Int. J. Advanced Design and Manufacturing Technology, 2023, Vol. 16, No. 2, pp. 1-11

DOI: 10.30486/admt.2021.1908459.1215 ISSN: 2252-0406 https://admt.isfahan.iau.ir

Frequency Analysis of Ring-Stiffened Composite Cylindrical Shell using Experimental, Analytical and Finite Element Methods

Hadi Salimi[1]
Department of Mechanical Engineering, Malek Ashtar University of Technology, Tehran, Iran.
E-mail: hadi.salimi@gmail.com

Ali Davar[2]
Composite Research Centre, Malek Ashtar University of Technology, Lavizan, Tehran, Iran.
E-mail: davar78@gmail.com

Mohsen Heydari Beni[3]
Department of Mechanical Engineering, Malek Ashtar University of Technology, Tehran, Iran.
E-mail: Mohsenheydari1371@gmail.com

Jafar Eskandari Jam[4, *]
Composite Research Centre, Malek Ashtar University of Technology, Lavizan, Tehran, Iran.
E-mail: jejaam@gmail.com , eskandari@mut.ac.ir
*Corresponding author

Majid Eskandari Shahraki[5]
Department of Aerospace Engineering,
Ferdowsi University of Mashhad, Mashhad, Iran.
E-mail: mjdeskandari@gmail.com

Received: 3 September 2020, Revised: 18 November 2020, Accepted: 1 December 2020

Abstract: In this paper, free vibration of laminated composite cylindrical shells reinforced with circumferential rings, are investigated with experimental, analytical and finite element methods and natural frequencies are obtained. The analysis is carried out for clamp-free and clamp-clamp boundary conditions and the results are compared with each other. To solve the problem, the equilibrium Equations of motions are written according to the classical shells theory and after simplification, the structural stiffness and mass matrices and the frequency Equation are derived using Galerkin method. The results obtained in this paper, are compared with the results available in the literatures, and the results of experimental and finite element methods and good agreement is observed.

Keywords: Boundary Conditions, Composite Cylindrical Shells, Free Vibration, Galerkin Method, Reinforcement Rings

Biographical notes: Hadi Salimi received his MSc in Mechanical Engineering from University of Malek Ashtar. His field of research is mechanical analysis of composite materaials. **Ali Davar** is currently Assistant Professor at the Department of Mechanical Engineering, at Malek ashtar University, Tehran, Iran. His current research interest includes composite manufacturing and composite structures. **Mohsen Heydari Beni** is currently a PhD student at Malek Ashtar University and his main reaserch interests are composite structures, plates and shell analysis and nanomechanics. **Jafar Eskandari Jam** is Professor of Mechanical engineering at Malek Ashtar University Tehran, Iran. His current research focuses on composite structures, plates and shell analysis and nanomechanics. **Majid Eskandari Shahraki** is currently a PhD student at Ferdowsi University of Mashhad, Iran and his main reaserch interests is nanomechanics.

Research paper

1 INTRODUCTION

Composite shells are widely used in various industries, such as aerospace, marine and oil industries. In most cases these shells are under dynamic loads. In addition to fatigue failure, dynamic loads can lead to resonance and sudden structural failure. Hence; it is necessary to determine the natural frequencies of composite shells. Frequency of composite structures is directly affected by the mass and stiffness. If the circumferential rings are used for structural reinforcements, these rings influence the stiffness and mass of the entire structure and consequently came the natural frequencies of the structure to be changed. Therefore, it is necessary to study the effect of reinforcement rings on the shell frequencies.

There are two methods to determine the natural frequencies of such structures. First, shell and reinforcements can be considered as a unit construction and then the natural frequencies are determined based on the average stiffness and mass. This method is applicable when the number of reinforcements is high and the distance between two adjacent rings is less than a longitudinal half-wave of mode shape of the cylindrical shell. In the second method which is more common, shell and reinforcements are considered as separate elements so stiffness and mass of reinforcements are locally entered into the Equations of motion of the system. Heretofore, several papers have been published on the vibration of composite shells which is mentioned below.

Marco Amabili [1] investigated nonlinear forced vibration of composite cylindrical shells for simply supported boundary condition at both ends, using various theories. He compared the results from Novazhilov classical theory (neglecting the effects of shear deformation and rotary inertia), higher order shear deformation theory and Amabili-Reddy shear deformation theory. The results showed that Amabili-Reddy theory and Novazhilov theory have good convergence for thin shells.

Hiruki Matsunaga [2] calculated the natural frequency and buckling load for a composite cylindrical shell with cross-ply layup and simply supported boundary condition at both ends using higher order theory. The displacement Equation of shell is expressed using power series. The study showed that the vibrational and buckling properties of composite shells are independent of the number of layers.

Jafari et al. [3] studied transient dynamic response of a composite cylindrical shell under axial compressive load. Their research showed that increasing the axial load reduces the natural frequencies. As the axial compressive load increases up to the critical buckling load, the natural frequency corresponding to mode shape in which the critical load occurs becomes zero.

Hafenbach et al. [4] studied the vibration and damping behaviour of composite cylindrical shell made of carbon-epoxy using analytical method. They investigated the effect of geometrical parameters such as the ratio of L/(R) and h/(R).

Lam et al. [5] studied the vibration of laminated composite cylindrical shell. In addition to the geometrical parameters such as L/(R) and R/(h), the effect of number of layers and lamination angle is investigated. Also, the natural frequencies for both angle-ply and cross-ply laminations are compared. The results showed that increasing the number of layers have a small effect on the frequency parameter. But changing layup from angle-ply to cross-ply can have a large impact on the frequency parameter.

Lam and Loy [6] derived the Equations of motion of the composite cylindrical shells using love theory. The composite layup is considered to be cross-ply and for all boundary conditions, except for the free-free boundary condition. The cylindrical shell is analysed and the Equations are solved by Galerkin method. The results show that the boundary conditions have a significant impact on the natural frequencies of composite cylindrical shell.

Cianssio et al. [7] studied free vibration of composite plate carrying a concentrated mass. The plate is made of boron-epoxy with a clamped edge and the other edges have free boundary conditions. Also concentrated mass is attached at the midpoint of the plate. They showed that in the first six frequencies, increasing the attached mass leads to reducing the natural frequencies.

Nallim et al [8] studied the natural frequencies of composite circular plate carrying concentrated mass with various boundary conditions. They investigated the effects of the ratio of the concentrated point mass to the total mass of the plate and also the effect of distance of the point mass from the centre of plate. The results show that increasing the concentrated mass led to reduce the natural frequencies also increasing the distance of concentrated mass from the fulcrum decreased the frequencies.

Amabili et al [9] investigated vibration of rectangular plate with effects of concentrated mass using analytical and experimental methods. They assumed the boundary conditions as elastic foundation and considered the rotary inertia of concentrated mass. They showed that placing a small mass on the diameter of a thin plate can change the mode shape from longitudinal to diagonal mode. In addition to reducing the natural frequencies, rotary inertia of the concentrated mass can create new structural mode shapes.

Khalili et al [10] studied free vibration of composite shells carrying concentrated masses considering the effect of stiffness of the concentrated masses. Composite shells with cross-ply layup and simply supported boundary conditions are analysed using shear

deformation theory. The effect of the various parameters such as shell thickness, thickness of the attached mass, variations of shell curvature and variations of elasticity modulus of the attached mass are studied. The results showed that increasing the thickness of the shell, decreases the effect of stiffness of the attached mass on the fundamental frequency.
Jafari and Bagheri [11] studied the free vibration of isotropic cylindrical shell reinforced with circumferential rings with non-uniform intervals. They used analytical Ritz method, finite element method and experimental approach. Finally, the results of these three methods are compared with each other.
Kim and Lee [12] carried out the vibration analysis and transient response of a composite cylindrical shell with circumferential rings. In their analysis, they considered the composite cylindrical shell made of graphite / epoxy with stacking sequence of $[(\pm 45/0/90)_2]_s$ with clamped boundary conditions at both ends. They investigated the effect of number of stiffeners and their cross-sectional dimensions on the natural frequencies. The results showed that sufficient number of reinforcement rings with suitable cross-sectional area can increase the natural frequency and decrease the amplitude of deformations. However, the excessive increase of the number of rings and ratio of width to height of the section of rings lead to decreasing the natural frequencies.
In this paper the natural frequencies of composite cylindrical shell with various boundary conditions are calculated using theoretical, experimental and finite element methods. The effect of reinforcement rings on the frequencies is studied.

2 GOVERNING EQUATIONS

Figure 1 shows a laminated composite cylindrical shell of radius R, Length L and thickness h. Parameters u, v and w, are displacement components along x, θ and z directions, respectively.

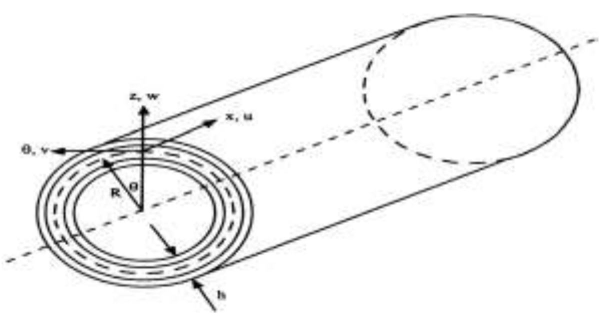

Fig. 1 Composite cylindrical shell with reference coordinate system.

Strain and curvature components in the curvilinear coordinate system are as follows [13]:

$$\begin{aligned}
\varepsilon_\alpha^0 &= \frac{\partial u}{\partial \alpha}\\
\varepsilon_\beta^0 &= \frac{\partial v}{\partial \beta} + \frac{w}{R}\\
\gamma_{\alpha\beta}^0 &= \frac{\partial v}{\partial \alpha} + \frac{\partial u}{\partial \beta}\\
k_\alpha &= -\frac{\partial^2 w}{\partial \alpha^2}\\
k_\beta &= \frac{\partial}{\partial \beta}\left(\frac{v}{R}\right) - \frac{\partial^2 w}{\partial \beta^2}\\
\tau &= \frac{\partial}{\partial \alpha}\left(\frac{v}{R}\right) - 2\frac{\partial^2 w}{\partial \beta\,\partial \alpha}
\end{aligned} \tag{1}$$

Also, the relationship between the force and moment resultants and the midpoint strain and curvature components are as follows [14]:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k \end{Bmatrix} \tag{2}$$

In “Eq. (2)”, A, B and D are extensional, coupling and bending stiffness matrices respectively and are defined as follows:

$$\begin{cases} A_{ij} = \sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)_k (h_k - h_{k-1}) \\ B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)_k ({h^2}_k - {h^2}_{k-1}) \quad i,j = \\ D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)_k ({h^3}_k - {h^3}_{k-1}) \end{cases} 1,2,6 \tag{3}$$

Where, h_k is the distance of k^{th} layer from the shell midplane, N is the total number of layers and $\bar{Q}$ is the transferred stiffness matrix and it is calculated from the following Equation:

$$\bar{Q}_{ij} = [T]^{-1}[Q_{ij}][T]^{-T} \qquad i,j = 1,2,6 \tag{4}$$

In the above Equation, Q_{ij} is the reduced stiffness matrix defined as follows:

$$\begin{aligned}
Q_{11} &= \frac{E_{11}}{1-\nu_{12}\nu_{21}}\\
Q_{22} &= \frac{E_{22}}{1-\nu_{12}\nu_{21}}\\
Q_{12} = Q_{21} &= \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_{22}}{1-\nu_{12}\nu_{21}}\\
Q_{16} = Q_{26} &= Q_{61} = Q_{62} = 0\\
Q_{66} &= G_{12}
\end{aligned} \tag{5}$$

T is the transfer matrix and calculated for each layer with fibre angle θ with respect to x axis:

$$T = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix} \quad (6)$$

The equilibrium Equations based on the classical shell theory are expressed as follows [15]:

$$\begin{aligned} &\frac{\partial N_\alpha}{\partial \alpha} + \frac{1}{R}\frac{\partial N_{\alpha\beta}}{\partial \alpha} = -\bar{I}_0 \frac{\partial^2 u}{\partial t^2} \\ &\frac{\partial N_\beta}{\partial \beta} + \frac{\partial N_{\alpha\beta}}{\partial \alpha} + \frac{1}{R}\left[\frac{\partial M_\beta}{\partial \beta} + \frac{\partial M_{\alpha\beta}}{\partial \alpha}\right] = -\bar{I}_0 \frac{\partial^2 v}{\partial t^2} \\ &-\frac{N_\beta}{R} + \frac{\partial^2 M_\alpha}{\partial \alpha^2} + 2\frac{\partial^2 M_{\alpha\beta}}{\partial \beta\, \partial \alpha} + \frac{\partial^2 M_\beta}{\partial \beta^2} == -\bar{I}_0 \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (7)$$

In “Eq. (7)”, $\bar{I}_0$ is the mass moment of inertia as follows:

$$\bar{I}_0 = \sum_{k=1}^{N} \rho^{(k)} (h_k - h_{k-1}) \quad (8)$$

Where, $\rho^{(k)}$ is the density of the k^{th} layer. By substituting “Eqs. (1 and 2)” in “Eq. (7)”, the following sets of differential Equations are obtained:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (9)$$

Components of differential Equations (L_{ij}) are as follows:

$$\begin{aligned} &L_{11} = A_{11}\frac{\partial^2}{\partial \alpha^2} + 2A_{16}\frac{\partial^2}{\partial \alpha\, \partial \beta} + A_{66}\frac{\partial^2}{\partial \beta^2} + \bar{I}_0 \frac{\partial^2}{\partial t^2} \\ &L_{12} = L_{21} = \left\{A_{16} + \frac{B_{16}}{R}\right\}\frac{\partial^2}{\partial \alpha^2} + \left\{A_{12} + A_{66} + \frac{B_{12}+B_{66}}{R}\right\}\frac{\partial^2}{\partial \alpha\, \partial \beta} + \left\{A_{26} + \frac{B_{26}}{R}\right\}\frac{\partial^2}{\partial \beta^2} \\ &L_{13} = L_{31} = -B_{11}\frac{\partial^3}{\partial \alpha^3} - B_{26}\frac{\partial^3}{\partial \beta^3} - 3B_{16}\frac{\partial^3}{\partial \alpha^2 \partial \beta} - (B_{12} + 2B_{66})\frac{\partial^3}{\partial \alpha\, \partial \beta^2} + \left(\frac{A_{12}}{R}\right)\frac{\partial}{\partial \alpha} + \left(\frac{A_{26}}{R}\right)\frac{\partial}{\partial \beta} \\ &L_{22} = \left\{A_{66} + \frac{2B_{66}}{R} + \frac{D_{66}}{R^2}\right\}\frac{\partial^2}{\partial \alpha^2} + \left\{2A_{26} + \frac{4B_{26}}{R} + \frac{4D_{26}}{R^2}\right\}\frac{\partial^2}{\partial \alpha\, \partial \beta} + \left\{A_{22} + \frac{2B_{22}}{R} + \frac{D_{22}}{R^2}\right\}\frac{\partial^2}{\partial \beta^2} + \bar{I}_0 \frac{\partial^2}{\partial t^2} \\ &L_{23} = L_{32} = \left\{-B_{16} - \frac{D_{16}}{R}\right\}\frac{\partial^3}{\partial \alpha^3} + \left\{-B_{22} - \frac{D_{22}}{R}\right\}\frac{\partial^3}{\partial \beta^3} + \left\{-(B_{12} + 2B_{66}) - \frac{D_{12}+2D_{66}}{R}\right\}\frac{\partial^3}{\partial \alpha^2 \partial \beta} + \left\{-3B_{26} - \frac{3D_{26}}{R}\right\}\frac{\partial^3}{\partial \alpha\, \partial \beta^2} + \left\{\frac{A_{26}}{R} + \frac{B_{26}}{R^2}\right\}\frac{\partial}{\partial \alpha} + \left\{\frac{A_{22}}{R} + \frac{B_{22}}{R^2}\right\}\frac{\partial}{\partial \beta} \\ &L_{33} = \left\{D_{11}\frac{\partial^4}{\partial \alpha^4} + 4D_{16}\frac{\partial^4}{\partial \alpha^3 \partial \beta} + 2(D_{12} + 2D_{66})\frac{\partial^4}{\partial \alpha^2 \partial \beta^2} + 4D_{26}\frac{\partial^4}{\partial \alpha\, \partial \beta^3} + D_{22}\frac{\partial^4}{\partial \beta^4}\right\} - 2\left\{\left(\frac{B_{12}}{R}\right)\frac{\partial^2}{\partial \alpha^2} + 2\left(\frac{A_{26}}{R}\right)\frac{\partial^2}{\partial \alpha\, \partial \beta} + \left(\frac{B_{22}}{R}\right)\frac{\partial^2}{\partial \beta^2}\right\} + \frac{A_{22}}{R^2} + \bar{I}_0 \frac{\partial^2}{\partial t^2} \end{aligned} \quad (10)$$

L_{ij} operators in above Equations are defined in the curvilinear coordinate system. In order to convert curvilinear coordinate system to the special cylindrical coordinate system, the following relations are used:

$$\frac{\partial}{\partial \alpha} = \frac{\partial}{\partial x}, \qquad \frac{\partial}{\partial \beta} = \frac{1}{R}\frac{\partial}{\partial \theta} \quad (11)$$

The displacement components u, v and w should be chosen so as to satisfy the boundary conditions of the problem. For this purpose, these functions are expressed as a double Fourier series:

$$\begin{aligned} u &= \sum_m \sum_n U f_{11}(x, \theta) f(t) \\ v &= \sum_m \sum_n V f_{21}(x, \theta) f(t) \\ w &= \sum_m \sum_n W f_{31}(x, \theta) f(t) \end{aligned} \quad (12)$$

Where, m and n respectively are the number of longitudinal half-waves and circumferential waves of vibrational mode shape of the structure. U, V and W are the coefficients of the natural mode shapes which are obtained by solving the free vibration. In order to solve the free vibration eigenvalue problem, $f_{(t)}$ is considered to be a periodic function of time as $f_{(t)} = e^{i\omega t}$. Also f_{11}, f_{21} and f_{31} are modal beam functions along longitudinal direction and trigonometric functions along circumferential direction, defined as follows:

$$\begin{aligned} f_{11}(x, \theta) &= \Psi'(x).\Phi_u(\theta) \\ f_{21}(x, \theta) &= \Psi(x).\Phi_v(\theta) \\ f_{31}(x, \theta) &= \Psi(x).\Phi_w(\theta) \end{aligned} \quad (13)$$

In “Eq. (13)”, the trigonometric functions along circumferential direction are as follows [3]:

$$\Phi_u = \Phi_w = \cos(n\theta) \quad \Phi_v = \sin(n\theta) \tag{14}$$

The modal beam function along longitudinal direction is as follows [12]:

$$\Psi(x) = \alpha_1 \cosh\frac{\lambda_m x}{L} + \alpha_2 \cos\frac{\lambda_m x}{L} - \sigma_m\left(\alpha_3 \sinh\frac{\lambda_m x}{L} - \alpha_4 \sin\frac{\lambda_m x}{L}\right) \tag{15}$$

In the above Equations α_i is a constant coefficient, λ_m is a root of a nonlinear Equation and the parameter σ_m is related to λ_m. All of them are determined according to the type of the boundary conditions.
In case of Clamped-Free boundary conditions [6]:

$$\begin{gathered}\cos\lambda_m . \cosh\lambda_m = -1 \\ \sigma_m = \frac{\sinh\lambda_m - \sin\lambda_m}{\cosh\lambda_m + \cos\lambda_m} \\ \text{at } x = 0: \frac{\partial\Psi(x)}{\partial x} = \Psi(x) = 0 \\ \text{at } x = L: \frac{\partial^2\Psi(x)}{\partial x^2} = \frac{\partial^3\Psi(x)}{\partial x^3} = 0\end{gathered} \tag{16}$$

In case of Clamped-Clamped boundary condition [6]:

$$\begin{gathered}\cos\lambda_m . \cosh\lambda_m = 1 \\ \sigma_m = \frac{\cosh\lambda_m - \cos\lambda_m}{\sinh\lambda_m - \sin\lambda_m} \\ \text{at } x = 0, x = L: \frac{\partial\Psi(x)}{\partial x} = \Psi(x) = 0\end{gathered} \tag{17}$$

Now the effect of reinforcement rings should be entered into the Equations. Reinforcement ring directly affects the system stiffness and mass matrices. However, it should be applied to the governing Equations. For this purpose, the Heaviside function is applied as follows [10]:

$$H(x - x_0) = \begin{cases} 1 & \text{at } x \geq x_0 \\ 0 & \text{at } x < x_0 \end{cases} \tag{18}$$

Thus, the Equations of motion are modified as follows [10]:

$$\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix}_s \begin{bmatrix} u \\ v \\ w \end{bmatrix} + H^*(x, x_0, c)\begin{bmatrix} L_{11} & L_{12} & L_{13} \\ L_{12} & L_{22} & L_{23} \\ L_{13} & L_{23} & L_{33} \end{bmatrix}_r \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

Where, subscript s refers to the shell and subscript r refers to the rings, x_0 is the distance between location of reinforcement ring and constrained end of the cylinder. The parameter c is width of the section of the ring and H^* is a combination of Heaviside functions defined as follows:

$$H^*(x, x_0, c) = [\ H(x - x_0) - H(x - x_0 - c)] \tag{20}$$

Analytical Galerkin method is used to solve the problem. By replacing “Eqs. (10 and 11)” in “Eq. (19)” and simplifying, the following Equations are obtained:

$$\left(\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}_s + H^*(x, x_0, c)\begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}_r\right) \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

Where the components k_{ij} are obtained from the following Equation [16]:

$$k_{ij} = \frac{\int_0^l \int_0^{2\pi} L_{ij} u_i f_{i1} dx d\theta}{\int_0^l \int_0^{2\pi} f_{i1}^2 dx d\theta} \tag{22}$$

After determining the modified stiffness components k_{ij}, to solve the characteristic Equation of the system, the determinant of the coefficients matrix is set to be zero, leading to a sixth order polynomial Equation in terms of ω as follows:

$$\beta_1\omega^6 + \beta_2\omega^4 + \beta_3\omega^2 + \beta_4 = 0 \tag{23}$$

3 EXPERIMENTAL METHOD

In order to provide the clamped boundary conditions at one or both ends of the composite cylindrical shell, first two steel disks are produced. Then a circular groove of 15 mm depth and same diameter of the shell is cut on each of these disks. The cylindrical shell is then placed inside the groove. Finally the groove is filled with plaster. Also, two steel rings with the section dimensions of $10 \times 10\ mm$ are produced. The inner diameter of the rings is equal to the outer diameter of the cylindrical shell. The rings are attached on the outside of the cylindrical shell using suitable binder.
A piezoelectric accelerometer is used to carry out the modal tests. Accelerometer is attached to the cylindrical shell by a suitable adhesive. Structure is stimulated by a hammer and the data is transmitted to an analyser device.

Analyser device is connected to a computer and the Frequency Response Function (FRF) is plotted in the computer. The peaks in the FRF curve represent the natural frequencies. In “Figs. 2, 3, 4 and 5”, the equipment and specimens used in the modal analysis tests are shown.

Fig. 2 Clamped-free composite cylindrical shell with hammer and piezoelectric accelerometer.

Fig. 3 Clamped-clamped composite cylindrical shell.

Fig. 4 Ring-stiffened clamped-free composite cylindrical shell

Fig. 5 Modal analyser device

4 FINITE ELEMENT METHOD

In order to ensure the accuracy of the analytical and experimental method and further verification of the results, finite element analysis is performed. For this purpose, dynamic and frequency analysis are achieved on the cylindrical shell and reinforcement rings using ABAQUS software. Natural frequencies and frequency response functions are obtained and compared with analytical and experimental results. Figure 6 shows the finite element model of the cylindrical shell with reinforcement ring.

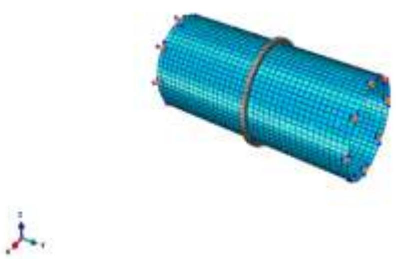

Fig. 6 Finite element model of the ring-stiffened cylindrical shell.

5 RESULTS AND DISCUSSION

5.1. Verifying the Results

In order to validate the analytical method, the results are compared with those of other references. "Table 1" presents the natural frequencies of an isotropic cylindrical shell with clamped-free boundary conditions at its ends [6]. Mechanical and geometrical properties of this shell are as follows:

$E = 210\ \text{GPa} \qquad v = 0.28 \qquad \rho = 7800\ \text{kg/m}^3$

$L = 502\ \text{mm} \qquad R = 63.5\ \text{mm} \qquad h = 1.63\ \text{mm}$

Table 1 Natural frequencies of isotropic cylindrical shell with clamped-free boundary condition (Hz)

m	n	Present	Lam & Loy [6]	Discrepancy %
1	3	759.81	759.9	0.5%
1	4	1458.49	1459.3	0.3%
1	5	2369.88	2360.9	0.9%
1	6	3489.67	3463.9	3.7%

The natural frequencies of a composite cylindrical shell with clamped-clamped boundary conditions at its ends [13] are presented in "Table 2".

Table 2 Natural frequencies of composite cylindrical shell with clamped-clamped boundary condition (Hz)

m	n	Present	Kim & Lee [13]	Discrepancy %
1	5	589.19	587.23	0.3
1	6	595.02	594.13	0.15
1	7	692.37	691.71	0.09
1	4	703.66	701.68	0. 3
1	8	853.21	848.47	0.5

Mechanical and geometrical properties of this shell are as follows:

$E_1 = 139.4\ \text{GPa} \quad E_2 = 8.7\ \text{GPa} \quad G_{12} = 3.1\ \text{GPa}$
$v_{12} = 0.268 \quad Stacking\ sequence: [(\pm 45°/0°/90°)_2]_S$
$L/R = 2.5 \quad R = 200\ \text{mm} \quad h = 2\ \text{mm}$

As can be seen in "Tables 1 and 2", in both cases the results of the present analytical analysis are in good agreement with the results presented in the references.

5.2. Effects of B.C on Non-Stiffened Composite Cylindrical Shell

In the following, the natural frequencies of a non-stiffened composite cylindrical shell made of glass/epoxy with clamped-free and clamped-clamped boundary conditions are obtained. Mechanical and geometrical properties of this shell are as follows:

$E_1 = 29.561\ \text{GPa} \quad E_2 = 8.77\ \text{GPa} \quad G_{12} = 2.81\ \text{GPa}$

$v_{12} = 0.26 \quad \text{Stacking Sequence}: [0°/90°/90°/0°]$

$L = 500\ \text{mm} \quad R = 100\ \text{mm} \quad h = 2\ \text{mm}$

Figure 7 presents FRF of this cylindrical shell with clamped-free boundary conditions which is obtained from experimental tests. In "Fig. 8", the same graph is plotted using finite element analysis. As can be seen, by comparing "Figs. 7 and 8", there is a high correspondence between experimental and finite element results. Also, in "Figs. 9 and 10", FRF of the cylindrical shell with clamped-clamped boundary conditions using experimental and finite element methods respectively is plotted. The results are also in good agreement.

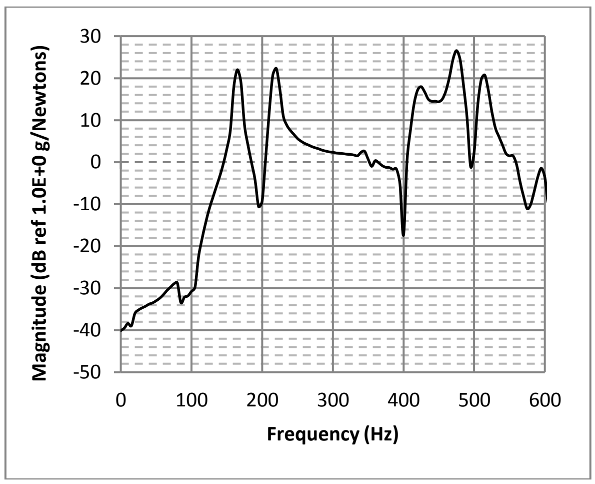

Fig. 7 FRF of clamped-free composite cylindrical shell obtained from experimental method.

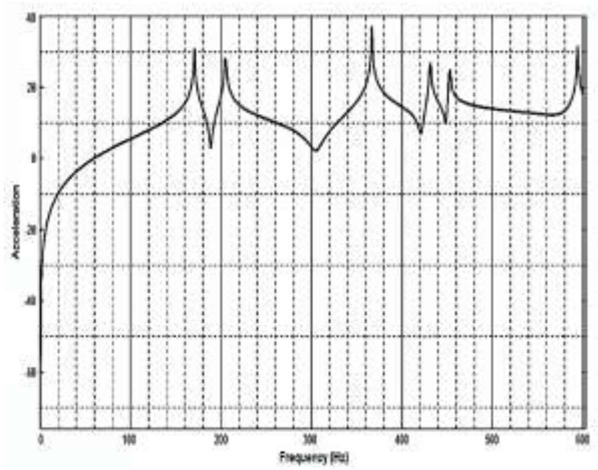

Fig. 8 FRF of clamped-free composite cylindrical shell obtained from FEM.

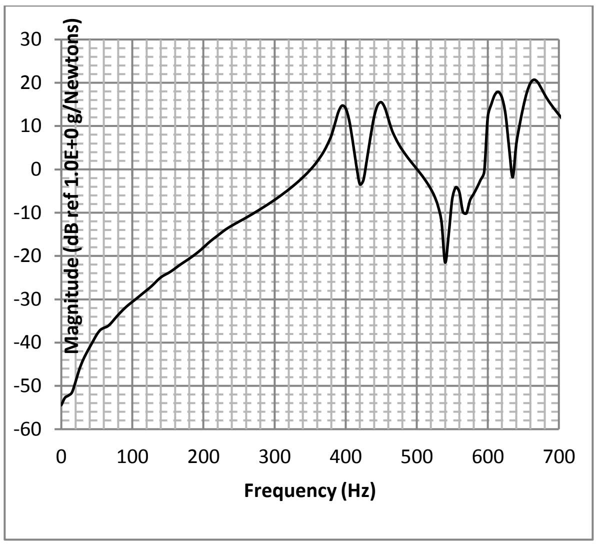

Fig. 9 FRF of clamped-clamped composite cylindrical shell obtained from experimental method.

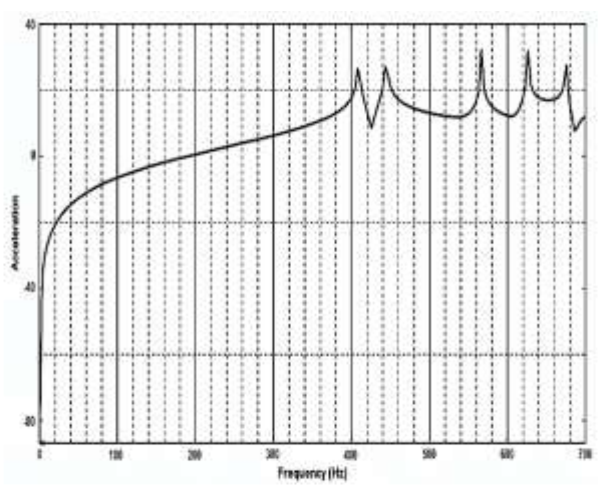

Fig. 10 FRF of clamped-clamped composite cylindrical shell obtained from FEM.

5.3. Effect of Attached Rings on Frequency of Composite Cylindrical Shell with Clamped-Free B.C

In this part, the steel reinforcement rings are attached on the cylindrical shell and the effect of rings on the natural frequencies is investigated. "Tables 3 and 4" present the effects of reinforcement rings on the natural frequencies of the composite cylindrical shell with clamped-free boundary conditions for mode numbers (m=1, n=2) and (m=1, n=1), respectively.

Table 3 Effect of attached rings on the natural frequencies of the clamped-free composite cylindrical shell for mode number (m=1, n=2)

Ring(s) Position	Theoretical	FEM	Experimental
Non stiffened shell	172.6 (4.6%)†	170.5	165
$\frac{x_0}{L} = 0.5$	274.23 (7.5%)	267.9	255
$\frac{x_0}{L} = 1$	401.37 (1.6%)	382.2	395
$\frac{x_0}{L} = 0.5, 1$	433.79 (2.1%)	428.7	425

† The numbers in the parenthesis show the discrepancies with respect to experimental method

Table 4 Effect of attached rings on the natural frequency of clamped-free composite cylindrical shell for mode number (m=1, n=1)

Ring(s) Position	Theoretical	FEM	Experimental
Non stiffened shell	357.8 (5.2%)†	366.8	340
$\frac{x_0}{L} = 0.5$	314.86 (8.5%)	312.2	290
$\frac{x_0}{L} = 1$	230.44 (12.3%)	230.4	205
$\frac{x_0}{L} = 0.5, 1$	220.51 (10.2%)	219.5	200

† The numbers in the parenthesis show the discrepancies with respect to experimental method

According to "Tables 3 and 4" for mode number (m=1, n=2), the natural frequency increased thereby attach reinforcement rings but for mode number (m=1, n=1) the rings lead to decrease the frequency. The reason is that the rings are structural reinforcements which increase the stiffness in the circumferential direction but have no effect on the longitudinal stiffness of the cylindrical shell. In other words, in case of longitudinal bending modes, the mass of reinforcement rings reduces the natural frequencies however the stiffness of the rings cause the circumferential bending frequencies to increase. In order to better assess this point, the natural frequencies of the composite cylindrical shell for circumferential and longitudinal bending modes are given in “Figs 5 and 6”, respectively. In these figures, the natural frequencies are obtained using analytical method. The values are compared with the case that a reinforcement ring is attached at the position $\frac{x_0}{L} = 1$.

The results in “Fig. 11” show that the frequencies of circumferential bending modes can be increased by adding reinforcement rings however, by increasing n the effect of reinforcement rings on the circumferential bending frequencies is decreased. Also, the results in “Fig. 12” indicate that reinforcement rings reduced the frequencies of longitudinal bending modes consistently.

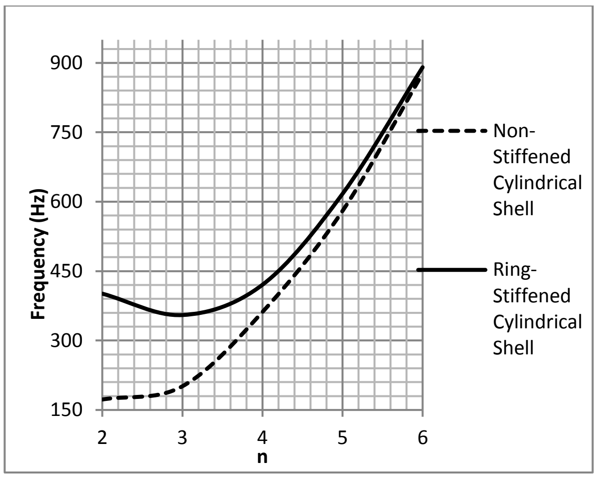

Fig. 11 Effect of attached rings on the natural frequencies of clamped-free composite cylindrical shell in circumferential mode shapes (m=1) that the ring is attached at the position $\frac{x_0}{L} = \mathbf{1}$.

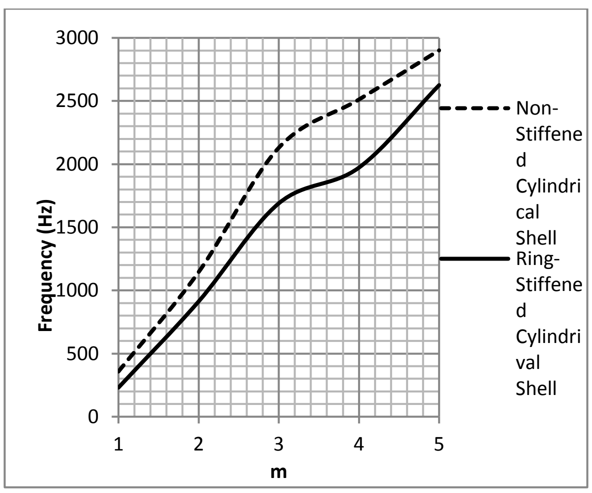

Fig. 12 Effect of attached rings on the natural frequencies of clamped-free composite cylindrical shell in longitudinal mode shapes (n=1) that the ring is attached at the position $\frac{x_0}{L} = \mathbf{1}$.

5.4. Effect of Attached Rings on Frequency of Composite Cylindrical Shell with Clamped-Clamped B.C

Similarly, for clamped-clamped boundary conditions, the effect of adding a steel ring on the first natural frequency of circumferential and longitudinal bending mode is given in “Table 5”. In this case the reinforcement ring is installed at the position $\frac{x_0}{L} = 0.5$.

Table5. Effect of attaching reinforcement ring on the natural frequencies of clamped-clamped composite cylindrical shell in mode shapes (m=1, n=3) and (m=1.n=1)

Mode No.	Ring(s) Position	Theoretical	FEM	Experimental
(m=1, n=3)	Non stiffened shell	411.2(4.1%)[†]	407.4	395
(m=1, n=3)	$\frac{x_0}{L} = 0.5$	810.36(3.2%)	801.04	785
(m=1, n=1)	Non stiffened shell	941.38(0.1%)	950.90	940
(m=1, n=1)	$\frac{x_0}{L} = 0.5$	639.12(0.6%)	644.18	635

[†] The numbers in the parenthesis show the discrepancies with respect to experimental method

According to “Table 5”, for clamped-clamped boundary conditions, adding reinforcement ring can affect the natural frequencies. In “Figs. 13 and 14”, this subject is investigated for circumferential and longitudinal bending frequencies using analytical method.

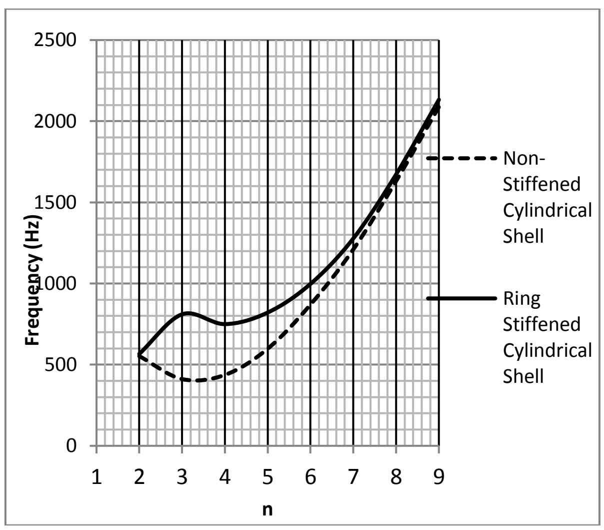

Fig. 13 Effect of attached rings on the natural frequencies of clamped-clamped composite cylindrical shell in circumferential mode shapes (m=1) that the ring is attached at the position $\frac{x_0}{L} = 0.5$.

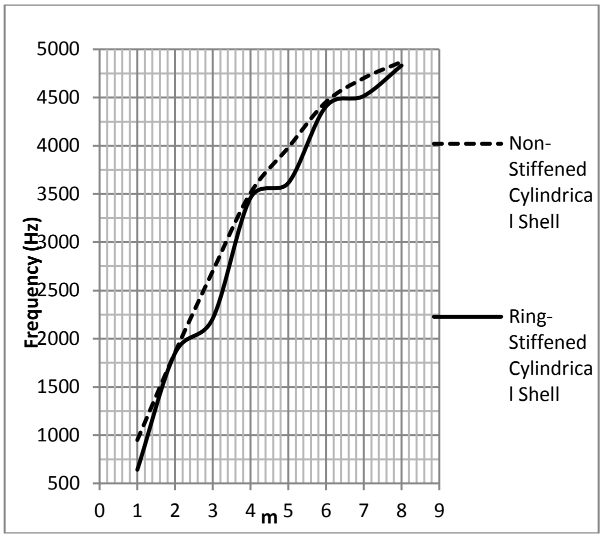

Fig. 14 Effect of attached rings on the natural frequencies of clamped-clamped composite cylindrical shell in longitudinal mode shapes (m=1) that the ring is attached at the position $\frac{x_0}{L} = 0.5$.

Similarly, as clamped-free boundary conditions, in case of the clamped-clamped boundary conditions, reinforcement rings increase the circumferential bending frequencies and decrease the longitudinal bending frequencies. Also, the results in “Fig. 14” show that for even values of m, the reinforcement ring has no effect on the longitudinal bending frequencies. The reason is that, the reinforcement ring is placed on the position $\frac{x_0}{L} = 0.5$ where for even values of m, it is located on the node of the longitudinal bending mode shapes.

6 CONCLUSION

In this paper, the natural frequencies of a composite cylindrical shell with clamped-free and clamped-clamped boundary conditions are determined using analytical, experimental and finite element methods. A good agreement achieved between the results of these three methods. Also, the effect of attaching reinforcement rings on the natural frequencies is investigated.

The results show that the reinforcement rings increase the circumferential bending frequencies and decrease the longitudinal bending frequencies. It is because of that the reinforcement rings increase the stiffness of the structure in the circumferential direction, but in the longitudinal direction, the rings have no effect on the stiffness of the structure even the mass of the rings leads to decrease the frequencies.

Another point is that for the circumferential bending mode shapes, the effect of attaching the reinforcement rings is decreased by increasing the value of n. Also, for the longitudinal bending mode shapes, the reinforcement ring has no effect on the natural frequency if it is located on the node of the mode shape.

REFERENCES

[1] Amabili, M., Nonlinear Vibrations of Laminated Circular Cylindrical Shells: Comparison of Different Shell Theories, Composite Structures, Vol. 94, 2011, pp. 207-220.

[2] Matsunaga, H., Vibration and Buckling of Cross-Ply Laminated Composite Circular Cylindrical Shells According to A Global Higher-Order Theory, International Journal of Mechanical Sciences, Vol. 49, 2007, pp. 1060-1075.

[3] Jafari, S. M, Khalili, R., Azarafza, R., Transient Dynamic Response of Composite Circular Cylindrical Shells Under Radial Impulse Load And Axial Compressive Loads, Thin-Walled Structures, Vol. 43, 2005, pp. 1763-1786.

[4] Hufenbach, W., Holste, C., Kroll, L., Vibration and Damping Behavior of Multi-Layered Composite Cylindrical Shells, Composite Structures, Vol. 58, 2002, pp. 165-174.

[5] Lam, K. Y., Wu, Q., Free Vibration of Symmetric Angle-Ply Thick Laminated Composite Cylindrical Shells, Composites, Part B, Vol. 31, 2000, pp. 345-354.

[6] Lam, K. Y., Loy, C. T., Influence of Boundary Conditions for A Thin Laminated Rotating Cylindrical Shell, Composite Structures, Vol. 41, 1998, pp. 215-228.

[7] Ciancio, P. M., Rossit, C. A., and Laura, P. A. A., Approximate Study of The Free Vibrations of a Cantilever Anisotropic Plate Carrying a Concentrated Mass, Journal of Sound and Vibration, Vol. 302, 2007, pp. 621-628.

[8] Nallim, L., Grossi, R. O., and Laura, P. A. A., Transverse Vibrations of Circular Plates of Rectangular Orthotropy Carrying a Central, Concentrated Mass, Journal of Sound and Vibration, Vol. 216, No. 2, 1998, pp. 337-341.

[9] Amabili, M., Pellegrini, M., Righi, F., and Vinci, F., Effect of Concentrated Masses with Rotary Inertia on Vibrations of Rectangular Plates, Journal of Sound and Vibration, Vol. 295, 2006, pp. 1-12.

[10] Khalili, S. M. R., Tafazoli, S., and Malekzadeh Fard, K., Free Vibrations of Laminated Composite Shells with Uniformly Distributed Attached Mass Using Higher Order Shell Theory Including Stiffness Effect, Journal of Sound and Vibration, Vol. 330, 2011, pp. 6355-6371.

[11] Jafari, A. A., Bagheri, M., Free Vibration of Non-Uniformly Ring Stiffened Cylindrical Shells Using Analytical, Experimental and Numerical Methods, Thin-Walled Structures, Vol. 44, No. 1, 2006, pp. 82-90.

[12] Kim, Y. W., Lee, Y. S., Transient Analysis of Ring-Stiffened Composite Cylindrical Shells with Both Edges Clamped, Journal of Sound and Vibration, Vol. 252, No.1, 2002, pp. 1-17.

[13] Qatu, M. S., Vibration of Laminated Shells and Plates, Elsevier, 2004.

[14] Kaw, K., Mechanics of Composite Materials, Second Edition, Published by CRC Press, 2006.

[15] Reddy, J. N., Mechanics of Laminated Composite Plates and Shells Theory and Analysis, Second Edition, Published by CRC Press, 2003.

[16] Soedel, W., Vibrations of Shells and Plates, Third Edition, Published by Marcel Dekker, 2004.