Dynamic Influence of Contraries Oscillators on Small-World Network Synchrony

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ABSTRACT:
Networks are all around us and can be in the Euclidean space of concrete objects such as power grids, the Internet, highways or subway systems, and neural networks, or in an abstractly defined space, such as networks of familiarity or cooperation between people. To express the general properties of such networks, their modeling is in the form of graphs that show the nodes as oscillators (the dynamic unit) and the edges as the existence of interaction between oscillators We applied the Kuramato model to networks of oscillators connected in a small-world network pattern and considered the influence of oscillators on each other as conformist and contrarian. Based on this, we examined the synchronization in the network. We showed that if the number of contrarian oscillators in the network reaches a certain value, it will cause more of the network, which is due to the weakening of defects created in compatible oscillators.

KEYWORDS: Kuramato Model ,Oscillators ,Random Networks, Synchronization.

1. INTRODUCTION

Networks are all around us and can be in the Euclidean space of concrete objects such as power grids, the Internet, highways or subway systems, and neural networks, or in an abstractly defined space, such as networks of familiarity or cooperation between people [1]. To express the general properties of such networks, their modeling is in the form of graphs that show the nodes as oscillators (the dynamic unit) and the edges as the existence of interaction between oscillators [2,3]. In most real networks, despite their large size, there is often a relatively short path between any two nodes. This feature is known as the small world property, which consists of a regular network and is rewired with a probability p of edges, which is p from 0.005 to 0.05, and has a large clustering coefficient and a small average path length, and between the regular network ($p = 0$) and the random network ($p = 1$) is located [4].

One of the main topics of network dynamics is synchronization. Synchronization can be seen in many different contexts. Including in engineering where synchronization or asynchrony is important, such as wireless communication networks [5] and electric grid networks [6]. Synchronization is the synchronization of a set of phase oscillators that interact weakly with each other.

The Kuramoto model analyzes a model of phase oscillators with natural frequency and coupled, whose interaction is a sinusoidal function of the phase difference [7,8]. This model is simple enough to show all kinds of synchronization patterns, and it is compatible with different conditions, and it also provides an acceptable description of synchronization.

In order for the Kuramoto model to be closer to real networks, two couplings can be assumed: if the coupling of two oscillators is positive, the interaction is convergent (in-phase) and if it is negative, the interaction is not convergent and is anti-phase [9- 11].

Hong and Strugatz considered a simple model for the network in which oscillators are divided into conformist and contrarian groups [12, 13]. In this model, conformists try to keep up with the majority of the population, so they have positive mating. While the contrarian oscillators tend to move in the opposite direction of the population and have negative coupling.

In this paper, we applied the Kuramoto model to two groups of conformists and contrarians of phase oscillators for the small world network and organized the

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paper as follows: In section II, we define the model and the numerical methods of quantifying the synchronization. Section III represents the results and discussion and section IV is devoted to the concluding remarks.

2. MODEL AND METHOD

In this work, we develop the Kuramato model for the small-world network assuming that there are two groups of oscillators, conformists and contrarians, in the network:

$$\frac{d\theta_i^s}{dt} = \omega_0 + \frac{\lambda_i^s}{k_i}\sum_{j=1}^{N} a_{ij}\sin(\theta_j - \theta_i^s), \quad i = 1, \ldots, N \tag{1}$$

Where θ_i denotes the phase of the oscillator sitting at node i, ω_0 is the intrinsic frequency of the oscillators and considered equal for all of them. a_{ij} denotes the elements of the adjacency matrix (i.e. $a_{ij} = 1$ if i and j are connected and $a_{ij} = 0$ otherwise) and k_i is the degree of node i. λ_i^s, where s = conformist; contrarian, is the coupling constant, which is positive for the conformist and negative for the contrarians. with assume $Q > 0$, $\lambda_i^f > 0, \lambda^t = -Q\lambda^f$. Rescaling the time variable as $\tau = \lambda_i^f t$ so in the stationary state we define the long time averaged order parameter as:

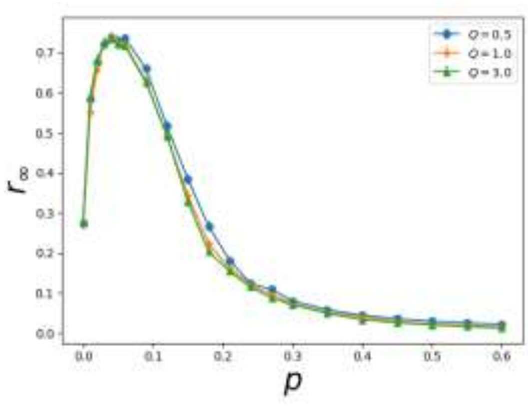
$$r_\infty = \lim_{\Delta\tau\to\infty} \frac{1}{\Delta\tau}\int_{\tau_s}^{\tau_s+\Delta\tau} r(\tau)d\tau \tag{2}$$

In which τ_s is the time of reaching a stationary state.

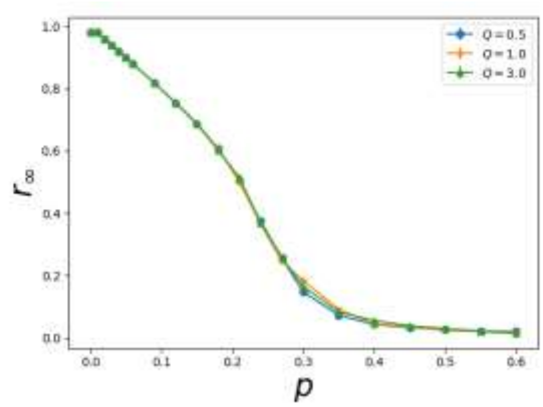
The findings of Hong and Strugats showed for this model [13], the system reaches one of four stable states: 1) Incoherent state: the total order parameter becomes zero. 2) π state: the phase distribution of conformist and contrarian oscillators is exactly equal to π, and conformist and contrarian oscillators are completely synchronized. 3) Soft π state: the order parameter of conformists and contrarians is less than unity ($r_t, r_f < 1$), but their difference is π. 4) Traveling wave state: conformist oscillators will have complete synchronization and contrarian oscillators will have partial synchronization ($r_f = 1$, $r_t < 1$), the peaks of the phase distribution have an angle less than π.

3. RESULTS AND DISCUSSION

We considered the number of oscillators in the grid to be 1000 and assumed the average degree of each node to be 10 (similar to real models) in small world and random networks. We selected the edges undirected and denoted the ratio of contrarian and conformist oscillators as p. The initial random phase of the oscillators is in $[0,2\pi]$ range and the intrinsic frequency distribution of the oscillators follows the Lorentz distribution function. The time to reach the steady state in the small world network (Strogats network) with the probability of rewiring (0.03) more than the random network (3500) is about 6×10^5 time steps. In the following, we express the obtained results.



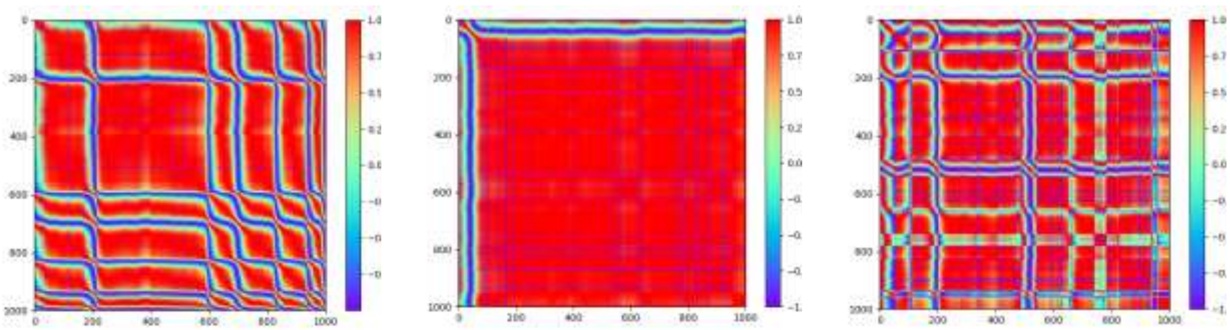
(a)



(b)

Fig. 1. (Color online) Stationary order parameters verses fraction of contrarians for the conformist-contrarian model for $Q = 0.5, Q = 1$ and $Q = 1.5$ for (a) small world (with the probability of rewiring 0.03) and (b) random (with the probability of rewiring 0) networks of $N = 1000$ oscillators and mean degree $\langle k \rangle = 10$. The error bars indicate the standard error of mean (SEM).

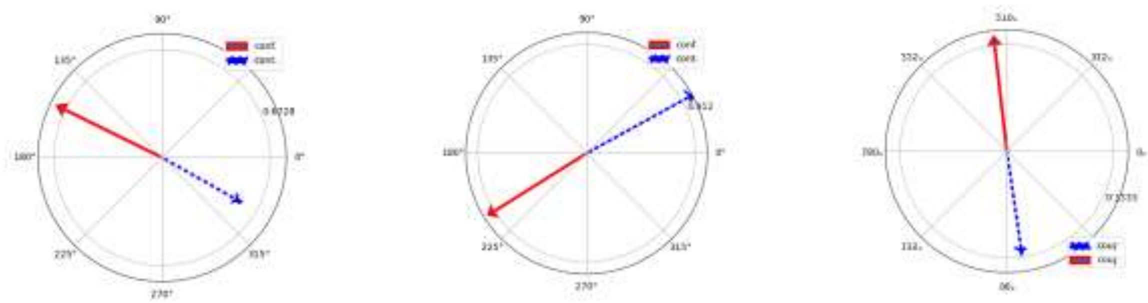
In Fig. 1a, the order parameter has a maximum at $p = 0.04$, and it shows that the order parameter of the network increases with the increase of contrarian nodes up to about 4% and the synchronization in the network increases. Then, with the increase of contrarian nodes, the synchronization decreases and finally reaches zero. For comparison, we examined the random network with the same characteristics of the small world network and drew a diagram in Fig. 1b. It can be seen that with the increase in the percentage of contrarian oscillators, the maximum order parameter remains at $r = 1$and then becomes zero with a smaller slope compared to the small world network. It can be seen in these graphs that it does not depend on the coupling strength like the small world network, so we will continue the graphs related to $Q = 0.5$.



(a) (b) (c)

Fig. 2. (Color online) correlation matrix for (a) $p = 0.03$, (b)$p = 0.0$ and (c)$p = 0.09$ in a small world network of $N = 1000$ oscillators, mean degree $\langle k \rangle = 10$ and $Q = 0.5$. p is the fraction of contrarians to conformists.

Fig. 2. correlation matrix is drawn for the point before the maximum, and after the maximum in Fig. 1a. At the beginning, the network is in a stable state and has network defects, where there are no inconsistent nodes [14]. By increasing the number of contrarian oscillators, network errors are reduced and the network becomes more synchronized. After reaching the maximum, the open defect until it disappears at $\backsim p = 0.1$and the network becomes incoherent.



(a) (b) (c)

Fig. 3. (Color online) the order parameter vector of conformist and contrarian oscillators for (a) $p = 0.03$, (b)$p = 0.0$ and (c)$p = 0.09$ in a small world network of $N = 1000$ oscillators, mean degree $\langle k \rangle = 10$ and $Q = 0.5$. p is the fraction of contrarians to conformists.

For detailed observation in Fig. 3, the order parameter vector of conformist and contrarian oscillators has been drawn for different ratios of contrarian after the stability of the network in the polar plane so that the phase difference of the design of the two groups of oscillators can be seen better. $Q = 0.5$ and random initial conditions are performed independently. The random network $p = 0$ is a structural system less than one, which is due to the existence of network defects in small world networks [6]. This is while the small world network depends on the initial phase of the oscillators and its value is between 0 and 1 and in most cases less than 1.

To ensure the results, the results were analyzed for 10 small world networks with the same number of vertices, degree of nodes and rewiring, but different adjacency matrices, the same results were obtained.

As a result, when enemies randomly enter the network, some of them will be located next to the network defects. The effect of these new factors on the oscillators within the defects, which are out of phase with the other oscillators in their group, gives them more freedom to deviate from their previous anti-phase state, thereby weakening them. For a given number of these counters, maximum freedom is given to errors, maximizing synchronization.

4. CONCLUSION

In summary, we have numerically investigated the Kuramoto model for a number of oscillators that include both conformist and contrarian groups located in the Strogatz small-world and Random networks.In random networks, the conformist and contrarian model is in the stable state of the network in $\pi\, state$, and it goes to blurred π state with the increase of incompatibilities. While for the small world (SW) network, the network is in a blurred π state state and remains in this state as mismatches increase. traveling wave state does not occur for any of the networks. In the SW network, by increasing the number of contrarians to an optimal level, the synchronization increases and then decreases by increasing their number in the network. The observed increase in synchrony is due to the interaction of contrarian oscillators with network defects and their attenuation.

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