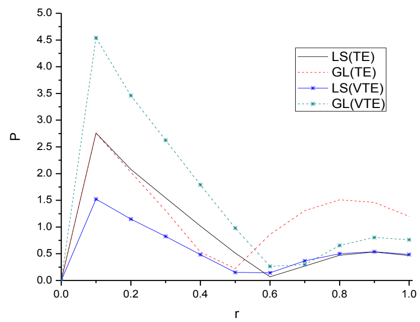

Fig.4
Variation of chemical potential with radial distance.

8 CONCLUSION

In this work, the problem of thick circular plate in visco-thermo-elastic diffusion is a significant problem of continuum mechanics. The resulting quantities depicted graphically in the absence and presence of viscosity. It is evident that the physical quantities are also effected by the different non-classical theories of thermoelastic diffusion. It is observed that effect of viscosity decrease the values of normal stress for both theories of thermoelastic diffusion. In addition, viscosity increase the values of chemical potential and decrease the values of temperature change for G-L theory in comparison for L-S theory. In the absence of viscosity, the values of tangential stress increase for L-S theory and decrease for G-L theory.

Nowadays, many people interested in the study of this phenomenon due to its application in geophysics and electronic industry. In integrated circuit fabrication, diffusion is used to introduce dopants in controlled amounts into the semiconductor substance. In particular, diffusion is used to form the base and emitter in bipolar transistors, integrated resistors, and the source/drain regions in metal oxide semiconductor (MOS) transistors and dope poly silicon gates in MOS transistors. The effect of diffusion is used to improve the conditions of oil extractions (seeking ways of more efficiently recovering oil from oil deposits). In addition, the study of thermal and diffusion effects plays an important role in understanding many seismological processes. The result obtained here are useful in engineering problems particularly in the determination state of stresses in a thick circular plate subjected to transient heat inside. Also, any particular case of special interest can be derived by assigning suitable values to the parameters and functions in the problem.

APPENDIX A

$$G_1 = b_6 + \delta b_8 \tau_{tt}^{22},$$

$$G_2 = \left\{ \begin{array}{l} -\left(\varsigma^2\delta + p^2\right) b_8 \tau_{tt}^{22} - \delta\left(\tau_{tt}^{66} + b_3 b_7 \tau_{tt}^{11}\tau_{tt}^{44} + \left(b_3\tau_{tt}^{33} + 2\varsigma^2\right) b_8 \tau_{tt}^{22}\right) - \tau_{tt}^{11}\left(b_5 b_8 \tau_{tt}^{22}\tau_{tt}^{55} + b_4 b_8 \tau_{tt}^{44}\right) \\ -b_8\tau_{tt}^{22}\left(b_7\tau_{tt}^{11}\tau_{tt}^{55} - b_3\tau_{tt}^{33}\right) - 3b_8\varsigma^2 \end{array} \right\},$$

$$G_3 = \left\{ \begin{array}{l} \left(\varsigma^2\delta + p^2\right)\left(\tau_{tt}^{66} + b_4 b_7 \tau_{tt}^{11}\tau_{tt}^{44} + \left(b_3\tau_{tt}^{33} + 2\varsigma^2\right) b_8\tau_{tt}^{22}\right) + \delta\left(\begin{array}{l} b_7\tau_{tt}^{22}\varsigma^4 + \\ \left(\tau_{tt}^{66} + b_3 b_6 \tau_{tt}^{11}\tau_{tt}^{44} + b_2 b_7 \tau_{tt}^{22}\tau_{tt}^{33}\right)\varsigma^2 + b_2\tau_{tt}^{33}\tau_{tt}^{66} \end{array} \right) + b_6\tau_{tt}^{11}\tau_{tt}^{55}\tau_{tt}^{66} \\ +2\varsigma^2\left(b_5 b_8 \tau_{tt}^{22}\tau_{tt}^{55} + b_4 b_6 \tau_{tt}^{44} + b_6\tau_{tt}^{22}\left(b_7\tau_{tt}^{11}\tau_{tt}^{55} - b_3\tau_{tt}^{33}\right)\right) + 3\varsigma^2 b_6 \end{array} \right\},$$

$$G_4 = \left\{ \begin{array}{l} -\left(\varsigma^2\delta + p^2\right)\left(b_8\tau_{tt}^{22}\varsigma^4 + \left(\tau_{tt}^{66} + b_4 b_7 \tau_{tt}^{11}\tau_{tt}^{44} + b_3 b_8 \tau_{tt}^{22}\tau_{tt}^{33}\right)\varsigma^2 + b_3\tau_{tt}^{33}\tau_{tt}^{66}\right) - b_6\tau_{tt}^{11}\tau_{tt}^{55}\tau_{tt}^{66}\varsigma^2 \\ -\varsigma^4\left(b_5 b_8 \tau_{tt}^{22}\tau_{tt}^{55} + b_4 b_6 \tau_{tt}^{44} + \tau_{tt}^{22}\left(b_5 b_6 \tau_{tt}^{11}\tau_{tt}^{55} - b_3 b_6 \tau_{tt}^{33} - b_6\varsigma^6\right)\right) \end{array} \right\},$$

$$B_1 = 2\varsigma^2,\ B_2 = \varsigma^4,\ \tau_{tt}^{11} = \left(1 + \tau_1 p\right),\ \ \tau_{tt}^{22} = \left(1 + \tau^1 p\right),\ \tau_{tt}^{33} = \left(p + \tau_0 p^2\right),$$

$$\tau_{tt}^{44} = \left(p + \gamma p^2\right),\ \tau_{tt}^{55} = \left(p + \eta_0\tau_0 p^2\right),\ \tau_{tt}^{66} = \left(p + \eta_0\tau^0 p^2\right),\ \delta = \left(b_1 + b_2\right), e = \nabla^2\phi.$$

APPENDIX B

$$A_1=\frac{\Delta_1}{\Delta},\ A_2=\frac{\Delta_2}{\Delta},\ A_3=\frac{\Delta_3}{\Delta},\ A_4=\frac{\Delta_4}{\Delta},\ A_5=\frac{\Delta_5}{\Delta},$$

$$\Delta=a_1\lambda_1 g_1\begin{bmatrix} M_2h_2\left(N_3K_5g_3h_5-N_5K_3g_5h_3+N_3K_4g_3h_4-N_4K_3g_4h_3\right)\\ -M_3h_3\left(N_2K_5g_2h_5-N_5K_2g_5h_2+N_2K_4g_2h_4-N_4K_2g_4h_2\right)\\ +M_4h_4\left(N_2K_3g_2h_3-N_3K_2g_3h_2\right)+M_5h_5\left(N_2K_3g_2h_3-N_3K_2g_3h_2\right)\end{bmatrix}$$

$$+a_2\lambda_2 g_2\begin{bmatrix} M_1h_1\left(N_3K_5g_3h_5-N_5K_3g_5h_3-N_3K_4g_3h_4+N_4K_3g_4h_3\right)\\ -M_3h_3\left(N_1K_5g_1h_5-N_5K_1g_5h_1-N_1K_4g_1h_4+N_4K_1g_4h_1\right)\\ +M_4h_4\left(N_3K_1g_3h_1-N_1K_3g_1h_3\right)+M_5h_5\left(N_1K_3g_1h_3-N_3K_1g_3h_1\right)\end{bmatrix}$$

$$+a_3\lambda_3 g_3\begin{bmatrix} M_1h_1\left(N_2K_5g_2h_5-N_5K_2g_5h_2-N_4K_2g_4h_2+N_2K_4g_2h_4\right)\\ -M_2h_2\left(N_1K_5g_1h_5-N_5K_1g_5h_1+N_1K_4g_1h_4-N_4K_1g_4h_1\right)\\ +M_4h_4\left(N_1K_2g_1h_2-N_2K_1g_2h_1\right)+M_5h_5\left(N_1K_2g_1h_2-N_2K_1g_2h_1\right)\end{bmatrix},$$

$g_1=\sinh(m_1d),\ g_2=\sinh(m_2d),\ g_3=\sinh(m_3d),\ g_4=\sinh(m_4d),\ g_5=\sinh(m_5d),$

$h_1=\cosh(m_1d),\ h_2=\cosh(m_2d),\ h_3=\cosh(m_3d),\ h_4=\cosh(m_4d),\ h_5=\cosh(m_5d),$

$\Delta_i\ \left(i=1,..........,5\right)$ are obtained by replacing 1st, 2nd, 3rd, 4th and 5th column by

$\left[\left(g_1\hat{F}(\varsigma z)\right),0,0,0,\left(\hat{F}(\varsigma)\right)\right]^T$ in Δ_i .

$$\sum_{i=1}^{3}M_i=\sum_{i=1}^{3}\left[b_9\left(\lambda_i^2-\varsigma^2\right)+2b_{10}\lambda_i^2-b_{11}\left(\tau_t^{11}a_i+\tau_t^{22}d_i\right)\right],\ \sum_{i=4}^{5}M_i=\sum_{i=4}^{5}\left[2\left(b_9+b_{10}\right)\lambda_i\varsigma^2\right],$$

$$\sum_{i=1}^{3}N_i=\sum_{i=1}^{3}2b_{10}\lambda_i,\ \sum_{i=4}^{5}N_i=\sum_{i=4}^{5}\left[b_{10}\left(\lambda_i^2+\varsigma^2\right)-b_{12}\left(\lambda_i^2-\varsigma^2\right)^2\right],$$

$$\sum_{i=1}^{3}K_i=\sum_{i=1}^{3}\left[-\left(\lambda_i^2+\varsigma^2\right)+b_{12}\tau_t^{22}d_i\right],\quad \sum_{i=4}^{5}K_i=\sum_{i=4}^{5}-2\varsigma^2\lambda_i .$$

REFERENCES

[1] Boit M., 1956, Thermoelasticity and irreversible thermodynamics, *Journal of Applied Physics of Solid*s **27**: 240-253.

[2] Daliwal R.S., Singh A., 1980, *Dynamical Coupled Thermoelasticity*, Hindustan Publishers, Delhi.

[3] Duhamel J., 1837, Une memoire sur les phenomenes thermo-mechanique, *Journal de l'École Polytechnique* **15**: 1-31.

[4] El-Maghraby N.M., Abdel-Halim A.A., 2010, A generalized thermos-elasticity problem for a half space with heat sources under axisymmetric distributions, *Australian Journal of Basic and Applied Sciences* **4**: 3803-3814.

[5] Ezzat M.A., Othman M.I., El-Karamany A.S., 2002, State space approach to generalized thermo-viscoelasticity with two relaxation times, *International Journal of Engineering Science* **40**: 283-302.

[6] Ezzat M.A., El-Karamany A.S., Samaan A.A., 2001, State-space formulation to generalized thermos-visco-elasticity with thermal relaxation, *Journal of Thermal Stresses* **24**: 823-846.

[7] Ezzat M.A., 2006, The relaxation effects of the volume properties of electrically conducting viscoelastic material, *Materials Science and Engineering B* **130**: 11-23.

[8] Green A.E., Lindsay K.A., 1972, Thermoelasticity, *Journal of Elasticity* **2**: 1-7.

[9] Kumar R., Kansal T., 2008, Propagation of lamb waves in transversely isotropic thermoelastic diffusion plate, *The International Journal of Solids and Structures* **45**: 5890-5913.

[10] Kumar R., Chawla V., Abbas I.A., 2012, Effect of viscosity on wave propagation in anisotropic thermoelastic medium with three-phase-lag model, *Theoretical and Applied Mechanics* **39**(4): 313-341.

[11] Kumar R., Sharma K.D., Garg S.K., 2015, Fundamental solution in micropolar viscothermoelastic solids with void, *International Journal of Applied Mechanics and Engineering* **20**(1): 109-125.

[12] Kumar R., Sharma K.D., Garg S.K., 2015, Reflection of plane waves in transversely isotropic micro polar visco-thermo-elastic solid, *Materials Physics and Mechanics* **22**: 1-14.
[13] Lord H.W., Shulman Y., 1967, A generalized dynamical theory of thermoelasticity, *Journal of Mechanics and Physics of Solids* **15**: 299-309.
[14] Marin M., 2008, Weak solutions in elasticity of dipolar porous materials, *Mathematical Problems in Engineering* **2008**: 158908.
[15] Marin M., 2010, Lagrange identity method for micro-stretch thermoelastic materials, *Journal of Mathematical Analysis and Applications* **363**: 275-286.
[16] Marin M., Stan G., 2013, Weak solutions in elasticity of dipolar bodies with stretch, *Carpathian Journal of Mathematics* **29**(1): 33-40.
[17] Mukhopadhyay S., Kumar R., 2010, Analysis of phase-lag effects on wave propagation in a thick plate under axisymmetric temperature distribution, *Acta Mechanica* **210**: 331-344.
[18] Neumann F., 1955, *Vorlesungen Uber die Theorie der Elasticitat*, Breslau, Meyer.
[19] Nowacki W., 1974, Dynamical problems of thermo diffusion in solids I, *Bulletin of the Polish Academy of Sciences Technical Sciences* **22**: 55-64.
[20] Nowacki W., 1974, Dynamical problems of thermo diffusion in solids II, *Bulletin of the Polish Academy of Sciences Technical Sciences* **22**: 129-135.
[21] Nowacki W., 1974, Dynamical problems of thermo diffusion in solids III, *Bulletin of the Polish Academy of Sciences Technical Sciences* **22**: 257-266.
[22] Nowacki W., 1976, Dynamical problems of thermo diffusion in solids, *Engineering Fracture Mech*anics **8**: 261-266.
[23] Othman M.I.A., Ezzat M.A., Zaki S.A., El-Karamany A.S., 2002, Generalized thermo-viscoelastic plane waves with two relaxation times, *International Journal of Engineering Science* **40**: 1329-1347.
[24] Podstrigach I.S., 1961, Differential equations of the problem of thermos diffusion in isotropic deformed solid bodies, *Dop Akad Nauk Ukr SSR* **1961**: 169-172.
[25] Sharma K., Kumar P., 2013, Propagation of plane waves and fundamental solution in thermos-visco-elastic medium with voids, *Journal of Thermal Stresses* **36**: 94-111.
[26] Sharma D.K., Parkash I., Dhaliwal S.S., Walia V., Chandel S., 2017, Effect of magnetic field on transient wave in visco-thermo-elastic half space, *International Journal of Computational and Applied Mathematics* **12**(2): 343-364.
[27] Sherief H.H., Saleh H., Hamza F., 2004, The theory of generalized thermoelastic diffusion, *International Journal of Engineering Sci*ences **42**: 591-608.
[28] Sherief H.H., Saleh H., 2005, A half-space problem in the theory of generalized thermoelastic diffusion, *International Journal of Solids and Structures* **42**: 4484-4493.
[29] Svanadze M.M., 2016, Plane waves and problems of steady vibrations in the theory of viscoelasticity for Kelvin-Voigt materials with double porosity, *Archive of Mech*anics **68**(6): 441-458.
[30] Tripathi J.J., Kedar G.D., Deshmukh K.C., 2014, Dynamic problem of generalized thermo-elasticity for a semi-infinite cylinder with heat sources, *Journal of Thermoelasticity* **2**: 1-8.
[31] Tripathi J.J., Kedar G.D., Deshmukh K.C., 2015, Generalized thermoelastic diffusion problem in a thick circular plate with axisymmetric heat supply, *Acta Mechanica* **226**(7): 2121-2134.