Journal of Solid Mechanics Vol. 12, No. 4 (2020) pp. 953-968
DOI: 10.22034/jsm.2020.1903452.1616

State Space Approach to Electro-Magneto-Thermoelasticity with Energy Dissipation

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Received 8 August 2020; accepted 10 October 2020

ABSTRACT

In this article a two-dimensional problem of generalized thermoelasticity has been formulated with state space approach. In this formulation, the governing equations are transformed into a matrix differential equation whose solution enables us to write the solution of any two-dimensional problem in terms of the boundary conditions. The resulting formulation is applied to an isotropic half space problem under Green-Naghdi type-III model i.e., with energy dissipation theory of thermoelasticity in the presence of a magnetic field. The bounding surface is traction free and subjected to a time dependent thermal shock. The solution for temperature distribution, displacements and stress components are obtained and presented graphically. The effect of magnetic field, electric field and phase velocity on the considered parameters is observed in the figures.

Keywords: State space approach; Normal mode analysis; Energy dissipation; Magnetic effect; Thermal shock.

1 INTRODUCTION

THE classical theory of thermoelasticity (Biot [1]) suffers from so called 'paradox of heat conduction' i.e. the heat equations for both theories of a mixed parabolic-hyperbolic type, predicting infinite speeds of propagation for heat waves contrary to physical observations. The generalized thermoelasticity theories in which the heat conduction equation is hyperbolic and do not suffer from this paradox. To remove this paradox, the conventional theories of thermoelasticity has been generalized, where the generalization is in the sense that these theories involve a hyperbolic type heat transport equation supported by experiments, which exhibit the actual occurrence of wave type heat transport in solids, called second sound effect. To eliminate the second sound paradox of classical thermoelasticity theory, Lord and Shulman [2] established a generalized thermoelasticity theory based on the CV heat conduction equation in 1960s, which is often referred to as LS model and widely used in the case of heat flux and low temperature. Green and Lindsay [3] introduced one more theory, called GL theory, which involves two relaxation times. Later Green and Naghdi ([4], [5], [6]) developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labeled as G-N models I, II, III. Detailed information regarding these

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theories can be found in Chandrasekharaiah [7]. State space methods are the cornerstone of modern control theory. The essential feature of state space methods is the characterization of the processes of interest by differential equations instead of transfer functions. This may seem like a throwback to the earlier, primitive period where differential equations are also constituted by the means of representing the behavior of dynamic processes. But in the earlier period the processes were simple enough to be characterized by a single differential equation of fairly low order. In the modern approach the processes are characterized by systems of coupled, first order differential equations. In principle, there is no limit to the order (i.e. the number of independent first order differential equations) and in practice the only limit to the order is the availability of computer software capable of performing the required calculations reliably. The importance of state space analysis is recognized in fields where the time behavior of any physical process is of interest. The state space approach is more general than the classical Laplace and Fourier transform theory. Consequently, state space theory is applicable to all systems that can be analyzed by integral transforms in time and is applicable to many systems for which transform theory breaks down. Furthermore, state space theory gives a somewhat different insight into the time behavior of linear systems. A method for solving coupled thermoelastic problems by state space approach has been developed by Bahar and Hetnarski [8]. The state space formulation for the problems that do not contain heat sources have been done by Anwar and Sherief [9]. Ezzat et al. [10] studied thermo-viscoelastic material. Ezzat et al. [11] considered two-temperature theory in generalized magneto-thermo-viscoelasticity. Ezzat et al. [12] investigated thermo-viscoelasticity with variable thermal conductivity and fractional-order heat transfer. Ezzat and El-Bary[13] studied magneto-thermoelectric viscoelastic materials with memory-dependent derivative involving two-temperature. Ezzat and El-Bary [14] considered fractional magneto-thermoelastic materials with phase-lag Green-Naghdi theories. Ezzat and El-Bary [15] proposed two-temperature theory of magneto-thermo-viscoelasticity with fractional derivative and integral orders heat transfer. Ezzat et al. [16] considered dual-phase-lag thermoelasticity theory with memory-dependent derivative. Ezzat et al. [17] discussed two-temperature theory in Green–Naghdi thermoelasticity with fractional phase-lag heat transfer. El-Karamany and Ezzat [18] considered thermal shock problem in thermoelastic medium in the context of four theories of generalized thermoelasticity. Sherief, El-Maghraby and Allam [19] studied stochastic thermal shock in generalized thermoelasticity. Ezzat and Youssef [20] discussed three dimensional thermal shock problem of generalized thermoelastic half-space Wang et al.[21]investigated thermoelastic behavior of elastic media with temperature-dependent properties under thermal shock. Baksi, Bera and Debnath [22] proposed a study of magneto-thermoelastic problems with thermal relaxation and heat sources in a three dimensional infinite rotating elastic medium, Said[23] investigated the influence of gravity on generalized magneto-thermoelastic medium for three-phase -lag model. Kalkal and Deswal [24] examined the effects of phase lags on three dimensional wave propagation with temperature dependent properties. Youssef et al. [25] considered vibration of gold nano beam in context of two-temperature generalized thermoelasticity subjected to laser pulse.

In this problem we have considered two dimensional generalized thermoelasticity with Green-Naghdi type-III model of thermoelasticity in the presence of a magnetic field. Normal mode analysis is employed to the governing equations and then the problem is solved using state space approach. To observe the nature of waves in a more clear way and illustrate the analytical results, we further perform numerical computations of the problem. The present study is believed to enhance the understanding of thermoelasticity for magneto-thermoelastic problems.

2 FORMULATION OF THE PROBLEM

We assume that the medium under consideration is a perfect electric conductor and the initial magnetic field vector $\vec{H}$ is oriented in such a way that propagation of plane waves in the $xz-$plane is possible. Under these assumptions we can obtain very simple expressions for the displacement, temperature and the electromagnetic quantities. We begin our consideration with the linearized equations of electromagnetism, valid for slowly moving media.

$$curl\vec{h} = \vec{J} + \varepsilon_0 \dot{\vec{E}} \tag{1}$$

$$curl\vec{E} = -\mu_0 \dot{\vec{h}} \tag{2}$$

$$\vec{E} = -\mu_0 (\dot{\vec{u}} \times \vec{H}) \tag{3}$$

$$div\vec{h} = 0 \tag{4}$$

The above equations are supplemented by the displacement equations of the theory of elasticity, taking into account the Lorentz force,

$$(\lambda + \mu)u_{j,ij} + \mu u_{i,jj} - \gamma T_{,i} + \mu_0(\vec{J} \times \vec{H})_i = \rho \ddot{u}_i \tag{5}$$

The constitutive equation

$$\tau_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \gamma(T - T_0)\delta_{ij} \tag{6}$$

and the strain-displacement relations

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \tag{7}$$

In the above equations $i = x,\ j = y$ and $k = z.$ In the above equations, a superposed dot denotes differentiation with respect to time, while a comma denotes material derivatives. We shall consider only the simplest case of the two-dimensional problem. We assume that all causes producing the wave propagation is independent of the variable y and that waves are propagated only in the $xz-$plane. Thus all quantities appearing in Eqs. (1)-(7) are independent of the variable y. Then the displacement vector has components $[u(x,z,t), 0, w(x,z,t)]$. Assume now that the initial conditions are homogeneous and the initial magnetic field has component $(0, H_0, 0)$. The relations (1)-(4) yield

$$\vec{J} = curl\vec{h} - \varepsilon_0\dot{\vec{E}} \tag{8}$$

$$\vec{E} = \mu_0 H_0(-\dot{w}, 0, \dot{u}) \tag{9}$$

$$\vec{h} = -H_0(0, 0, e) \tag{10}$$

We shall consider a thermoelastic medium governed by the equations of generalized electro-magneto thermoelasticity whose state depends on the space variables x', y' and the time variable t'. The initial conditions are taken to be homogeneous. The heat conduction equation of Green –Naghdi theory type-III is given by (Pramanik and Biswas [26])

$$K\nabla^2\dot{T} + K^*\nabla^2 T = \left(\rho C_e\ddot{T} + \gamma T_0\ddot{e}\right) \tag{11}$$

We use the following non-dimensional variables:

$$x' = c_0\eta_0 x, z' = c_0\eta_0 z, u' = c_0\eta_0 u, w' = c_0\eta_0 w, T' = \frac{\gamma(T - T_0)}{\rho c_0^2}, \tau'_{ij} = \frac{\tau_{ij}}{\mu}, \eta_0 = \frac{\rho C_e}{K^*}$$

where the dashed quantities denote non-dimensional variables. In terms of these non-dimensional variables, the equations of motion has the form (dropping primes)

$$\beta^2 u_{,xx} + u_{,zz} + (\beta^2 - 1)w_{,xz} - \beta^2 T_{,x} = \alpha_0\ddot{u} \tag{12}$$

$$(\beta^2 - 1)u_{,xz} + \beta^2 w_{,zz} + w_{,xx} - \beta^2 T_{,z} = \alpha_0\ddot{w} \tag{13}$$

and the components of the stress are

$$\tau_{xx} = \beta^2 u_{,x} + (\beta^2 - 2) w_{,z} - \beta^2 T \tag{14a}$$

$$\tau_{xz} = u_{,z} + w_{,x} \tag{14b}$$

$$\tau_{zz} = (\beta^2 - 2) u_{,x} + \beta^2 w_{,z} - \beta^2 T \tag{14c}$$

The Eq. (11) in non-dimensional form is obtained as:

$$\bar{K}\left(\dot{T}_{,xx} + \dot{T}_{,zz}\right) + \left(T_{,xx} + T_{,zz}\right) = (\ddot{T} + \varepsilon \ddot{e}) \tag{15}$$

where $\bar{K} = \dfrac{K c_0 \eta_0}{K^*}$.

3 NORMAL MODE ANALYSIS

For harmonic wave propagation in $x-$direction, we seek solution of Eqs. (12), (13) and (15) in the following form:

$$\left(u, w, e, T\right)\left(x, z, t\right) = \left(\bar{u}, \bar{w}, \bar{e}, \bar{T}\right)\left(z\right) \exp\left[ik\left(x - ct\right)\right] \tag{16}$$

where k is wave number and c is the phase velocity. Applying normal mode analysis to both sides of Eqs. (12), (13) and (15) we obtain

$$-\beta^2 k^2 \bar{u} + D^2 \bar{u} + ik(\beta^2 - 1) D\bar{w} - ik\beta^2 \bar{T} = -\alpha_0 k^2 c^2 \bar{u} \tag{17}$$

$$ik(\beta^2 - 1) D\bar{u} + \beta^2 D^2 \bar{w} - k^2 \bar{w} - \beta^2 D\bar{T} = -\alpha_0 k^2 c^2 \bar{w} \tag{18}$$

$$D^2 \bar{T} - k^2 \bar{T} = P(\bar{T} + \varepsilon \bar{e}) \tag{19}$$

where $D \equiv \dfrac{d}{dz}$, $P = \dfrac{k^2 c^2}{\left(ikc\bar{K} - 1\right)}$

4 STATE –SPACE FORMULATION

We take the quantities e, T, De, DT as state variables. Now

$$\bar{e} = ik\bar{u} + D\bar{w} \tag{20}$$

Eliminating $\bar{u}$ and $\bar{w}$ between Eqs. (17), (18) and (19) with the help of Eq. (20), we obtain the following equation

$$D^2 \bar{e} = (-\alpha k^2 c^2 + k^2 + P\varepsilon)\bar{e} + P\bar{T} \tag{21}$$

$$D^2 \bar{T} = P\varepsilon \bar{e} + (k^2 + P)\bar{T} \tag{22}$$

Eqs. (21) and (22) can be written in matrix differential equation form as follows:

$$\frac{d\tilde{V}\left(z\right)}{dz} = \tilde{A}\tilde{V}\left(z\right) \tag{23}$$

where $\tilde{A}=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\alpha k^2c^2+k^2+P\varepsilon & P & 0 & 0 \\ P\varepsilon & k^2+P & 0 & 0 \end{bmatrix}$, $\tilde{V}=\begin{bmatrix} \overline{e} \\ \overline{T} \\ D\overline{e} \\ D\overline{T} \end{bmatrix}$. The formal solution of system (24) can be written in the form

$$\tilde{V}(z)=\exp(\tilde{A}z)\tilde{V}\left(z_0\right) \tag{24}$$

where z_0 denotes any arbitrarily chosen initial value for z. The characteristic equation for the matrix $\tilde{A}$ is

$$\lambda^4-(-\alpha k^2c^2+2k^2+P\varepsilon+P)\lambda^2+k^4+k^2(-\alpha k^2c^2+\varepsilon P+P)-P\alpha k^2c^2=0 \tag{25}$$

The roots of the Eq. (25) satisfy the relations

$$k_1^2+k_2^2=-\alpha k^2c^2+2k^2+P\varepsilon+P \tag{26a}$$

$$k_1^2k_2^2=k^4+k^2(-\alpha k^2c^2+P\varepsilon+P)-P\alpha k^2c^2 \tag{26b}$$

The Maclaurin series expansion of $\exp\left(\tilde{A}z\right)$ is given by $\exp\left(\tilde{A}z\right)=\sum_{n=0}^{\infty}\frac{\left[\tilde{A}.z\right]^n}{n!}$. Using Cayley-Hamilton theorem, the infinite series representing $\exp\left(\tilde{A}z\right)$ can be truncated to the following form:

$$\exp(\tilde{A}z)=\tilde{L}=b_0\tilde{I}+b_1\tilde{A}+b_2\tilde{A}^2+b_3\tilde{A}^3 \tag{27}$$

where $\tilde{I}$ is the unit matrix of order 4 and $b_0,....,b_3$ are some parameters depending on z,k and t.

We shall stress here that the above expressions for the matrix exponential is a formal one. In the actual physical problem, the space is divided into two regions accordingly as; $z\geq 0$ or $z\leq 0$. By Cayley-Hamilton theorem, the characteristic roots $\pm k_1$ and $\pm k_2$ of the matrix $\tilde{A}$ must satisfy the equations

$$\begin{aligned}
\exp(k_1z)&=b_0+b_1k_1+b_2k_1^2+b_3k_1^3\\
\exp(-k_1z)&=b_0-b_1k_1+b_2k_1^2-b_3k_1^3\\
\exp(k_2z)&=b_0+b_1k_2+b_2k_2^2+b_3k_2^3\\
\exp(-k_2z)&=b_0-b_1k_2+b_2k_2^2-b_3k_2^3
\end{aligned}$$

The solution of the above system is given by

$$\begin{aligned}
b_0&=\frac{1}{k_1^2-k_2^2}[k_1^2\cosh(k_2z)-k_2^2\cosh(k_1z)]\\
b_1&=\frac{1}{k_1^2-k_2^2}[\frac{k_1^2}{k_2}\sinh(k_2z)-\frac{k_2^2}{k_1}\sinh(k_1z)]\\
b_2&=\frac{1}{k_1^2-k_2^2}[\cosh(k_1z)-\cosh(k_2z)]\\
b_3&=\frac{1}{k_1^2-k_2^2}[\frac{1}{k_1}\sinh(k_1z)-\frac{1}{k_2}\sinh(k_2z)]
\end{aligned} \tag{28}$$

Substituting the expressions (28) into (27) and computing $\tilde{A}^2$ and $\tilde{A}^3$, we obtain after repeated use of Eqs. (26a) and (26b), the elements $l_{ij}(i,j=1,2,3,4)$ of the matrix $\tilde{L}$ as:

$$\begin{aligned}
l_{11} &= \frac{1}{k_1^2-k_2^2}[(k_1^2-k^2-P)\cosh(k_1 z)-(k_2^2-k^2-P)\cosh(k_2 z)] \\
l_{12} &= \frac{P}{k_1^2-k_2^2}[\cosh(k_1 z)-\cosh(k_2 z)] \\
l_{13} &= \frac{1}{k_1^2-k_2^2}[\frac{(k_1^2-k^2-P)}{k_1}\sinh(k_1 z)-\frac{(k_2^2-k^2-P)}{k_2}\sinh(k_2 z)] \\
l_{14} &= \frac{P}{k_1^2-k_2^2}[\frac{1}{k_1}\sinh(k_1 z)-\frac{1}{k_2}\sinh(k_2 z)] \\
l_{21} &= \frac{P\varepsilon}{k_1^2-k_2^2}[\cosh(k_1 z)-\cosh(k_2 z)] \\
l_{22} &= \frac{1}{k_1^2-k_2^2}[(k_1^2-k^2-P)\cosh(k_2 z)-(k_2^2-k^2-P)\cosh(k_1 z)] \\
l_{23} &= \frac{P\varepsilon}{k_1^2-k_2^2}[\frac{1}{k_1}\sinh(k_1 z)-\frac{1}{k_2}\sinh(k_2 z)] \\
l_{24} &= \frac{1}{k_1^2-k_2^2}[\frac{(k_1^2-k^2-P)}{k_2}\sinh(k_2 z)-\frac{(k_2^2-k^2-P)}{k_1}\sinh(k_1 z)] \\
l_{31} &= \frac{1}{k_1^2-k_2^2}\left\{\left[k_1(k_1^2-k^2-P)\right]\sinh(k_1 z)-\left[k_2(k_2^2-k^2-P)\right]\sinh(k_2 z)\right\} \\
l_{32} &= \frac{P}{k_1^2-k_2^2}[k_1\sinh(k_1 z)-k_2\sinh(k_2 z)] \\
l_{33} &= \frac{1}{k_1^2-k_2^2}[(k_1^2-k^2-P)\cosh(k_1 z)-(k_2^2-k^2-P)\cosh(k_2 z)] \\
l_{34} &= \frac{P}{k_1^2-k_2^2}[\cosh(k_1 z)-\cosh(k_2 z)] \\
l_{41} &= \frac{P\varepsilon}{k_1^2-k_2^2}[k_1\sinh(k_1 z)-k_2\sinh(k_2 z)] \\
l_{42} &= \frac{1}{k_1^2-k_2^2}\left\{[k_1(k^2+P-k_2^2)]\sinh(k_1 z)-[k_2(k^2+P-k_1^2)]\sinh(k_2 z)\right\} \\
l_{43} &= \frac{\varepsilon P}{k_1^2-k_2^2}[\cosh(k_1 z)-\cosh(k_2 z)] \\
l_{44} &= \frac{1}{k_1^2-k_2^2}[(k_1^2-k^2-P)\cosh(k_2 z)-(k_2^2-k^2-P)\cosh(k_1 z)]
\end{aligned} \tag{29}$$

It should be noted that we have repeatedly used Eqs. (26a) and (26b) in order to write (29) in the simplest possible form. Furthermore, the corresponding expressions for generalized thermoelasticity in the absence of magnetic field can be deduced by setting $\alpha=1$ in Eqs. (26a) and (26b). Using Eq. (24), upon equating Matrices we obtain

$$\bar{e}(z)=l_{11}e_0+l_{12}\theta_0+l_{13}e_0'+l_{14}\theta_0' \tag{30}$$

$$\bar{T}(z)=l_{21}e_0+l_{22}\theta_0+l_{23}e_0'+l_{24}\theta_0' \tag{31}$$

where $e_0 = \overline{e}\left(z_0\right),\ \theta_0 = \overline{T}\left(z_0\right),\ e_0' = D\overline{e}\left(z_0\right), \theta_0' = D\overline{T}\left(z_0\right)$

Using Eq. (29) into Eqs. (30) and (31) we obtain

$$\overline{T} = \sum_{i=1}^{2}\left[M_i \cosh\left(k_i z\right) + \frac{M_i'}{k_i}\sinh\left(k_i z\right)\right] \tag{32}$$

$$\overline{e} = \sum_{i=1}^{2}\left[N_i \cosh\left(k_i z\right) + \frac{N_i'}{k_i}\sinh\left(k_i z\right)\right] \tag{33}$$

where

$$\begin{aligned}
M_i &= \frac{\left(-1\right)^{i-1}}{k_1^2 - k_2^2}\left\{\varepsilon P e_0 + \left(k_1^2 - k^2 - P\right)\theta_0\right\} \\
M_i' &= \frac{\left(-1\right)^{i-1}}{k_1^2 - k_2^2}\left\{\varepsilon P e_0' + \left(k_1^2 - k^2 - P\right)\theta_0'\right\} \\
N_i &= \frac{\left(-1\right)^{i-1}}{k_1^2 - k_2^2}\left\{\left(k_i^2 - k^2 - P\right)e_0 + P\theta_0\right\} \\
N_i' &= \frac{\left(-1\right)^{i-1}}{k_1^2 - k_2^2}\left\{\left(k_i^2 - k^2 - P\right)e_0' + P\theta_0'\right\}
\end{aligned} \tag{34}$$

Using Eq. (20) in the Eq. (17), we obtain

$$\left(D^2 - k_3^2\right)\overline{u} = \sum_{i=1}^{2} ik\left\{\left[\left(1-\beta^2\right)N_i + \beta^2 M_i\right]\cosh\left(k_i z\right) + \frac{\left[\left(1-\beta^2\right)N_i' + \beta^2 M_i'\right]}{k_i}\sinh\left(k_i z\right)\right\}$$

where $k_3^2 = -\alpha_0 k^2 c^2 + k^2$. Now solving the above equation, we get

$$\overline{u} = C\cosh\left(k_3 z\right) + ik\sum_{i=1}^{2}\left\{\frac{\left(1-\beta^2\right)N_i + \beta^2 M_i}{\left(k_i^2 - k_3^2\right)}\cosh\left(k_i z\right) + \frac{\left(1-\beta^2\right)N_i' + \beta^2 M_i'}{k_i\left(k_i^2 - k_3^2\right)}\sinh\left(k_i z\right)\right\} \tag{35}$$

Substituting (35) into (20) and integrating the resulting equation, we get

$$\overline{w} = \frac{-ikC}{k_3}\sinh\left(k_3 z\right) + \sum_{i=1}^{2}\left\{\left[\frac{N_i}{k_i} + \frac{k^2\left(\left(1-\beta^2\right)N_i + \beta^2 M_i\right)}{k_i\left(k_i^2 - k_3^2\right)}\right]\sinh\left(k_i z\right) + \left[\frac{N_i'}{k_i^2} + \frac{k^2\left(\left(1-\beta^2\right)N_i' + \beta^2 M_i'\right)}{k_i^2\left(k_i^2 - k_3^2\right)}\right]\cosh\left(k_i z\right)\right\} \tag{36}$$

Maxwell's electromagnetic stress tensor $\overline{\tau}_{ij}$ is given by

$$\overline{\tau}_{ij} = \mu_0\left[H_i h_j + H_j h_i - \left(\vec{H}.\vec{h}\right)\delta_{ij}\right]$$
$$\overline{\tau}_{zz} = \mu_o H_0^2 e,\ \ \overline{\tau}_{xz} = 0$$

5 BOUNDARY CONDITIONS

We consider the case where the surface of the half space is subjected to a time dependent thermal shock and the surface is traction free.

Thermal boundary condition that the surface of the half-space is subjected to a time dependent thermal shock i.e. exponentially decaying with respect to time

$$T(x,0,t)=F(t)H\left(a-|x|\right) \tag{37}$$

where a is constant and if we take $t=0$, $F(t)$ becomes constant.

Mechanical boundary condition that the surface to the half-space is traction free

$$\begin{aligned}
&\tau_{zz}(x,0,t)+\overline{\tau}_{zz}\left(x,0,t\right)=0\\
&\tau_{xz}(x,0,t)+\overline{\tau}_{xz}\left(x,0,t\right)=0
\end{aligned} \tag{38}$$

6 APPLICATION

We shall apply our results to solve a problem for a half-space$\left(z\geq 0\right)$. Inside the region $0\leq z\leq\infty$, the positive exponential terms, not bounded at infinity, must be suppressed. Thus, for $z\geq 0$ we should replace each $\sinh\left(kz\right)$ by $-\frac{1}{2}\exp\left(-kz\right)$ and each $\cosh\left(kz\right)$ by $\frac{1}{2}\exp\left(-kz\right)$. The solution of the problem is given by Eq. (24) with z_0 chosen as zero for convenience. Thus, the two components of the initial vectors $V_0=V\left(0\right)$ are known, i.e., $e_0'=\theta_0'=0$. Now replacing each $\sinh\left(kz\right)$ by $-\frac{1}{2}\exp\left(-kz\right)$ and each $\cosh\left(kz\right)$ by $\frac{1}{2}\exp\left(-kz\right)$, we obtain

$$\begin{aligned}
\overline{T}&=\frac{1}{2}\sum_{i=1}^{2}M_i\exp\left(-k_iz\right)\\
\overline{e}&=\frac{1}{2}\sum_{i=1}^{2}N_i\exp\left(-k_iz\right)\\
\overline{u}&=\frac{C}{2}\exp\left(-k_3z\right)+\frac{ik}{2}\sum_{i=1}^{2}\frac{\left(1-\beta^2\right)N_i+\beta^2M_i}{\left(k_i^2-k_3^2\right)}\exp\left(-k_iz\right)\\
\overline{w}&=\frac{ikC}{2k_3}\exp\left(-k_3z\right)-\frac{1}{2}\sum_{i=1}^{2}\left\{\frac{N_i}{k_i}+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{k_i\left(k_i^2-k_3^2\right)}\right\}\exp\left(-k_iz\right)
\end{aligned} \tag{39a}$$

The displacement components are obtained as:

$$u=\left[\frac{C}{2}\exp\left(-k_3z\right)+\frac{ik}{2}\sum_{i=1}^{2}\frac{\left(1-\beta^2\right)N_i+\beta^2M_i}{\left(k_i^2-k_3^2\right)}\exp\left(-k_iz\right)\right]\exp\left[ik\left(x-ct\right)\right] \tag{39b}$$

$$w=\left[\frac{ikC}{2k_3}\exp\left(-k_3z\right)-\frac{1}{2}\sum_{i=1}^{2}\left\{\frac{N_i}{k_i}+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{k_i\left(k_i^2-k_3^2\right)}\right\}\exp\left(-k_iz\right)\right]\exp\left[ik\left(x-ct\right)\right] \tag{39c}$$

Using (39) in Eqs. (14a-14c), the stress components are obtained as:

$$\tau_{xx}=[\frac{-\beta^2k^2}{2}\sum_{i=1}^{2}\frac{\left(1-\beta^2\right)N_i+\beta^2M_i}{\left(k_i^2-k_3^2\right)}\exp\left(-k_iz\right)+ikC\exp\left(-k_3z\right)+\frac{\left(\beta^2-2\right)}{2}\sum_{i=1}^{2}\left\{N_i+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}\right\}$$
$$\exp\left(-k_3z\right)-\frac{\beta^2}{2}\sum_{i=1}^{2}M_i\exp\left(-k_iz\right)]\exp\left[ik\left(x-ct\right)\right] \tag{40}$$

$$\tau_{zz}=[\frac{-k^2\left(\beta^2-2\right)}{2}\sum_{i=1}^{2}\frac{\left(1-\beta^2\right)N_i+\beta^2M_i}{\left(k_i^2-k_3^2\right)}\exp\left(-k_iz\right)+\frac{\beta^2}{2}\sum_{i=1}^{2}\left\{N_i+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}\right\}\exp\left(-k_iz\right)$$
$$-ikC\exp\left(-k_3z\right)-\frac{\beta^2}{2}\sum_{i=1}^{2}M_i\exp\left(-k_iz\right)]\exp\left[ik\left(x-ct\right)\right] \tag{41}$$

$$\tau_{xz}=[\frac{-ikk_i}{2}\sum_{i=1}^{2}\frac{\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}\exp\left(-k_iz\right)-\frac{ik}{2}\sum_{i=1}^{2}\left\{\frac{N_i}{k_i}+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{k_i\left(k_i^2-k_3^2\right)}\right\}\exp\left(-k_iz\right)$$
$$-\frac{C}{2k_3}\left(k^2+k_3^2\right)\exp\left(-k_3z\right)]\exp\left[ik\left(x-ct\right)\right] \tag{42}$$

Now applying boundary conditions (37) and (38), we obtain

$$\frac{1}{2}\sum_{i=1}^{2}M_i=F\left(t\right)H\left(a-|x|\right)\exp\left[-ik\left(x-ct\right)\right] \tag{43}$$

$$\frac{-k^2\left(\beta^2-2+\mu_0H_0^2\right)}{2}\sum_{i=1}^{2}\frac{\left(1-\beta^2\right)N_i+\beta^2M_i}{\left(k_i^2-k_3^2\right)}+\frac{\left(\beta^2+\mu_0H_0^2\right)}{2}\sum_{i=1}^{2}\left\{N_i+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}\right\}-ikC-\frac{\beta^2}{2}\sum_{i=1}^{2}M_i=0 \tag{44}$$

$$\frac{ik}{2}\sum_{i=1}^{2}\frac{\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}+\frac{ik}{2}\sum_{i=1}^{2}\left\{N_i+\frac{k^2\left[\left(1-\beta^2\right)N_i+\beta^2M_i\right]}{\left(k_i^2-k_3^2\right)}\right\}+\frac{C}{2k_3}\left(k^2+k_3^2\right)=0 \tag{45}$$

From (34), we see that M_i and N_i are expressed in terms of e_0 and θ_0. By solving the Eqs. (43), (44) and (45), e_0, θ_0 and C can be obtained. This completes the solution of the problem.

7 SPECIAL CASES

Now we discuss some special cases as follows:

(a) If we take $K=0$ then Eq. (11) reduces to without energy dissipation i.e. Green-Naghdi type-II (GN-II).

(b) If we take $H_0=0$ then the study reduces to a problem without magnetic field.

8 NUMERICAL DISCUSSION

In order to illustrate the above results graphically the time dependent thermal shock $F\left(t\right)$ is taken in the following form:

$$F(t) = 10\exp(-bt)$$

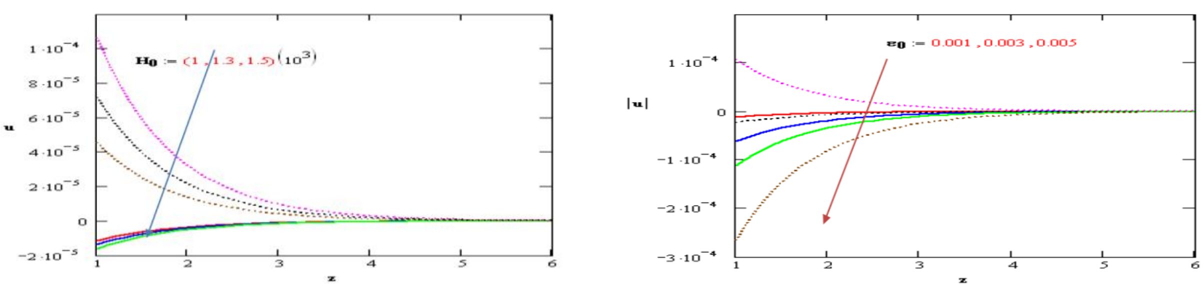
The copper material is chosen for the purpose of numerical computation (Pramanik and Biswas [26])

$$\lambda = 7.76\times10^{10}\,N/m^2,\ \mu = 3.86\times10^{10}\,N/m^2,\ \alpha_t = 1.78\times10^{-5}\,K^{-1},\ \rho = 8954Kg/m^3, C_e = 383.1m^2/K,$$
$$K = 386W/mK, K^* = 124W/mKs, T_0 = 293K.$$

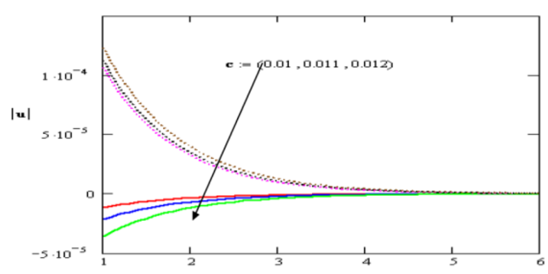
Further for numerical purpose we take $x = 0, a = 2m, t = 1s, b = 0.1, k = 1.2, \mu_0 = 1.2Hm^{-1}$.

In the figures red color represents the first value, blue color represents second value and green color represents third value.

In Fig. 1(a) the effect of magnetic field on u with respect to z is presented. It is observed that u decreases with the increase of magnetic field. So the value of u in the presence of a magnetic field will be less than the value of u in the absence of a magnetic field. The value of u for GN-II model is greater than the value of u for GN-III model. In Fig. 1(b) the effect of electric field on u with respect to z is presented. It is observed that u decreases with the increase of electric field. The value of u for GN-II model is greater than the value of u for GN-III model. u is converging towards zero with the increase of z. In Fig. 1(c) the effect of phase velocity on u with respect to z is presented. It is observed that u decreases with the increase of phase velocity. The value of u for GN-II model is greater than the value of u for GN-III model. u is converging towards zero with the increase of z.



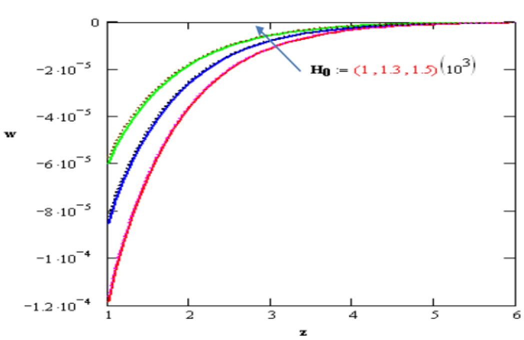
(a) (b)



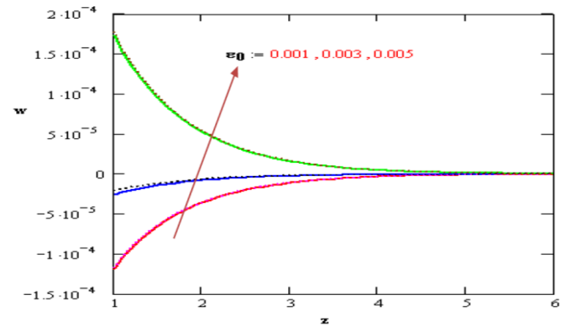
(c)

Fig.1
Variation of magnetic field, electric field, and phase speed on Horizontal displacement ***u*** with respect to ***z***: (GN III ___, GN II ……..).

In Fig. 2(a) the effect of magnetic field on w with respect to z is presented. It is observed that w decreases with the increase of magnetic field. So the value of w in the presence of a magnetic field will be less than the value of w in the absence of a magnetic field. w is converging towards zero with the increase of z. The value of w for GN-II model is almost same with the value of w for GN-III model. In Fig. 2(b) the effect of electric field on w with respect to z is presented. It is observed that w remains same with the increase of electric field. The value of w for GN-II model is almost same with the value of w for GN-III model. w is converging towards zero with the increase of z. In Fig. 2(c) the effect of phase velocity on w with respect to z is presented. It is observed that w increases with the increase of phase velocity. The value of w for GN-III model is greater than the value of w for GN-II model.



(a)



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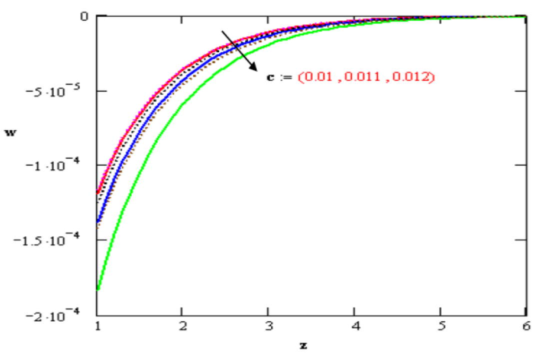
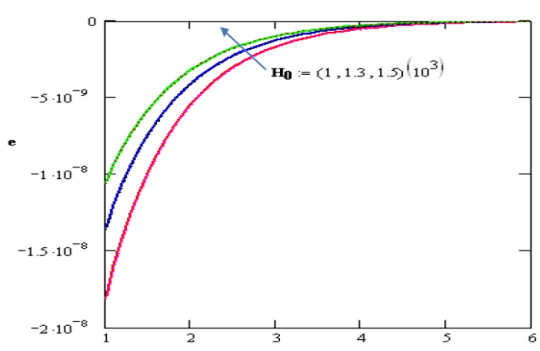


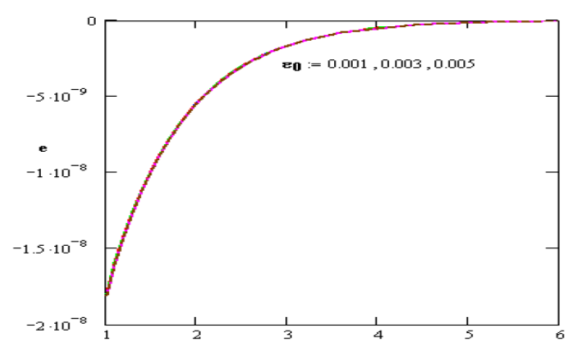
(c)

Fig.2
Variation of magnetic field, electric field and phase speed on vertical displacement ***w*** with respect to ***z***: (GN III ___, GN II ……..).

In Fig. 3(a) the effect of magnetic field on e with respect to z is presented. It is observed that e decreases with the increase of magnetic field. So the value of e in the presence of a magnetic field will be less than the value of e in the absence of a magnetic field. e is converging towards zero with the increase of z. The value of e for GN-II model is almost same with the value of e for GN-III model. In Fig. 3(b) the effect of electric field on e with respect to z is presented. It is observed that e remains same with the increase of electric field. The value of e for GN-II model is almost same with the value of e for GN-III model. e is converging towards zero with the increase of z. In Fig. 3(c) the effect of phase velocity on e with respect to z is presented. It is observed that e increases with the increase of phase velocity. The value of e for GN-II model is greater than the value of e for GN-III model.



(a)



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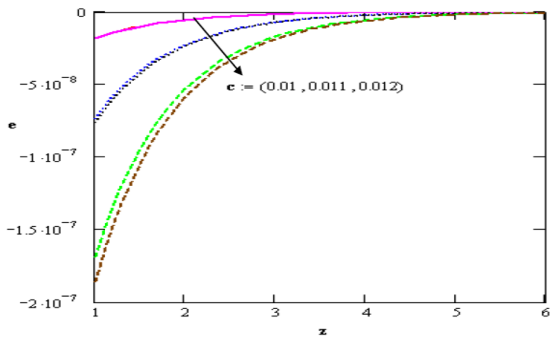
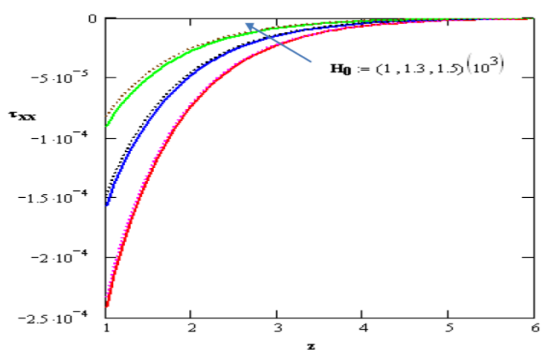


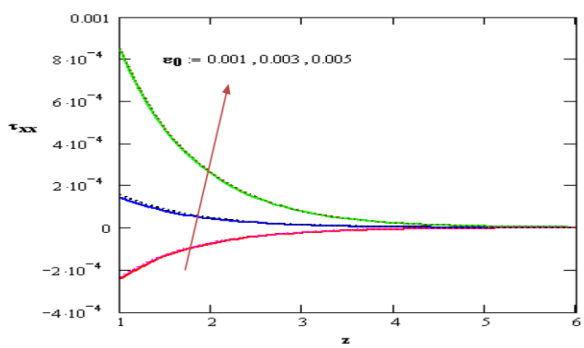
(c)

Fig.3
Variation of magnetic field, electric field and phase speed on volume expansion ***e*** with respect to *z*: (GN III ___, GN II ……..).

In Fig. 4(a) the effect of magnetic field on τ_{xx} with respect to z is presented. It is observed that τ_{xx} decreases with the increase of magnetic field. So the value of τ_{xx} in the presence of a magnetic field will be less than the value of τ_{xx} in the absence of a magnetic field. τ_{xx} is converging towards zero with the increase of z. The value of τ_{xx} for GN-III model is greater than the value of τ_{xx} for GN-II model. In Fig. 4(b) the effect of electric field on τ_{xx} with respect to z is presented. It is observed that τ_{xx} remains same with the increase of electric field. The value of τ_{xx} for GN-II model is almost same with the value of τ_{xx} for GN-III model. τ_{xx} is converging towards zero with the increase of z. In Fig. 4(c) the effect of phase velocity on τ_{xx} with respect to z is presented. It is observed that τ_{xx} increases with the increase of phase velocity. The value of τ_{xx} for GN-III model is greater than the value of τ_{xx} for GN-II model.



(a)



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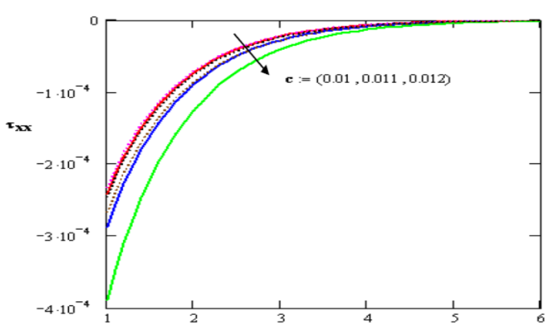
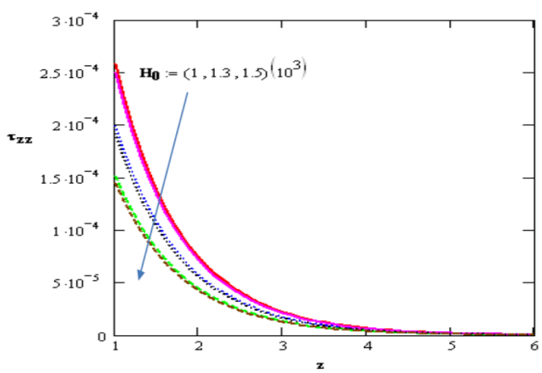


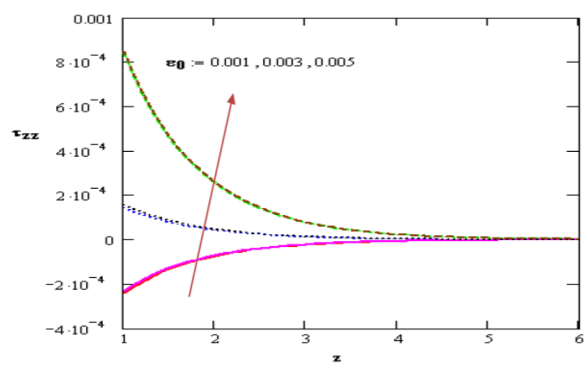
(c)

Fig.4
Variation of magnetic field, electric field and phase speed on normal stress $\boldsymbol{\tau}_{xx}$ with respect to *z*: (GN III ___, GN II ……..).

In Fig. 5(a) the effect of magnetic field on τ_{zz} with respect to z is presented. It is observed that τ_{zz} decreases with the increase of magnetic field. So the value of τ_{zz} in the presence of a magnetic field will be less than the value of τ_{zz} in the absence of a magnetic field. τ_{zz} is converging towards zero with the increase of z. The value of τ_{zz} for GN-III model is almost same with the value of τ_{zz} for GN-II model. In Fig. 5(b) the effect of electric field on τ_{zz} with respect to z is presented. It is observed that τ_{zz} increases with the increase of electric field. The value of τ_{zz} for GN-II model is almost same with the value of τ_{zz} for GN-III model. τ_{zz} is converging towards zero with the increase of z. In Fig. 5(c) the effect of phase velocity on τ_{zz} with respect to z is presented. It is observed that τ_{zz} increases with the increase of phase velocity. The value of τ_{zz} for GN-II model is greater than the value of τ_{zz} for GN-III model.



(a)



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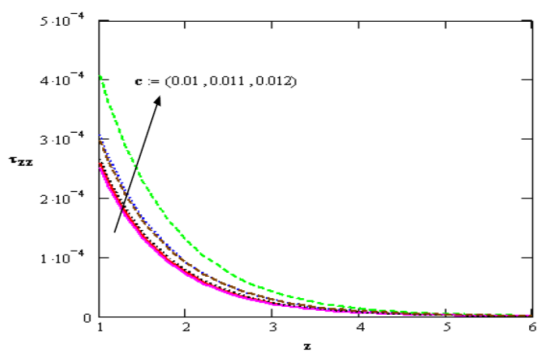
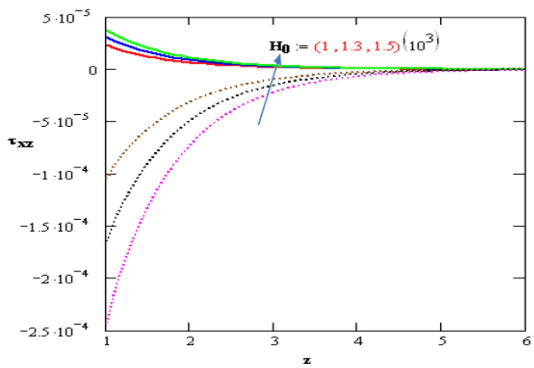


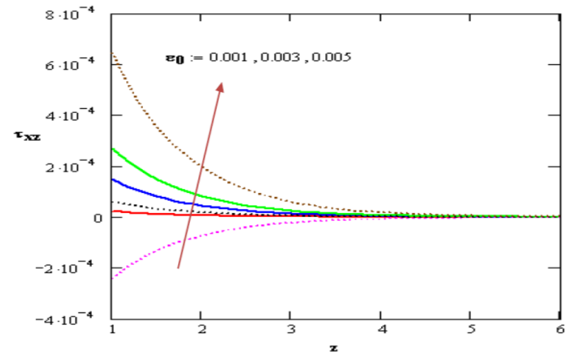
(c)

Fig.5
Variation of magnetic field, electric field and phase speed on normal stress $\boldsymbol{\tau}_{zz}$ with respect to z: (GN III ___, GN II ……..).

In Fig. 6(a) the effect of magnetic field on τ_{xz} with respect to z is presented. It is observed that τ_{xz} increases with the increase of magnetic field. So the value of τ_{xz} in the absence of a magnetic field will be less than the value of τ_{xz} in the presence of a magnetic field. τ_{xz} is converging towards zero with the increase of z. The value of τ_{xz} for GN-III model is almost same with the value of τ_{xz} for GN-II model. In Fig. 6(b) the effect of electric field on τ_{xz} with respect to z is presented. It is observed that τ_{xz} increases with the increase of electric field. The value of τ_{xz} for GN-II model is greater than the value of τ_{xz} for GN-III model. τ_{xz} is converging towards zero with the increase of z. In Fig. 6(c) the effect of phase velocity on τ_{xz} with respect to z is presented. It is observed that τ_{xz} increases with the increase of phase velocity. The value of τ_{xz} for GN-III model is greater than the value of τ_{xz} for GN-II model.



(a)



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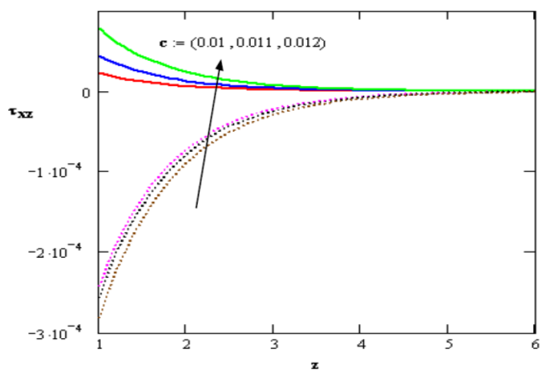

(c)

Fig.6
Variation of magnetic field, electric field and phase speed on shear stress τ_{xz} with respect to z: (GN III ___, GN II ……..).

In Fig. 7 the effect of phase velocity on T with respect to z is presented. It is observed that T increases with the increase of phase velocity. The value of T for GN-II model is greater than the value of T for GN-III model. T is converging towards zero with the increase of z

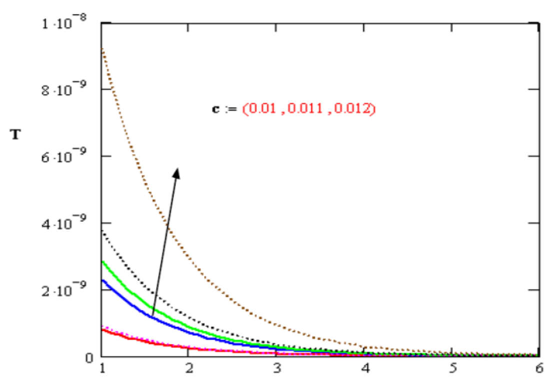

Fig.7
Variation of phase speed on Temperature T with respect to z: (GN III ___, GN II ……..).

9 CONCLUSIONS

Two-dimensional problem of generalized thermoelasticity has been formulated with state space approach and the problem is treated under Green-Naghdi type-III model i.e., with energy dissipation theory of thermoelasticity in the presence of a magnetic field. The effect of magnetic field, electric field and phase velocity on the considered parameters is observed in the figures.

From theoretical and numerical discussion the following concluding remarks can be drawn:

All the considered parameters decrease with the increase of z and they are converging towards zero.

(a) Displacements, dilatation, τ_{xx} and τ_{zz} decrease with the increase of magnetic field.
(b) All parameters except u increase with the increase of phase velocity.
(c) w, e and τ_{xx} remain same with the increase of electric field but τ_{zz} and τ_{xz} increase with the increase of electric field.
(d) There is significant change in some parameters for GN-III and GN-II models. The value of τ_{xx} and τ_{xz} for GN-III model is greater than the value of τ_{xx} and τ_{xz} for GN-II model. The value of τ_{zz} for GN-II model is greater than the value of τ_{zz} for GN-III model.
(e) The value of T for GN-II model is greater than the value of T for GN-III model.

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