Detection of Blood Vessels in Retina Images using Gray Level Grouping Method

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ABSTRACT:
The most important part of the eye is the retina that covers the entire back part of the eye. Eye diseases are one of the most important causes of disability and even death in developed countries as well as in developing countries. Disorders created in the retina that occur due to special diseases can be detected by specific retinal images. Studying the changes in retinal images in a specific time period can help physicians to diagnose the related diseases. In the present paper, the detection of blood vessels in retina images has been investigated. For this purpose, first a new method is proposed to improve the quality of retina images by combining the histogram adjustment and gray level grouping. We use the feature vector to classify the pixels. Next, a method for classifying the images based on the feature extraction vector is required. The use of neural networks is one of the best and most widely used methods of machine learning for classification. We used a 3-layer Perceptron to classify pixels.

KEYWORDS: Retinal Images, Histogram Modulation, Gray Level Grouping, Feature Extraction Vector, Perceptron Neural Network.

1. INTRODUCTION

Eye diseases are one of the most important causes of disability and even death in developed countries as well as in developing countries. By examining the eye and the retina, in addition to eye diseases such as ocular diabetes, cataracts, age-related changes in vision cells (AMD and retinal vein occlusion), other illnesses like hypertension and obesity can be easily diagnosed. The retina of eye covers the entire back part of the eye, and all light receptors are distributed on it. Examining the changes made in retinal images in a specific time period can help the physician to diagnose the related diseases. Retinal blood extraction is sometimes performed manually and sometimes by the physician, which is time-consuming and difficult and has a lot of mistakes due to the high dependence on the doctor's skill level. [1]

Most existing image processing technicians either use gray display of images or process the color channels individually. After receiving the digital images (either in color or as gray), they will be used for various transmissions or more processing by using various computational procedures in automated medical analysis systems. Fortunately, the advent of digital imaging technology in the medical field and the possibility of transmission of images and communication with distant points has also increased the use of these techniques in clinical decision making on retina images of people referred to health centers. Obtaining the images at specified intervals from the patient and recording these images also provide an opportunity to study the procedure of pattern changes and various stages of the progression of the disease on the retina [2]. In these systems, medical images from different areas, especially rural areas that do not have access to a specialist, have been sent to well-equipped medical centers where the pictures are analyzed by relevant specialists in order to diagnose the intended diseases. The result is then communicated through computer networks to the sick person to track her/his condition. It should be noted that in cases where a general screening and monitoring is required, the analysis of a massive amount of medical images is automatically difficult and even impossible by an expert user. In the recent years, with the increasing the computing and processing power of computers, as well as with the development of image processing techniques, the idea of using computers to analyze medical images and auto-detection of diseases has been considered by governments, medical professionals, and computer science professionals.

This idea is very practical and inexpensive for the analysis of retina images and the detection of ocular and vascular diseases, because the retina is the only point in the body that both it and its vessels can be viewed and imaged without the need to enter or even inject the specific substances. This article is along with this direction and has provided an automated method for analyzing retina images and diagnosing the veins and their features.

2. DIGITAL IMAGING IN OPHTHALMOLOGY

The first step in analyzing the digital images is taking photos of the retina area. Fundus digital cameras are used for this purpose. This type of camera can provide the possibility of taking retina photos without having to inject radioactive material or enter into the body. In general, the purpose of processing the medical images is in one of the following three categories [3]:

Image upgrade

Images retrieval

Segment the images

The quality of the taken images is usually influenced by factors such as the intensity of non-uniform lighting, the lack of image alignment at the appropriate focal length and the failure of the sensors (Fig. 1).

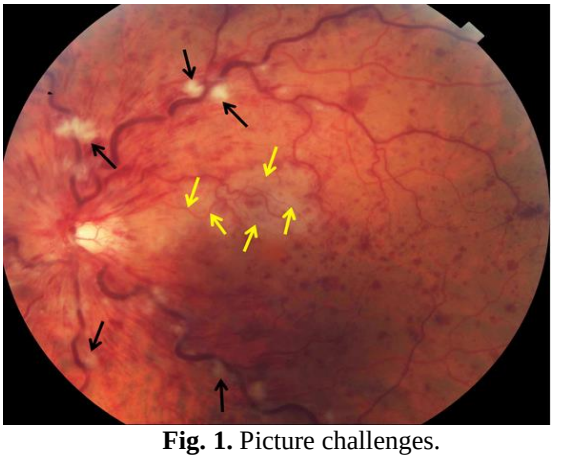

Fig. 1. Picture challenges.

Therefore, image upgrade should be considered to provide a proper analysis and processing. Typically, in the algorithms, it is tried to increase the resolution of the input image through manipulation of pixel amounts. In retrieval of images, it is tried to extract the original image from the image corrupted due to factors such as blur and noise. Segmentation operations are used to divide the image into sub-areas to find and process various structures in them. The images obtained from the segmentation operation will be binary. In these images, the intended points and areas are 1 and the other points will be zero.

3. INVESTIGATING THE EXTRACTION OF VESSELS METHODS

So far, many efforts have been made in this regard, and various methods have been proposed to extract the veins from retina images, all of which can be divided into four main groups:

1-Model-based methods, 2- Pixel classification-based methods, 3- Vessel tracing-based methods, 4- Multi-scale analysis-based methods.

In model-based methods, a model for the vessels is considered and then, according to that model, the same patterns in the image are extracted. First-order Gaussian models, Second-order Gaussian models, and Ridge models [6] are among the models used for vessels. Of course, these methods also referred as window-based methods too.

In pixel classification-based methods, a monitoring classification technique is used to assign pixels to the vein and non-vein classes, and the feature vector is composed of several properties such as the brightness of the pixels with conversion factors such as wavelet transform with Moorlet wavelet [7] Or Stribel wavelet [8]. The results of Moorlet wavelet method are shown in Fig. 2.

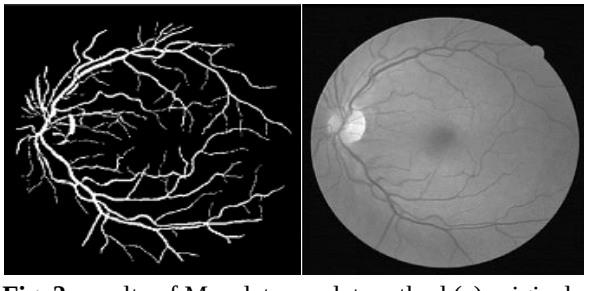

Fig. 2. results of Moorlet wavelet method (a) original image, (b) extraction of vessels.

In vessel tracing-based methods, tracing is used to achieve the structure of the vessels [11-9]. Tracing in this method is done with the help of local information and tries find a path to achieve the best fit with the profile model. The tracking method is also used to reveal thin veins of the retina. Large vessels are distinguishable by using a good contrast threshold alignment. To distinguish the thin veins with a poor contrast from vein length tracing method, supported vector is used that can prevent heavy computing and manual operations [12].

In the multi-scale feature analysis-based methods, the local maximum applied on the gradient domain scales and the maximum main curvature of the Hesin tensor has been used. Ultimately, the area's growth gradually extracts blood vessels from the retinal image, taking into account the features mentioned above along with spatial information.

4. PREPROCESSING

The first step in processing the retina images is preprocessing section, which consists of several sections. In this section, we will explain the different stages of preprocessing.

4.1. Selecting the Image Display

Images in the Retina Image Database are color images for their processing, it has been tried to convert them to gray scale images with the best possible contrast. For this purpose, among three bands as red, green, and blue, in the green band there is the best contrast between the blood vessels and the background. The red-band images have a saturated state and blue band images are almost blank.

As you can see, the image of this band is selected for processing due to the proper contrast of the image in the green band. But the red band is brighter and, in comparison with the green band, extends beyond the gray levels, resulting in lower contrast between the background and abnormalities. Therefore, both the green and red bands can be used simultaneously in an image. Therefore, it is better to use the green band histogram adjustment using the content of the red band histogram. In this case, the resulting image have the benefits of both image bands. However, for high-brightness images, using the red band histogram to correct the green band histogram causes the contrast of the blood vessels and backgrounds in the green band to decrease earlier than before. To solve this problem, a criterion for implementing or not implementing the histogram is considered. This criterion is called $C_{\mu r}$ and is defined as the relation (1).

$$C_{\mu r} = \sum_{j=0}^{\mu r} P_r(r_j) \tag{1}$$

P_r is the density function related to the subband G and μ_r is the mean gray levels of R sub-band images. $C_{\mu r}$ is the sum of the probability density function from the zero gray level to μ_r and is related to the sub-band G images. Now, if for example $C_{\mu r} \geq 0.45$, it means that the brightness of the image below the G band is appropriate and we only use this sub-band to continue the work, otherwise, by matching the histogram and using the image of R subband, we increase the dynamic range of the gray sub-band surfaces G related to the desired retina image [14].

4.2. Uniform Lighting of the Image Background

One of the problems with retina images that seriously affects the result and causes errors in the results of the work is the lack of uniformity of the background image. Different approaches have been proposed to counter this problem, including the correction of background lighting by the proposed method in [15]. In this method, the image of improper brightness I is considered to be multiplied by a destructive function L in the initial image of I_o. Suppose L is a function of low variation that can be estimated by supposing an inappropriate brightness image. If Is is the smooth image, L is equal to:

$$L(x,y) = \begin{cases} \dfrac{I_S(x,y)}{l_0} & if \quad I_S(x,y) < l_0 \\ I(x,y) & if \quad I_S(x,y) \geq l_0 \end{cases} \tag{2}$$

The l_0 is the level of optimal brightness, which can be considered as half the brightness variation of the image pixels. Using this estimate, the image with a corrected brightness with the same image is obtained from equation (3):

$$I_0(x,y) = \begin{cases} I(x,y) \times \dfrac{l_0}{I_S(x,y)} & if \quad I_S(x,y) < l_0 \\ I(x,y) & if \quad I_S(x,y) \geq l_0 \end{cases} \tag{3}$$

Another suggested approach to address this issue is the proposed approach [16]. In this method, the brightness of each pixel is obtained from the corrected image from relation (4).

$$I_{eq}(i,j) = I(i,j) + \mu_{desired} - W_\mu(i,j) \tag{4}$$

$W(i,j)$ is a part of the image with $N \times N$ dimensions (N persons) centered on the pixel (i,j), $I(i,j)$ is the initial brightness of the pixel (i,j) and $\mu_{desired}$ **is** the desired amount of average brightness and $W_\mu(i,j)$ is the actual value of the average brightness in the centered window in pixels (i,j).

5. COMBINED HISTOGRAM ADJUSTMENT AND GRAYSCALE GROUPING

Since some of the features in the image are hardly recognizable with the eye, we often convert the images before the display. This is a combination of adjustment of the histogram and gray level grouping (GLG). The proposed method has been carried out through various

steps. Histogram is an image with a surface intensity between zero and L-1 in a discrete function $h(r_k) = n_k$, where r_k is kth intensity level and n_k is the number of pixels in the image is intensity r_k. This method is commonly used to normalize a histogram by dividing each of its components by the total number of pixels in the image, by the total pixels in the specified row and bottom, where, typically, M and N are the dimensions of the row and column of the image. Therefore, a normal histogram is defined by $P(r_k) = \frac{n_k}{MN}$, for ks from zero to L-1, where $P(r_k)$ is an estimate of the probability of occurrence of surface intensity in an image. Assume that an input image with a surface intensity is in the range from 0 to L-1, the proposed method is as follows:

5.1. Histogram Separation

First, the aim is to find the highest point in the domain histogram (P_{hist}) on a gray scale. If the highest component of the domain histogram is in the left part of the non-zero area of histogram , but not in the first part of the non-zero area of histogram, the histogram can be divided into two sub-curves of histograms: the first half of the intensity from zero to P_{hist}-1 and the second half with starting from the highest component of histogram to the maximum intensity level of L-1. On the other hand, if the highest histogram component is located inside the right-hand side in the non-zero histogram area or in the first part of the non-zero area of histogram, then we need to use gray level grouping to enhance the low-contrast image.

5.2. Single-conversion Function

After histogram separation based on the position of the highest component of the histogram of our domain, we can directly apply it to the first sub histogram from zero to P_{hist}-1 and quickly group the gray surface to the second half under the histogram from the highest histogram component to the L-1. The conversion function using the histogram can be expressed as follows:

$$T_{HE}(rk) = (L-1)\sum_{j=0}^{k} P_r(r_j) = \frac{(L-1)}{MN}\sum_{j=0}^{k} n_j \qquad (5)$$

k = 0,1,2, ..., Phist-1

For gray surface transformation function, we have:

$$T_{FGLG}(rk) = (L-1)\sum_{i=Phist}^{k} P_r(r_j) = \frac{(L-1)}{MN}\sum_{i=Phist}^{k} n_j \qquad (6)$$

k = P_{hist}, P_{hist} + 1 P_{hist} + 2, ..., L-1,

So the single(piece)-conversion function can be written as follows:

$$T_{rk} = T_{HE}(r_k) + T_{FGLG}(r_k) \qquad (7)$$

k = 0,1,2, ..., L-1

Finally, the single- conversion function is applied to the original image, which increases the optimum of image.

6. CHARACTERISTIC VECTOR AND PIXEL CLASSIFICATION

The purpose of the extraction phase is to examine the pixel properties using a characteristic vector. A pixel represents some of the measurements and evaluations that, at the stage of classification, may easily decide whether a pixel belongs to the true blood vessel. In this paper, a series of the following features were selected.

Gray level-based features: Features based on the difference between the gray level in the intended pixel and the statistical value of a representative of the surrounding environment.

Moment specification-based features: Features based on the momentum specification to describe the small image areas formed by the gray level displayed in the center of the pixel window.

Gray level-based features: since blood vessels are always darker than their surrounding environment, it seems that this feature is a good choice for describing the gray level variety around the intended pixel.

A 9-dimensional feature vector including analysis for vector field gradient (a feature) is used to eliminate the dark and light waste with Improvement of vessel quality, morphological function(one of features) to eliminate waste bright , line strength criteria (two features), and Gabor filter on various scales (four features) to eliminate dark waste, respectively.

6.1. Analysis of Vector Field Gradient

The blood vessels are analyzed locally with the gradient vector field. We have used a single gradient vector in the discontinuous image along the double symmetric regions, showing the linear structure of blood vessels. The extraction of a feature depends on the correct orientation of the gradient vector, not its value, which is why it is resistant to low contrast and even uniform light.

The approximate gradient vector for the image I (x, y) is based on the first derivative in the horizontal (Kx) and vertical (Ky) direction.

$$g_x(x,y) = I(x,y) \times K_x \qquad (8)$$

$$g_y(x,y) = I(x,y) \times K_y \qquad (9)$$

The gradient vector g_x (x, y) and g_y (x, y) normalized by dividing by the gradient vector of the unit u_x (x, y) and u_y (x, y) has been calculated.

$$u_X(x,y) = \frac{g_X(x,y)}{\sqrt{g_x^2(x,y) + g_y^2(x,y)}} \quad (10)$$

$$u_y(x,y) = \frac{g_y(x,y)}{\sqrt{g_x^2(x,y) + g_y^2(x,y)}} \quad (11)$$

If the gradient size is too small (<3 of 255), the unit vectors are assigned to zero. The first derivative of the calculated unit vector to find the discontinuity in the gradient direction is as follows:

$$d_{XX}(x,y) = u_X(x,y) \times k_X \quad (12)$$

$$d_{XY}(x,y) = u_X(x,y) \times k_Y \quad (13)$$

$$d_{YX}(x,y) = u_y(x,y) \times k_X \quad (14)$$

$$d_{YY}(x,y) = u_y(x,y) \times k_Y \quad (15)$$

The amount of discontinuation in the direction of the gradient D (x, y) is expressed in terms of the first derivatives of the unit vector.

$$D(x,y) = d_{xx}^2(x,y) + d_{xy}^2(x,y) + d_{yx}^2(x,y) + d_{yy}^2(x,y) \quad (16)$$

D (x, y) includes gradient direction analysis (GOA) of mapping of blood vessels. There is a variance across the vessel that can be related to the radial motion of the optical disc. Therefore, the operator of first-order derivative, which is used at various scales $(\sigma = \{1, \sqrt{2}, 2\sqrt{2}, 4\})$, is used to generate multiple maps of GOA from various blood vessels. The mapping of the final GOA, which acts as one of the selected characteristic vectors, is obtained by aggregating individual mappings generated at various scales.

6.2. Gabor Multiple-scale Filter

The Gabor filter is a linear filter and is widely used for multi-scale and multi-directional edge detection. The Gabor filter can operate at a specific frequencies, size, and direction and also as a noise-reducing and low-level feature extraction. The hit response of the Gabor kernel filter is defined as a Gaussian and sinusoidal kernel definition complex. It can be expressed in the form (17):

$$g(x,y) = \exp\left\{-0.5\left(\frac{x'^2 + \gamma y'^2}{2\sigma^2}\right)\right\}\exp\left\{i\left(2\pi\frac{x'}{\lambda} + \Psi\right)\right\} \quad (17)$$

Where:
λ: sinusoidal wave length factor
Θ: Direction
ψ: Phase offset
σ: A Gaussian Cover Scale
γ: Spatial dimensional ratio

The Gabor filter response in the green canal is the reverse image of the retinal color image obtained by a two-dimensional convolutional operator and calculated in the frequency domain. The maximum response of the filter above the angle Θ, range [0, π] in step / 18π for each pixel in the image is calculated on different scales. The maximum response has been considered as a pixel attribute vector in width scale and in one direction. The normalized feature distance is the normalized mean of the source and standard deviation using the normal function.

6.3. Line Power Feature

Retinal arteries appear to be fragmented as a feature, which can be seen in the retinal image with variations in their width and branches. The mean gray level along the length of the special lines of each pixel is considered in 12 different directions and measured at a distance of 15 degrees. The line with the highest mean value is specified as gray. The line power in one pixel is calculated by calculating the difference in the mean gray value of the axis of a square window in the target pixel with the mean gray value of the specified line. The calculated line power for each pixel is considered as the pixel attribute vector.

6.4. Morphological function

An open morphology was used for an element with a linear directed structure at a particular angle to remove a vessel or part of that vessel when that vessel is not within the vessel's range. This happens when the vessel and its components are in the right direction and its components are longer than the width of the vein.

$$I_{th}^{\theta} = I - (IoS_e^{\theta}) \quad (18)$$

$$Is_{th} = \sum_{\theta \in A} I_{th}^{\theta} \quad (19)$$

The tophat morphological function has been shown in (18). Where θ is Tophat function of converted image, I is the processed image, and 0 represents the morphology opening operator. If it is again considered along a class of elements of linear structure, then Tophat's result along any direction of the vessel's brightness irrespective of the direction of the vessel is to extract the largest diameter of the vessel, provided that the length of the structure element is large enough. Therefore, 21 long pixels for each member, a pixel width and zero up to π at a spin angle of π /8 for the

spin has been selected. Its size is approximately equal to the range of the largest diameter of the vessel in the retina. The Tophat series is shown in (19). The increase of the Tophat function is described in (19). The set A consists of the angular orientation of the elements and can be limited to 0 <X <π. The recovery of all vessels in each direction, including small veins or indirect removal of areas is revealed from Tophat result.

7. PERCEPTRON MULTILAYER NEURAL NETWORK

In the process of classifying each pixel from the retina image, it can take one of two labels: a vein and non-vein. At this stage, a method for classifying the images based on the feature extraction vector is required. The use of neural networks is one of the best and most widely used methods of machine learning for classification. These networks are trained by special algorithms so that they can achieve the best output according to its input. We use the perceptron multilayer neural network method here.

The post-propagation learning algorithm is a more appropriate than forward-looking networks approach.
In this study, we used 3 layers for the perceptron multi-layer network where the number of hidden layers has been calculated empirically. As the active function of the hidden layer neuron, we chose the sigmoid tangent function (tanh) described in equations (21) and (22), which is an asymmetric function and in the interval (-1, 1). The two main activation functions used in this paper are described in Equations (20), (21) and (22).

$$y = \frac{1}{1+e^{-x}} \tag{20}$$

$$y = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \tag{21}$$

$$y = \tanh(x) \tag{22}$$

In addition, tanh improves the learning speed of the perceptron multilayer neural network and other sigmoid functions in the output layer called the active sigmoid function (logsig), which is in the range (0, 1) defined in the equation (4-20).

Using these features as inputs in the neural network, it is classified as a vessel or non-vessel. These features train the network, the purpose of stage of training is to set the free parameters of the multi-layer perceptron neural network. At this stage, 9 characteristics have been calculated for vessel and non-vessel classification. To determine the number of hidden neurons and to set the parameter using the back prepagation gradient as the training algorithm with the mean squared function, the error gradient has been minimized and the training has ended after 179 repetitions. Training network parameters are shown in Fig. 3

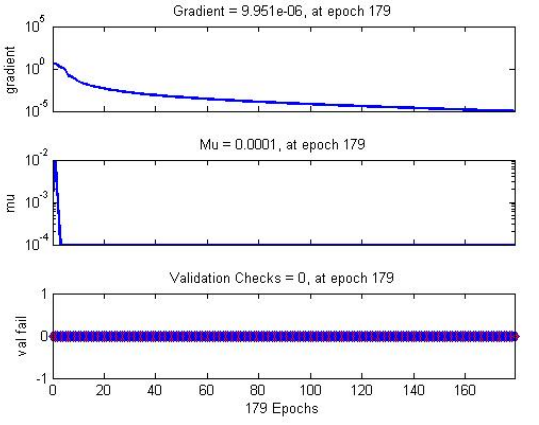

Fig.3. Training network parameters.

Moreover, some of the matching results for the STARE database are in Fig. 3. In Fig. 3, the left images are related to the input images and the right images are detection of the blood vessels. Detection of blood vessels for STARE data is also shown in Fig. 4.

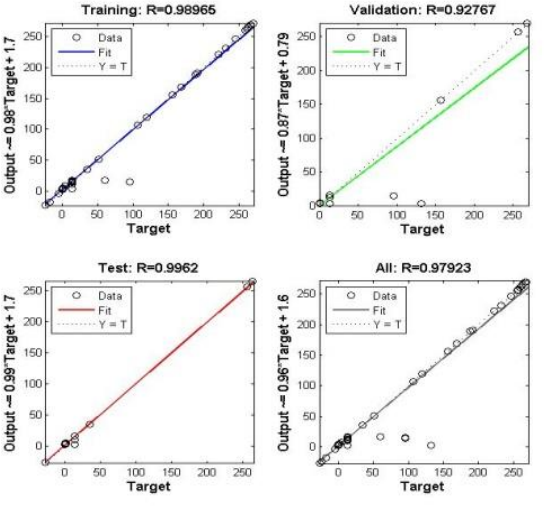

Fig. 3. regression diagrams.

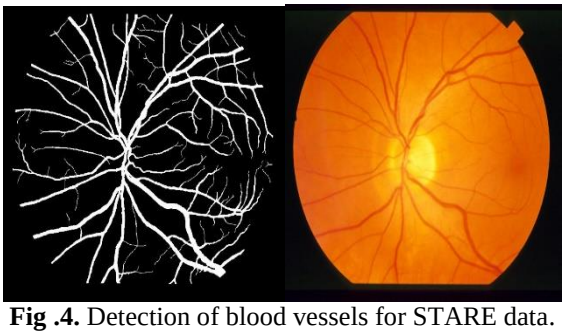

Fig .4. Detection of blood vessels for STARE data.

8. EVALUATION OF BLOOD VESSEL EXTRACTION ALGORITHM

In order to evaluate the proposed algorithm for the detection of blood vessels in retina images, a few quantitative criteria are introduced as in the previous section. True Positive Rate (TPR) and False Positive Rate (FPR) are two quantitative criteria for examining the function of image segmentation algorithm.

TPR: represents the number of pixels whose algorithm is diagnosed as related to blood vessels, and these pixels are actually related to blood vessels.

FPR: represents the number of pixels whose algorithms are related to the blood vessels, but in fact these pixels have not be related to blood vessels.

Among the few other quantitative criteria that can be used to evaluate the partitioning algorithm is the accuracy of the algorithm representing the number of pixels that the algorithm correctly detects their independence or dependence on the blood vessels. In order to evaluate the images with this criterion, two other qualities, such as True Negative Rate (TNR) and False Negative Rate (FNR), have first been defined.

TNR: represents the number of pixels that their algorithm does not count as blood vessels, and these pixels did not actually belong to blood vessels.

The FNR represents the number of pixels whose algorithm belongs to the background, but in fact these pixels belong to blood vessels.

With respect to this definition, the accuracy of the algorithm is defined as (23).

$$Acc = \frac{TPR + TNR}{TPR + FNR + TNR + FPR} \quad (23)$$

Table 1 represents the mean values of the mentioned criteria for all images in the DRIVE database. As seen, the proposed algorithm can extract blood vessels with high precision from the background of retina images.

Table 1. The results of the evaluation of the blood vessel extraction algorithm.

Algorithm	TPR	FPR	Accuracy(%)
MLP	0/8026	0/0195	97/71

8.1. Comparison of the Results of Blood Vessel Extraction Algorithms

In Table 2, a comparison between the proposed algorithm in this paper with other methods available in this field and based on the criteria described in the previous section is used to diagnose the blood vessels of network images in the DRIVE database.

Table 2. Comparison of proposed algorithm with some existing methods.

Method	TPR	FPR	Average Accuracy(%)
A.M.Mendonca et al.[13]	0/7344	0/0236	94/63
J.staal Et al.[6]	0/6780	0/0170	94/41
D.Marin Et al.[17]	0/7068	0/0305	94/52
MLP	0/8026	0/0195	97/71

9. CONCLUSION

In this paper, human retina digital images were examined and processed. An aim of this study was to improve the quality of extraction of features such as blood vessels from retina images.

The features of our proposed method in this article include:

Using the histogram adjustment method and the rapid gray level grouping results in an acceptable contrast enhancement as well as highlighting our favorite components in retina images.

Using the neural network in addition to providing high speed and precision, in other words, the use of the neural network in this algorithm simplifies implementation, which speeds up the system, and also greatly increases the persistent accuracy.

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