# Analysis and Control of Chaos in Nonlinear Gear System using Predictive Sliding Mode Control

**Nima Valadbeigi**
Department of Mechanical Engineering,
Industrial and Mechanics Faculty,
Qazvin Islamic Azad University, Iran
E-mail: n.valadbeigi@qiau.ac.ir

**Seyed Mahdi Abtahi***
Department of Mechanical Engineering,
Industrial and Mechanics Faculty,
Qazvin Islamic Azad University, Iran
E-mail: m.abtahi@qiau.ac.ir
*Corresponding author

**Received: 13 January 2022, Revised: 9 April 2022, Accepted: 15 April 2022**

**Abstract:** This paper presents a control system for elimination of chaotic behaviors in spur gear system. To this end, at first different aspects of chaos are investigated by means of numerical tools including time series response, phase plane trajectories, bifurcation diagram, Poincare' section, Lyapunov exponent and power spectrum density. The nonlinear dynamic model encompasses constant mesh stiffness and damping along the line of action, static transmission error and backlash. In order to suppress the chaotic oscillations, a novel controller on the basis of the Predictive Sliding Mode Control (PSMC) is proposed in which the sliding surface is predicted by the use of model predictive control theory and the control input is obtained. Consequently, the control system takes advantage of the both approaches in developing a robust controller. The simulation results of the feedback system depict the effectiveness of the controller in elimination of the chaotic vibrations along with reduction of settling time, overshoots, and energy consumption. Furthermore, stability and robustness of the system are guaranteed.

**Keywords:** Bifurcation Diagram, Chaos Analysis, Chaotic Vibration, Gearbox System, Predictive Sliding Mode Control

**How to cite this paper:** Nima Valadbeigi, Seyed Mahdi Abtahi, "Analysis and Control of Chaos in Nonlinear Gear System using Predictive Sliding Mode Control", Int J of Advanced Design and Manufacturing Technology, Vol. 15/No. 2, 2022, pp. 97–104. DOI: 10.30495/admt.2022.1949735.1333.

**Biographical notes: Nima Valadbeigi** received his M.Sc. in Mechanical Engineering from Karaj Islamic Azad University, Iran in 2016 and B.Sc. in Mechanical engineering from Karaj Islamic Azad University, Iran, in 2013. He is currently Ph.D. student in Qazvin Islamic Azad University, Iran. His current research interest includes Dynamical system and control, Nonlinear control, Chaos Analysis and Control. **Seyed Mahdi Abtahi** received his B.Sc.in Mechanical Engineering from Guilan University, Iran, in 2004, an M.Sc. in Mechanical Engineering in 2007, and Ph.D. in Mechanical Engineering from K.N. Toosi University of Technology, Iran, in 2012. He is currently Assistant Professor at the Department of Mechanical Engineering, Qazvin Islamic Azad University, Qazvin, Iran. His research interest includes Dynamical system and control, Nonlinear control, Chaos analysis and control.

Research paper 

## 1 INTRODUCTION

Gearbox systems play an important role in disparate applications. Thus, developing an optimal gear transmission system, necessitates a proper understanding about the main characteristics and nonlinear phenomena that affect the system. One of the most important phenomena that occurs in gear spur systems is chaos that culminates in harmful vibrations. The chaotic behavior of gearbox systems was studied by Hongler and Streit in 1988. [1] aimed to discuss the origin of chaos from a theoretical point of view. According to this research, Melnikov method could be applied to calculate the values of the parameters which lead to chaos. Moreover, Fokker-Planck description of the system dynamics could be a valuable approach to apply on actual systems. By analytical integration of the Equations of motion, some dynamical behaviors were observed in [2].

The research aimed to describe some nonlinear features which were not explored in previous works such as Pfeiffer. De Souza tried to investigate the dynamical effects of noise in gear-rattling model [3]. Effects of flash temperature of tooth surface on the dynamics of the spur gear system considering backlash, bearing clearance and time-varying stiffness were studied in [4]. Consequently, complicated phenomena such as periodic bifurcation and chaos were observed. Optimization of gearbox geometric design parameters such as module, axial clearance and backlash is presented in [5] to reduce gear rattle noise. The optimization culminated in reducing the vibrations and noise levels. The Melnikov analysis was extended in [6] to develop a practical model in order to control the chaotic behavior of gearbox system in the presence of backlash, static transmission error and time-varying stiffness. Consequently, the non-feedback control method was used by applying an additional control excitation.

In [7], After investigating the chaotic behavior of gearbox system, the adaptive terminal sliding mode control approach was presented. In 2003, a controller was presented in [8] such that by combination of Model Predictive Control (MPC) and Sliding Mode Control (SMC), future control of movement for the sliding surface became possible. This led to precise prediction and process control with dead time. In [9] a constrained finite-time optimal controller was designed and feasibility and closed loop stability were discussed.

An overview of some hierarchical control schemes was presented in [10] based on the realization of considered SMC. Disturbances which affect the plant were rejected and both continuous and discrete time solutions were discussed. A MPC-SMC method for nonlinear systems was presented in [11] which was resulted from a new sliding manifold definition. The dynamics of the system could be conducted to sliding surface faster by using this controller. [12] aimed to present an extended MPC-SMC controller. Using an increasing number of voltage vectors for prediction and cost function minimization led to great computation delay.

A table-based implementation process was used to reduce the whole execution time. Tube model predictive control with a SMC as an auxiliary controller was studied in [13]. Spasic used a nominal tube MPC to highlight the robustness improvement. To adaptively tune the switching gain of an SMC, a linear MPC scheme based on the closed loop system dynamics was presented [14].

In this paper, a generalized nonlinear dynamic for spur gear system is formulated such that takes constant mesh stiffness and damping along the line of action, static transmission error and backlash into account. In order to study different aspects of chaos, the system is investigated by the use of time series response, phase plane trajectories, bifurcation diagram, Poincare′ section, Lyapunov exponent and power spectrum density methods. After chaos analysis, a novel controller based on the Predictive Sliding Mode Control is presented such that with prediction of the sliding surface, the control law is obtained.

The results obtained from the novel feed-back control system are compared with a conventional model predictive controller (MPC) to show superiority of the PSMC controller in elimination of chaotic vibrations, reduction of settling time and energy consumption. The results can be compared with open-loop system too, in which chaos occurred. The results illustrate the effectiveness of the controller in terms of robustness and stability due to taking advantage of the both control approaches. The paper is organized as follow: in section 2 dynamic model of the system is described. In section 3 by means of analytical tools, chaotic behavior of the nonlinear system is investigated. In section 4, predictive sliding mode control is explored in details and in section 5 simulation results of feedback system are presented.

## 2 DESCRIPTION OF THE MODEL

The dynamic model of a spur gear system is considered in this paper. The generalized model is shown in "Fig. 1".

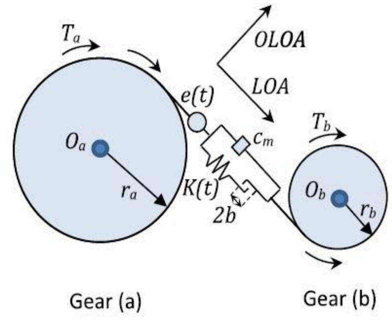


**Fig. 1** A spur gear pair model.

The gear mesh is modeled as a pair of rigid disks which are considered to be connected by a set of spring and damper with a constant damping C along the line of action. $r_a$ and $r_b$ represent the radius of the base circles of the gears while $I_a$ and $I_b$ are the mass inertia moment of the gears, $k_m$ is mesh stiffness, and the external torques acting on gears are $T_a$ and $T_b$, respectively. In order to represent the gear clearances, the backlash $f_h$ is used. Internal excitation *e(t)* is used to represent the static transmission error. It is worth to mention that the static transmission error is applied to represent any manufacturing errors and teeth deformations from the perfect involute form [6]. Considering the mentioned parameters, the Equation of torsional motion is:

$$m\frac{d^2\tilde{x}}{dt^2}+c\frac{d\tilde{x}}{dt}+kf_h(\tilde{x})=\hat{F}_m+\hat{F}_e(t) \tag{1}$$

Where:

$$f_h(\tilde{x})=\begin{cases}\tilde{x}-(1-\alpha)b\ ; & b<\tilde{x}\\ \alpha\tilde{x} & ; -b\le\tilde{x}\le b\\ \tilde{x}+(1-\alpha)b; & b<-\tilde{x}\end{cases}$$

And:

$$\hat{F}_m=m(\frac{T_a r_a}{I_a}+\frac{T_b r_b}{I_b}),\hat{F}_e(t)=-m\frac{d^2 e(t)}{dt^2},$$

$$m=\frac{I_a I_b}{I_a r_a^2+I_a r_b^2}$$

Where, $\tilde{x}$ is the difference between the dynamic and static transmission errors and is written as: $\tilde{x}=r_a\theta_a-r_b\theta_b$. $\theta_a$ and $\theta_b$ represent the torsional displacements of the gears [6]. Static transmission error is a function of mesh frequency as a harmonic function in the form of $e(t)=e(t+2\pi/\omega_e)=ecos(\omega_e+\varphi_e)$.

By defining:

$$x=\frac{\tilde{x}}{b},\qquad \omega_n=\sqrt{\frac{k_m}{m}},\qquad \tau=\omega_n t,$$

$$\Omega_k=\frac{\omega_k}{\omega_n}$$

$$\Omega_e=\frac{\omega_e}{\omega_n},\qquad \tilde{\mu}=\frac{c}{2m\omega_n},\qquad \tilde{k}_p=\frac{k_p}{m\omega_n^2}$$

$$\tilde{F}_m=\frac{\hat{F}_m}{bk_m},\ \tilde{F}_e=e/b$$

So, the dimensionless form of Equation 1 is:

$$\frac{d^2x}{d\tau^2}+2\tilde{\mu}\frac{dx}{d\tau}+f_h(x)=\tilde{F}_m+\tilde{F}_e\Omega_e^2\cos(\Omega_e t+\phi_e) \tag{2}$$

Where:

$$f_h(\tilde{x})=\begin{cases}\tilde{x}-1-\alpha\ , & 1<\tilde{x}\\ \tilde{x} & , -1\le\tilde{x}\le 1\\ \tilde{x}+(1-\alpha)b, & 1<-\tilde{x}\end{cases}$$

And:

$$\tilde{F}_m=\varepsilon f_m, \tilde{F}_e=\varepsilon f_e, \tilde{\mu}=\varepsilon\mu, 0<\varepsilon<1$$

A 3-order approximation polynomial was suggested by [6] to express the backlash function. Therefore, by defining $f_h(x)=-0.1667x+0.1667x^3$ the Equation of motion is given by:

$$\frac{d^2x}{d\tau^2}+2\tilde{\mu}\frac{dx}{d\tau}+(-0.1667x+0.1667x^3)=\tilde{F}_m+\tilde{F}_e\Omega_e^2\cos(\Omega_e t+\phi_e) \tag{3}$$

The state-space could be given as:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=-2\varepsilon\mu x_2+(0.1667x_1-0.1667_1^3)\\ \quad +\varepsilon(f_m+f_e\Omega_e^2\cos(\Omega_e t+\varphi_e))\end{cases} \tag{4}$$

## 3 NONLINEAR ANALYSIS OF THE GEAR SYSTEM

The nonlinear dynamics of the system is investigated by using time series response, phase plane trajectories, Poincare′section, Bifurcation diagram, Lyapunov exponent and power spectrum. According to “Figs. 2 and 3” in which time series response of the system for 300 seconds are presented, the behavior of the system is neither periodic nor quasi periodic. Figure 4 suggests that basin of attraction the system is obvious. Furthermore, the dynamics of the system is analyzed using the Poincare′ section. The section is a hyper-surface in the state space, transverse to the flow of the considered system.

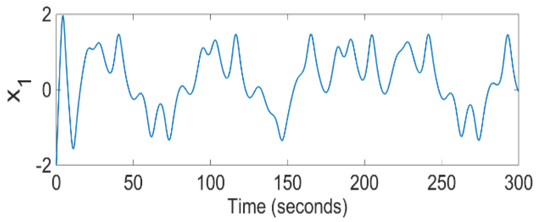


**Fig. 2** Time series response of the $x_1$.

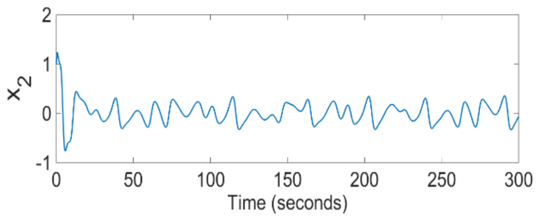


**Fig. 3** Time series response of the $x_2$.

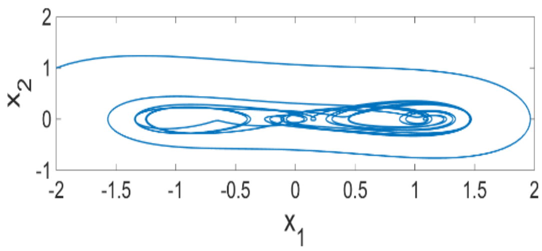


**Fig. 4** Trajectory of the system.

Poincare′ section for the system is shown on "Fig. 5". Return points form a fractal structure comprise many irregularly-distributed points. The bifurcation diagram of the nonlinear system is generated using $f_e$ as the control parameter. The bifurcation control parameter is varied from 10 to 40 with a constant step. According to "Fig. 6", for values $f_e > 25$ chaos occurs.

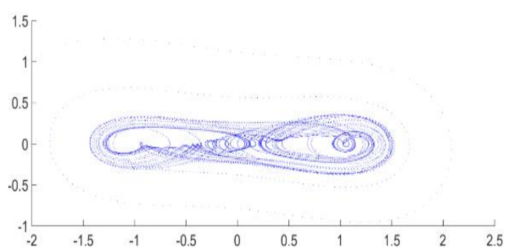

**Fig. 5** Poincare′ section.

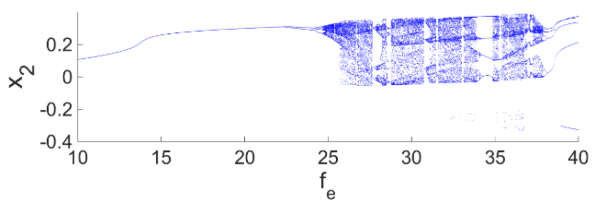


**Fig. 6** Bifurcation diagram.

Beside the above-mentioned numerical methods, Lyapunov exponent can prove the chaotic behavior in the trajectory space of the system. Lyapunov exponent of a dynamic system is defined by:

$$\lambda_j = \lim_{t\to\infty} \frac{1}{t} \log \frac{\varepsilon_j(t)}{\varepsilon_0} \tag{5}$$

Where, $\lambda_j$ denotes the divergence of the nearby trajectories [15]. Lyapunov exponent of the system is calculated and plotted in "Fig. 7". Positive value of the maximum Lyapunov exponent in "Fig. 7" could be taken as an indication of chaotic behavior. The spectrum components of the system are analyzed using Welch algorithm. Wide range of frequencies in the power spectrum density diagrams is clear. Based on the results achieved from the analytical tools, chaotic behavior of the gear system is validated ("Figs. 8-9").

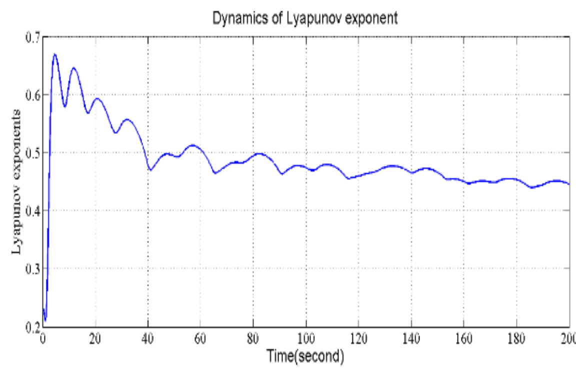


**Fig. 7** Lyapunov exponent.

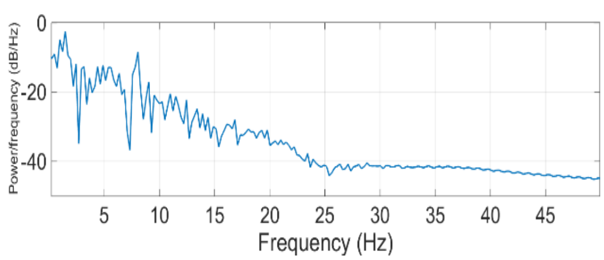


**Fig. 8** Power spectrum density for $x_1$.

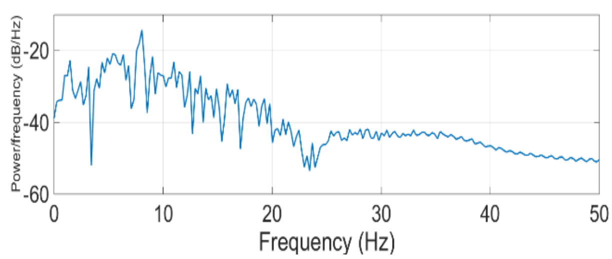


**Fig. 9** power spectrum density for $x_2$.

## 4 CHAOS CONTROL SYSTEM

### 4.1. MPC-SMC Algorithm

Consider the nonlinear system:

$$\boldsymbol{x}^{(n)} = f(x,t) + g(x,t)u \tag{6}$$

Where, $\boldsymbol{x} \in R^n$ is the output of interest and $x \in R^m$ is the state vector and $u \epsilon R^m$ is the control input vector. *f(x)* is considered to be the given function vector and *g(x)* is control gain matrix [16]. SMC design entails two phases. The first phase in Sliding mode control is the sliding surface design. Let $\tilde{x} = x - x_d$ be the tracking error, a surface in the state-space could be defined as:

$$s(x,t) = (\frac{d}{dt} + \lambda)^{n-1} \overline{x} \tag{7}$$

Where, $\lambda$ is a strictly positive constant. For the considered system in this paper *n=2*. Thus, the sliding surface can be:

$$s = \tilde{x}^{\cdot} + \lambda \tilde{x} \tag{8}$$

Defining:

$$z = \lambda \tilde{x}^{\cdot} \tag{9}$$

Differentiating the sliding surface results in:

$$\dot{s}(t) = \ddot{x} - \ddot{x}_d + z \tag{9}$$

Within the moving time frame, the sliding surface $\hat{S}(t+T)$ at the time T is predicted by [17]:

$$\hat{s}(t+T) = s(t) + T\dot{s}(t) \tag{11}$$

According to the predictive control approach, a cost function is defined. Consequently, the control signal is obtained by optimizing the cost function within a time domain called control horizon [18]. Optimization is repeated in every time step. In order to derivate the control law, the cost function at time T is given by:

$$J(x,u,t) = \frac{1}{2} \hat{s}^T (t+T) \hat{s}(t+T) \tag{12}$$

In the second phase of sliding mode control, control input is derived as follows. The requisite condition for the optimal control to minimize (12) with respect to u is given by:

$$\frac{\partial J}{\partial u} = 0 \tag{13}$$

By substituting (11) in (12) and solving (13), the predictive sliding mode control law would be:

$$u = [\hat{u} - K sign(s)] / \hat{g} \tag{14}$$

Where;

$$K = (F+\eta)G + (G-1)|\hat{u}| \tag{15}$$

In which *F* and *G* are approximations of *f* and g and:

$$\hat{u} = -\lambda(x_2 - x_{d_2}) - \hat{f} + x_{d_2} \tag{16}$$

Where, $\hat{f}$ and $\hat{g}$ are bounds for uncertainties and gain matrices respectively.

### 4.2. Control of the Gearbox Transmission System via the MPC-SMC Algorithm

In Equation (6), *n=2*. According to Equation (4) we have:

$$f = -2\mu\varepsilon x_2 + \varepsilon(f_m + f_e \Omega_e^2 \cos(\Omega_e + \phi_e)) \tag{17}$$

And:

$$g = 0.1667 x_1 - 0.1667 x_1^3 \tag{18}$$

To find the upper bounds for uncertainties and gain matrix we may write:

$$\hat{f} = |2\varepsilon\mu x_2| + |\varepsilon(f_m + f_e \Omega_e^2 \cos(\Omega_e + \phi_e)| \tag{19}$$

And:

$$\hat{g} = |0.1667(x_1 - x_1^3)| \tag{20}$$

Sliding surface is:

$$s = (x_2 - x_{d_2}) + \lambda(x_1 - x_{d_1}) \tag{21}$$

Predicted sliding surface could be defined as:

$$\hat{s} = (x_2 - x_{d_2}) + \lambda(x_1 - x_{d_1}) + T(-2\varepsilon\mu x_2 + 0.1667(x_1 - x_1^3) + \varepsilon(f_m + f_e \Omega_e^2 \cos(\Omega_e t + \phi_e) - \dot{x}_{d_2} + z) \tag{22}$$

Defining:

$$F = 0.8\hat{f} \tag{23}$$

And:

$$G = 0.8\hat{g} \tag{24}$$

Finally control signal is obtained as:

$$u = [\hat{u} - K sign(s)] / \hat{g} \tag{25}$$

## 5 SIMULATION OF THE FEEDBACK SYSTEM

In order to assess performance of the control system, simulation studies are conducted. The system is simulated based on values of parameters listed in "Table 1". Also, the parameters of the controller are considered as "Table 2".

**Table 1** Values of parameters

| e | μ | $f_e$ | $\Omega_e$ | $\varphi_e$ | $f_m$ | $x_1(0)$ | $x_2(0)$ |
|---|---|---|---|---|---|---|---|
| 0.01 | 9 | 0.5 | 30 | 1 | 0 | -2 | 1 |

**Table 2** Parameters of the controller

| η | λ | $x_{d1}$ | $x_{d2}$ | Prediction horizon | Control horizon |
|---|---|---|---|---|---|
| 2 | 5 | -3 | 0 | 10 | 2 |

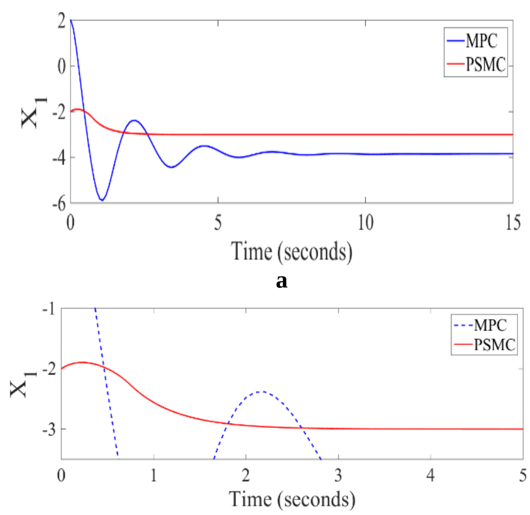


**Fig. 10** Closed loop response for the state $x_1$ : (a): Response for 15 seconds, and (b): response for 5 seconds in more details.

Simulation is done for 15 seconds. Time series responses of the states $x_1$ and $x_2$ are presented in "Figs. 10 and 11", respectively. The figures illustrate the results for PSMC controller and conventional MPC controller to assess performance of the suggested feed-back system. It is noteworthy that the results can be compared with uncontrolled responses which are plotted in "Figs. 2 and 3". Figures 10-11 clearly depict faster response and less overshoot for the PSMC controller in comparison with MPC controller. Furthermore, comparing closed-loop system results with open-loop system results, shows that suggested controller succeeded to remove chaotic vibrations adequately. Fast convergence for the predicted sliding surface is obvious in Fig. 12. Finally, from Fig. 13 less energy consumption for PSMC controller is clear compared with SMC controller.

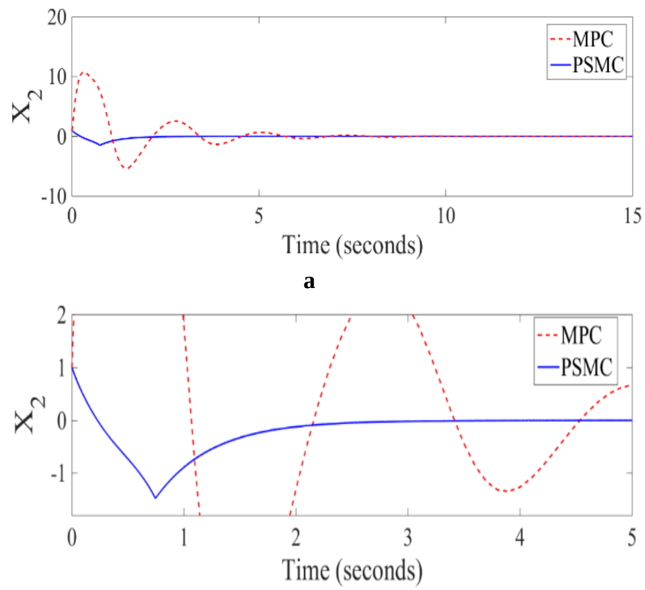


**Fig. 11** Closed loop response for the state $x_2$ : (a): Response for 15 seconds, and (b): response for 5 seconds in more details.

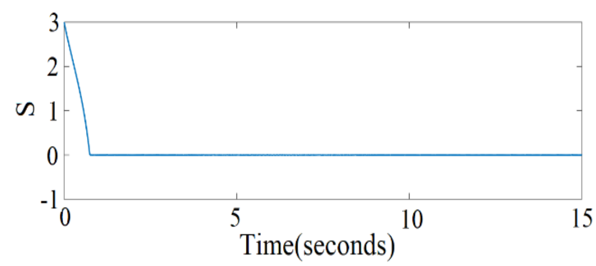


**Fig. 12** Sliding surface.

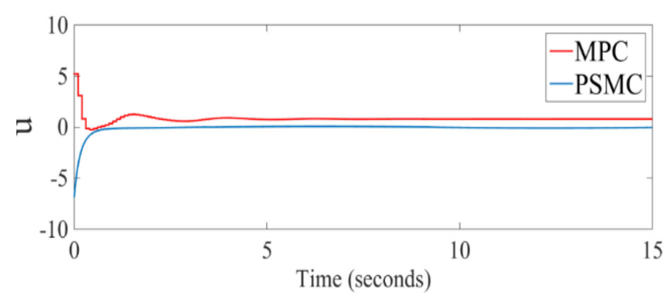


**Fig. 13** Control input.

## 6 CLOSED-LOOP STABILITY ANALYSIS

### 6.1 Lyapunov Stability

In this section closed-loop stability of the system is investigated by means of Lyapunov linearization method. **linearization theorem**: If the linearized system is stable, which means $\Re\lambda_i(A) < 0$ *for* $i = 1,...,n,$ then the nonlinear system is locally asymptotically stable [20]. Where $\lambda_i$ are the eigenvalues of the linearized system. On the contrary, if for some $i, \Re\lambda_i(A) > 0,$ the nonlinear system is not locally asymptotically stable. the operating point for the system is $x_e = (1.195, 0)$ thus the linearized system regarding to the operating point is obtained. Eigen values of the linearized system are:

$$\lambda_1 = -0.0231$$
$$\lambda_2 = -23.1068$$

Since the eigenvalues are negative, closed-loop stability is proved according to Lyapunov stability theory.

### 6.2 Popov Criterion Based on Development of Nyquist Stability Theorem

The nonlinear system is presented in "Fig. 14", which includes a static part, (N), and the plant, (G).

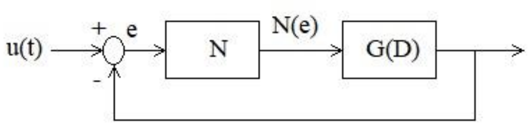


**Fig. 14** System with nonlinearity.

**Popov criterion**: The system is absolutely stable if for $1 \le i \le p,\ N_i \in [0, k_i]$ and there exists a constant $\gamma_i \ge 0,$ with $(1+\lambda_k\gamma_i) \ne 0$ for every eigenvalue $\lambda_k$ *of* $A,$ such that $\frac{1}{k} + (1+s\gamma)G(s)$ is strictly positive real. Thus if $\frac{1}{k} + (1+s\gamma)G(s)$ is strictly positive real and G(s) is Hurwitz:

$$\begin{aligned} &\frac{1}{k} + \mathrm{Re}[G(j\omega) - \gamma\omega\,\mathrm{Im}[G(j\omega)] > 0,\ \forall\omega \in [0,\infty) \\ &if \\ &\lim_{\omega\to\infty} \left\{\frac{1}{k} + \mathrm{Re}[G(j\omega) - \gamma\omega\,\mathrm{Im}[G(j\omega)]\right\} = 0 \end{aligned} \quad (26)$$

We also need [21]:

$$\begin{aligned} &\lim_{\omega\to\infty} \omega^2 \left\{\frac{1}{k} + \mathrm{Re}[G(j\omega) - \gamma\omega\,\mathrm{Im}[G(j\omega)]\right\} > 0 \\ &\frac{1}{k} + \mathrm{Re}[G(j\omega) - \gamma\omega\,\mathrm{Im}[G(j\omega)] > 0,\ \forall\omega \in [0,\infty) \end{aligned} \quad (27)$$

The Popov diagram of the system is:
According to "Fig. 15", regarding to the Popov criterion, the system is stable.

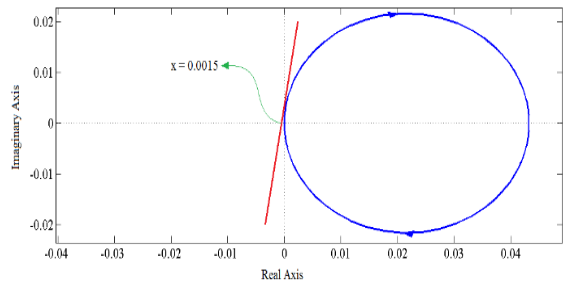


**Fig. 15** Popov diagram.

## 7 ROBUSTNESS ANALYSIS

In order to evaluate robustness of the control system, in case of parametric uncertainties, value of some of the system parameters are changed by 10 to 20%. For instance, meshing stiffness, $k_m$, from a constant value turned into a harmonic parameter as $k_m = 1 + k\sin\omega t$. Eventually no change in performance of the controller is observed and analogous results like the ones achieved in section 5 were observed. Thus, the controller is robust against the nonlinear parametric uncertainties, although the open-loop is chaotic and inherently sensitive to any simple change in parameters.

## 8 CONCLUSIONS

The dynamics of a spur gear system and the chaotic behavior of the system is investigated in this paper by the use of numerical tools including time series response, phase plane trajectories, bifurcation diagram, Poincare′ section, Lyapunov exponent, and power spectrum density. According to the results of open loop simulation, chaotic behavior of the system is quite obvious and different aspects of chaos are studied. In order to control the chaotic behavior, a chaos controller based on the predictive sliding mode control is implemented such that the sliding surface is predicted and the control law is obtained.

The simulation results of the closed-loop system suggest that the PSMC controller improves the settling time for $x_1$ by 67% and for $x_2$ by 81% in comparison with the MPC controller. On the other hand, PSMC controller declined the overshoot for $x_1$ by 80% and for $x_2$ by 88%. These results achieved while the energy consumption is declined by 10%. Moreover, PSMC controller shows better results and performance for $x_1$, $x_2$ and input signal in terms of eliminating chaotic vibrations, reducing the settling time and overshoot compared to [19].
Finally, fast convergence to the sliding surface is counted as another good attribute of PSMC controller in comparison with [19]. Thus, well performance of the PSMC controller in eliminating the chaotic behavior and stabilizing the system is adequately verified.

## REFERENCES


[1] Hongler, M., Streit, M., On the Origin Of Chaos In Gearbox Models, Physica D: Nonlinear Phenomena, Vol. 29, No. 3, 1988, pp. 402-408, doi.org/10.1016/0167-2789(88)90038-3.

[2] De Souza, S., Caldas, I., Viana, R., and Balthazar, J., Sudden Changes in Chaotic Attractors and Transient Basins in A Model for Rattling in Gearboxes, Chaos, Solitons & Fractals; Vol. 21, No. 3, 2004, pp. 763-772, doi.org/10.1016/j.chaos.2003.12.096.

[3] De Souza, S., Caldas, I., Viana, R., Batista, A., and Kapitaniak, T., Noise-Induced Basin Hopping in A Gearbox Model, Chaos, Solitons & Fractals, Vol. 26, No. 5, 2005, pp. 1523-1531, doi.org/10.1016/j.chaos.2005.04.033.

[4] Gou, X., Zhu, L., and Qi, C., Nonlinear Dynamic Model of a Gear-Rotor-Bearing System Considering the Flash Temperature, Journal of Sound and Vibration; Vol. 410, No. 8, 2017, pp. 187-208, doi.org/10.1016/j.jsv.2017.08.014.

[5] Bozca, M., Transmission Error Model-Based Optimization of the Geometric Design Parameters of An Automotive Transmission Gearbox to Reduce Gear-Rattle Noise, Applied Acoustics, Vol. 130, No. 15, 2017, pp. 247-259, doi.org/10.1016/j.apacoust.2017.10.005.

[6] Farshidianfar, A., Saghafi, A., Identification and Control of Chaos in Nonlinear Gear Dynamic Systems Using Melnikov Analysis, Physics Letters A, Vol. 378, No. 46, 2014, pp. 3457-3463, doi.org/10.1016/j.physleta.2014.09.060.

[7] Delavari, H., Mohadeszadeh, M., Chaos Control in a Gear Transmission System via a Novel Adaptive Terminal Sliding Mode Control Approach with Nonlinearity in the Control Input, Journal of Control, Vol. 10, No. 4, 2016, pp. 45-35, joc.kntu.ac.ir/article-1-311-fa.html (in Persian).

[8] Garcia Gabin, W., Camacho, E., Sliding Mode Model Based Predictive Control for Non-Minimum Phase Systems, European Control Conference (ECC), Cambridge, UK, 2003, pp. 904-909, 10.23919/ECC.2003.7085073.

[9] Incremona, G., Ferarra, A., and Magni, L., Hierarchical Predictive/Sliding Mode Control of Nonlinear Constrained Uncertain Systems, IFAC-Papers Online, Vol. 48, No. 23, 2015, pp. 102-109, doi.org/10.1016/j.ifacol.2015.11.268.

[10] Temel, T., Ashrafiuon, H., Nataraj, C., and Yildirim, S., A New Sliding-Mode Model Predictive Controller for Output Tracking of Nonlinear Systems with Online Optimization, International Journal of Advanced Research, Vol. 3, No. 9, 2015, pp. 660-664.

[11] Hassine, I., Naouar, M., Extended Model Predictive-Sliding Mode Control for Three-Phase Grid Connected Converters, IEEE Transactions on Industrial Electronics, Vol. 64, No. 2, 2017, pp. 1341-1349, 10.1109/TIE.2016.2618867.

[12] Spasic, M., Hovd, M., Matic, D., and Antic, D., Tube Model Predictive Control with An Auxiliary Sliding Mode Controller, Modeling, Identification and Control, Vol. 37, No. 3, 2016, pp.181-193, 10.4173/mic.2016.3.4.

[13] Haus, B., Rohl, J., Mercorelli, P., and Aschemann, H., Model Predictive Control for Switching Gain Adaptation in A Sliding Mode Controller of a DC Drive with Nonlinear Friction, 22nd International Conference on System Theory, Control and Computing (ICSTCC), Sinaia, Romania, 2018, 10.1109/ICSTCC.2018.8540681.

[14] Chang Jian, C., Chang, S., Bifurcation and Chaos Analysis of Spur Gear Pair with And Without Nonlinear Suspension, Nonlinear Analysis: Real World Applications, Vol. 12, No. 2, 2011, pp. 979-989, doi.org/10.1016/j.nonrwa.2010.08.021.

[15] Slotin, J., Li, W., Applied Nonlinear Control, Eglewood Cliffs, NJ, USA: Prentice Hall, 1991.

[16] Cheng, X., Wang, P., Liu, L., and Tang, G., Predictive Sliding Mode Control Using Feedback Linearization for Hypersonic Vehicle, Procedia Engineering, Vol. 99, 2015, pp. 1076-1081, doi.org/10.1016/j.proeng.2014.12.642.

[17] Camacho, E., Bordons, C., Model Predictive Control, 2nd ed. London, UK: Springer-Verlag London, 2007.

[18] Gharagozloo, M., Shahmansoorian, A., Chaos Control in Gear Transmission System using GPC and SMC Controllers, Journal of Applied and Computational Mechanics, 2020.

[19] Khalil, H. K., Lyapunov Stability, Control Systems, Robotics and AutomatioN–Volume XII: Nonlinear, Distributed, and Time Delay Systems-I, Vol. 11, 2009, pp. 115.

[20] Muller, P., Popov and Circle Criterion, Control, Systems Robotics, and Automation, Vol. 3.

[21] Haddad, W., Chellaboina, V., Nonlinear Dynamical Systems and Control, Princeton University Press, 2011.