A New Model of Pull-in Voltage for MEMS Variable Capacitive with Fully-clamped Diaphragm

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ABSTRACT:
In this paper a new model of Pull-in Voltage for MEMS capacitor with fully clamped diaphragm is presented. This model not only makes it possible to calculate the exact value of pull-in voltage in a capacitor but also provides the ability to detect the accurate deflection of a fully clamped diaphragm. By introducing this model, the precise value of pull-in voltage at the border, between stable and un-stable state, can be calculated. As an advantage of this new achievement, it is exhibited that the all theoretical equations are fully compatible with simulation results using finite element analysis (FEA).

KEYWORDS: MEMS capacitive, fully-clamped diaphragm, analytical model, pull-in voltage, strain energy.

1. INTRODUCTION
MEMS-based capacitive instruments offer advantages due to their small size, relatively high sensitivity, batch fabrication capability, inherently low power consumption and low noise features. Serious research efforts have been made to improve the design methodology and performance of MEMS-based capacitive instruments [1-3]. As it is well known, bias voltage is the voltage applied to a capacitor at normal working condition. Any voltage exceeding this voltage will reduce the air gap thickness between the two electrodes of the capacitor by forcing the movable electrode which generally works as a diaphragm closer to the ground plate which is fixed. The voltage being high enough to pull the movable plate to a distance equal to one-third of the original air gap thickness is called Pull- In voltage. This is known as the highest acceptable voltage which can be applied to a capacitor without causing any damage. Consequently any increase above Pull- In voltage is considered destructive and should be avoided.

In MEMS based capacitive structures, the pull-in voltage or the collapse voltage is a highly critical factor in the design process as it can provide a destructive working condition and result the collapse of the structure. Previously in a capacitive structure with clamped diaphragm, the small deflection model of diaphragm deformation [4] did not account for nonlinearities associated with the presence of in-built residual stress in the diaphragm and predicted unrealistically high deformation values. In [5], an empirical method is provided to approximate pull-in voltage for cantilevers, fixed-fixed beams and circular diaphragms under electrostatic actuation. Naturally, this factor was neglected that whenever the bias voltage is applied to a capacitor, an excitation of electrostatic attraction force is formed which causes a non-linear behavior in the structure of the capacitor. This factor has been considered in [6] so as a result, the large deflection model for a clamped square diaphragm deflection was introduced. But still none of mentioned methods could fulfill the accuracy in calculating the precise pull-in voltage in capacitive structures. So essentially it is necessary to formulate a method to determine the exact pull-in voltage that accounts for the nonlinear nature of the design parameters.

In this paper, we try to design and introduce a new method that able to detect and evaluate the exact pull-in voltage of MEMS capacitor with fully clamped diaphragm. This method also provides the ability to calculate the amount of diaphragm deflection.

2. DESIGN CONSIDERATIONS

In capacitive structures, the electrostatic attraction force pulls the capacitor electrodes towards each other and thus causes the diaphragm to deflect up to its maximum limit. This maximum allowable limit is defined by the elasticity of the diaphragm material. Any forces exceeding pull-in voltage will drive the movable plate to the instability range and moves it to destructive state. This is defined as pull-in condition. In this new design, the destructive limit is proved not to be one-third of air gap thickness.

In parallel plate capacitors, as the sides are not fixed and can freely move towards the fixed plate, the boundary condition is changeable. Whereas in this new type of capacitor all four sides of the top diaphragm are fixed. Therefore the diaphragm is stationary and has no movement. Only the center portion of the diaphragm gets attracted towards the fixed plate and gets stretched under electrostatic attraction force. When a suitable diaphragm with a high quality material having high elastic limit is chosen, the deflection will extend beyond the one third of air gap and it is allowed to get closer to the fixed plate. With this difference, the previous methods which were applied to parallel type capacitors are no longer applicable for this fully clamped diaphragm. Thus employing new methods are essential.

Calculating the exact value of pull-in voltage at the border between stable and unstable state, is achievable by using MATLAB software in conjunction with this new mechanical model. In addition, the graph of deflection versus voltage indicates that the results are more accurate and more realistic than the results of previous methods.

3. ANALYTICAL MODEL OF PULL-IN VOLTAGE

A typical capacitive structure with a fully clamped diaphragm is shown in Fig 2. The air gap between parallel plates is a distance called d_0. The lower plate is a fixed plate and has no movement, whereas the upper one despite being fixed its periphery can deflect towards the lower plate. By applying electrostatic force, the central region of movable diaphragm will be attracted to the fixed plate. By this attraction, the air gap thickness between two plates of the capacitor is reduced. This movement stops at a point which is $\frac{1}{3}d_0$ from the upper plate. This is the ultimate point of motion which can take place in any capacitor that leaves a final distance of $\frac{2}{3}d_0$ between the two plates. Any force beyond that can cause destruction. But the capacitor has been introduced in this paper, which its movable plate is fully fixed at all sides. By applying any electrostatic force, the central region of movable diaphragm will be attracted to the fixed plate. By this attraction, the air gap thickness between the two plates of the capacitor is reduced to less than 2/3 of d_0.

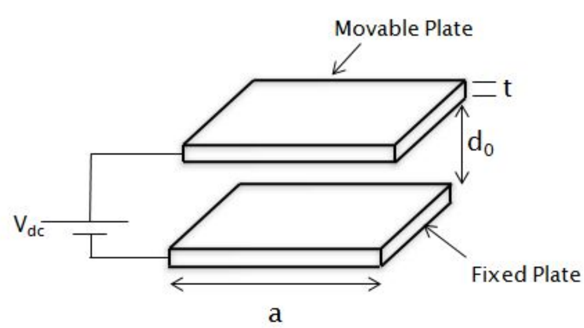

Fig 2. Capacitive structure

In parallel plate capacitors when voltage is applied, the central capacity is defined as:

$$C = \frac{\varepsilon_0 A}{d_0 - w} \quad (1)$$

Where ε_0 is permittivity of free space or electric constant which is equal to 8.854×10^{-12}, A is the area of capacitor plate, d_0 is the distance between the two plates and w is the central deflection of diaphragm

[7].Electrostatic energy in capacitor is expressed as:

$$U_c = \frac{1}{2} C V^2 \quad (2)$$

Where C is the central capacity and V represents the bias voltage between two electrodes (the diaphragm and the backplate).The electrostatic attraction force F_E between the plates due to the charges on the plates can be found by differentiating the stored energy of the capacitor respect to the position of the movable plate and is expressed as [6]:

$$F_E = -\frac{dU_c}{dw} = -\frac{d}{dw}\left(\frac{1}{2}CV^2\right) = \frac{\varepsilon_0 AV^2}{2\,(d_0-w)^2} = \frac{\epsilon_0 AV^2}{2(d_0-w)^2} \quad (3)$$

Where ε_0 is permittivity of free space, A is the area of capacitor plate, d_0 is the distance between two plates and w is the central deflection of diaphragm. Generally, Euler – Bernoulli theory is used to analyze diaphragm deflection. Whenever this theory is used for plates with small thicknesses, the effects of rotational inertia and shear deformations can be neglected [8].In this method only the bending force is considered and above mentioned factors are totally ignored and have no effect on the results. In this theory the plate is assumed to be under pure bending force. Therefore the other forces including shear and tensional forces are zero. Furthermore, the diaphragm material is isotropic and homogeneous. The following assumptions are made in this paper:

1. The thickness of the plate (h) is small compared to its lateral dimensions.
2. The influence of transverse shear deformation is neglected. This assumption implies that the transverse shear strains, ε_{xz} and ε_{yz} are negligible, where z denotes the thickness direction.
3. The transverse normal strain ε_{zz}under transverse loading can be neglected. The transverse normal stress $\sigma_{zz}$is small and hence can be neglected compared to the other components of stress.

The strain energy of the system (U$_s$) can be expressed as:

$$U_s = \frac{1}{2}\iiint_{V_p}\left[\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{zz}\epsilon_{zz} + \sigma_{xy}\epsilon_{xy} + \sigma_{yz}\epsilon_{yz} + \sigma_{zx}\epsilon_{zx}\right]dxdydz \quad (4)$$

where$\epsilon_{xx}$is axial strain in a bar or beam. $\epsilon_{xx}, \epsilon_{yy}, \epsilon_{zz}, \epsilon_{xy}, \epsilon_{yz}$and ϵ_{zx} are components of strain. In fact, ϵ_{xx} andϵ_{yy} are normal strains parallel to the x and y axes respectively, and ϵ_{xy} is the shear strain in the *xy* plane. On the other hand, σ or σ_{xx} is axial stress in a bar or beam. $\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{zx}$ are components of stress. For thin plates subjected to bending forces (i.e., transverse loads and bending moments), the direct stress in the z direction (σ_{zz}) is usually neglected. Thus, the nonzero stress components are $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}, \sigma_{yz}$ and σ_{zx}.

It can be noted that in beams, which can be considered as one-dimensional analogs of plates, the shear stress σ_{xy}will not be present. As in beam theory, the stresses $\sigma_{xx}, \sigma_{yy}, \sigma_{xz}and\sigma_{yz}$are assumed to vary linearly and parabolically, respectively, over the thickness of the plate. The shear stress $\sigma_{xy}$is assumed to vary linearly over the thickness of the plate. Because of assumptions 2 and 3, the state of stress in a thin plate can be assumed to be plane stress. Thus, the nonzero stresses induced in a thin plate are given by σ_{xx},σ_{yy}, and σ_{xy}.As a result, the strain energy density (U_{s_0}) of the plate can be expressed as:

$$U_{s_0} = \frac{1}{2}\,(\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{xy}\epsilon_{xy}) \quad (5)$$

Integrating (5) over the volume of the plate (V), the strain energy of bending can be obtained as:

$$U_s = \iiint U_{s0}\, dV \quad \rightarrow U_s = \frac{1}{2}\iiint_{V_p}\left[\sigma_{xx}\epsilon_{xx} + \sigma_{yy}\epsilon_{yy} + \sigma_{xy}\epsilon_{xy}\right]dV \quad (6)$$

Where *dV= dAdz* denotes the volume of an infinitesimal element of the plate. The strain components can be expressed in terms of the transverse displacement of the middle surface of the plate, w(x,y),as follows [9]:

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} = -z\frac{\partial^2 w}{\partial x^2} \quad (7)$$

$$\varepsilon_{yy} = \frac{\partial v}{\partial y} = -z\frac{\partial^2 w}{\partial y^2} \quad (8)$$

$$\varepsilon_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = -2z\frac{\partial^2 w}{\partial x \partial y} \quad (9)$$

$$\varepsilon_{zz} = \frac{\partial w}{\partial z} \approx 0, \quad \varepsilon_{xy} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0, \; \varepsilon_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \quad (10)$$

Where *u, v,* and *w* denote the components of displacement parallel to x, y, and z directions, respectively. On the other hand, Stress and strain relations are expressed as [10]:

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left[\epsilon_{xx} + \nu\epsilon_{yy}\right], \; \sigma_{yy} = \frac{E}{1-\nu^2}\left[\epsilon_{yy} + \nu\epsilon_{xx}\right], \; \sigma_{xx} = \frac{E}{2(1+\nu)}\epsilon_{xy} \quad (11)$$

Where E is Young's modulus, ν is Poisson's ratio, σ is stress, and ε is the strain. As a result, the stress-strain relations permit stresses to be expressed in terms of the transverse displacement, w(x,y) as:

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left[\epsilon_{xx} + \nu\epsilon_{yy}\right] = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right) \quad (12)$$

$$\sigma_{yy} = \frac{E}{1-\nu^2}\left[\epsilon_{yy} + \nu\epsilon_{xx}\right] = -\frac{Ez}{1-\nu^2}\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right) \quad (13)$$

$$\sigma_{xy} = \frac{E}{2(1+\nu)}\epsilon_{xy} = -\frac{Ez}{(1+\nu)}\frac{\partial^2 w}{\partial x \partial y} \quad (14)$$

With substituting σ_{xx}, σ_{yy}, and σ_{xy} in (6) by stresses from (11), the following equation is achieved:

$$U_s = \frac{E_s}{2(1-\nu_s^2)}\iiint_{V_s}\left[\epsilon_{xx}^2 + 2\nu_s\epsilon_{xx}\epsilon_{yy} + \epsilon_{yy}^2 + \frac{1-\nu_s}{2}\epsilon_{xy}^2\right]dxdydz \quad (15)$$

The index of "s" in above equation denotes the strain energy. Using Eqs.((7),(8),(9)) and integrating the equation in (15) over the plate thickness, (16) will be obtained. As mentioned before, all lengths of the plate are equal (a = b).

$$U_s = \frac{E_s}{2(1-\nu_s^2)}\int_0^a\int_0^b\left\{h\left[\frac{1}{4}\left(\frac{\partial w_0}{\partial x}\right)^4 + \frac{1}{4}\left(\frac{\partial w_0}{\partial y}\right)^4 + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\left(\frac{\partial w_0}{\partial y}\right)^2\right] + \frac{h^3}{12}\left[\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + 2\nu_s\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} + \left(\frac{\partial^2 w_0}{\partial y^2}\right)^2 + 2(1-\nu_s)\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right]\right\}dxdy = \frac{E_s h}{2(1-\nu_s^2)}\int_0^a\int_0^b\left[\frac{1}{4}\left(\frac{\partial w_0}{\partial x}\right)^4 + \frac{1}{4}\left(\frac{\partial w_0}{\partial y}\right)^4 + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\left(\frac{\partial w_0}{\partial y}\right)^2\right]dxdy + \frac{D}{2}\int_0^a\int_0^b\left[\left(\frac{\partial^2 w_0}{\partial x^2}\right)^2 + 2\nu_s\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} + \left(\frac{\partial^2 w_0}{\partial y^2}\right)^2 + 2(1-\nu_s)\left(\frac{\partial^2 w_0}{\partial x \partial y}\right)^2\right]dxdy \quad (16)$$

Where E_s is Young's modulus, ν_s is Poisson's ratio, and *z* was replaced by *h* (height or the thickness of the plate).

Each point of the membrane is assumed to move only in the z direction, and the displacement, w(x, y, t) is assumed to be very small compared to the dimensions of the membrane. The above equation leads to equation of motion. Since the structure is considered at equilibrium, the time dependent term can be neglected. As this equation, involves fourth-order partial derivatives with respect to *x* and *y*, we need to specify four conditions in terms of each of *x* and *y*(i.e., two conditions for any edge) and two conditions in terms of *t* (usually, in the form of initial conditions) to find a unique solution of the problem. If the displacement and velocity of the plate at *t* = 0 are specified as $w_0(x,y)$and $\dot{w}_0(x,y)$, the initial conditions can be expressed as:

$$w(x,y,0) = w_0(x,y) \quad (17)$$

$$\frac{\partial w}{\partial t}(x,y,0) = \dot{w}_0(x,y) \quad (18)$$

The general boundary conditions that are applicable for any type of geometry of the plate can be stated as follows. Let *n* and s denote the coordinates in the directions normal and tangential to the boundary. At a fixed edge, the deflection and the slope along the normal direction must be zero: ($w=0$ and $\frac{\partial w}{\partial n} = 0$). For a simply supported edge, the deflection and the bending moment acting on the edge about the s direction must be zero. In this equation D is the flexural rigidity of the plate which can be calculated as [11]:

$$D = \frac{Eh^3}{12(1-\nu^2)} \quad (19)$$

Where ν is Poisson's ratio, h is the plate thickness, and E is Young's modulus. The work done by axial forces can be evaluated as:

$$W_a = -\iiint_{V_p} [N_x\epsilon_{xx} + N_y\epsilon_{yy}]dxdydz = -\iiint_V \left\{N_x\left[\frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 - z\frac{\partial^2 w_0}{\partial x^2}\right] + N_y\left[\frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 - z\frac{\partial^2 w_0}{\partial y^2}\right]\right\}dxdydz = -\frac{1}{2}\iint_0^a \left\{N_x h\left(\frac{\partial w_0}{\partial x}\right)^2 + N_y h\left(\frac{\partial w_0}{\partial y}\right)^2\right\}dxdy \quad (20)$$

Where $N_x.h$ and $N_y.h$ represent the membrane forces. Consequently, N_x and N_y are the result of h multiplied by σ (h.σ). Furthermore, h is the distance between the two sheets of the upper plate (m), σ is stress and the dimension of stress is (N / m^2).Each material has residual stress. Since work is equal to the integration of force with respect to displacement, therefore work done by electrostatic force is expressed by (21) which is achieved by substituting electrostatic force (F_E) by (3):

$$W_E = \iint_0^a F_E w_0 dxdy = \iint_0^a \frac{\epsilon_0 V^2}{2(d_0 - w_0)^2} w_0 dxdy \quad (21)$$

Where the displacement of the plate at $t = 0$ is specified as $w_0(x, y)$ and W_E is the mentioned work. To derive the equation of motion of a membrane using the extended Hamilton's principle, the expressions for the strain and kinetic energies as well as the work done by external forces are needed. The generalized Hamilton's principle can be stated as [12]:

$$\delta \int (T - U_s + W_{E,a})dt = 0 \quad (22)$$

Where the strain and kinetic energies of the beam can be found as U_s and T, respectively. δ denotes the variation of the above function. According to Hamilton's principle in mechanical science, energy consumption should be minimized. Thus, T is set equal to zero. The substitution of Eqs. (16), (20) and (21) into (22) yields the equation of motion. The result of this expression is sum of two equations, one of them refers to "structure" and another one is "boundary conditions". Therefore, all types of works are included in Hamilton's equation. Thus the forces such as electrostatic, axial, forces related to strain and stress can be accounted. W_a denotes axial forces and $W_E$indicateselectrostatic forces. To get the accurate equations of motion, Hamilton's principle is used. In this condition the strain energy is considered by partial differential equations. As the aim is to get the details of strain energy, therefore the extended Hamilton's principle in form of (23) will be resulted:

$$\delta U_s = \frac{E_s h}{2(1-\nu_s^2)}\iint_0^a \left[\left(\frac{\partial w_0}{\partial x}\right)^3 \frac{\partial(\delta w_0)}{\partial x} + \left(\frac{\partial w_0}{\partial y}\right)^3 \frac{\partial(\delta w_0)}{\partial y} + \frac{\partial w_0}{\partial x}\left(\frac{\partial w_0}{\partial y}\right)^2 \frac{\partial(\delta w_0)}{\partial x} + \left(\frac{\partial w_0}{\partial x}\right)^2 \frac{\partial w_0}{\partial y}\frac{\partial(\delta w_0)}{\partial y}\right]dxdy + D\iint_0^b \left[\frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2(\delta w_0)}{\partial x^2} + \nu_s \frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2(\delta w_0)}{\partial x^2} + \nu_s \frac{\partial^2 w_0}{\partial x^2}\frac{\partial^2(\delta w_0)}{\partial y^2} + \frac{\partial^2 w_0}{\partial y^2}\frac{\partial^2(\delta w_0)}{\partial y^2} + 2(1-\nu_s)\frac{\partial^2 w_0}{\partial x\partial y}\frac{\partial^2(\delta w_0)}{\partial x\partial y}\right]dxdy \quad (23)$$

By separating the different components of above equation, two new equations $\delta U_{s_1}$and δU_{s_2} will be resulted ($\delta U_s = \delta U_{s_1}$+$\delta U_{s_2}$). In Hamilton's principle, the target is to get the values of work done by axial forces (W_a) and electrostatic forces (W_E) in the form of partial equations. To achieve it, (21) has been used. Consequently (24) has been formed as:

$$\delta W_E = \iint_0^a \frac{\epsilon_0 V^2}{2(d0 - w_0)^2}\delta w_0 dxdy \quad (24)$$

Similarly, δW_a can be expressed as:

$$\delta W_a = -\iint_0^a \left[N_x h\frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial x} + N_y h\frac{\partial w_0}{\partial y}\frac{\partial \delta w_0}{\partial y}\right]dxdy = -\int_0^a N_x h\frac{\partial w_0}{\partial x}\delta w_0\Big|_0^a dy - \int_0^b N_y h\frac{\partial w_0}{\partial y}\delta w_0\Big|_0^b dx + \iint_0^a \left[N_x h\frac{\partial^2 w_0}{\partial x^2} + N_y h\frac{\partial^2 w_0}{\partial y^2}\right]\delta w_0 dxdy \quad (25)$$

Considering (22) in a partial form, and the axial forces as well as work related to stress and electrostatic forces (Eqs.(23),(24),(25)), the equation of movement will be expressed as:

$$D\left[\frac{\partial^4 w_0}{\partial x^4} + 2\frac{\partial^4 w_0}{\partial x^2\partial y^2} + \frac{\partial^4 w_0}{\partial y^4}\right] - \frac{Eh}{2(1-\nu^2)}\left[3\frac{\partial^2 w_0}{\partial x^2}\left(\frac{\partial w_0}{\partial x}\right)^2 + 3\frac{\partial^2 w_0}{\partial y^2}\left(\frac{\partial w_0}{\partial y}\right)^2 + \frac{\partial}{\partial x}\left(\frac{\partial w_0}{\partial x}\left(\frac{\partial w_0}{\partial y}\right)^2\right) + \frac{\partial}{\partial y}\left(\left(\frac{\partial w_0}{\partial x}\right)^2\frac{\partial w_0}{\partial y}\right)\right] - N_x h\frac{\partial^2 w_0}{\partial x^2} - N_y h\frac{\partial^2 w_0}{\partial y^2} = \frac{\epsilon_0 V^2}{2(d_0 - w_0)^2} \quad (26)$$

Where w_0 is the displacement of movable plate at t=0and is no longer equal to d_0. E is Young's modulus, h is thickness, ν is Poisson's ratio, and D is flexural rigidity of plate. On the other hand, by applying a voltage between the two electrodes of the capacitor, movable electrode will be attracted towards the fixed electrode. Therefore, a deflection will occur in movable diaphragm. This deflection (W) that satisfies the boundary conditions can be expressed as:

$$W=\sum_{m=1}^{\infty=4}\sum_{n=1}^{\infty=4} q_{mn}W_{mn}(x,y) = \sum_{m=1}^{\infty=4}\sum_{n=1}^{\infty=4} q_{mn}\sin(\frac{m\pi}{a}x)\sin(\frac{n\pi}{b}y) \quad (27)$$

Where $W_{mn}(x,y)$ are the natural modes of vibration and $q_{mn}$are the corresponding generalized coordinates. This equation denotes the double Fourier sine series expansions of the function W(x,y,t) or W. The fundamental or lowest mode shape of the membrane corresponds to m =n = 1. In this modal pattern, the deflected surface of the membrane will consist of one half of a sine wave in each of the x and y directions. The higher values of m and n correspond to mode shapes with m and n half sine waves along the x and y directions, respectively. A specific natural frequency is associated with each combination of m and n values.

As the displacement takes place at the center portion of the structure, W ($x=\frac{a}{2}, b=\frac{b}{2}$) is the basic parameter in above calculation, thussin($\frac{\pi}{2}$)will be resulted in (27).Consequently the value of W is equal to q_{mn}. The mentioned structure works in a multi-mode manner, but only one mode has been considered in this study (first mode).So by considering m =n =1, the above equation changes to (28):

$$W=\sum_{m=1}^{\infty=4}\sum_{n=1}^{\infty=4} q_{11}\sin(\frac{1\pi}{a}\frac{a}{2})\sin(\frac{1\pi}{b}\frac{b}{2}) = q_{11} \quad (28)$$

N_{xy} is set to zero ($N_{xy}=0$), and as a result, this factor does not exist in the equation of motion. To reduce the amount of errors, dimensionless elements should be selected. No dimensional parameters defined as:

$$\xi=\frac{x}{a}, \eta=\frac{y}{b}, \quad \overline{W}=\frac{w}{d_0}, \overline{V}\pi^4=\frac{\epsilon_0 a^2 V^2}{2\sigma h d_0^3}, \quad \overline{D}=\frac{D}{\sigma h a^2}, \overline{N}_x=\frac{E_p d_0^2}{2(1-\nu_p^2)a^2\sigma} \quad (29)$$

Where W is the displacement and $\overline{W}$represents the dimensionless quantity. $\xi$and η will be used instead of x and y. Also, $\overline{D}$can be expressedin terms of flexural rigidity (D), stress (σ), thickness (h) and length (a). Similarly, $\overline{N}_x$ can be expressed in terms of Young's modulus (E_p), Poisson's ratio (ν_p),stress (σ),length (a), and the distance between the two electrodes(d_0).By substituting above dimensionless elements in the equation of motion (26), the following equation is obtained:

$$D\left[\frac{d_0}{a^4}\frac{\partial^4\overline{W}}{\partial\xi^4}+2\frac{d_0}{a^2b^2}\frac{\partial^4\overline{W}}{\partial\xi^2\partial\eta^2}+\frac{d_0}{b^4}\frac{\partial^4\overline{W}}{\partial\eta^4}\right] -\frac{E_p h d_0^3}{2(1-\nu_p^2)}\left[\frac{3}{a^4}\frac{\partial^2\overline{W}}{\partial\xi^2}\left(\frac{\partial\overline{W}}{\partial\xi}\right)^2 +\frac{3}{b^4}\frac{\partial^2\overline{W}}{\partial\eta^2}\left(\frac{\partial\overline{W}}{\partial\eta}\right)^2+\frac{1}{a^2b^2}\frac{\partial}{\partial\xi}\left(\frac{\partial\overline{W}}{\partial\xi}\left(\frac{\partial\overline{W}}{\partial\eta}\right)^2\right) +\frac{1}{a^2b^2}\frac{\partial}{\partial\eta}\left(\left(\frac{\partial\overline{W}}{\partial\xi}\right)^2\frac{\partial\overline{W}}{\partial\eta}\right)\right]-N_x\frac{d_0}{a^2}h\frac{\partial^2\overline{W}}{\partial\xi^2} -N_y\frac{d_0}{b^2}h\frac{\partial^2\overline{W}}{\partial\eta^2}=\frac{\epsilon_0V^2}{2d_0^2(1-\overline{W})^2} \quad (30)$$

After reorganizing above elements, and dividing equation by σ and h, and then after multiplying both sides to a^2, a new equation is obtained. Furthermore, by defining $\overline{D}=\frac{D}{\sigma h a^2}$ and consider other elements of (29), the equation appears in its new form. Then it is continued by inserting $\overline{W}$as dimensionless displacement, and inserting derivatives of $\overline{W}$. The final equation is simplified as follows:

(31)

$$\sum_{m=1}^{2}\sum_{n=1}^{2} q_{mn}\left[\overline{D}m^4+2\overline{D}\left(\frac{a}{b}\right)^2 m^2n^2+\overline{D}\left(\frac{a}{b}\right)^4 n^4+\left(\frac{m}{\pi}\right)^2 +\frac{a^2}{b^2}\left(\frac{n}{\pi}\right)^2\right]\sin m\pi\xi\sin n\pi\eta$$

$$+\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} q_{mn}q_{rs}q_{jk}\left[\overline{D}m^4+2\overline{D}\left(\frac{a}{b}\right)^2 m^2n^2 +\overline{D}\left(\frac{a}{b}\right)^4 n^4+\left(\frac{m}{\pi}\right)^2 +\frac{a^2}{b^2}\left(\frac{n}{\pi}\right)^2\right]\sin m\pi\xi\sin r\pi\xi\sin j\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta$$

$$-2\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2} q_{mn}q_{rs}\left[\overline{D}m^4+2\overline{D}\left(\frac{a}{b}\right)^2 m^2n^2 +\overline{D}\left(\frac{a}{b}\right)^4 n^4+\left(\frac{m}{\pi}\right)^2 +\frac{a^2}{b^2}\left(\frac{n}{\pi}\right)^2\right]\sin m\pi\xi\sin r\pi\xi\sin n\pi\eta\sin s\pi\eta$$

$$-\overline{N}_x\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} q_{mn}q_{rs}q_{jk}\left[(m\pi)^2+\frac{a^2}{b^2}(n\pi)^2\right]$$

$($
$3m^2rj\sin m\pi\xi\cos r\pi\xi\cos j\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta+$
$3\frac{a^4}{b^4}n^2sk\sin m\pi\xi\sin r\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta$

$$+\frac{a^2}{b^2}(m^2sk\sin m\pi\xi\sin r\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta$$
$$+mjsk\cos m\pi\xi\sin r\pi\xi\cos j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta$$
$$+mrnk\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\cos n\pi\eta\sin s\pi\eta\cos k\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta$$
$$+mrk^2\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta)$$
$$)$$

$$-\bar{N}_x\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{o=1}^{2}\sum_{p=1}^{2}\sum_{e=1}^{2}\sum_{f=1}^{2} q_{mn}q_{rs}q_{jk}q_{op}q_{ef}\left[(m\pi)^2+\frac{a^2}{b^2}(n\pi)^2\right]$$
$$($$
$$3m^2rj\sin m\pi\xi\cos r\pi\xi\cos j\pi\xi\sin o\pi\xi\sin e\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta\sin p\pi\eta\sin f\pi\eta$$

$$+3\frac{a^4}{b^4}n^2sk\sin m\pi\xi\sin r\pi\xi\sin j\pi\xi\sin o\pi\xi\sin e\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta$$

$$+\frac{a^2}{b^2}($$
$$m^2sk\sin m\pi\xi\sin r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin e\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta\sin f\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin e\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta\sin f\pi\eta$$
$$+mrnk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin e\pi\xi\cos n\pi\eta\sin s\pi\eta\cos k\pi\eta\sin p\pi\eta\sin f\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin e\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta\sin f\pi\eta$$
$$+mrk^2\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\sin o\pi\xi\sin e\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta\sin p\pi\eta\sin f\pi\eta)$$
$$)$$

$$+2\bar{N}_x\sum_{m=1}^{2}\sum_{n=1}^{2}\sum_{r=1}^{2}\sum_{s=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2}\sum_{o=1}^{2}\sum_{p=1}^{2} q_{mn}q_{rs}q_{jk}q_{op}\left[(m\pi)^2+\frac{a^2}{b^2}(n\pi)^2\right]$$

$$(3m^2rj\sin m\pi\xi\cos r\pi\xi\cos j\pi\xi\sin o\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta\sin p\pi\eta)$$

$$+3\frac{a^4}{b^4}n^2sk(\sin m\pi\xi\sin r\pi\xi\sin j\pi\xi\sin o\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta)$$

$$+\frac{a^2}{b^2}($$
$$m^2sk\sin m\pi\xi\sin r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta$$
$$+mjsk\cos m\pi\xi\sin r\pi\xi\sin o\pi\xi\cos j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta$$
$$+mrnk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\cos n\pi\eta\sin s\pi\eta\cos k\pi\eta\sin p\pi\eta$$
$$+mrsk\cos m\pi\xi\cos r\pi\xi\sin o\pi\xi\sin j\pi\xi\sin n\pi\eta\cos s\pi\eta\cos k\pi\eta\sin p\pi\eta$$
$$+mrk^2\cos m\pi\xi\cos r\pi\xi\sin j\pi\xi\sin o\pi\xi\sin n\pi\eta\sin s\pi\eta\sin k\pi\eta\sin p\pi\eta)$$
$$=\bar{V}$$

The final equation (31) is pull-in voltage and $q_m = q_{11}$ ($q_m$is either q_{11} or the displacement, w).

4. RESULTS AND DISCUSSION

Since the instrument's sensitivity, frequency response, noise properties, etc. depend on the bias voltage, it is necessary to obtain the pull-in voltage. To illustrate the mentioned model of pull-in voltage evaluation, a fully-clamped capacitive structure with device parameters in Table 1 has been used. Since the instrument's sensitivity, frequency response, noise properties, etc. depend on the bias voltage, it is necessary to obtain the pull-in voltage. To illustrate the mentioned model of pull-in voltage evaluation, a fully-clamped capacitive structure with device parameters in Table 1 has been used.

Table 1. Device parameters

(a)	Diaphragm side length	2400 μm
(t)	Diaphragm thickness	2 μm
(d_0)	Air gap thickness	4 μm
(E)	Young's modulus	170 GPa
(σ)	Residual stress	13 MPa
(υ)	Poisson's ratio	0.22

The graph of voltage versus deflection (Eq.(31)) is plotted using MATLAB software (see Fig 3). This curve shows the exact pull- in point which it is an important factor. As can be seen in Fig 3, despite q_{11}(w) is $\frac{1}{3}$of air gap, the pull-in phenomena has not yet taken place. This curve indicates clearly that the pull-in voltage occurs at 13 volt. As can be seen, the deflection is more than $\frac{1}{3}$of air gap. By simulating this structure using Intellisuite software, the pull-in voltage is 13.58 V (see Fig 4). It can be seen that the simulation outcome matches with the theoretical results.

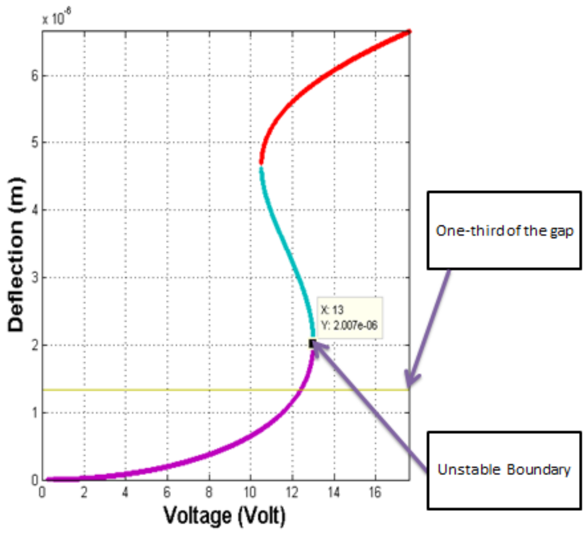

Fig 3. Voltage versus deflection in MATLAB (theoretical results).

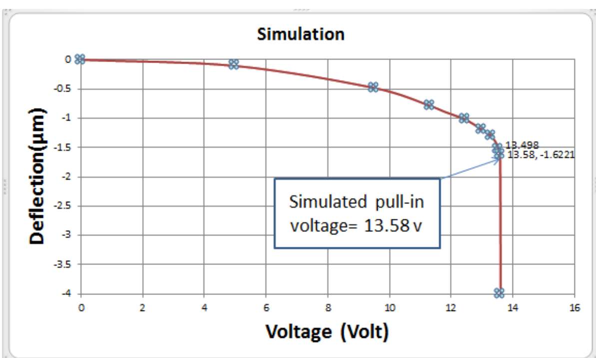

Fig 4. Voltage-Deflection graph

In earlier researches, the criterion which the structure collapses is at one - third of air gap($\frac{4micrometer}{3}$ = 1.33micrometer).As can be seen in Fig 3, which utilizes MATLAB software, the boundary between stable and unstable zone occurs where the movement is more than $\frac{1}{3}$of the air gap (1.7 micrometer). So the former criterion is not applicable any more. In this case the value of Pull-in Voltage is 13 volt. This structure has also been simulated with Intellisuite software as shown in Fig 4. In this illustration, the value of voltage which causes the structure go to unstable state is 13.58 volt. Therefore, the proposed theory is considered to be quite close to simulated results.

5. CONCLUSION

In this paper, an analytical model for calculating deflection and Pull-in voltage of MEMS capacitor with fully-clamped membrane is presented. The calculated results using MATLAB are quite compatible with simulation results. This model shows that the critical point where the collapse happens is not at one-third of air gap. In previous works, when the diaphragm is moving towards the fixed plate, the movable electrode is collapsed to the fixed plate where the distance between them becomes tow third of original gap. In this research, it has been proved that the rules which were previously denoted for borders are not applicable. When four sides of the structure are fixed, the structure goes under a tensile force. This interpretation is shown in Fig 3, and proves that the distance of one- third of air gap is not criteria, since the mentioned structure will be unstable at a distance more than one- third of air gap.

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