

# Increasing Spectral Efficiency of GFDM with Adaptive Modulation and Coding for Next Generation Cellular Networks

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## ABSTRACT:

As a result of resource constraints in each generation of wireless systems, new technologies should be used in order to obtain maximum system efficiency. The combination of Adaptive Modulation and Coding (AMC) with Generalized Frequency Division Multiplexing (GFDM) is a promising technique which efficiently optimizes the data rate, increases the capacity and reduces Bit Error Rate (BER) of the systems, which are primary goals of each generation of wireless networks. In paper, we primarily analyze multicarrier GFDM performance based on adaptive modulation using a discrete rate, then examine a variable power transmission scheme and finally find the form of rate and power which maximizes the spectral efficiency. The results show that significant improvement in terms of spectral efficiency and BER can be achieved demonstrating the superiority of the proposed AMC scheme in comparison with nonadaptive transmission schemes. Therefore, the presented system can be used in next generation cellular networks with higher capacity.

**KEYWORDS:** Next Generation Cellular Network, Spectral Efficiency, AMC, GFDM, Convolutional Coding.

## 1. INTRODUCTION

Emerging new applications and services in mobile networks, such as tactile internet [1], Machine to Machine (M2M) communication [2] and other of this kind, will increase the number of wireless devices and sensors considerably. 6G wireless networks need to be capable of handling this highly increasing the number of networks a lot participants as well as significant growth in data volume. Sporadic random access from devices can be reached by the use of Cognitive Radio (CR). In CR systems, flexibility in structuring and shaping of the transmit signal is vital to exploitation of the available spectrum white spaces. The Out of Band (OOB) leakage of CR systems should be minimized in order to decrease interface between adjacent frequency bands [3], [4].

Current cellular transmission schemes such as fifth generation (5G) and Long-Term Evolution (LTE) [5] employ Orthogonal Frequency Division Multiplexing (OFDM) scheme [6] for its advantages like robustness in fading channels [7] and implementation based on Fast Fourier Transform (FFT) algorithms [8]. Despite these benefits, OFDM is not well suited for future requirement regarding its disadvantage such as reduced spectral

efficiency owing to requirement of a cyclic prefix (CP) [9] and demanding a fine synchronization for its sensitivity in Carrier Frequency Offset (CFO) [10]; It is also unattractive for CR application for its large OOB leakage [11]. Consequently, a new multicarrier transmission scheme has been proposed in order to reduce the disadvantages of OFDM and spectral efficiency. One of the most promising candidate waveforms in the next generation of wireless network systems is GFDM [12].

GFDM is a block-based multicarrier transmission scheme which is taken from the filter bank technique [13] and was first introduced as a solution to be used in white spaces, E.g. across the bands of TV for CR systems [14]. The transmitted data of each block in GFDM is decomposed into frequency and time domains and each subcarrier is pulse-shaped with an adjustable filter. In this system the length of the CP is reduced by implementing alternative cyclic pulse shaping filters instead of orthogonal filters [14]. The insertion of CP in each block instead of each symbol increases the spectral efficiency of GFDM compared to OFDM. OOB leakage and self- interference can be controlled by a pulse

shaping filter in this system [15]. The effect of pulse shaping on the OOB was studied in [16], then the closed form of SER, OOB, and Noise Enhancement Factor (NEF) and spectral efficiency were obtained when using Zero-Forcing (ZF) receiver.

The purpose of this paper is to increase the spectral efficiency of the GFDM system by optimally varying combination rate and power in Additive White Gaussian Noise AWGN channel. This goal is obtained when the channel state information of wireless communication systems is available with feedback from a receiver to the transmitter. The channel state information allows us to send symbols based on the Channel Quality Indicator (CQI). We assume that a discrete finite set of constellations and code rates are available. The analysis is done in both an instantaneous and an average power constraint subject to an average BER and power constraint.

Then this paper is organized as follows: In Section two, the system model is presented, then the BER approximation used to drive the optimal solution of adaptive modulation is described. In Section three, derive the optimal modulation mode switching, and power adaption under restricted target BER and average transmit power. Numerical results and plots of BER in both uncoded and convolutional coded systems along with the spectral efficiency of them are illustrated in section four. Ultimately conclusions are given in section five.

## 2. SYSTEM MODLE

In this section the GFDM transmission block diagram is introduced and then the proposed AMC system model to achieve the main goals in this paper is described.

### 2.1. GFDM Block Diagram

Consider the block data bits vector  $\vec{b}$  are encoded to obtain  $\vec{b}_c$ . Then a mapper, E.g.  $2^\mu$ -QAM, maps the bits to the symbols where  $\mu$  is modulation order. The resulting vector  $\vec{d}$  denotes a data block with  $N$  elements. Each element can be distributed into  $K$  subcarriers with  $M$  subsymbols according to  $\vec{d} = (\vec{d}_0^T, \dots, \vec{d}_{M-1}^T)^T$  and  $\vec{d}_m = (\vec{d}_{0,m}, \dots, \vec{d}_{K-1,m})^T$ . The total number of symbols follows as  $N = KM$ . Therein, each  $d_{k,m}$  corresponds to the data symbol that has to be transmitted on the  $k^{th}$  subcarrier in  $m^{th}$  time slot. Each symbol is carried by the waveforms given by [17]:

$$g_{k,m}[n] = g[(n - mK) \bmod N] e^{-j2\pi n \frac{k}{K}} \quad (1)$$

Where  $g_{k,m}[n]$  is the time and frequency shifted type of the prototype filter  $g[n]$ . The superposition of all modulated subcarriers and subsymbols leads to the GFDM signals [18]:

$$x[n] = \sum_{k=0}^{K-1} \sum_{m=0}^{M-1} g_{k,m}[n] d_{k,m}, \quad n = 0, 1, \dots, N-1 \quad (2)$$

Equation (2) can be modeled as a matrix operation given by [17]:

$$\vec{x} = A \vec{d} \quad (3)$$

Where  $A$  is a  $N \times N$  transmitter matrix [17] with a structure according to:

$$A = (\vec{g}_{0,0} \dots \vec{g}_{K-1,0} \vec{g}_{0,1} \dots \vec{g}_{K-1,M-1}) \quad (4)$$

$\vec{g}_{m,n}$  is circularly frequency and time shifted types of  $\vec{g}_{0,0}$ . So  $\vec{x}$  includes the transmit samples that correspond to the GFDM data block  $\vec{d}$ .

By transmission through a wireless channel, received counterpart of  $\vec{x}$  is modelled by  $\vec{y} = H\vec{x} + \vec{w}$  where  $\vec{w} \sim CN(0, \sigma^2 I_{N \times N})$  is AWGN noise, and  $H$  denotes the channel matrix. In this paper we assume  $H = I_{N \times N}$  in corresponding to AWGN channel model.

At the receiver side, time and frequency synchronization is applied. By utilizing a ZF equalizer to remove self-generated interference, the signal after linear GFDM demodulation can be given by  $\vec{\tilde{d}} = B\vec{y}$  where  $B = A^{-1}$ . The received symbols  $\vec{\tilde{d}}$  are demapped to produce a sequence of bits  $\vec{\tilde{b}}_c$  then passed to a decoder to obtain  $\vec{\tilde{b}}$ . The SER performance of GFDM over AWGN channel is achieved by [17]:

$$SER = 2\left(\frac{L-1}{L}\right) \text{erfc}(\sqrt{\beta}) - \left(\frac{L-1}{L}\right)^2 \text{erfc}^2(\sqrt{\beta}) \quad (5)$$

Where  $L = \sqrt{2^\mu}$  and  $\beta$  is a function of average energy per symbol ( $E_s$ ) and noise power density ( $N_0$ ) which is defined as:

$$\beta = \frac{3}{2(2^\mu - 1)} \frac{E_s}{\xi N_0} \quad (6)$$

$\xi$  denotes NEF which determines Signal to Noise Ratio (SNR) decrease when using the ZF receiver and it is equal to  $\sum_{i=0}^{MK-1} | [B_{ZF}]_{k,n} |^2$ .

### 2.2. AMC GFDM

Block diagram for proposed AMC GFDM system model is displayed in Fig. 1. Discrete-time channel with ergodic and stationary time-varying gain  $\sqrt{h[i]}$  for each GFDM symbol is assumed.  $\bar{s}$  denotes average transmit signal power,  $B$  denotes the received signal bandwidth, and  $\bar{h}$  denotes the average channel power gain. The instantaneous channel state information of each GFDM block is known at the transmitter with a feedback path from the receiver to the transmitter which is supposed to be error-free and instantaneous, so the channel gain estimation  $\hat{h}[i] = h[i]$ . The transition rate and power can be adapted at the receiver. Channel quality estimation of the each GFDM symbols is measured at the receiver. At the transmitter, the AMC block comprises of different modulator modes and coding rates. The best modes and rates can be chosen for the next transmission GFDM symbol by the channel quality estimate E.g. SNR of the receiver based on the current GFDM symbol.

$\gamma[i]$  is defined as the instantaneous received SNR at time  $i$ . The rate region boundaries  $\{\gamma_i\}_{i=0}^{Z-1}$  define the ranges of  $\gamma$  values over which the variation modes and code rates are transmitted. One modulation mode and code rate is used for each region  $[\gamma_i, \gamma_{i+1})$  for  $0 \leq i \leq Z-1$ , where  $\gamma_Z = \infty$ . When the instantaneous SNR  $\gamma$  falls in a given region, the corresponding signal modulation order and code rate are transmitted.  $\gamma_0$  is a cutoff SNR which the channel is not used within ranges.

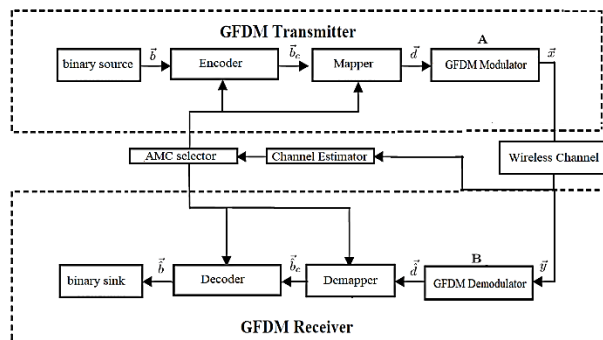


Fig. 1. GFDM system with AMC

To obtain optimal rate and power adaptations, we require an expression for BER in AWGN channel for GFDM system that should be easily converted to rate and power. Unfortunately, for the majority non-binary

modulation techniques an exact expression for BER is not easy to find. Often the BER with Gray bit mapping at high SNR is approximated as SER divided by the number of bits per symbol and  $E_s/N_0$  is defined as  $SNRT_{sym}/T_{samp}$ , where  $T_{sym}$  is signal's symbol period and  $T_{samp}$  is signal's sampling period [19]. Therefore, we introduce new tight BER approximation as a function of SNR for the system which can be differentiated and inverted. By substituting SNR in (6),  $\Gamma$  is defined as  $3SNRT_{sym}/2\xi(2^u-1)T_{samp}$ . We approximate  $erfc(\sqrt{\Gamma})$  to  $\exp(-\Gamma)/(\sqrt{\pi\Gamma})$  for high SNR's [18] and rewrite (5). The BER expression of GFDM system with Gray bit mapping in AWGN channel can be approximated by:

$$BER = \frac{(L-1)^2}{\mu\pi L^2} \left( \frac{2L\sqrt{\pi}}{L-1} u - u^2 \right) \quad (7)$$

Where  $L$  is a function of the modulation order and  $u$  is a function of SNR and is equal to  $\exp(-\Gamma)/\sqrt{\Gamma}$ . In the following, (7) is used for BER of the system to get the boundaries level and choose the optimum transmission modes.

### 3. SWITCHING THRESHOLDS

In this section, we derive the optimal solution for power adaption and optimum adaptive modulation mode switching levels under assigned target BER and average transmit power conditions so that the spectral efficiency of the system is maximized. In this paper the analysis is apply for AWGN channel, assuming uniform distribution for SNR. The spectral efficiency of the system is its average data rate per unit bandwidth (R/B). System sends  $\mu MK$  bits in  $MT_s$  second, where  $T_s$  is symbol time. By assuming RC pulse shape filter which is a Nyquist data pulse, the bandwidth of the received signal is equal to  $B = K/T_s$ . Consequently, the average spectral efficiency of discrete rate GFDM is the summation of data rates in each region  $\mu_i$  correlated with the each of the  $Z$  regions, weighted by the probability of  $\gamma$  when it falls in  $(Z-1)^{th}$  region [19].

$$\eta_B = \frac{R}{B} = \sum_{i=0}^{Z-1} \mu_i \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma \quad \text{bit/sec/Hz} \quad (8)$$

We also assumed an average transmit power( $\bar{s}$ ) limitation between modes is determined by [20]

$$\sum_{i=0}^{Z-1} \int_{\gamma_i}^{\gamma_{i+1}} s_i(\gamma) p(\gamma) d\gamma \leq \bar{s} \quad (9)$$

Where  $S_i$  is instantaneous transmit power for each region.

### 3.1. Optimal Rate

We assign the rate  $\mu_i$  to the rate region  $[\gamma_i, \gamma_{i+1})$  and the rate set is changeable within the limited set  $\{\mu_i\}_{i=0}^{i=Z-1}$ . The maximizing spectral efficiency problem is consider through the optimal rate, under instantaneous BER constrain  $BER(\gamma) = \overline{BER}$  and constant transmit power. We rewrite (7) as a function of  $u$ . By solving (7) in terms of  $u$  the roots of second order are obtained:

$$u_{1,2} = \frac{L\sqrt{\pi}}{L-1} \left(1 \pm \sqrt{1 - \mu BER}\right) \quad (10)$$

BER of the transmitted system should be very small, so  $L\sqrt{\pi}(1 + \sqrt{1 - \mu BER}) / L - 1$  is acceptable. By replacing  $\beta$  instead of  $u$  and substituting (10) in (6) the optimal rate region boundaries are obtained:

$$\gamma_i = \frac{(L_i^2 - 1)\xi}{3} W\left(\frac{2}{u_i^2}\right) \quad (11)$$

Which is equal to SNR.  $W(*)$  Denotes the real-valued Lambert's  $W$ -function that is defined as the inverse of function  $f(W) = we^w$  for  $w \geq 0$ . As a result, the unique solution for  $\gamma_i$  can be found. Assuming the uniform distribution of  $\gamma$  for AWGN channel, the spectral efficiency is Cumulative Distribution Function (CDF) in (8) with the obtained boundaries in (11).

### 3.2. Optimal Rate and Power

With the same discrete rate set as a previous section, now we want to optimize spectral efficiency through optimal rate and power subject to average power and instantaneous BER constraints  $BER(\gamma) = \overline{BER}$ . For a stable transmit power  $\bar{s}$ , the instantaneous received SNR is  $\gamma[i] = \bar{s}h[i] / \sigma^2$ . We define the transmit power at time  $i$ , by  $s(\gamma[i])$  which is a function of  $\gamma[i]$ . The received SNR at time  $i$  is then  $\gamma[i](s(\gamma[i]) / \bar{s})$ . As  $h[i]$  is stationary, the distribution of  $\gamma[i]$  is not related to  $i$ . In the following we ignore the time reference  $i$  relative to  $\gamma$  and  $s(\gamma)$ . Under these conditions the maximize power adaptation is obtained with substituting received  $SNR = \gamma(s(\gamma) / \bar{s})$  in (11):

$$\frac{s(\gamma)}{\bar{s}} = \frac{y(\mu_i)}{\gamma} \quad (12)$$

Where  $y(\mu_i)$  is equal to optimal rate region boundaries in constant power (11). We find the optimal rate region boundaries that optimal spectral efficiency for power and rate constraint using the Lagrangian method. Under the above power adaptation strategy, the Lagrange equation is given as:

$$J(\gamma_1, \gamma_2, \dots, \gamma_Z) = \sum_{i=0}^{Z-1} \mu_i \int_{\gamma_i}^{\gamma_{i+1}} p(\gamma) d\gamma + \lambda \left[ \sum_{i=0}^{Z-1} \int_{\gamma_i}^{\gamma_{i+1}} \frac{y(\mu_i)}{\gamma} p(\gamma) d\gamma - 1 \right] \quad (13)$$

Where  $\lambda$  is the Lagrange multiplier. The general function of maximized mode switching levels can be obtained by solving the following equation for  $\gamma_i$ .

$$\frac{\partial J}{\partial \gamma_i} = 0 \quad 0 \leq i \leq Z-1 \quad (14)$$

which results

$$\begin{cases} \gamma_i = \frac{y(\mu_i)}{\mu_i} \lambda & i = 0 \\ \gamma_i = \frac{y(\mu_i) - y(\mu_{i-1})}{\mu_i - \mu_{i-1}} \lambda & 1 \leq i < Z-1 \end{cases} \quad (15)$$

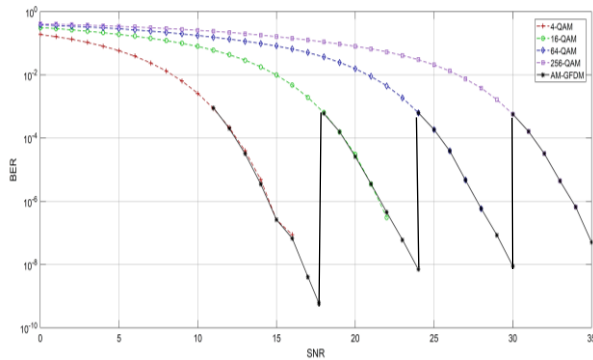
Where the constant  $\lambda$  can be determined numerically such that the AMC mode switching levels achieve the average power limitation in (9). With these obtained boundaries the spectral efficiency is given by:

$$\eta = \sum_{i=0}^{Z-1} \frac{\mu_i}{(\gamma_{Z-1} - \gamma_0)} \gamma \Big|_{\gamma_i}^{\gamma_{i+1}} \quad (16)$$

## 4. NUMERICAL RESULTS

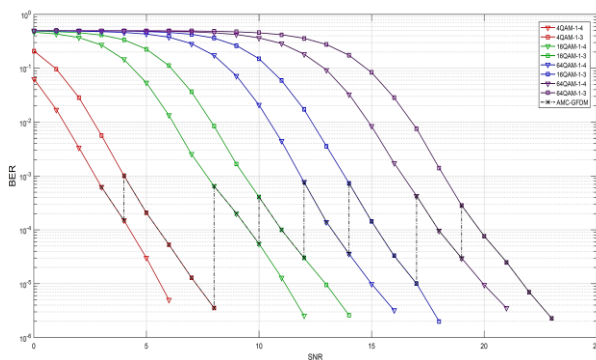
Our derivations are for general distributions however, we calculate our numerical results for uniform distribution in the AWGN. We suppose BER requirement is  $10^{-3}$ . and assume that 4 different modulation order responding to 4-QAM, 16-QAM, 64-QAM and 256-QAM bit/symbol with 2 different convolutional coding rates  $\frac{1}{3}$  and  $\frac{1}{4}$  are available. The generation matrixes are assumed [133 171 165] and [133 171 165 155] for  $\frac{1}{3}$  and  $\frac{1}{4}$  code rates respectively. Fig.2 shows the BER of GFDM transmitter for  $K = 64$ ,  $M = 9$  and a RC filter by roll-off 0.5 for GFDM system. Fig.2 shows, the BER performance adaptive modulation compare fixed modulation order system is the best. It can be seen that

the BER performance of the adaptive scheme is equal to 4-QAM for SNR ranges (11-18 dB), for SNR ranges (18-24 dB) the BER performance in adaptive scheme is like 16-QAM, also for SNR ranges (24-29 dB) BER performance in adaptive modulation is like 64-QAM. After 29 dB BER performance in adaptive scheme is like 256-QAM. Instantaneous BER in each region is  $10^{-3}$ .



**Fig. 2.** BER for fixed mode and adaptive modulation systems

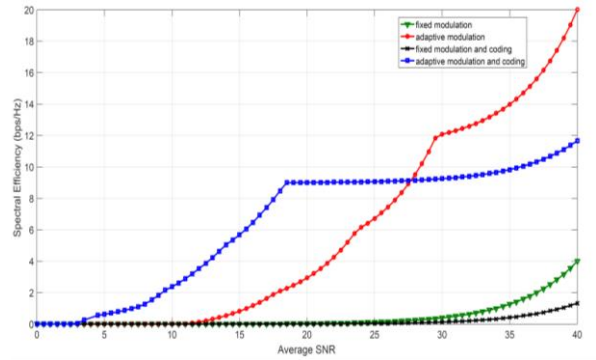
Fig.3 shows, the BER performance by employing convolutional coding in GFDM modulation and compares it to AMC system. As can be seen utilization of coding can improve the BER performance and provide a considerable coding gain. Additionally, the figure shows that the AMC system has the best BER performance compared to others systems.



**Fig. 3.** BER for fixed mode and adaptive modulation and coding systems

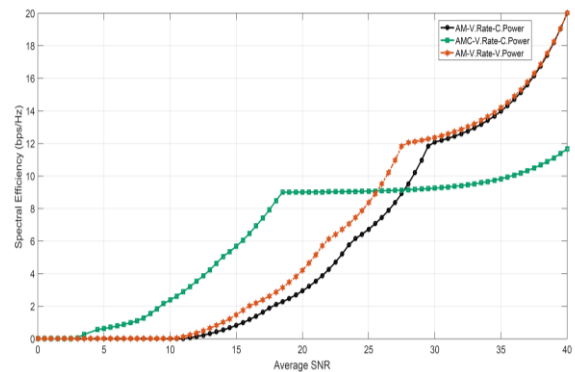
Fig.4 shows the average spectral efficiency of the system owing to optimal rate. According to LTE standard, for uncoded fixed system 64-QAM modulation and for coded fixed system  $\frac{1}{3}$  code rate and 64-QAM modulation is employed. The results show that the spectral efficiency for adaptive modulation has the best gain. In adaptive modulation the spectral efficiency is increased around 11 dB in comparison to a fixed system. In addition, it shows that the spectral efficiency for AMC

is a bit more than 2 bps/Hz than uncoded adaptive modulation for low SNR ranges (11-20 dB). Although, the spectral efficiency of AMC reduces at 25 dB SNR values when compared to adaptive modulation. It shows that using the adaptive transmission cause increasing the system in terms of spectral efficiency significantly.



**Fig.4** Spectral efficiency for variable rate systems

As a previous section Fig.5 illustrated the average spectral efficiency of the system cause optimal power and rate. In particular, it is observed in Fig. 5 that the spectral efficiencies in the two cases, one case A and the other case B, are very close to each other under the same BER constraints even the less SNR is required.



**Fig. 5** Spectral efficiency for variable rate and power systems

**5. CONCLUSION**

In this study, the performance of applying adaptive modulation and AMC for multicarrier GFDM systems has been analyzed. After proving and simulating the performance of the proposed scheme, it is clear that having some knowledge about the channel state helps to reduce the average transmission power and BER of the system. The performance of the adaptive transmission scheme is compared with the fixed system. It is resulted that AMC scheme decrease the SNR required for reaching a given target BER at the cost of a deducted

spectral efficiency performance. Furthermore, it is observed that by utilizing one or two degrees of freedom in adaptive transmission, the maximum spectral efficiency will be acquired. So, the parameters of adaption chosen must be selected on implementation considerations.

## REFERENCES

- [1] S.K. Sharma, I. Woungang, A. Anpalagan, S. Chatzinotas, "Toward tactile internet in beyond 5G era: recent advances, current issues, and future directions," *IEEE Access* vol. 12, no. 8, pp.56948-56991, Mar 2020.
- [2] R. Prasad and V. Rohokale, V, "Internet of Things (IoT) and machine to machine (M2M) communication. In *Cyber security: The lifeline of information and communication technology*," Springer, Cham, pp. 125-141, 2020.
- [3] E. Hossain, D. Niyato, and Z. Han, "Dynamic spectrum access and management in cognitive radio networks," *Cambridge university press*, Jun 2009.
- [4] HF Wang, FB Ueng, YS Shen, KX Lin, "Low-complexity receivers for massive MIMO-GFDM communications," *Transactions on Emerging Telecommunications Technologies* vol. 32, no. 4, pp. 4219 Apr, 2021.
- [5] MN Patwary, SJ Nawaz, MA Rahman, SK Sharma, MM Rashid, SJ Barnes, "The potential short-and long-term disruptions and transformative impacts of 5G and beyond wireless networks: Lessons learnt from the development of a 5G testbed environment," *IEEE Access*, vol. 7, no. 8, pp. 11352-79, Jan 2020 .
- [6] E. Başar, U. Aygözü, E. Panayırçı, HV, Poor, "Orthogonal frequency division multiplexing with index modulation," *IEEE Transactions on signal processing*, vol. 61, no. 22, pp.5536-49, Aug. 2013.
- [7] M. Mirahmadi, A. Al-Dweik, and A. Shami, "BER reduction of OFDM based broadband communication systems over multipath channels with impulsive noise," *IEEE transactions on communications* vol. 61, no. 11, pp.4602-4615, Nov. 2013.
- [8] S. A. Fechtel and A. Blaickner, "Efficient FFT and equalizer implementation for OFDM receivers," *IEEE Transactions on Consumer Electronics* vol. 45, no. 4, pp. 1104-1107, Nov. 1999.
- [9] H. Kim, J. Kim, S. Yang, M. Hong, Y. Shin, and S. Briefs, "An effective MIMO-OFDM system for IEEE 802.22. WRAN channels," *IEEE Transactions on Circuits and Systems II: Express Briefs* vol. 55, no. 8, pp. 821-825, Jun. 2008.
- [10] G. Wunder, P. Jung, M. Kasparick, T. Wild, F. Schaich Y. Chen, S. Ten Brink, I. Gaspar, N. Michailow, A. Festag, L. Mendes, "5G NOW: non-orthogonal, asynchronous waveforms for future mobile applications," *IEEE Communications Magazine* vol. 52, no. 2, pp. 97- 105, Feb. 2014.
- [11] J. Van De Beek and F. Berggren, "Out-of-band power suppression in OFDM," *IEEE communications letters*, vol. 12, no. 9, pp. 609-611, Sep. 2008.
- [12] A. Hammoodi, L. Audah and M.A. Taher, "Green coexistence for 5G waveform candidates: a review," *IEEE Access*, vol. 7, pp.10103-10126, Jan. 2019.
- [13] R. W. Chang, "Synthesis of band-limited orthogonal signals for multichannel data transmission," *Bell system technical journal* vol. 45, no. 10, pp. 1775-1796, Dec. 1966.
- [14] G. Fettweis, M. Krondorf, and S. Bittner, "GFDM-generalized frequency division multiplexing," *IEEE 69th Vehicular Technology Conference*, pp. 1-4 Apr. 2009.
- [15] R. Datta, N. Michailow, M. Lentmaier, and G. Fettweis, "GFDM interference cancellation for flexible cognitive radio PHY design," *IEEE Vehicular Technology Conference (VTC Fall)*, pp. 1-5, Sep. 2012.
- [16] N. Michailow, I. Gaspar, S. Krone, M. Lentmaier, and G. Fettweis, "Generalized frequency division multiplexing: Analysis of an alternative multi-carrier technique for next generation cellular systems," *International symposium on wireless communication systems (ISWCS)*. IEEE pp. 171-175, Aug 2012.
- [17] N. Michailow, M. Maximilian, S. G. Ivan, N. C. Ainoa, L. M. Luciano, F. Andreas and F. Gerhard, "Generalized frequency division multiplexing for 5th generation cellular networks," *IEEE Transactions on Communications* vol. 62, no. 9, pp. 3045- 3061, Aug. 2014.
- [18] N. Michailow, S. Krone, M. Lentmaier, and G. Fettweis, "Bit error rate performance of generalized frequency division multiplexing," *IEEE Vehicular Technology Conference (VTC Fall)*, pp. 1-5, Sep 2012.
- [19] E. Prokis, McGraw-Hill International, "Digital communications," 2001.
- [20] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Transactions on Communications* vol. 49, no. 9, pp. 1561-1571, Sep, 2001.