

Evaluation of Sperling's Index in Passenger and Freight Trains Under Different Speeds and Track Irregularities

Sajjad Sattari

Department of Mechanical Engineering,
Najafabad Branch, Islamic Azad University, Najafabad, Iran
E-mail: sajjad.sattari@gmail.com

Mohammad Saadat*, Sayed Hasan Mirtalaie, Mehdi Salehi, Ali Soleimani

Department of Mechanical Engineering,
Najafabad Branch, Islamic Azad University, Najafabad, Iran
E-mail: saadat@pmc.iaun.ac.ir, mirtalaie@pmc.iaun.ac.ir,
mehdi.salehi@pmc.iaun.ac.ir, soleimani@pmc.iaun.ac.ir

*Corresponding author

Received: 13 July 2022, Revised: 24 September 2022, Accepted: 25 September 2022

Abstract: The two factors of track irregularity and train speed affect the dynamic behavior of rail vehicles and can lead to an increase in dynamic forces, a decrease in ride comfort, and derailment in some cases. In this paper, the effect of train speed increase and different conditions of track irregularity on ride comfort and ride quality are investigated. For this purpose, first, two freight and passenger train models have been modeled in UM software, and then the effect of train speed increase and track irregularities (different US federal classes) have been studied with Sperling's index. A freight train with the model of 18-100 and 3-piece bogie and a TGV high-speed train with 10 wagons were simulated. The results showed that in Sperling's index, with the increase in the train speed and irregularity amplitude, the value of ride comfort and ride quality generally increased. For example, in the passenger train and irregularity classes 5 and 4, with the increase in train speed from 10 to 100 m/s, the Sperling's index values changed from 0.66 to 1.99 and from 0.78 to 2.25, and increased 200% and 188%, respectively. In other words, at a speed of 10 m/s, passengers' comfort is just noticeable, while at a speed of 100 m/s and class 4, the situation is more pronounced but not unpleasant and the system should be monitored.

Keywords: Numerical Simulation, Railway Vehicles, Ride Comfort, Ride Quality, Sperling's Index, Track Irregularities

Biographical notes: **Sajjad Sattari** is a PhD candidate of Mechanical Engineering at Islamic Azad University of Najafabad (IAUN), Iran. His current research focuses on railway engineering and solid mechanics. He received his MSc in Mechanical Engineering from IAUN in 2016. **Mohamad Saadat** is Assistant Professor of mechanical engineering at IAUN. His current research interest includes control, mechatronics, and hybrid electric vehicles. He received his PhD in Mechanical Engineering from IUT in 2016. **Sayed Hasan Mirtalaie** is Assistant Professor of mechanical engineering at IAUN. His current research interest includes dynamic and vibration. He received his PhD in Mechanical Engineering from UK in 2017. **Mehdi Salehi** is Assistant Professor of Mechanical Engineering at IAUN. He received his PhD in Mechanical Engineering from IUT in 2011. **Ali Soleimani** is Assistant Professor of mechanical engineering at IAUN. He received his PhD in Mechanical Engineering from TMU in 2016.

Research paper

COPYRIGHTS

© 2022 by the authors. Licensee Islamic Azad University Isfahan Branch. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution 4.0 International (CC BY 4.0)

(<https://creativecommons.org/licenses/by/4.0/>)



1 INTRODUCTION

All industries, including the railway industry, need accurate modeling and simulations. The dynamic performance of railway vehicles, which generally includes two parts of comfort and safety, is evaluated with indicators of ride quality, wagon stability, etc. Ride comfort is one of the most important indicators of a railway vehicle, which has a relatively complex concept, and its general definition is the ability to suspend a railway vehicle to maintain movement within the range of human comfort or within the range necessary to ensure no damage. Ride comfort during movement is affected by various factors including vibration, temperature, sound, seat characteristics, etc. But in the dynamic criteria of checking ride comfort, only part of the comfort that is affected by the vibrations of the vehicle movement is considered. Passenger trains usually consist of a carbody and two (and/or three) bogies. In a railway system, the most effective factors in ride comfort are the suspension system between the carbody and the bogie (secondary suspension system) and the suspension system between the bogie and the wheelsets (primary suspension system), and the effect of structure flexibility of the carbody and bogies is much less than the effect of suspension systems. Also, due to being metal, the wheels are almost rigid (compared to the suspension system) [1-6]. On the other hand, with the rapid development of the rail transport industry, determining the working range at high speed and away from hunting instability is one of the important issues. Hunting in a critical state, along with wheel wear and fatigue, can lead to derailment and accidents. Knowing the dynamic behavior of the rail vehicle is not only necessary for the designer, but it can also represent the performance of the rail vehicle and applied forces to the rail. The consequences of hunting can be mentioned as the lack of comfort sense for passengers, increasing the cost of repairs, and large lateral forces that cause damage to the train, track, and derailment. Therefore, in the current research, one of the consequences of the hunting phenomenon, which means passenger discomfort, was evaluated [5].

After modelling a train in universal mechanism (UM), Nasr et al. [1] investigated the dynamic behavior of the model and the Sperling's index in the speed range of 10 to 25 m/s by applying the track irregularity. Their results showed that increasing the speed increases the Sperling's index and decreases passengers' comfort. Halladin et al. [4] compared the comfort methods of a tramway system on different tracks with a field (experimental) test. Younesian et al. [7] optimized and checked the life of the wagon suspension system, considering the ride comfort as the limiting factor. Gangadharan et al. [8] also conducted research on analytical and laboratory evaluation of ride comfort.

Goga et al. [9] optimized the vehicle suspension system using 4-DOF model, they ignored the vibrations around the longitudinal axis of the carbody. Sun et al. [10] investigated the effect of the suspension system of the carbody on the ride comfort of high-speed trains. Suarez et al. [11-12] analyzed the mutual influence of track quality, wheel-rail contact characteristics, and elastic characteristics of the train suspension system on safety, ride comfort, and rail fatigue. Baghmisheh et al. [13] optimized the car suspension system with the genetic algorithm, they used 2-DOF model to analyze car vibrations and by ignoring the effect of longitudinal and transverse accelerations, they reduced the average value of vertical accelerations (without calculating the ride comfort index). Ebadi et al. [2] analyzed wagon vibrations and the effect of track irregularities on ride comfort (MATLAB/Simulink). Various methods have been developed to evaluate passenger comfort, usually methods based on ISO-2631, and Sperling, known as Sperling's method or Wz comfort index method, are often used. The Wz comfort index method was presented by Sperling in Germany in the middle of the 20th century, and this method is still the most well-known evaluation method for passenger comfort and quality of railway vehicles. The UIC and the CEN, along with ISO, have published standards for the evaluation of the ride comfort of railway vehicles based on ISO-2631, UIC brochure 513R, standards EN-12299 and ISO-10056 through the ERRI [14-19].

In this paper, UM software has been used to simulate the 3D dynamics of rail vehicles. First, two freight and passenger train models were modeled, and then the Sperling's index was examined as one of the parameters for the evaluation of railway vehicles. To evaluate ride comfort, different researchers have used two experimental and simulation methods, each of which has advantages and limitations. In similar studies, researchers generally investigated the effect of one parameter, such as train speed, track irregularity, and/or other effective parameters. For example, in reference [1], an irregularity class has been investigated at several different speeds to evaluate the ride comfort (with UM), and/or in reference [2], the effect of irregularity in two irregularity classes 3 and 6 of the US has been checked (with MATLAB/Simulink). In this research, a wide range of speed changes as well as a wide range of track irregularities with different amplitudes have been investigated to evaluate the ride comfort for both freight and passenger train models.

2 EVALUATION OF RIDE COMFORT

One of the methods of evaluating the ride quality and comfort of railway vehicles is the ride index Wz method, which was introduced by Sperling. Ride comfort should

be evaluated according to the effect of mechanical vibrations of the vehicle on passengers. Constants and different speeds can be evaluated to show ride quality and ride comfort in different track irregularities. Appropriate validity, accurate numbers, and easy interpretation are the advantages of Sperling's index. The Sperling's index is a function of the vehicle's vibration level and provides information about the dynamic behavior of the system, which enables the diagnosis and creation of solutions to improve the dynamic performance in terms of ride quality and ride comfort. This method is often used in all standard railway tracks (ballasted and slab tracks). The ranges of ride quality and ride comfort are presented in "Tables 1 and 2" [3-4].

Table 1 Scale for the ride quality

Ride quality	Vibration sensitivity
1.0	Very good
2.0	Good
3.0	Satisfactory
4.0	Acceptable for running
4.5	Not acceptable for running
5.0	Dangerous

Table 2 Scale for the ride index

Ride index	Vibration sensitivity
1.0	Just noticeable
2.0	Clearly noticeable
2.5	More pronounced but not unpleasant
3.0	Strong, irregular, but still tolerable
3.25	Very irregular
3.5	Extremely irregular, unpleasant, annoying, prolonged exposure intolerable
4.0	Extremely unpleasant; prolonged exposure harmful

Ride index W_z is weighted on the frequency range based on the following Equations (1) and (2):

$$W_z = \sqrt[10]{a^3 \cdot B^3} \quad (1)$$

$$W_z = \sqrt[6.67]{a^2 \cdot B^2} \quad (2)$$

The ride quality weighting factor B is calculated according to Equation (3):

$$B(f) = 1.14 \cdot \sqrt{\frac{[(1 - 0.056f^2)^2 + (0.645f)^2 \cdot (3.55f^2)]}{[(1 - 0.252f^2)^2 + (1.547f - 0.00444f^3)^2] \cdot (1 + 3.5f^2)}} \quad (3)$$

The ride comfort weighting factor B is calculated according to Equation (4):

$$B(f) = k \cdot \sqrt{\frac{1.911f^2 + (0.25f^2)^2}{(1 - 0.277f^2)^2 + (1.563f - 0.0368f^3)^2}} \quad (4)$$

Where, $k = 0.588$ for vertical vibrations (B_s), and $k = 0.737$ for lateral vibrations (B_w). Values of weighting curves are plotted in the 0.5 to 30 Hz frequency range in 1/3 octave bands ("Fig. 1"), for further signal processing of peak acceleration values a peak [4], [20-22].

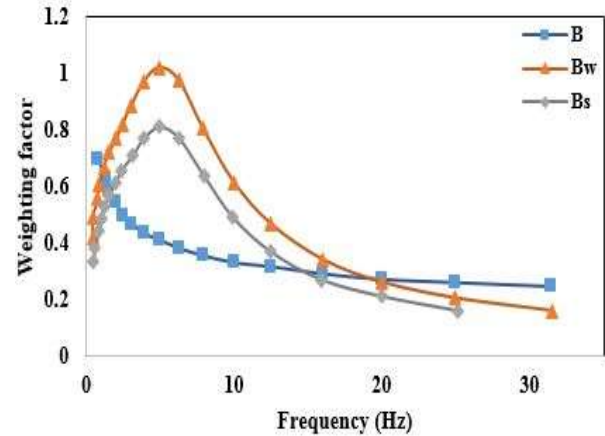


Fig. 1 B , B_s , and B_w weighting curves for Sperling's ride index.

3 SIMULATIONS

The study of train dynamic behavior is one of the most important studies in the railway industry. Considering that one of the goals of this industry is the safe transportation of passengers, dynamic simulator tools of the train are used to evaluate the safety of rolling stock. One of the tools used to simulate train dynamics is the use of Multi-Body Dynamics (MBDs) software. Among the dynamic software available in the industry, Universal Mechanism (UM), ADAMS, SIMPACK, ABAQUS, etc. can be mentioned [23-25]. Figure 2 showed a view of passenger and freight trains modelled in UM in detail. Also, the parameters and values of both trains are presented in "Tables 3 and 4". The main components of trains include the carbody, primary and secondary suspension systems, and wheelsets. The passenger train has a primary suspension system including vertical springs and dampers, which are located in "Fig. 2". In order to simulate the train's secondary suspension system, an air spring has been used between the bogie frame and the carbody (Nishimura model). In some studies, this train model has been used for simulations [1], [17-19].

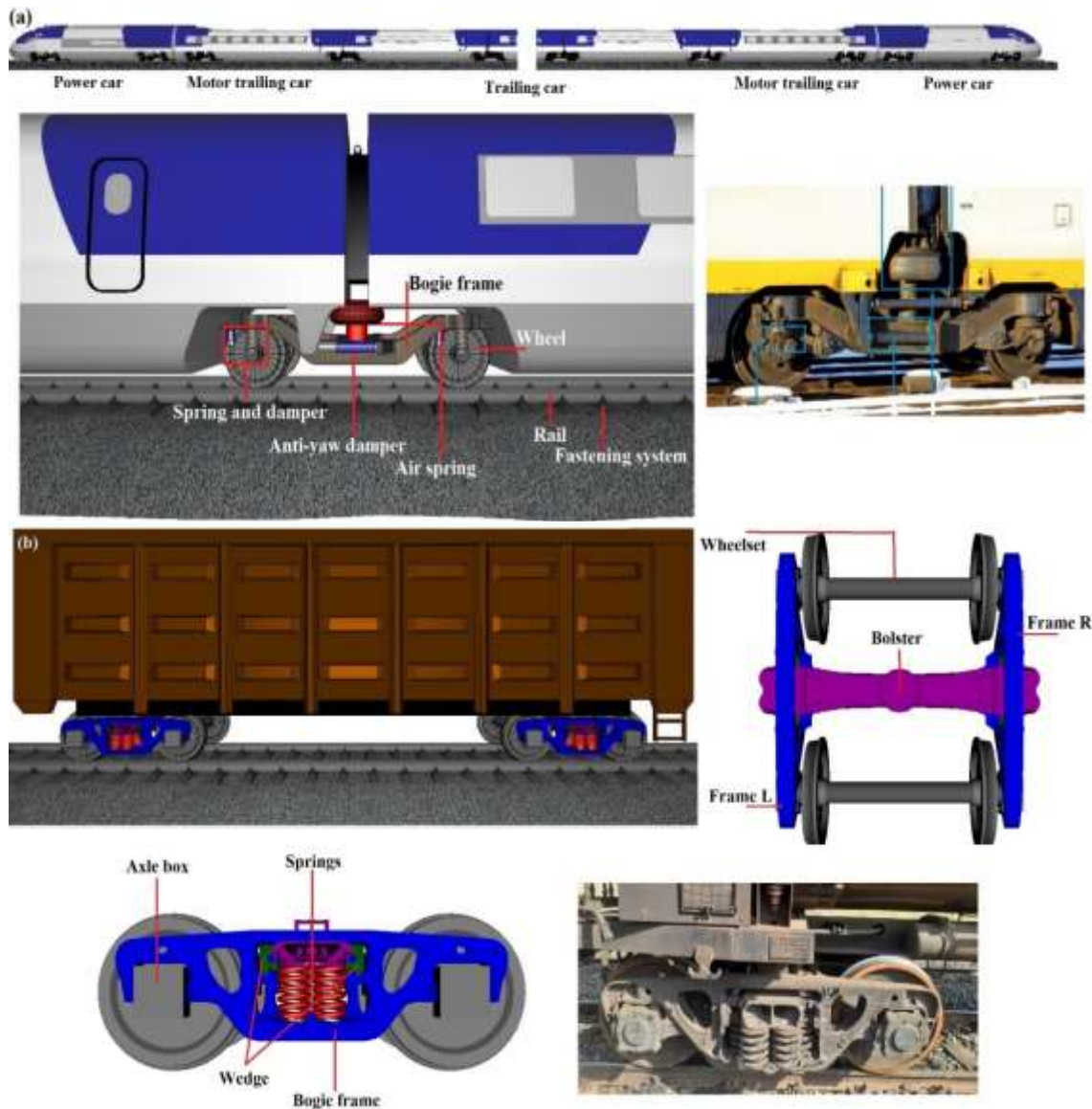


Fig. 2 A view of passenger and freight trains.

Table 1 Train-track parameters in passenger train

Parameters	Value and unit	Track	Parameters	Value and unit
Carbody mass	33250 (kg)	Track	vertical and lateral stiffness	$44 \times 10^6, 18 \times 10^6$ (N/m)
Bogie frame mass	22963 (kg)		vertical and lateral damping	$4 \times 10^5, 1 \times 10^5$ (Ns/m)
Wheelset mass	2150 (kg)	Primary suspension	longitudinal, lateral, and vertical stiffness	$4.08 \times 10^7, 4.56 \times 10^6, 1.40 \times 10^6$ (N/m)
Wheel radius	0.460 (m)	Secondary suspension	longitudinal, lateral, and vertical stiffness	$3.50 \times 10^5, 3.50 \times 10^5, 1.71 \times 10^6$ (N/m)

Table 4 Train parameters in freight train

Parameters	Value and unit	Parameters	Value and unit
Mass of carbody	90000 (kg)	Lateral spring stiffness (one/double spring)	6.43×10^5 (N/m)
Mass of bolster	596.2 (kg)	Vertical spring stiffness (one/double spring)	6.32×10^5 (N/m)
Mass of wedge	21.6 (kg)	Contact stiffness (axle box)	1×10^8 (N/m)
Mass of side frame (+springs)	526.3 (kg)	Contact damping (axle box)	3×10^4 (Ns/m)
Wheel radius	0.475 (m)	Coefficient of friction in axle box contact	0.3
Coefficient of friction (wedge)	0.3		

In order to simulate a freight train, the three-piece bogie type 18-100 is used in the countries of America, Russia, China, Canada, India, Australia, Brazil, etc. Generally, three-piece bogies simulations are based on models of a wedge frictional system. The inertia properties of wedges are ignored and the linear model of tangential contact forces is used. The bogie has rigid contacts between side frame and axleboxes, carbody and bolster in the center plate and side bearings including clearances, between frictional wedges and bolster or side frame. The rail is considered a massless force element, the stiffness and damping of the rail are considered, but the inertial properties are not considered. Hertzian solution and FASTSIM algorithm by Kalker as well as modified non-elliptic multipoint contact model are used. Some researchers used this train model for dynamic simulations [26-30]. After completing the simulation of the train's dynamic characteristics in the UM input

section, the UM simulation section has been used to simulate the dynamic behaviour of the system. In order to solve the wheel and rail contact equations, Kalker nonlinear theory has been used, which calculates the geometric parameters of the wheel and rail using the FASTSIM solver algorithm. Power Spectral Density (PSD) function is the most important and commonly used statistical function to express the track irregularity taken as a stationary random process. Based on a large number of field measured data, the Federal Railroad Administration of America (FRA) obtained the irregularity power spectral density function of rail track, which was fitted to a function expressed by cutoff frequency and roughness constant. The track irregularity in the US can be divided into six levels [31-34]. The parameters of this function are presented in "Table 5". In the next part of the paper, the irregularity diagrams and the amplitude of them are presented.

Table 5 Parameters of American track irregularity PSD function

Parameter	Parameter values for different line levels					
	1 (worst)	2	3	4	5	6 (best)
A_v (cm ² rad/m)	1.2107	1.0181	0.6816	0.5376	0.2095	0.0339
A_a (cm ² rad/m)	3.3634	1.2107	0.4128	0.3027	0.0762	0.0339
(rad/m) Ω_s^2	0.6046	0.9308	0.8520	1.1312	0.8209	0.4380
(rad/m) Ω_c^2	0.8245	0.8245	0.8245	0.8245	0.8245	0.8245

4 RESULT AND DISCUSSION

The irregularity of rail surface is one of the most influential parameters in the efficiency of the rolling stock systems and especially passenger type. In "Fig. 3", the track irregularities in different classes can be seen for simulation. Classes 4, 5, and 6 have been added in common for both trains, class 1 for freight trains, and irregularity of Tehran metro for passenger trains. The irregularity amplitude for classes 1, 4, 5, 6, and Tehran's metro is about 35, 15, 10, 5, and 3 mm, respectively. One of the advantages of numerical simulation compared to field test is that in order to evaluate the ride comfort and ride quality of a railway system, it is possible to change various values of a railway system, including train speed, track conditions, defects, etc., and evaluate the results. Figure 4 shows the acceleration-time diagram of carbody and bogie frame vibration (vertical accelerations). These charts are an important indicator for evaluating ride comfort with Sperling's index. Similar to "Fig. 4", after applying the desired parameters as input, the acceleration of the carbody is taken as the output of the system, and then, depending on the desired goal, the data is used to calculate the ride comfort. In "Figs 5, 6, and 7" the simultaneous effect of train speed and different conditions of random track irregularities with various severities is presented. At different speeds

and different irregularities of the track, the vertical Sperling's index has been reported the irregularities significantly excite the strong vibration between the vehicle and the track. This not only reduces the ride quality of the train but also causes the failure of parts of the wheel and railway system and reduces the quality of the track. In "Figs. 5, 6, and 7", Sperling's method is presented as an important indicator to evaluate passenger comfort. In the passenger train and the Sperling's index, with the increase of train speed and irregularity amplitude, the value of ride comfort has generally increased. For example, in the irregularity of classes 6 and 5, with the increase in speed from 10 to 100 m/s, the Sperling's index values changed from 0.61 to 1.86 and from 0.66 to 1.99 and increased about 200%. In a constant irregularity amplitude, for example, class 4, with the increase in speed from 10 to 100 m/s, Sperling's index has increased by about 188% and changed from clearly noticeable to more pronounced but not unpleasant. According to Sperling's index, in all the studied speeds and irregularities, carbody acceleration amplitude is in a suitable range and passenger comfort is in a good condition. As for motions and vibrations, Sperling's index is often considered an acceptable value for ride comfort on trains [6]. The studies of Dumitriu et al. [3], Haladin et al. [4], and Sadeghi et al. [20-21] also have similar results to the current paper. In general, ride comfort has increased by increasing train speed.

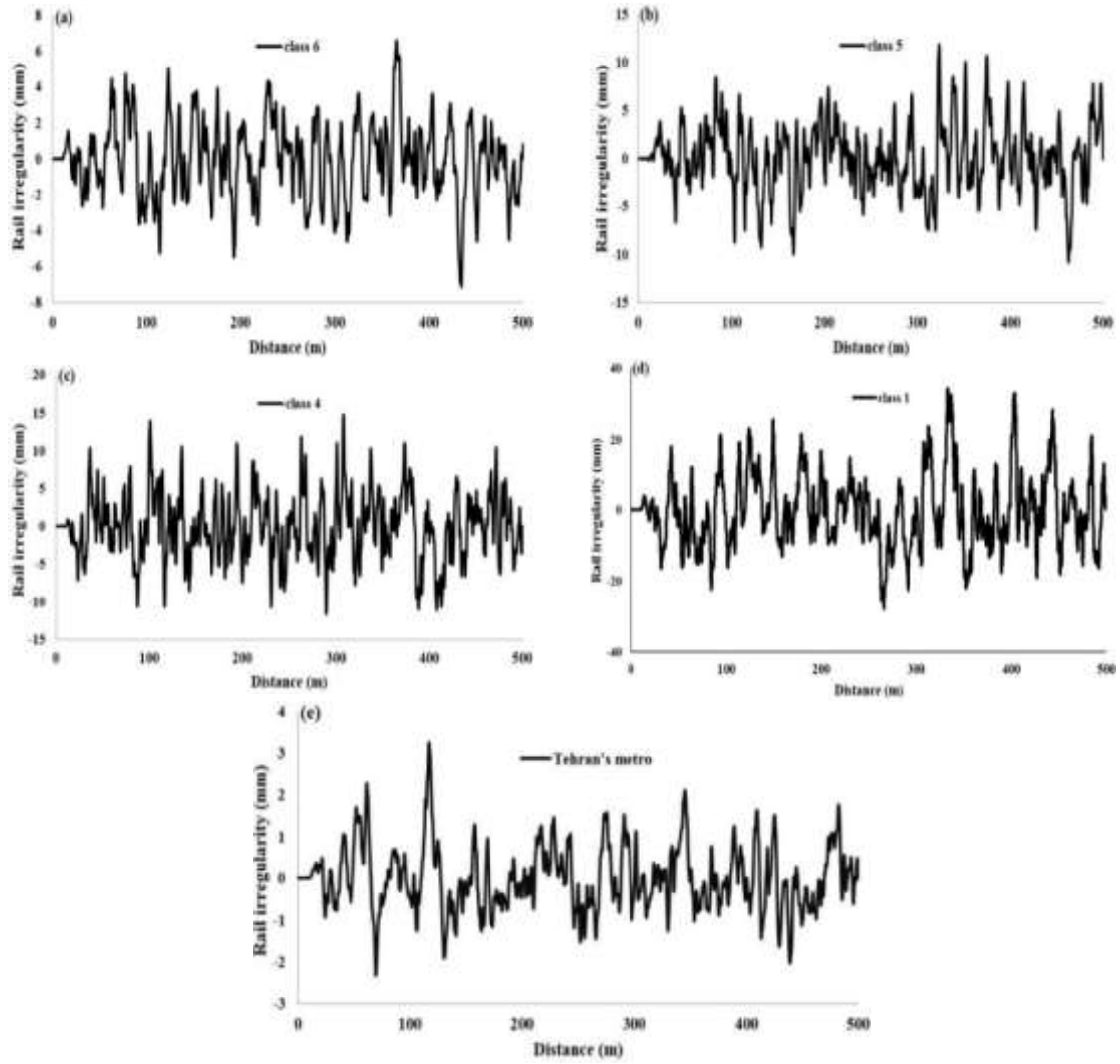
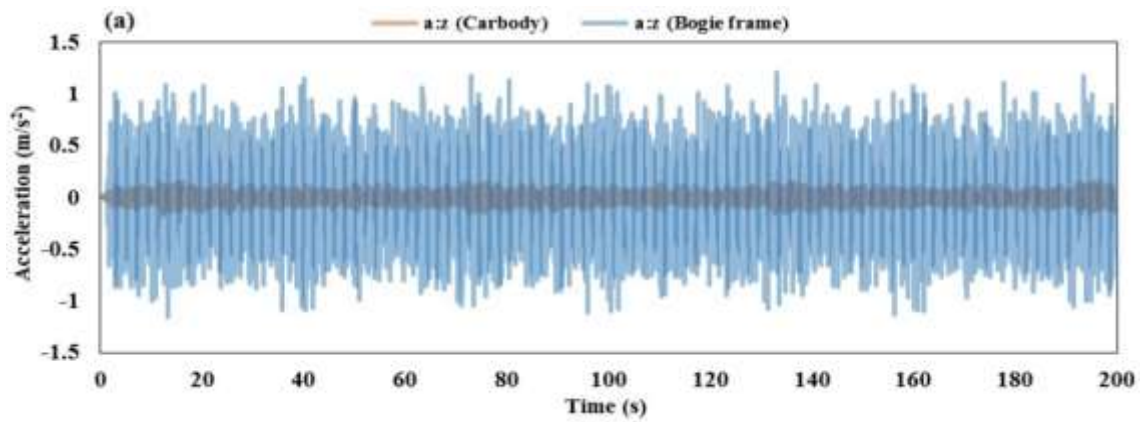


Fig. 3 Track irregularities of different classes.



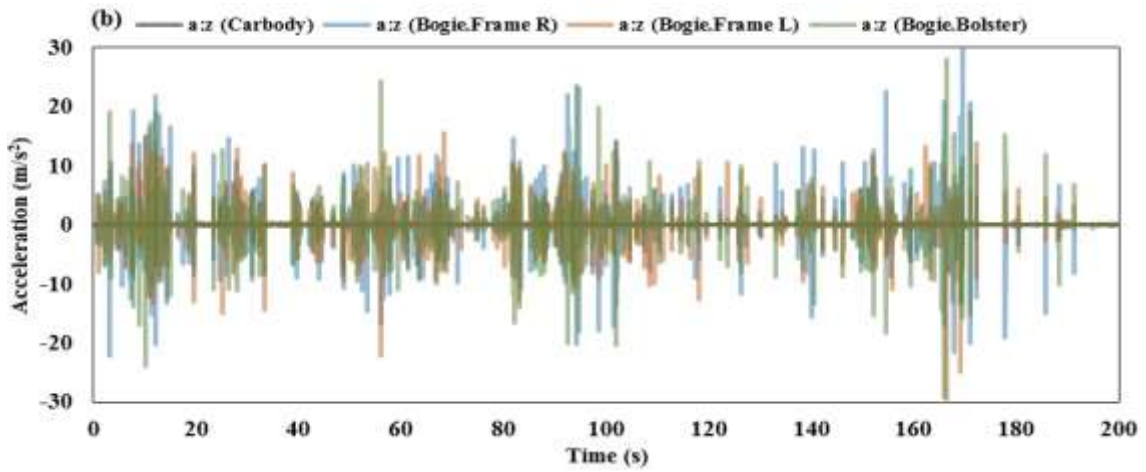
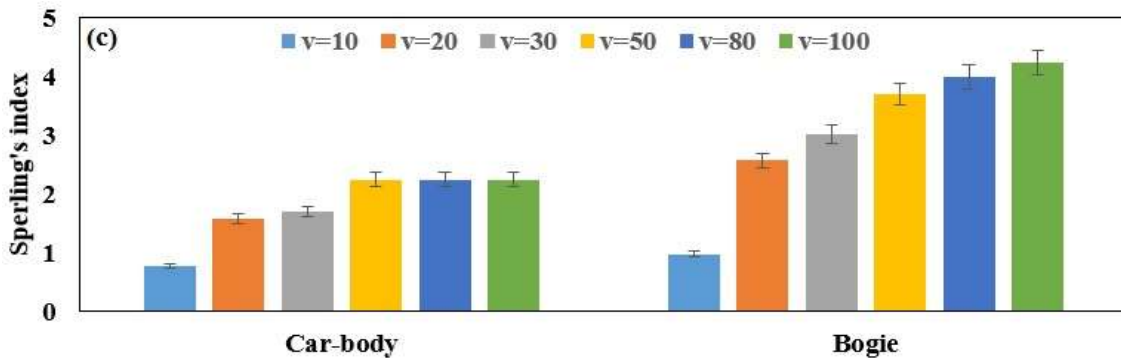
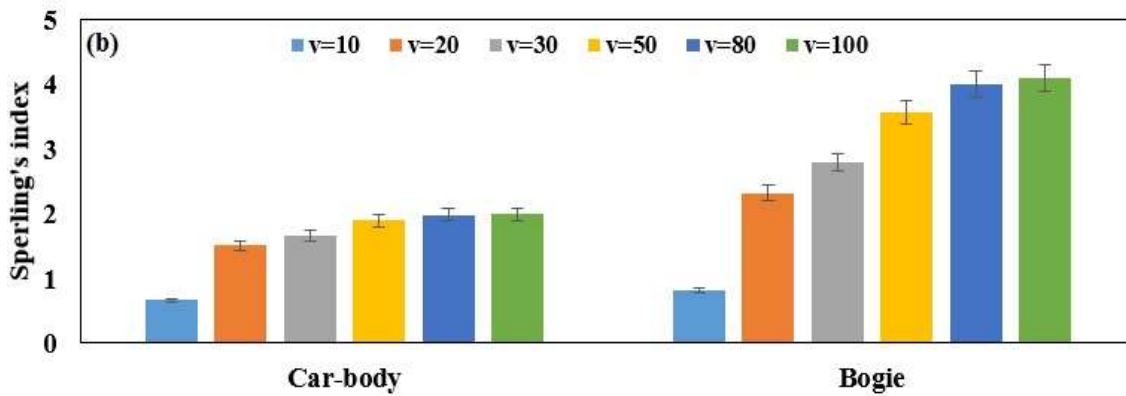
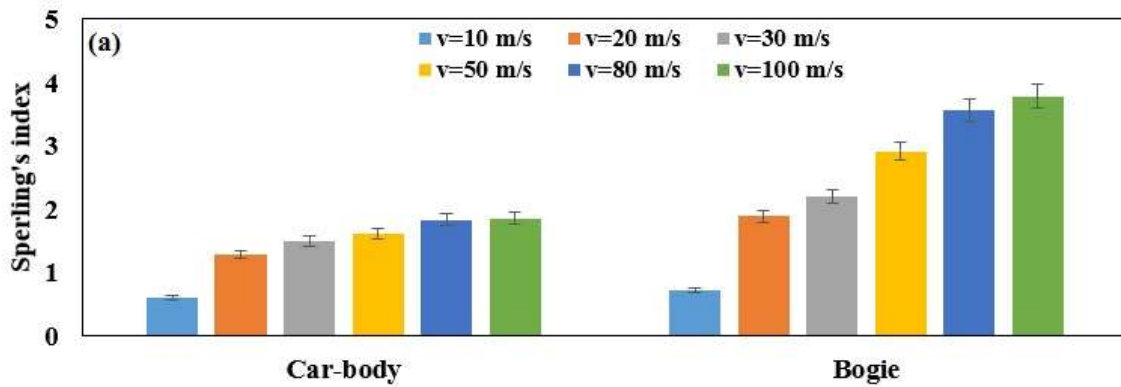


Fig. 4 Acceleration-time diagram: (a): passenger train, and (b): freight train.



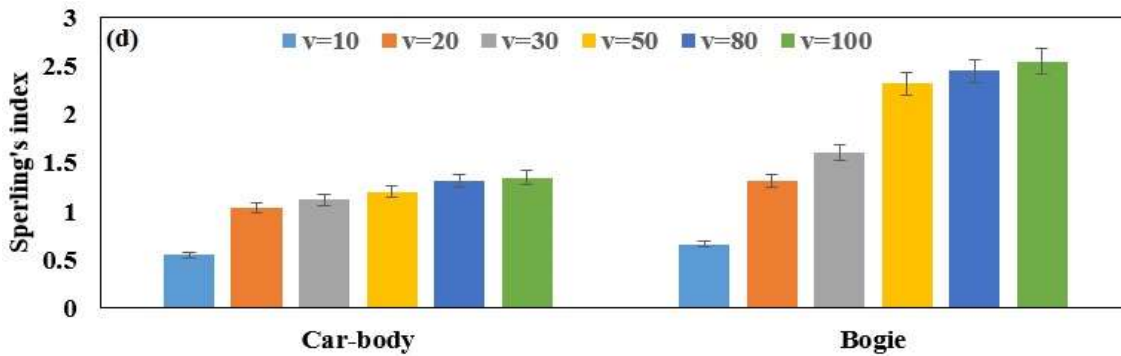


Fig. 5 Sperling's index in passenger train: (a): class 6, (b): class 5, (c): class 4, (d): Tehran's metro.

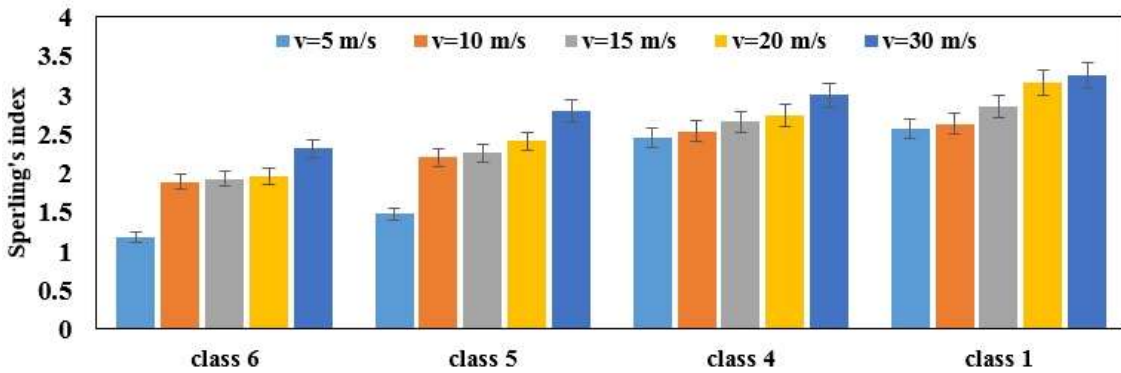
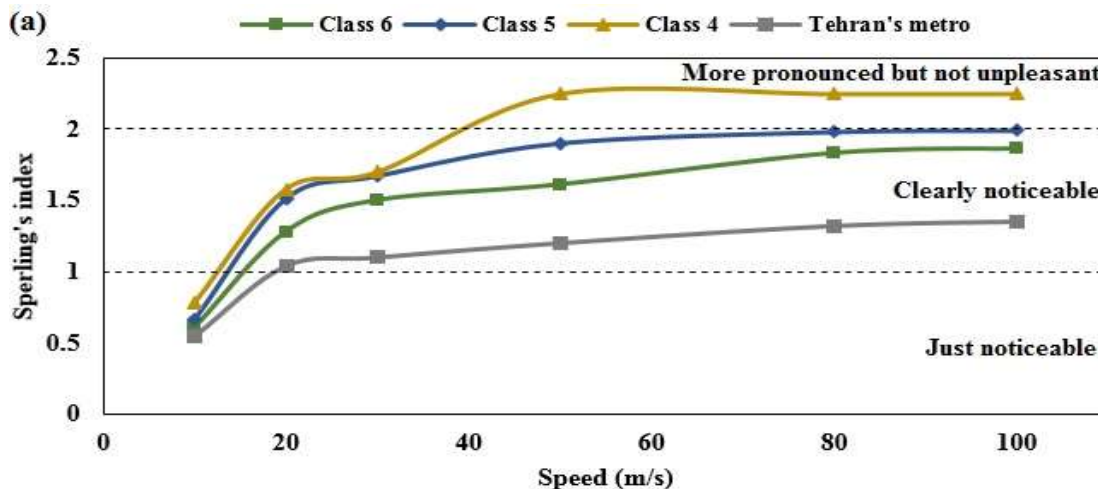


Fig. 6 Sperling's index in freight train.

In the freight train and the Sperling's index, with the increase in the train speed and irregularity amplitude, the value of ride comfort has increased. For example, in the irregularity of classes 6 and 5, with an increase in speed from 5 to 30 m/s, the Sperling's index values changed from 1.18 to 2.32 and from 1.48 to 2.80 and increased about 96% and 89%, respectively. Sperling's index shows that the ride comfort is suitable in classes 6 and 5, and in high irregularity amplitudes (classes 4 and 1) and at high speeds, it is sometimes an inappropriate situation. For example, in class 1 and at speed 5 m/s, the state of

ride comfort is almost suitable (strong, irregular, but still tolerable mode), while at speed 30 m/s, the ride comfort is not suitable and changes to extremely irregular mode. In freight trains, it is better to compare the ride quality index, in which case the situation is good (less than 4). Of course, the lower the train speed and irregularity amplitude, the better the ride quality index will be, and the vehicle and track will have fewer defects and dangers. The simulation results showed that in different classes, with an increase in speed higher than 35 m/s, the freight train has an accident and derailment.



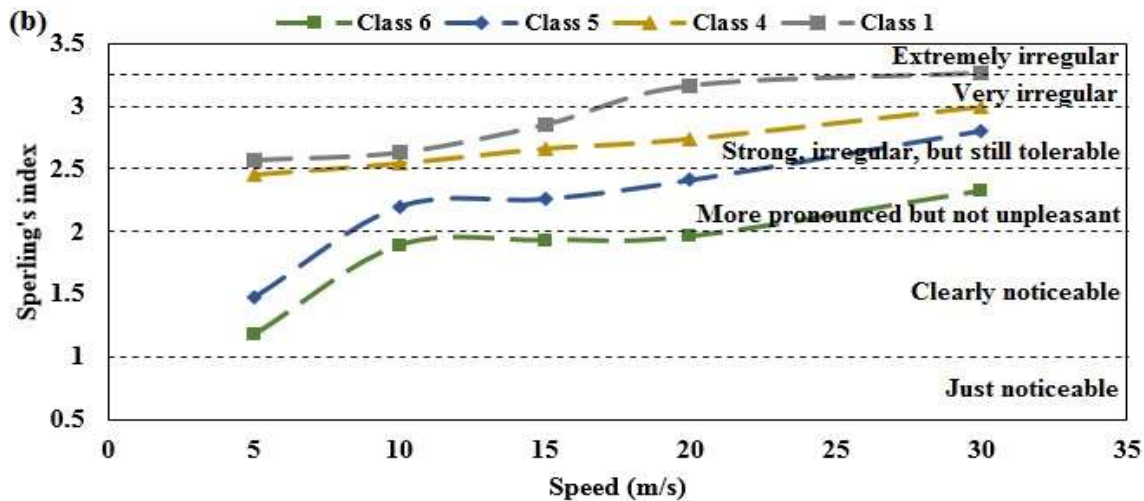


Fig. 7 Comparison of Sperling's index at different conditions: (a): passenger and (b): freight trains.

5 CONCLUSIONS

In this paper, the dynamic behavior of a passenger train and a freight train while passing over an irregular track was investigated. For this purpose, UM multibody dynamic software was used for modeling and simulation. The irregularity of the rail surface was simulated for several different track classes using random data (Using the irregularity spectrum of US tracks). An important dynamic index, namely the ride comfort, was analyzed. During a parametric study, the effect of track irregularity and train speed was investigated. The results showed that the ride comfort is strongly affected by the train speed and track irregularity. The irregularities significantly excite the strong vibration between the vehicle and the track. This not only reduces the quality of the train but also causes the failure of parts of the wheel and rail system and reduces the quality of the track.

REFERENCES

- [1] Nasr, A., Taheri, M. M. N., and Shahravi, M., Tehran Subway Rolling Stock Dynamic Analysis, *Transportation research*, Vol. 17, No. 1, 2020, pp. 61-70.
- [2] Ebadi Rajoli, J., Molatefi, H. A., Analysis of Wagon Vibration and Investigation of Track Irregularities Effect on the Ride Comfort, *Quarterly Journal of Transportation Engineering*, Vol. 7, No. 2, 2015, pp. 251-261.
- [3] Dumitriu, M., Stănică, D. I., Study on the Evaluation Methods of the Vertical Ride Comfort of Railway Vehicle—Mean Comfort Method and Sperling's Method, *Applied Sciences*, Vol. 11, No. 9, 2021, pp. 3953.
- [4] Haladin, I., Lakušić, S., and Bogut, M., Overview and Analysis of Methods for Assessing Ride Comfort on Tram Tracks, *Gradevinar*, Vol. 71, 2019, pp. 901-921.
- [5] Ghazavi, M., Azari Nejad, M., and Rahmanian, S., Dynamic Analysis of the Derailment of High-Speed Railway Vehicle on A Curved Path with Longitudinal Displacement, *Modares Mechanical Engineering*, Vol. 15, No. 5, 2015, pp. 309-318.
- [6] Jiang, Y., Chen, B. K., and Thompson, C., A Comparison Study of Ride Comfort Indices Between Sperling's Method and EN 12299, *International Journal of Rail Transportation*, Vol. 7, No. 4, 2019, pp. 279-296.
- [7] Younesian, D., Nankali, A., Spectral Optimization of High-Speed Train Suspension Systems, *International Journal of Vehicle Structures & Systems*, Vol. 1, 2009.
- [8] Gangadharan, K. V., Sujatha, C., and Ramamurti, V., Experimental and Analytical Ride Comfort Evaluation of a Railway Coach, *Mechanical Engg. Deptt., National Institute of Technology Karnataka, Surathkal*, 2004.
- [9] Goga, V., Klůčik, M., Optimization of Vehicle Suspension Parameters with use of Evolutionary Computation, *Procedia Engineering*, Vol. 48, 2012, pp. 174-179.
- [10] Sun, X., Research of Simulation on the Effect of Suspension Damping on Vehicle Ride, *Energy Procedia*, Vol. 17, 2012, pp. 145-151.
- [11] Suarez, B., Influence of the Track Quality and of the Properties of the Wheel–Rail Rolling Contact on Vehicle Dynamics, *Vehicle System Dynamics*, Vol. 51, No. 2, 2013, pp. 301-320.
- [12] Suarez, B., Assessment of the Influence of the Elastic Properties of Rail Vehicle Suspensions on Safety, Ride

- Quality and Track Fatigue, Vehicle System Dynamics, Vol. 51, 2012.
- [13] Vakil Baghmisheh, M. T., Application of Genetic Algorithms in Optimal Design of a Passive Suspension System a Vehicle Subjected to Random Excitations of Actual Road, Modares Mechanical Engineering, Vol. 10, No. 4, 2010, pp. 1-12.
- [14] Vongierke, H. E. The ISO Standard: Guide for the Evaluation of Human Exposure to Whole-Body Vibration, in NASA. Langley Res. Center The 1975 Ride Quality Symp., 1975.
- [15] Standardization, I. O. f., Mechanical Vibration and Shock: Evaluation of Human Exposure to Whole-body Vibration, International Organization for Standardization, 1985.
- [16] Khalid, H., Turan, O., and Bos, J. E., Guide to Measurement and Evaluation of Human Exposure to Whole-Body Mechanical Vibration and Repeated Shock Guide to Measurement and Evaluation of Human Exposure to Whole-Body Mechanical Vibration and Repeated Shock 30, 1987, Journal of Marine Science and Technology, Vol. 16, No. 2, 2011, pp. 214-225.
- [17] Railways, I. U. O., UIC Code 513, R: Guidelines for Evaluating Passenger Comfort in Relation to Vibration in Railway Vehicles, International Union of Railways, 1995.
- [18] CEN, E., Railway Applications–Ride Comfort for Passengers–Measurement and Evaluation, Railway Applications–Ride Comfort for Passengers–Measurement and Evaluation, 2009, pp. 12299.
- [19] Standardization, I. O. f., Mechanical Vibration: Measurement and Analysis of Whole-Body Vibration to Which Passengers and Crew Are Exposed in Railway Vehicles, International Organization for Standardization Geneva, 2001.
- [20] Sadeghi, J., Rabiee, S., and Khajehdezfuly, A., Effect of Rail Irregularities on Ride Comfort of Train Moving Over Ballast-Less Tracks, International Journal of Structural Stability and Dynamics, Vol. 19, No. 6, 2019, pp. 1950060.
- [21] Sadeghi, J., Rabiee, S., and Khajehdezfuly, A., Development of Train Ride Comfort Prediction Model for Railway Slab Track System, Latin American Journal of Solids and Structures, Vol. 17, No. 7, 2020, pp. 1-22.
- [22] Kumar, V., Rastogi, V., and Pathak, P. M., Simulation for Whole-Body Vibration to Assess Ride Comfort of a Low–Medium Speed Railway Vehicle, Simulation, Vol. 93, No. 3, 2017, pp. 225-236.
- [23] Zakeri, J., Impact of Heavy Urban Rail Vehicles Running Over Light Rail Turnouts, Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit, Vol. 235, No. 3, 2021, pp. 300-312.
- [24] Aziznia, M., Owhadi, A., and Shadfar, M., Analysis of wheel and rail Hertzian and Non-Hertzian Contact Theories Using UM Software Considering the Effect of Rail Inclination on Wheel Wear, International Journal of Railway Research, Vol. 8, No. 2, 2021, pp. 21-32.
- [25] Kalker, J. J., Three-Dimensional Elastic Bodies in Rolling Contact, Springer Science & Business Media, Vol. 2, 2013.
- [26] Pogorelov, D., Simulation of Freight Car Dynamics: Mathematical Models, Safety, Wear. 2009.
- [27] Kovalev, R., Freight Car Models and Their Computer-Aided Dynamic Analysis, Multibody System Dynamics, Vol. 22, 2009, pp. 399-423.
- [28] Zakharov, S., Computer-Aided Simulation of The Influence of Track and Vehicle Parameters on Wheel/Rail Interaction Characteristics, 2009.
- [29] Bernal, E., Spiriyagin, M., and Cole, C., Onboard Condition Monitoring Sensors, Systems and Techniques for Freight Railway Vehicles: A Review. IEEE Sensors Journal, Vol. 19, No. 1, 2019, pp. 4-24.
- [30] Yazykov, V., Numerical Simulation of Railway Vehicle Derailments, 2010.
- [31] Kargarnovin, M. H., Ride Comfort of High-Speed Trains Travelling Over Railway Bridges, Vehicle System Dynamics, Vol. 43, No. 3, 2005, pp. 173-197.
- [32] Kargarnovin, M. H., Nonlinear Vibration and Comfort Analysis of High-Speed Trains Moving Over Railway Bridges, Vol. 2, 2004.
- [33] Yang, H., Z. Chen, and Zhang, H., Vibration of Train-Rail-Bridge Interaction Considering Rail Irregularity with Arbitrary Wavelength, International Journal of Engineering, Vol. 28, No. 4, 2015, pp. 516-522.
- [34] Au, F., Wang, J., and Cheung, Y., Impact Study of Cable-Stayed Railway Bridges with Random Rail Irregularities, Engineering Structures, Vol. 24, No. 5, 2002, pp. 529-541.