Int. J. Advanced Design and Manufacturing Technology, 2025, Vol. 18, No. 4, pp. 25-30

DOI: *ISSN: 2252-0406* *https://admt.isfahan.iau.ir*

Sensitivity Analysis of AFM Piezoelectric MC in Electromagnetic Excitation

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Received: 15 July 2025, Revised: 1 August 2025, Accepted: 16 September 2025

Abstract: Atomic Force Microscopes (AFM) are reliable and accurate tools for surface imaging, mechanical properties detection, and measuring particle motion at the nanoscale. The vibration behavior of the microcantilever (MC) in an AFM is a highly crucial factor for its performance. Also, the dimensions of the MC contribute to its vibratory behavior. The exact surface topography of the sample, and determining its mechanical properties and behavior require thorough knowledge of the effects of different geometric parameters on the coefficients of the interaction forces and the vibration of the MC. In this paper, the authors analyze the dynamic behavior of an air piezoelectric MC under electromagnetic actuation. For this purpose, at first, a dynamic model of the system was developed using the Equation of motion of a continuous beam under vibrations. Then, the effects of the surface interaction force on the behavior of the MC under nonlinear vibrations are investigated. Also, a sensitivity analysis is carried out using the Sobol method to study how the dimensions of an MC affect its nonlinear frequency.

Keywords: Atomic Force Microscopes (AFM), Microcantilever Beams, Sensitivity Analysis

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Research paper

1 INTRODUCTION

AFMs are strong tools for nanoscale surface imaging [1-2], mechanical properties detection [3-4], and measuring particle motion [5]. It is possible to measure mechanical properties on the nanoscale using AFM. It has a wide range of applications, including measurement of intermolecular forces and mechanical properties of materials, pharmacology and biology research, biosensor design, DNA studies, etc [6-8], emphasizing the importance of AFM technology and its development. Several studies have been carried out on modelling and simulation of the MC of an AFM. Park et al. [9] expanded Euler-Bernoulli beam theory using couple stress theory and solved the Equations of equilibrium. Kong et al. [10] studied the Euler-Bernoulli theory for a beam with a circular profile and its boundary conditions, using the modified couple stress (MCS) theory and considering the impact of dimensions. Simsek [11] introduced Euler-Bernoulli theory for hollow beams with circular profiles and confirmed boundary conditions for a dense mass based on MCS theory. Lee et al. [12] presented a V-shaped model for an Euler-Bernoulli MC, using MSC theory. Ansari et al. [13] introduced a piezoelectric MC for Euler-Bernoulli and Timoshenko theories, using MCS theory and voltage. Utilizing MSC theory, Li et al. [6] derived the Equations of a piezoelectric Euler-Bernoulli micro beam. Kahrobaiyan et al. [14] investigated AFM for the Euler-Bernoulli beam, considering MCS theory, the impact of dimensions, and Hamilton's principle. Replacing the laser with a piezoelectric layer in AFMs was first proposed by Tortonise et al. and raised interest due to simplicity, accuracy, and low costs [15]. Fung et al. [16] then modeled the vibrations of an MC with a triangular tip and a piezoelectric layer. Mahmoodi et al. [17] investigated an MC with a piezoelectric layer in AFM and calculated its natural frequency for the first three modes of vibration. Shibata et al. [18] studied diamond MCs in AFM using piezoelectric material as both sensor and actuator and calculated d_{31} coefficient for piezoelectric Zinc oxide (ZnO). Mahmoodi et al. [19] investigated two types of piezoelectric MC in AFM with base excitation and self-excitation variations. In this paper, dynamic modeling of a piezoelectric MC with electromagnetic actuation close to the surface of the sample is achieved. Additionally, Sobol sensitivity analysis is conducted to examine the impact of the MC layers' dimensions.

2 DYNAMIC MODELLING OF MC NEAR THE SAMPLE

A non-homogeneous MC with a piezoelectric layer wrapped between two electrodes on the tip of it is considered for modeling purposes, as is shown in "Fig. 1". The MC is fixed at one end at an angle, and the other end is free. It is affected by interaction forces from the probe and the surface. Theoretical analysis of the structure is conducted using the Euler-Bernoulli theory, with the assumption that the impacts of shear deformation and moment of inertia are negligible.

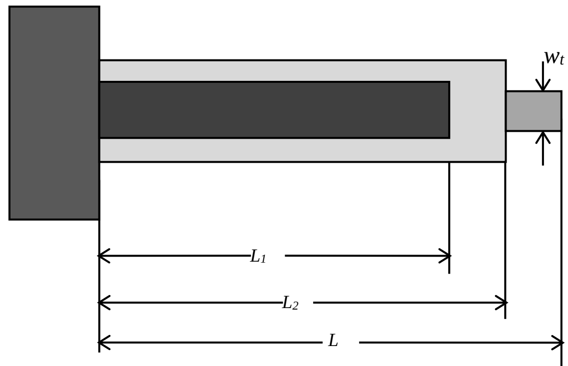


Fig. 1 Schematic of piezoelectric MC.

Using Hamilton's principle, the differential Equation of vibration of the MC is stated below [16]:

$$\rho A\ddot{u} + \left[K\left(x\right)u''\right]'' + c\dot{u} = F_{ts} + F_m \tag{1}$$

Where ρ, A, and c are density, cross-sectional area, and damping coefficient, respectively. F_{ts} and F_m are tip-sample and electromagnetic forces which act on the MC tip. K(x), ρA can be written as:

$$\rho A = \sum_{i=1}^{4}\rho_i h_i w_i\left(H_0 - H_{L_1}\right) + \rho_1 h_1 w_1\left(H_{L_1} - H_{L_2}\right) + \rho_1 h_1 w_t\left(H_{L_2} - H_L\right) \tag{2}$$

h_i and w_i are the thickness and width of each layer of MC. H is the Heavisine function. The MC can be considered equivalent to three homogenous beams with two different conditions of continuity in each stage. In order to analytically solve "Eq. (1)", Galerkin method is used to separate the variables of "Eq. (1)":

$$v\left(x,t\right) = \sum_{n=0}^{\infty} U_n\left(x\right) q_n\left(t\right) \tag{4}$$

In this Equation, $q_n(t)$ and $U_n(x)$ are global coordinates and the comparison function of the nth vibration mode, respectively. Since the MC is replaced with three homogeneous beams, the comparison function is as shown:

$$U_n(x)=\begin{cases}A_{n1}\sin\beta_n x+B_{n1}\cos\beta_n x+C_{n1}\sinh\beta_n x+D_{n1}\cosh\beta_n x & 0<x<L_1\\ A_{n2}\sin\beta_n x+B_{n2}\cos\beta_n x+C_{n2}\sinh\beta_n x+D_{n2}\cosh\beta_n x & L_1<x<L_2\\ A_{n3}\sin\beta_n x+B_{n3}\cos\beta_n x+C_{n3}\sinh\beta_n x+D_{n3}\cosh\beta_n x & L_2<x<L\end{cases} \quad (5)$$

Here $\beta_n^4=\omega_n^2\rho A/EI$ and A_{n1}, B_{n1}, C_{n1}, D_{n1} are unknown variables that are calculated knowing continuity and boundary conditions, deformation, slope, bending moment, shear force, and also by normalizing relative to mass. By substituting "Eqs. (2) and (3)" into "Eq. (1)" and then calculating the dot product of the result with $U_n(x)$, the ordinary differential Equation below results:

$$\ddot{q}_n+\omega_n^2 q_n+\mu\dot{q}_n-g_{1n}q_n^2-g_{2n}q_n^3+g_{3n}F_m=0 \quad (6)$$

Where:

$$\omega_n^2=\int_0^L U_n(K(x)U_n'')''-F_{ts}'U_nH(x-L)\,dx \quad (7)$$

$$g_{1n}=\int_0^L U_n^3\,F_{ts}H(x-L)dx \quad (8)$$

$$g_{2n}=\int_0^L U_n^4\,F_{ts}H(x-L)dx \quad (9)$$

$$g_{3n}=\int_0^L U_n\,H(x-L)dx \quad (10)$$

3 FREQUENCY RESPONSE ANALYSIS

Multiscale method can be used to solve "Eq. (4)", a nonlinear differential Equation:

$$O(\varepsilon):\quad D_0^2\,q_{n1}+\omega_n^2 q_{n1}=0 \quad (11)$$

$$O(\varepsilon^2):\quad D_0^2 q_{n2}+\omega_n^2 q_{n2}+2D_0D_1q_{n1}-g_{1n}q_{n1}^2=0 \quad (12)$$

$$O(\varepsilon^3):\quad D_0^2 q_{n3}+\omega_n^2 q_{n3}+2D_0D_1q_{n2}+2D_0D_1q_{n1}+\mu D_0q_{n1}+g_{3n}F_m-2g_{1n}q_{n1}q_{n2}-g_{2n}q_{n1}^3=0 \quad (13)$$

The solution to "Eq. (9)" appears below:

$$q_{n1}=A_n(T_1,T_2)e^{i\omega_n T_0}+C_c \quad (14)$$

In this Equation, A_n is the amplitude of a complex number and C_c is the complex conjugate of the previous expressions. Substituting "Eq. (12)" in "Eq. (9)" yields:

$$D_0^2 q_{n2}+\omega_n^2 q_{n2}+2i\omega_n D_1A_ne^{i\omega_nT_0}-g_{1n}(A_n^2e^{2i\omega_nT_0}+A_nA_n^*)+C_c=0 \quad (15)$$

In which, A_n^* is the complex conjugate of A_n. Omitting monic expressions results in the Equation below:

$$2i\omega_n D_2A_1+i\mu\omega_nA_1+\frac{1}{2}g_3F_me^{i\sigma T_2}-8A_1^2A_1^*\gamma_f=0 \quad (16)$$

In which:

$$\gamma_f=\frac{\frac{10}{3\omega_n^2}g_{1n}^2+3g_{2n}}{8} \quad (17)$$

γ_f is a criterion for the effects of the nonlinearities in the system and, thus, it can be introduced as a nonlinear coefficient. If negative, this coefficient suggests hardening and if positive, the softening phenomena. Since, in this case it is always positive, it can be concluded that the nonlinear force between the tip of the probe and the sample always causes softening in the frequency response of the system.

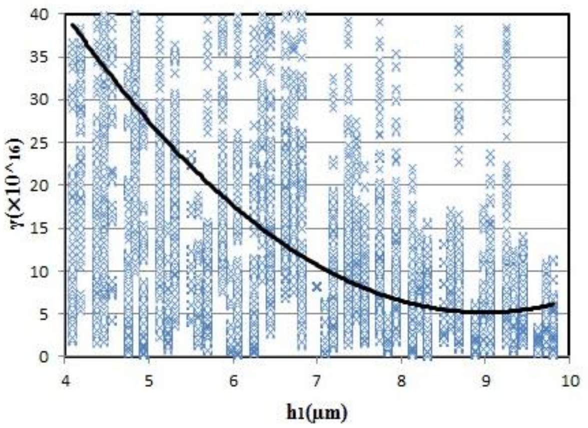
4 SENSITIVITY ANALYSIS OF NONLINEARITY OF VIBRATION

A silicon MC is considered as sample to analyze the sensitivity of the vibrations. Also, a layer of a variety of piezoelectric materials is chosen for further analysis. The piezoelectric layer is placed between two electrodes made of Ti/Au and 0.25 μm thick. The piezoelectric layer goes on top the MC. The necessary geometric and physical properties of the system are stated in "Table 1".

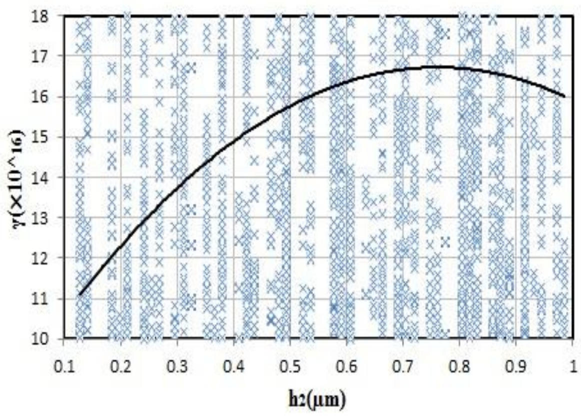
Table 1 Mechanical properties of the piezoelectric MC

	Material	E (Gpa)	ρ (Kg/m^3)
Base Layer	Si	180	2330
Piezoelectric Layer	Zno	130	6390
tip	Si	180	2330

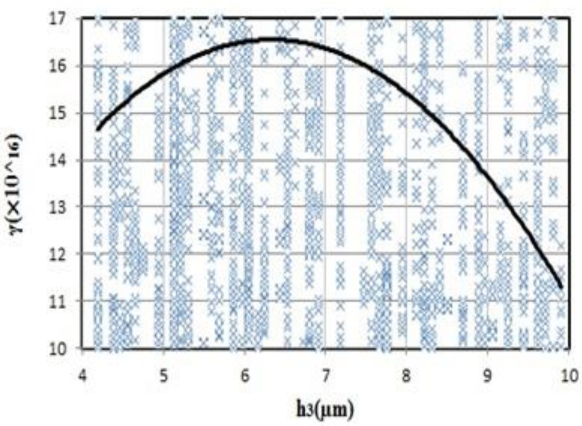
Since scientific and accurate research on the behavior of the MC on the surface requires sensitivity analysis, Sobol sensitivity analysis is conducted near the surface of the sample to determine the effects of the force coefficients on the vibratory motion of the MC. Therefore, the effects of thickness, width, and length are investigated during sensitivity analysis.



(a)



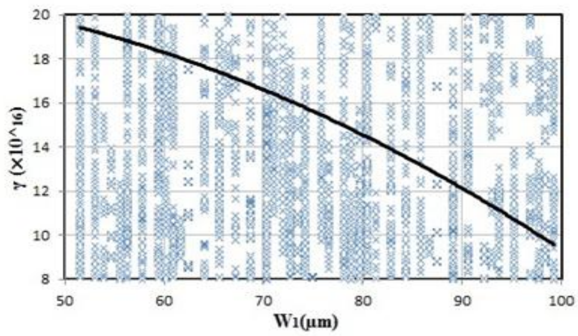
(b)



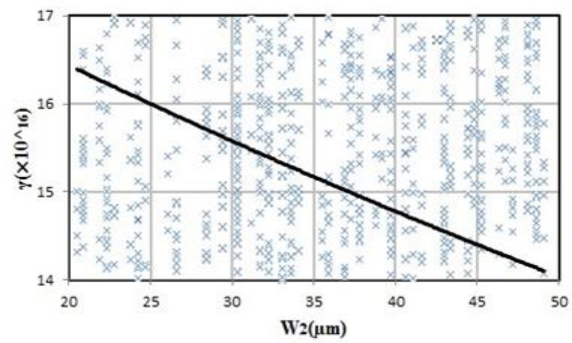
(c)

Fig. 2 Effect of MC thickness of layers on nonlinearity: (a): base layer thickness, (b): electrode thickness, and (c): piezoelectric layer thickness.

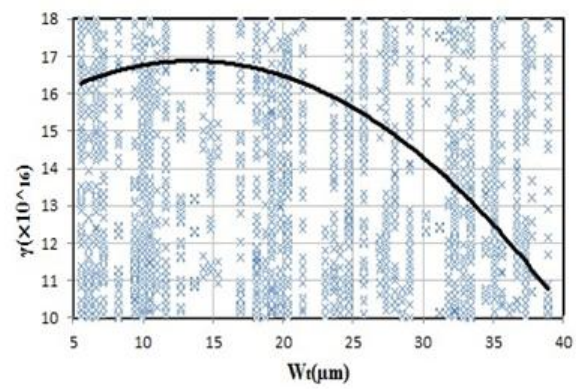
Figure 2 illustrates the changes in the magnitude of the coefficient of nonlinearity of the system with an increase in the thickness of the MC, electrodes, and the piezoelectric layer. The results in "Fig. 2a" demonstrate that the coefficient of nonlinearity of the system reduces for an increase in the thickness of the MC. This trend continues until a thickness of 8.97 μm, which yields a coefficient of nonlinearity of 5.06 × 10^16.



(a)



(b)



(c)

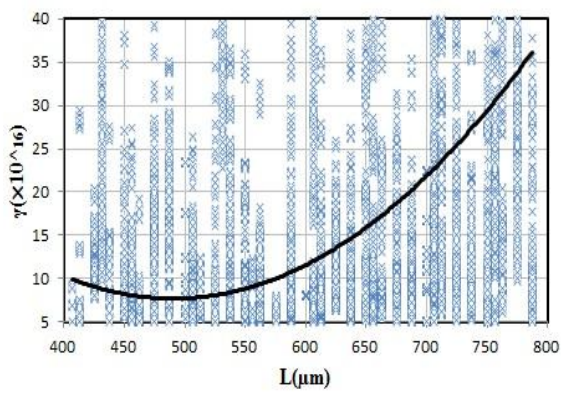
Fig. 3 Effect of width of layers on nonlinearity: (a): base layer width, (b): electrode width, and (c): piezoelectric layer width.

The effect of the thickness of the electrode layer on the coefficient of nonlinearity is presented in "Fig. 2b". According to the analysis results, an increase in the thickness of the electrode layer leads to an increase in the system's coefficient of nonlinearity. The maximum coefficient of nonlinearity in this case is 16.7×10^{16}, resulting from 0.761µm of electrode layer thickness. The coefficient of nonlinearity of the system decreases with increasing thickness. According to “Fig. 2c”, the coefficient of nonlinearity also increases with an increase in the thickness of the piezoelectric layer. The maximum amount results in 6.3 µm of thickness of the piezoelectric layer and equals 16.6×10^{16}. By comparing these three graphs, it is concluded that an increase in the thickness of the electrode layer or the piezoelectric layer initially leads to a rising trend in the coefficient of the nonlinearity of the system, but is followed by a decreasing trend. On the contrary, increasing the thickness of the MC initially decreases the coefficient of nonlinearity, only to be followed by an increasing trend after the minimum point of the graph.

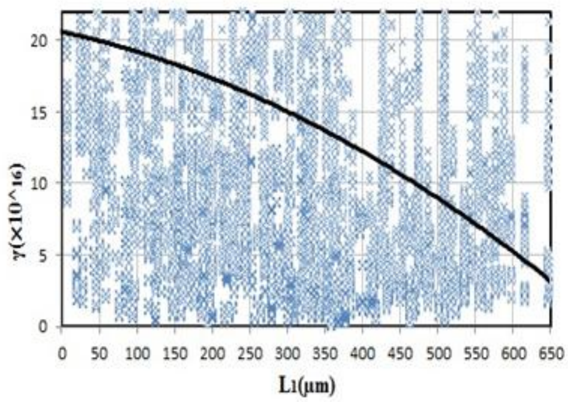
Figure 3 demonstrates how an increase in the thickness of the MC, the piezoelectric layer, and the tip of the probe affects the coefficient of nonlinearity of the system.

Figure 3a shows a decrease in the coefficient of nonlinearity for an increase in the width of the MC. In other words, increasing the width of the MC dampens nonlinear vibrations of the system. Similarly, based on “Fig. 3b”, increasing the width of the piezoelectric layer decreases the coefficient of nonlinearity. On the other hand, “Fig. 3c” suggests an initially decreasing but eventually rising trend for the coefficient of nonlinearity as a result of an increase in the width of the tip of the probe. The maximum coefficient of nonlinearity in this case is 16.6×10^{16}, which is achieved for a width of 13.7µm. By comparing these three graphs, it can be concluded that an increase in the width of the MC and the piezoelectric layer reduces the coefficient of nonlinearity. In contrast, such a change in the width of the beam tip leads to an increase in the coefficient of nonlinearity until a maximum width of 13.7 µm is reached, followed by a decrease thereafter.

Figure 4 demonstrates the influence of increasing the length of the MC and the piezoelectric layer on the coefficient of nonlinearity.



(a)



(b)

Fig. 4 Effect of: (a): MC length, and (b): piezoelectric length on nonlinearity.

According to “Fig. 4a”, increasing the length of the MC initially leads to a decrease in the coefficient of nonlinearity. The minimum value is 7.49×10^{16}, achieved for a value of 487.5µm. Further increasing the length of the MC brings about an ascending trend in the coefficient of nonlinearity. Figure 4b shows that increasing the length of the piezoelectric layer reduces the coefficient of nonlinearity. This coefficient is 3.47×10^{16} for a piezoelectric layer with a length of 648.5µm, which is 83% less than 20.4×10^{16}, calculated when there is no piezoelectric layer.

5 CONCLUSIONS

To achieve a dynamic model of a vibrating MC in air with electromagnetic excitation, first, contact forces between the tip of the probe and the surface of the sample and the excitation force, concentrated at the tip of the MC, are applied. Then the differential Equations of motion are derived using Hamilton’s principle and based on Euler-Bernoulli theory for a vibrating continuous beam. Since the interaction force between the tip of the probe and the surface of the sample is nonlinear and a function of the deformation of the MC, the differential Equations are solved using the Galerkin method, followed by the multiscale method. The impact of the MC dimensions on system nonlinearity is also studied using Sobol sensitivity analysis. The results have shown that:

- An increase in the thickness of the MC leads to an initial increase and eventual decrease in the coefficient of nonlinearity after the minimum point. An increase in the thickness of the electrodes leads to an initial decrease and eventual increase in the coefficient of nonlinearity with a maximum point. Similarly, the resulting graph for the changes in the coefficient of nonlinearity as a result of increasing the thickness of the piezoelectric layer follows a rising trend initially, only to start falling after reaching its maximum point.

- The coefficient of nonlinearity reduces as a result of increasing the width of the MC or the piezoelectric layer. However, the graph of changes in the coefficient of nonlinearity for increasing the width of the beam's tip is initially descending and then ascending after the minimum point.
- Increasing the length of the MC yields a descending-ascending curve with a minimum point for the changes of the coefficient of nonlinearity. On the other hand, increasing the length of the piezoelectric layer results in a reduction of the system's coefficient of nonlinearity.

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