Existence Of Excitatory and Inhibitory Oscillators in The Small World Network and Its Dynamic Effect on Network Synchronization

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ABSTRACT:
Synchronization was investigated in Watts-Strogats small world network with inhibited and excitable oscillators. According to the Kuramoto model in the small world network, with the increase in the limited number of inhibited oscillators, the synchronization in the system will be accompanied by network defects, and with their increase, the synchronization will also increase, and after reaching its maximum value, it will begin to decrease. That is, with a certain ratio of inhibitory oscillators to excitation depending on the coupling strength, network synchronization is maximum. As the coupling strength of the oscillators increases, the interval of the number of inhibitions for which the network is in synchronization decreases. This result is not related to a specific small world network and has been observed by repeating it in different small world networks. Excitatory and inhibitory oscillators are in phase up to a certain percentage of inhibitory oscillators in the network (depending on the coupling strength).

KEYWORDS: Kuramoto, Synchronization, Small World Network, Inhibitory Oscillator, Excitatory Oscillator.

1. INTRODUCTION

Since 1950, network science has become a living and interdisciplinary field. Today, networks play an important role for research in various fields, including social sciences, economics and psychology, biology, physics and mathematics [1,2] and as a forward-looking concept, it is used to describe the interactions of many systems. Several network models have been developed that have statistical properties consistent with real-world networks. In particular, we can mention random networks or René camp, small world network in network science.

Real-world networks such as brain networks, electrical networks, etc. [3] are characterized by a high clustering coefficient. Also, despite the large size, there is often a relatively short path between both nodes. Strugats presented a model with small-world network [3] that exhibits both features, small shortest path length and high clustering factor. These features are known as the small world feature, which consists of a regular network and is rewired with the probability p of edges, which is from 0.005 to 0.05 and is between the regular network ($p = 0$) and the random network ($p = 1$).

One of the main topics of network dynamics is synchronization [4,5]. Synchronization can be seen in many different contexts. In computer science, such as synchronization has been used to extract data in a large database [6]. Other applications in engineering where synchronization or asynchrony are important, such as wireless communication networks [7] and electric networks [8].

By simulating his model, Winfrey [9] found that spontaneous synchronization appears as a threshold process, a phenomenon similar to phase transition, and in his studies and Kuramoto [10], it is stated that the start of synchronization

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in Oscillating populations represent phase transitions; Below the transition point, the individual movement of oscillators in a group is uncorrelated. As their interactions become stronger, the connections between the dynamic modes of the oscillators in one part of the set are established and the frequencies of these oscillators become the same. Near the transition point, the size of the coherent oscillator group is small, but the group grows and the number of interacting oscillators increases. The size of this group can be chosen as a synchronization parameter.

Based on Winfrey's method, The Kuramoto model consists of a population of phase oscillators whose interaction is determined by differential equations [11,12] and expresses the rotation of oscillators with heterogeneous natural frequency that are coupled in the form of phase difference sinusoids.

The paper is organized as follows. In Sect. II, we define the model and the numerical methods of quantifying the synchronization. Sect. III represents the results and discussion and sect. IV is devoted to the concluding remarks.

2. MODEL AND METHOD

We used the Kuramoto model in a network with N oscillators at the nodes of the network, which include two groups. One group (excitatory oscillators) has positive coupling and tries to be in phase with its neighbor, and another group (inhibitory oscillators) tries to be in the opposite phase (π) with it.

Therefore, in the Kuramoto equations:

$$\frac{d\theta_i^s}{dt} = \omega_0 + \frac{1}{k_i}\sum_{j=1}^{N} a_{ij}\lambda_j^s \sin\left(\theta_j - \theta_i^s\right), \quad i = 1, \dots, N \qquad (1)$$

In this equation, θ_i is the phase of the ith oscillator, ω_0 is the intrinsic frequency of the oscillators, which are equal and zero without losing any generality. a_{ij} are the elements of the adjacency matrix, where $a_{ij} = 1$, if oscillator i and j are connected, otherwise $a_{ij} = 0$, and k_i is the degree of node ith. λ_i^s is the coupling constant of the ith oscillator (s indicates excitatory and inhibitory) which λ_j^{excit} is positive if the oscillator is excitatory and λ_j^{inhib} is negative if the oscillator is inhibitory. Assuming $Q > 0$ and λ_j^{excit}, we will have: $\lambda_i^{inhib} = -Q\lambda_i^{excit}$, also $\tau = \lambda_i^f t$.

We determine the degree of synchronization of all oscillators in each time interval by the order parameter:

$$\mathrm{r}(\tau) = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j(\tau)} \qquad (2)$$

we define the longtime averaged order parameter in the stationary state as:

$$r_\infty = \lim_{\Delta\tau\to\infty} \frac{1}{\Delta\tau}\int_{\tau_s}^{\tau_s+\Delta\tau} r(\tau)d\tau \qquad (3)$$

in which τ_s is the time of reaching a stationary state.

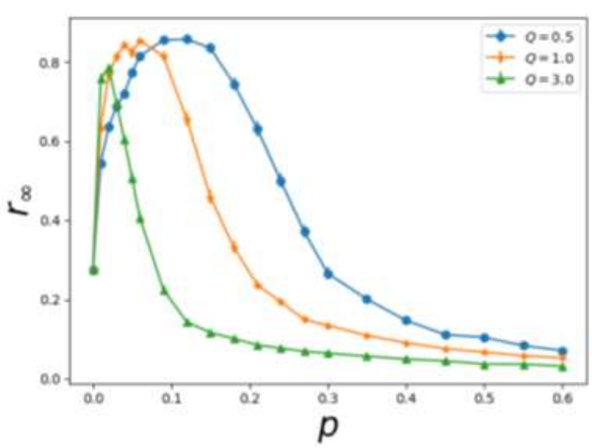


Fig. 1. (Color online) Stationary order parameters verses fraction of inhibitory oscillators for the excitatory-inhibitory model for Q=0.5,Q=1 and Q=1.5 for small world (with the probability of rewiring 0.03) networks of N=1000 oscillators and mean degree ⟨k⟩=10. The error bars indicate the standard error of mean (SEM).

3. RESULTS AND DISCUSSION

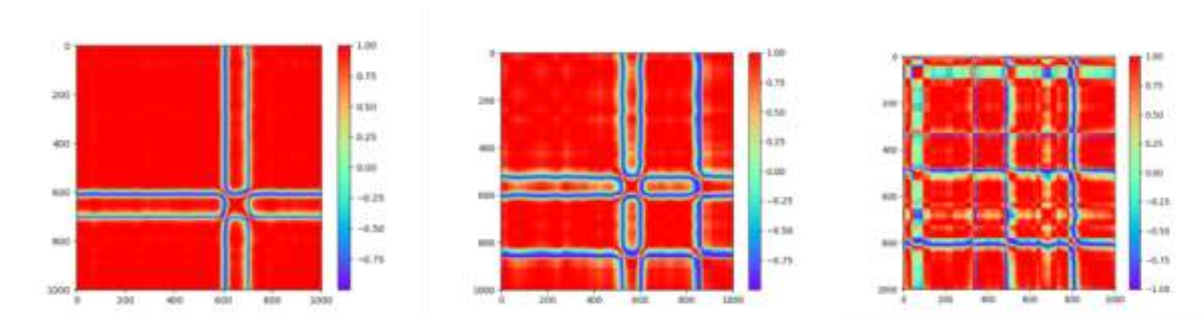
We considered the model for a small world network with 1000 nodes and an average degree of 10. Note that all edges are bidirectionally selected and we denote the ratio of inhibitory oscillators to excitatory oscillators by p. To obtain the time evolution of the oscillators, we used the fourth-order Rangkota method with a time step of 0.1 and considered the initial phase of the oscillators from the box diagram in the interval [π, -π], and the natural frequency distribution of the oscillators follows the Lorentz distribution function.

We obtained the average order parameter for 150 runs for the small world network and different initial conditions and 30 runs for 10 different networks and different initial conditions. ∽8×10^5time step calculations have been done and from this number, 1000 final steps have been kept and averaged. In the calculations, it can be seen that the time step of the network is about ∽$6x10^5$ and reaches a stable state.

As it is thought, the power of coupling of inhibition and excitation oscillators can be effective in the order parameter and thus network synchronization by increasing the number of inhibition oscillators in the network up to a certain percentage. In Fig. 1, three states are considered for Q: $Q < 1$, $Q = 1$, $Q > 1$. By setting γ=0 in the model, the order parameter is drawn in terms of p. For all three modes, the initial phase of the oscillators is the same. With the increase of inhibitory nodes in the network, the order parameter increases and then decreases, and the higher the coupling strength of the nodes, the faster this decrease and the resistance of the network for synchronization is lower. This turning point depends on the value of Q and decreases with increasing Q.

It can also be seen in Fig. 1 when there is no inhibitory oscillator in the network, the network has not reached full synchronization and what is shown in this figure is the average of 10 initial conditions. By increasing the inhibited oscillators in the small world network in a certain range, not only the order parameter does not decrease, but the order parameter increases up to a certain percentage of the inhibited oscillators.

For a better review, the correlation matrix for $Q = 0.5$ is drawn in Fig. 2. In this Figure, it can be seen that network defects are seen for percentages of the inhibited oscillator which is the maximum order parameter. At first, when the percentage of inhibited oscillators in the network increases from zero, the network defects decrease and in fact the network becomes more regular. Then, with the increase of inhibitory oscillators, network defects increased and for higher percentages (for network with $Q = 0.5$, for $p > 0.18$, for network with $Q = 1$, for $p > 0.1$ and for network with $Q = 3$, for $p > 0.03$) disappears. In fact, around the maximum order parameter, the oscillators are divided into two groups that are in opposite phase (π).

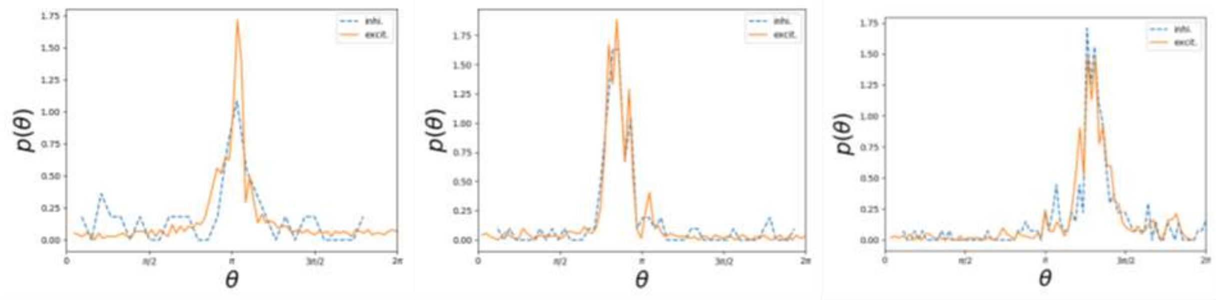


(a) (b) (c)

Fig. 2. (Color online) correlation matrix of inhibitory and excitatory oscillators for (a) p 0.03,(b)p=0.09 and (c)p=0.18 in a small world network of N=1000 oscillators, mean degree ⟨k⟩=10 and Q=0.5. p is the fraction of inhibitory to excitatory oscillators.

With the investigation, we came to the conclusion that the group of oscillators that are in opposite phase with other oscillators and cause network defects, are not only inhibited oscillators and include both inhibited and excited oscillators. The presence of a small percentage of the inhibited oscillator causes a number of oscillators to be in opposite phase with the rest of the oscillators and even for a certain percentage, they create a higher order in the network.

The phase density of the oscillators after the network reached a stable state, separately (inhibitory and excitatory) is drawn in Fig. 3 for Q=0.5 in the small world network for the percentages presented respectively in Fig. 2. As can be seen in these figures, for the percentage of inhibition oscillators that we observed network defects, the phase density diagrams of inhibition and excitation oscillators are in phase.



(a) (b) (c)

Fig. 3. (Color online) The probability density function of the phase of inhibitory and excitatory oscillators for (a) p= 0.03,(b)p=0.09 and (c)p=0.18 in a small world network of N=1000 oscillators, mean degree ⟨k⟩=10 and Q=0.5. p is the fraction of inhibitory to excitatory oscillators.

4. CONCLUSION

In summary, By using the Kuramato model in the small world network and defining inhibitory and excitatory oscillators, we found that the excitatory and inhibitory oscillators are always in phase. We also observed an increase in synchrony by increasing the fraction of inhibitors in the SW network, where the number of inhibitory oscillators to maximize synchrony depends on the coupling strength of the oscillators.

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