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Nonlinear Mechanical Properties of Random Networks Composed of Nonlinear Fibers

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Abstract: The disordered fibrous networks provide load-bearing and main structural to different biological materials such as soft tissues. These networks display a highly nonlinear stress-strain relationship behavior when subjected to mechanical loads. This nonlinear strain-stiffening behavior is dependent on the network microstructure and properties of constituting fiber. We conduct a comprehensive computational study to characterize the importance of material properties of individual fibers as well as the local connectivity or coordination number and bending rigidity in the overall nonlinear mechanical response of a 3D random fiber network. The presented model shows the nonlinear stiffening with increasing applied shear strain more than critical shear strain. We determine the amount of strain-stiffening as a function of network microstructure parameters and the amount of nonlinearity of the fibers. The results show that the constitutive behavior of fibers displays much more strain-stiffening than networks made up of linear fibers. We find that the importance of the nonlinear reaction of individual fiber materials in the general mechanical behavior of networks becomes more important with increasing network connectivity. Furthermore, the amount of stress created in the network under shear increases with the enhanced connectivity of the network due to an increase in the network stiffness. Our model points to the important role of the mechanical response of individual fiber as well as the microstructure of the network in determining the overall mechanical properties of the 3D random network, which could be used to design and better understand the complex biomimetic network systems such as biological tissues and artificial engineering networks.

Keywords: Athermal Fibers, Biopolymer Networks, Lattice Structure, Mechanical Properties, Nonlinear Fiber, Random Networks

Biographical notes: Reyhane Mirkhani is a PhD student in Textile Engineering at Yazd University. Her field of research is the Mechanical Behavior of fiber networks**.** **Aliasghar Alamdar Yazdi** is Assistant Professor of Textile Engineering at Yazd University. His research field of study focuses on the Mechanical Behavior of Yarns and Fabrics. **Saeed Ebrahimi** is currently Associate Professor of Mechanical Engineering at Yazd University and his main research interests are Multibody Systems Dynamics, Robotics, Vibration Analysis of Mechanical Systems and Mechanism Analysis.

Research paper

1 INTRODUCTION

Biopolymer networks as the major part of the structure of living materials need to be recognized and evaluated for their mechanical properties. These networks are present in a wide range such as collagen and the actin cortex of eukaryotic cells [1]. In order to evaluate the shape and structure of the cell as well as its function, it is necessary to know the mechanical properties of the cytoskeleton and the extracellular matrix [2]. The most abundant fibrous protein, which is mainly found in various tissues such as articular cartilage, ligaments, cornea, and tendons, is collagen which shows the nonlinear elastic response [3-4]. This material has been considered as a major component in the performance and mechanical properties of the extracellular matrix and plays a major role in the fiber network [5-6]. Due to the widespread presence of fiber networks in various vital processes such as cellular motility [7], mechanical cell-cell communication [8-9], and stress management in articular cartilage [10], understanding the nonlinear mechanical response of fibrous networks and the effect of architecture parameter on it, are always been considered as challenging issues by researchers.

Various experimental and numerical studies have been performed on biopolymer networks; the results of these studies have shown nonlinear elastic behavior as well as large deformations in the network under different loading conditions [11-14]. In general, based on the elastic properties and network architecture, these types of networks are classified as semi-flexible biopolymer networks [15-16]. Researchers have mainly evaluated the behavior and performance of the fiber network based on affine models. In affine models, it is assumed that fiber segments are deformed based on far-field strain [5]. In these models, the mechanical behavior of the fiber network has been influenced by microstructural parameters such as fiber orientation and cross-link density [11-12]. But in reality, the behavior and mechanical responses of fiber networks, especially biological materials, are nonaffine. This issue has been important because in order to conduct detailed studies on these networks, nonlinear behavior and properties must be considered [17]. To date, various numerical studies have been performed in the form of affine and non-affine models with lattice-based and off-lattice network structures for assessing the origins of the nonlinear elasticity of fibrous networks [18-23].

Previous studies using numerical models have investigated that nonlinear strain-stiffening behavior is related to their architecture and mechanical properties of their fibrous constituents [24–32]. Moreover, the specific structural parameter of networks has been shown to have a significant influence on certain aspects of their mechanical response [33-34]. However, according to the real structure of the fiber network, there is a need to study and evaluate the network in 3D space, and little attention has been paid to the 3D random networks constituting nonlinear fibers that could play in controlling the nonlinear elasticity of random networks. The primary objective of this study is to provide a thorough investigation of how material properties of individual fibers along with various geometrical network parameters, such as network connectivity, and bending rigidity, affect the nonlinear mechanics of 3D random networks. For this purpose, we present the microstructure of 3D networks as disordered and diluted faced-cantered cubic lattices with different connectivity. In the model, the stress-strain response of individual fibers is represented by an exponential function to study the nonlinear mechanical response of 3D random networks, in which nonlinearity is varied from low to high. The computer simulation is used to study the mechanical response of fibrous networks subjected to simple shear. The effects of geometrical parameters of the 3D random networks being composed of nonlinear fibers and their relation to the nonlinearity of the network mechanical response are also characterized. The influence of the nonlinear stress-strain behavior of fibers on the network shear modulus is also studied. The numerical results show that the mechanical properties of constituting fibers and geometrical network parameters have important effects on the mechanical response of 3D random polymer networks.

2 PROCEDURES FOR PAPER SUBMISSION

In this research, by evaluating the research background and the results of previous research, the main parameters in evaluating the mechanical behavior and responses of the fiber network are identified and categorized. Based on the extracted parameters, research scenarios are created to implement the studies. Under the research scenarios, parametric studies are performed using numerical modeling. In this section, research variables are evaluated, as well as research scenarios and numerical models are presented.

2.1. Numerical Model Specifications

In this research, in order to evaluate the mechanical properties and responses of the 3D fiber network, the network microstructure (network connectivity, bending rigidity) and properties of constituting fiber are examined, and the models are implemented based on the change in these parameters and the results are obtained.

Network Structure

Various structures such as lattice-based and off-lattice network structures can be used to perform numerical studies on fiber networks. In this study, because of the superior features of the lattice structure in modeling network architecture, we use this structure to conduct studies. Due to the conditions and 3D modeling space, we used the faced-cantered cubic lattice (FCC) structure to model the fiber network. Figure 1a shows a schematic view of an FCC lattice network.

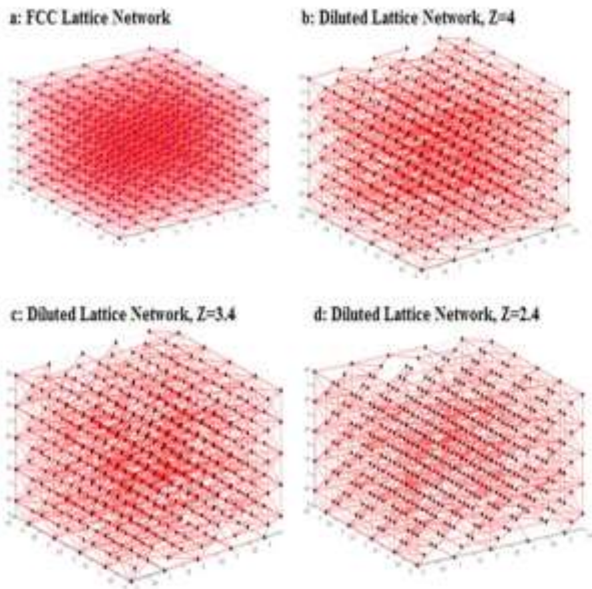

Fig. 1 Fiber network structure: (a): FCC Lattice Network, (b): Diluted Lattice Network Z=4, (c): Diluted Lattice Network Z=3.4, and (d): Diluted Lattice Network Z=2.4.

Network Connectivity

Biopolymer materials mainly form cross-linked network structures. The network connectivity indicates the average number of cross-links in a polymer network. In natural tissues, the average cross-links (Z) of biopolymer materials are between 2 and 4. [20], [35-37]. The network connectivity parameter has been one of the effective criteria in the stabilization and stability of polymer networks, which has an important role in the mechanical properties and network responses under different loading conditions [36]. In this research, to create 3D polymer networks with average variable connectivity (2<Z<4), first a complete faced-cantered cubic lattice network with $\boldsymbol{W} \times \boldsymbol{W} \times \boldsymbol{W}$ dimensions has been created. The maximum number of cross links in 3D networks is 12 (Zmax=12). The next step is to dilute the network by randomly removing the components with a q=1-p (p is a Possibility of existence) probability in order to adjust the average network connectivity. In this research, we have considered the dimensions of the network as 4.5*4.5*4.5 (w= 4.5 mm). We have also considered the values of network connection (Z) in 3 conditions (Z = 2.4, 3.6, 4) in the numerical studies (“Fig. 1. b-d”).

Bending Rigidity

One of the effective parameters in evaluating fiber network responses and their mechanical behavior is dimensionless bending rigidity. This parameter is defined based on the physical and mechanical properties of the fiber segment and is considered as an effective factor in tensile, shear, and bending deformations. In order to numerical model the fiber network composed of elastic athermal fiber, the beam element is used. Assuming the physical properties of the beam element as follows, the dimensionless bending rigidity relationship can be formulated [37].

- A: cross-sectional area of the beam element
- I: second moment of inertia
- E: Young’s modulus
- μ: stretching modulus - $\mu = EA$
- κ: bending rigidity - $\kappa = EI$
- $\bar{\kappa}$: dimensionless bending rigidity - $\bar{\kappa} = \kappa/\mu l^2$

The $\bar{\kappa}$ parameter represents the flexibility of the fibers. Because of its significant effect on the properties of the fiber network, we have considered it as one variable of this research.

Nonlinear Fiber Behavior

The fibers are modeled as Timoshenko beams taking into account their stretching, shear, and bending deformations. For networks composed of linear elastic fibers, the beam segments are assumed to have cross-sectional area A, second moment of inertia I, Young’s modulus E, stretching modulus $\mu = EA$, and bending rigidity $\kappa = EI$, and thus, dimensionless bending rigidity $\bar{\kappa} = \kappa/\mu l^2$ [5]. The dimensionless bending rigidity quantifies the flexibility of the fibers and is varied from 0. 001 to 0.1 in the present study reported in the literature [1], [5]. To evaluate the nonlinear behavior of fibers in a 3D network, the exponential function to represent the stress-strain response of nonlinear fibers has been used based on (H. Marbini & M. Rohanifar, 2020) [5]. According to the proposed model, the fibers have an initial linear elastic response followed by an exponential hardening reaction [5], [36]. The proposed relationship between the stress-strain of the fibers is formulated according to Equation (1).

$$\sigma_f = \begin{cases} E\varepsilon & \varepsilon \leq \varepsilon_y \\ EB\left(e^{\frac{\varepsilon-\varepsilon_y}{B}} - 1\right) + E\varepsilon_y & \varepsilon > \varepsilon_y \end{cases} \tag{1}$$

Where ε_y represents the strain at which the linear behavior of the material switches to exponential form, B is a parameter for controlling the nonlinearity of the material behavior, which varies between 1 and 0.1.

The major purpose of this study is to evaluate the performance and mechanical properties of the fiber networks under shear loading. In this regard, as shown in "Fig. 2", to apply the shear strain γ to random fiber network, all fibers intersecting the vertical boundary are only fixed in the horizontal and vertical directions, and those attaching to the opposite side boundary are constrained to translate vertically down. The finite shear strain γ is applied incrementally from 0 to 100%. Once the finite element simulation results are obtained, we calculate the shear stress by dividing the summation of forces in the fibers intersecting the upper lattice boundary by $\boldsymbol{W}$. The differential shear modulus of the networks, also referred to as stiffness herein, at each increment is defined as the slope of the stress-strain response, i.e. $K = \partial \sigma / \partial \gamma$, where γ and σ are the applied shear strain and calculated shear stress, respectively. In the following, the stress and stiffness are given in units of μ / l.

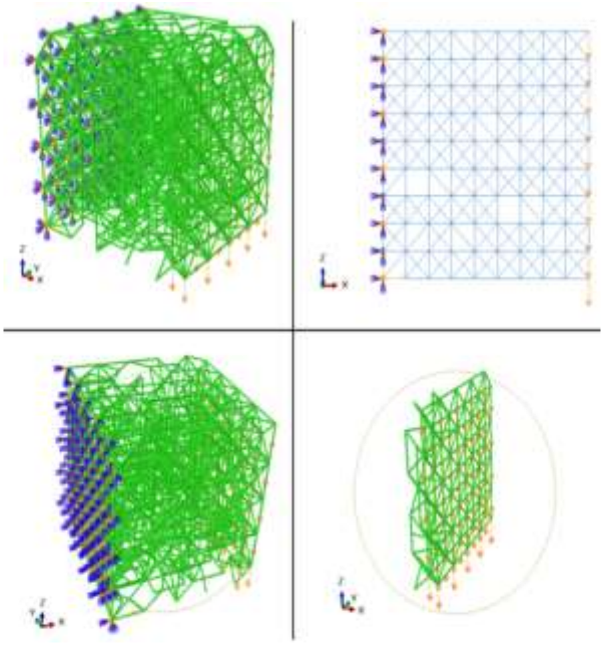

Fig. 2 Initial and boundary conditions of the research models.

2.2. Research Scenarios

In order to evaluate the mechanical behavior and responses of the fiber network accurately, after extracting the main effective variables and parameters, numerical modeling scenarios are defined.

According to the research variables that were introduced in the previous section, the modeling scenarios have been as follows.

- Network connectivity variable: we consider three different values Z=2.4, 3.4, and 4 [20], [35-37] for the average fiber network connectivity.
- Bending rigidity variable: In order to evaluate the performance of the network and its flexibility, the properties of the beam element that represents the fiber segment are defined in such a way that three different values for the dimensionless bending rigidity ($\bar{\kappa}$) are studied (0.1, 0.01 and 0.001).
- Nonlinear fiber variable: Based on the explanations provided in the previous section, it is clear that parameter B in equation 1 is considered to control the nonlinear performance of the material, therefore we considered three different values B=1, 0.2, and 0.1 for slightly nonlinear, nonlinear and highly nonlinear for this parameter as mentioned in literature [5]. Figure 3 shows the nonlinear behavior of the fiber material (normalized stress-strain response curve) based on different values of parameter B [5].

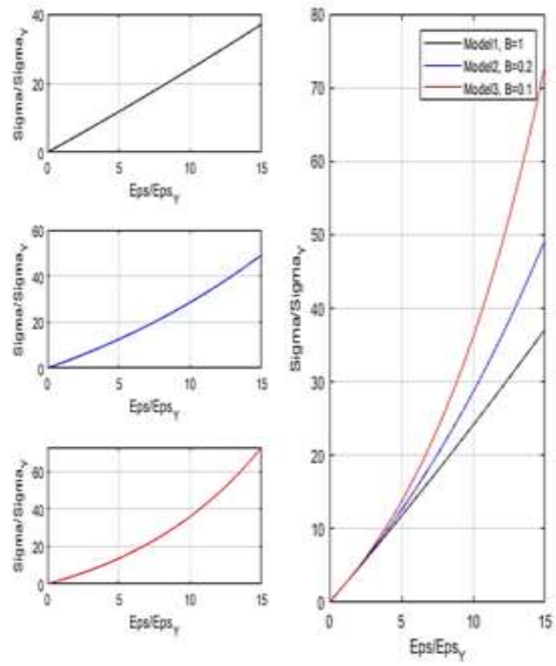

Fig. 3 The normalized stress-strain response of fibers Based on changes in B.

According to the proposed variables and to evaluate the impact of each variable, the numerical modeling scenario is defined taking into account all possible cases and 27 models are planned to model comprehensive parametric studies ("Table 1"). In this research, parametric studies and numerical modeling based on defined scenarios have been performed in Abacus finite element software. In this regard, Python programming language has been used to define the various structures of the fiber network as well as the nonlinear properties of the fiber material.

Table 1 Research modeling scenarios

scenarios NO.	Model Name	Network connectivity (Z)			Non-Linear Material Properties (B)			Dimensionless bending rigidity ($\bar{k}$)		
		Z =2.4	Z =3.4	Z =4	B=1	B=0.2	B=0.1	$\bar{k}=0.1$	$\bar{k}=0.01$	$\bar{k}=0.001$
1	M1	✓			✓			✓		
2	M2	✓			✓				✓	
3	M3	✓			✓					✓
4	M4	✓				✓		✓		
5	M5	✓				✓			✓	
6	M6	✓				✓				✓
7	M7	✓					✓	✓		
8	M8	✓					✓		✓	
9	M9	✓					✓			✓
10	M10		✓		✓			✓		
11	M11		✓		✓				✓	
12	M12		✓		✓					✓
13	M13		✓			✓		✓		
14	M14		✓			✓			✓	
15	M15		✓			✓				✓
16	M6		✓				✓	✓		
17	M17		✓				✓		✓	
18	M18		✓				✓			✓
19	M19			✓	✓			✓		
20	M20			✓	✓				✓	
21	M21			✓	✓					✓
22	M22			✓		✓		✓		
23	M23			✓		✓			✓	
24	M24			✓		✓				✓
25	M25			✓			✓	✓		
26	M26			✓			✓		✓	
27	M27			✓			✓			✓

3 RESULTS AND DISCUSSION

In this section, the results of numerical modeling and parametric studies performed according to research scenarios (“Table 1”) are presented. The outputs of numerical modeling include a wide range of results, which are categorized as follows because of the accurate evaluation of mechanical properties and 3D random network fiber responses.

- Assessment of von Mises stresses
- Assessment of the network shear modulus
- Investigation of bending and stretching energy to total energy
- Differential shear modulus vs. shear stress
- Effect of network connectivity on the mechanical response of 3D random fiber networks

3.1. Assessment of Von Mises Stresses

One of the useful parameters for evaluating the performance of 3D random networks under shear is von Mises stress, the results are presented in “Figs. 4 to 6.”

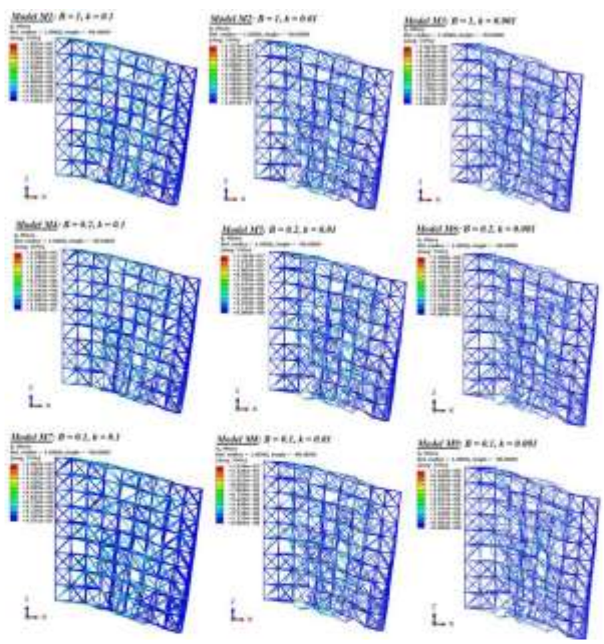

Fig. 4 Von Mises stress results, 3D random fiber network with Z=2.4 (M1 to M9 models).

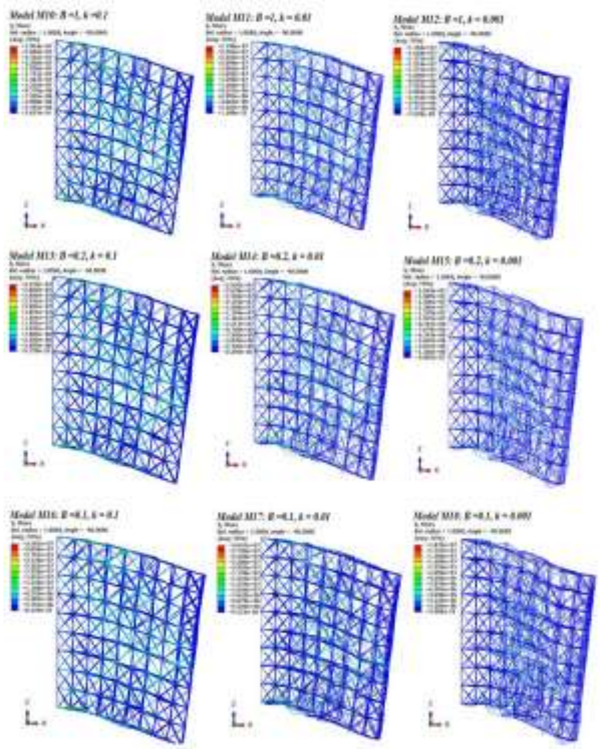

Fig. 5 Von Mises stress results, 3D random fiber network with Z=3.4 (M10 to M18 models).

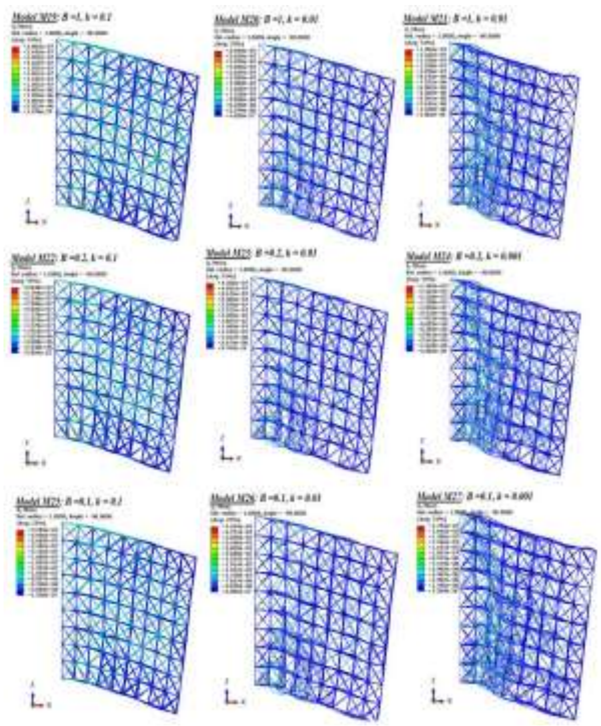

Fig. 6 Von Mises stress results, 3D random fiber network with Z=4 (M19 to M27 models).

Models M_1 to M_9 have a random 3D network structure with an average connectivity (Z=2.4). Figure 4 shows the results of von Mises stress for this network structure under shear. Models M_{10} to M_{18} have a random network with an average connectivity Z=3.4 and models M_{19} to M_{27} have a network with Z=4. Figures 5 and 6 show the Von Mies stress under shear for the network with Z=3.4 and Z=4, respectively. Figures 4 to 6 show the von Mises stresses created in the fiber network with Z = 2.4, 3.4, 4, respectively. According to the results, it is clear that by reducing the amount of the dimensionless bending rigidity (k) from 0.1 to 0.001, the stress in the network under shear has decreased. Because of the nature of the k parameter, it is clear that by reducing this parameter, the flexural stiffness of the network is decreased and the stress in the network is reduced. Also, by evaluating the change in parameter B, which represents the degree of nonlinearity of the fiber material, it is determined that with decreasing B from 1 to 0.2, the values of stress are increased. In addition, in the network with highly nonlinear fibers (B = 0.2), the number of plastic distortions and deformations of the fibers is clearly visible.

3.2 Assessment of the Network Shear Modulus

Based on the results of various numerical and experimental studies, it was found that random fiber networks under large shear, exhibit notable strain-stiffening. Therefore, the shear stiffness of these networks increases significantly with strain. This stiffening behavior mainly depends on the fiber bending rigidity and the connectivity of random networks [37-39].

To evaluate the effect of nonlinear fibers on the mechanical behavior of three-dimensional random fiber networks, the shear stiffness of networks composed of nonlinear fibers based on normalized shear strain for different values of the dimensionless bending rigidity is shown in “Fig. 7”. In these diagrams, the shear stiffness K is normalized by the stiffness of a similar network composed of linear elastic fibers. In addition, the applied shear strain normalized by γ_d. γ_d is the far-field shear strain at which the strain in at least one of the fiber sections reaches the yielding strain ε_y [5].

Figure 7 plots the normalized shear stiffness of networks composed of nonlinear fibers with different values of the dimensionless bending rigidity $\overline{(\kappa)}$ as a function of normalized shear strain. In these plots, the shear stiffness K and shear strain γ is normalized with the stiffness of similar networks composed of linear elastic fibers and by γ_d, which is the far-field shear strain at which the strain in at least one of the fiber segments reaches the yielding strain ε_y, equation 1, respectively [5]. As shown in “Fig. 7” with various $\bar{\kappa}$, they display nonlinear mechanical response. Moreover, the mechanical properties of networks with increased nonlinear fiber variable become different from networks composed of only linear elastic fibers, the random 3D fiber network has average network connectivity equal to 2.4 (Z=2.4). According to the results, the shear stiffness changes proportional to the nonlinear performance of the material.

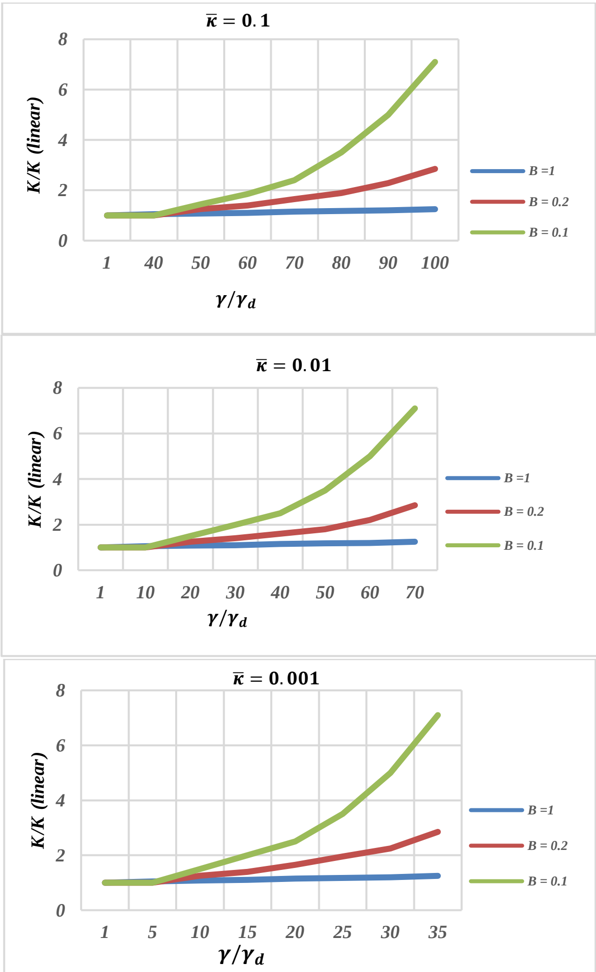

Fig. 7 The effect of nonlinear elastic properties of individual fibers on the mechanical response of fibrous networks, shear stiffness of networks composed of nonlinear fibers based on normalized shear strain for different values of the dimensionless bending rigidity ($\bar{\kappa}$).

In addition, the changes in the first section appear linearly and then exponentially. In addition, the changes in the first section appear linearly and then exponentially. According to the results, with decreasing bending rigidity from 1 to 0.1, the linear region changes from 40% γ to 7% γ, respectively.

3.3. Investigation of Bending and Stretching Energy to Total Energy

Based on the results obtained from modeling in this study, it is clear that with increasing applied shear strain and also with increasing bending rigidity of fibers, the nonlinear behavior of the fiber material becomes more apparent. In this regard, the evaluation of changing in the bending energy (H_b) and stretching energy (H_s) relative to the total applied energy to the random 3D fiber network, can provide a correct understanding of the mechanical behavior of the 3D random networks. It should be noted that the total energy is equal to the sum of bending energy and stretching energy in the network.

$$H_t = H_b + H_s \tag{2}$$

The stiffening corresponds to a transition from bending- to stretching-dominated behaviour [22]. Figure 8 shows the relative contributions of stretching energy (H_s) and bending energy (H_b) versus the applied shear strain for M_7 to M_9 Models. Models M_7 to M_9 have a highly nonlinear fibre behavior (B = 0.1).

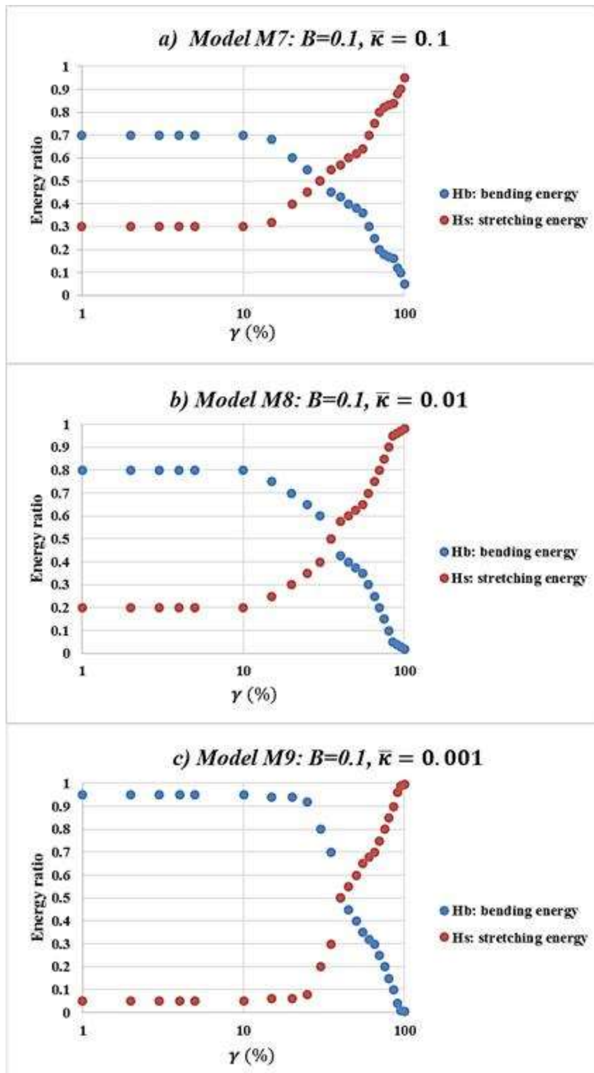

Fig. 8 Relative contributions of stretching energy (Hs) and bending energy (Hb) versus the applied shear strain for M7 to M9 Models.

According to "Fig. 8", it is clear that the bending rigidity ($\bar{\kappa}$) has been an effective component in changing stretching energy H_s and bending energy H_b. By decreasing this parameter from 0.1 to 0.001, the critical shear strain γ_c changes from 30% γ to 41% γ, respectively. These results indicate that with increased network connectivity and bending stiffness more energy is stored in stretching modes. Therefore, the random network will move from the bending-dominated regime towards an affine, stretch-dominated

3.4. Differential Shear Modulus vs. Shear Stress

Stress-stiffening has been one of the important parameters in studying the mechanical properties of random 3D fiber networks. In this section, variation of the differential shear modulus of nonlinear fiber networks versus the shear stress is presented for different bending rigidity regimes ("Fig. 9").

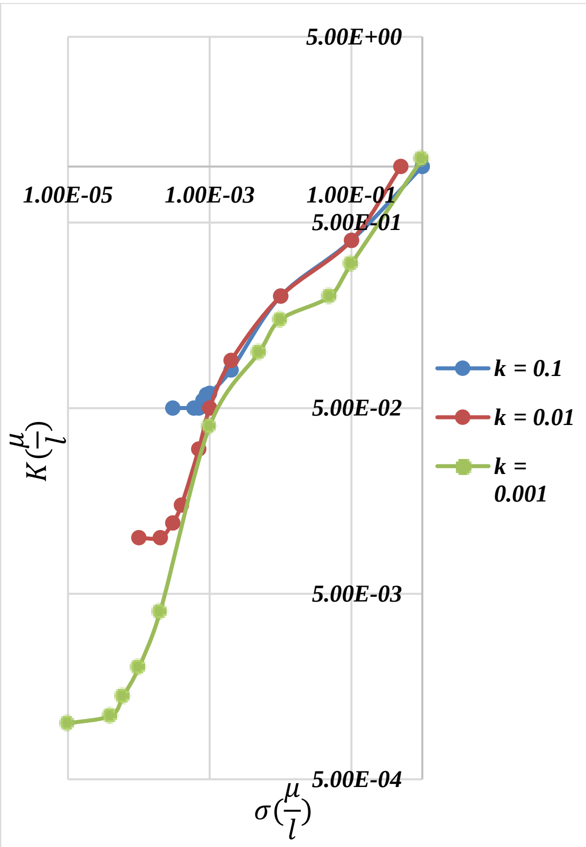

Fig. 9 The variation of the differential shear modulus of nonlinear fiber networks versus the shear stress for different values of the dimensionless bending rigidity ($\bar{\kappa}$).

In general, the relationship between K and stress (σ) is as follows.

$$K \propto \sigma^{\alpha} \tag{3}$$

In relation 3, α varies between 0.6 and 1.5 for biopolymer materials, which according to the results of this study, this value is also true for the random 3D fiber networks. The α parameter depends on the bending rigidity of fibers and varies from about 1.1 (K = 0.001) to 0.7 (K = 0.1). These findings agree with previous numerical, experimental, and theoretical reports that show that the exponent increases from 0.7 to 1.1 with a decline K for networks [1], [5], [22-23].

3.5. Effect of Network Connectivity on Mechanical Response of 3D Random Fiber Networks

To evaluate the effect of the network connectivity parameter (Z), The shear modulus K is plotted as a function of the applied shear strain for networks composed of nonlinear fiber and based on variation of the network connectivity (Z).

Figure 10 plots the shear modulus as a function of the applied shear strain for networks with various connectivity. An increase in network connectivity enhances the effect of the material nonlinearity of fibers on the network stiffness. Furthermore, the critical strain shifts towards smaller strains with increasing Z, which means that fibers reach their yielding strain at a smaller applied shear strain. The results also show good compatibility between the nonlinear material model and the shear model variation.

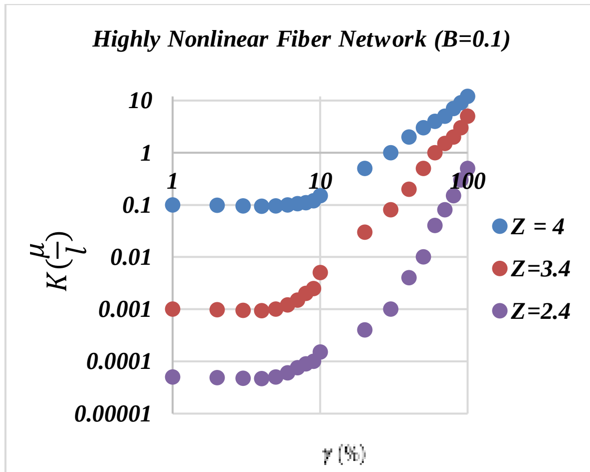

Fig. 10 The shear modulus K vs the applied shear strain for different network connectivity.

4 CONCLUSIONS

In this research, comprehensive parametric studies have been defined and implemented in order to evaluate the mechanical properties and investigate the responses of 3D random fibrous networks under shear loading.

In the first step, the effective parameters in 3D random networks were identified and based on the research variables, numerical modeling scenarios have been developed. At this stage, 4 parameters (1- network structure, 2- average network connectivity, 3- bending rigidity and 4- nonlinear fiber behavior) were selected as the main variables of the research. Based on these parameters, 27 different models have been modeled for conducting studies in Abacus software.

The results of the present study indicate the effect and sensitivity of each of the research variables on the mechanical behavior and responses of a 3D random fibrous network. The general results of this research can be classified as follows:

- Based on the results of the study, it is determined that the parameters of network structure, average network connectivity, bending rigidity, and nonlinear behavior of fiber material are effective factors in the mechanical behavior of 3D random fibrous networks.
- Based on von Mises stress evaluation in 3D fiber networks, it showed that with increasing network connectivity, the amount of stress created in the network under shear increases. This indicates an increase in the stiffness of the network, also a decrease in dimensionless bending rigidity reduces the stresses created in the 3D fiber network.
- The network connectivity parameter (Z) plays an important role in how the nonlinearity of single fibers affects the overall strength of 3D networks.
- The results of the present research show that in the 3D random network with increasing applied shear strain more than gamma, significant strain-stiffening is observed. This phenomenon is more pronounced in highly nonlinear fiber networks.
- Examining the relationship between shear modulus and shear stress, it is clear that the variation of the differential shear modulus of nonlinear fiber networks versus the shear stress in 3D random networks has been consistent with theoretical relationships and the results of numerical and experimental research.
- The results of applying the nonlinear fiber material model in 3D random networks show that the exponential function to represent the stress–strain response of nonlinear fibers is well adapted to evaluate the nonlinear behavior of 3D random fibrous networks. The results of the present research show that using the lattice network structure for modeling and evaluating the mechanical behavior of the 3D random fibrous network can be appropriate and efficient.

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