Optimal Trajectory Planning for Flexible Mobile Manipulators under Large Deformation Using Meta-heuristic Optimization Methods

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Abstract: In present paper, a point to point optimal path is planned for a mobile manipulator with flexible links and joints. For this purpose, a perfect dynamic modeling is performed for mobile manipulators considering large deformation in links, shear effects, elastic joints, effect of gravitation, and non-holonomic constraints. To study large deformation of links, non-linear relation of displacement-strain and Green's strain tensor are used. Optimal path is planned based on direct methods and applying meta-heuristic optimization methods. In order to get an optimal path profile, maximum load carried by manipulator and minimum transmission time are considered as the objective functions for optimization problem. To provide the parameters of optimization problem, parametric optimization problem is solved by using Harmony Search (HS) and Simulated Annealing (SA) efficient methods. In order to investigate the efficiency of the proposed method, simulation studies are performed considering two-link flexible manipulator with wheeled base. The results indicate that the method proposed has a suitable power and performance when facing dynamics non-linear system. Moreover, the results of path planning for manipulators by small and large deformation models are also compared. The effect of flexibility in joints is studied when planning a point to point path.

Keywords: Elastic joints, Mobile flexible manipulator, Meta-heuristic optimization

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1 INTRODUCTION

Modeling and planning the path of mobile flexible manipulators has been recently proposed as one of challenging subjects in the field of robotics. Because of the complexities in solving the problem of path planning in point to point case, different methods have been proposed for solving this problem. All methods applied are a subset of two main methods, namely direct and indirect methods. In general, indirect methods are based on application of optimal control theory and Pontryagin's minimum principle. Korayem et al. have studied planning an optimal path for mobile manipulators with elastic joints. Optimal path has been designated based on Pontryagin's minimum principle. For dynamic modeling of system, the links are considered rigid and the joints are elastic. Dynamic equations of the system are used in optimal control method without any simplification and planning variables contain system state variables [1].

Korayem et al. have used indirect method to plan optimal path for two-link manipulator with mobile base. The researchers try to present a perfect dynamic model for the system considering flexibility of the links and non-holonomic constraints of the base and to examine the efficiency of the optimal control method considering perfect dynamics of system. Investigations indicate that implementing optimal control method by considering non-linear perfect dynamics of system is not simple due to computation of Hamiltonian gradient, presence of two-point boundary value problem and the use of numerical multiple shooting method [2]. In another paper, Korayem et al. have studied optimal path planning for mobile manipulator with elastic joints. They have tried to investigate optimal control method considering a perfect dynamics of the system. By considering the effect of flexibility in joints and non-holonomic constraints of the base, complex equations have been obtained through optimal control method [3]. Korayem and Rahimi have solved the problem of planning the path of two-link manipulator with mobile base using optimal control method. Dynamic equations of system have been obtained based on small deformation model and assumed mode method. By applying different weight functions, different paths are obtained based on minimum torque [4].

Rahimi et al. have studied planning an optimal path for manipulators with flexible links and joints. Optimal values of the problem variables including flexibility, rigidity, and derivatives are achieved through solving boundary value problem by the help of numerical methods [5]. Planning an optimal path for two-link rigid manipulator with fixed base is performed based on optimal control method by Korayem and Nikoobin. The researchers have considered maximum load carrying capacity of manipulator as the goal of path planning problem. By using Pontryagin optimality conditions, the problem of maximum load carrying capacity determination is converted to standard form of two-point boundary value problem and used to solve boundary value problem by BVP4C command and MATLAB software [6]. Korayem et al. have investigated planning optimal path of flexible manipulator to calculate maximum load carrying capacity. To get the answer of the problem, sinusoidal shape functions are used. The researchers have achieved different paths by using different weight functions and investigated the effect of weight functions through optimal control method [7].

Implementing optimal control method to plan mobile manipulator path is studied considering flexibility of links and joints [8]. It is indicated that the problem of the final optimization problem contains many planning variables due to dynamic variables of flexible system. Moreover, investigation of complete dynamics of the system changes optimization problem to a complicated concern.

In all activities mentioned in which indirect methods and optimal control theory are used for planning optimal path, there are following limitations. To solve the problem of path planning by optimal control, non-linear numerical techniques such as non-linear multiple shooting method are needed. These techniques need to initial guess and are sensitive to it. In these methods, an analytical form of Hamiltonian gradient is needed and optimal value of state variables are obtained through the gradient. State variables are used to solve the problem and in general, (particularly by considering the flexibility in the problem) the estimation will be very difficult. In addition, determination of the exact value of weight functions for different objective functions creates some limitations when implementing these methods. For solving these problems, direct methods are proposed for path planning problem. Direct methods are based on discretization of dynamic variables of system (state and control variables) that, for providing problem parameters, path planning problem is converted to a parametric optimization problem.

In these methods, the profile of joints motion is presented directly by polynomial, Spline and B-spline functions. By the profile of joints' motion, the problem of path planning is converted to optimization problem to determine fixed coefficients of path profile function through an optimal manner. Therefore, in these methods, there is no need to Hamiltonian gradient and the data of objective function will be directly applied. Planning variables in these methods are the coefficients of path profile and there is no need to use state variables of system. To solve optimization problem, meta-heuristic methods by high convergence speed could be used that are not sensitive to initial guess. To implement these methods, there is no need to linearizing and simplify dynamic equations of system and complete dynamics of the system can be considered. The number of optimization problem variables in direct methods is less than that of optimal control method.

Gasparetto et al. have reported fully about all activities performed related to robots path planning. They have presented them in two parts: direct and indirect solution. Objective functions in direct method are divided into three distinct groups including the minimization of transmission time from the beginning to the end, the minimization of the energy consumed in motors and the minimization of the jerk in joints. For each part, all activities performed are presented [9]. In [10], Sequential Quadratic Programming (SQP) method is used for solving the problem of path planning. It is indicated that SQP may obtain local optimal conditions and for the convergence of this method to global optimized solving, it is necessary to select a suitable initial guess. Haddad et al. have planned a path for wheeled mobile robot considering minimum transmission time as the objective function. The optimization problem resulted is solved through B-Spline interpolation functions and random optimization method [11].

Haddad et al. have investigated planning a point to point path for manipulator with mobile base. For planning the path, they have used direct method and random optimization. Planning the path is performed based on objective function of minimum transmission time and kinematic constraints of velocity, acceleration and dynamic constraint of motors torque. In addition, planning the path based on stability constraint of manipulator is done by Zero Moment Point (ZMP) method [12]. Saravanan and Remabalan also have dealt with planning a point to point path using direct method and evolutionary technique. For approximating path profile, they have used cubic Spline interpolation functions, genetic algorithm method and Differential Evolution (DE) to solve non-linear parametric optimization problem. The results of simulation have indicated that DE yields better results than genetic algorithm method and it has more convergence [13]. Saravanan et al. have investigated planning the path for Puma 560 robot using evolutionary optimization as a direct method. They have used B-Spline smooth functions and converted path planning problem to a parametric optimization problem. For solving optimization problem, they have used genetic algorithm and DE methods.

The results presented through both methods have demonstrated that DE yields better and more rapid results than genetic algorithm. Also, computation time in DE is less [14]. Tangpattanakul and Artrit have considered path planning for manipulator based on objective function of minimum transmission time in point to point case. The problem of optimization is solved by considering kinematic constraints of velocity, acceleration and jerk using HS method. Simulation is performed based on a manipulator with 6 degrees of freedom via two methods including HS and SQP. The results of simulation have indicated that HS method is converged to optimal response faster and sensitivity of this method to initial guess is less [15].

Devendra et al. have used direct method for solving the problem of planning point to point path for manipulators. They have solved optimization problem for finding optimal profile of joints motion by two methods, namely genetic algorithm and simulated annealing methods considering cubic polynomial for path profile. The results have indicated that SA method has more convergence speed than genetic algorithm method [16]. Tangpattanakul et al. have investigated planning a point to point path using HS optimization method. A comparison has been also made between HS and SQP methods. The results of simulation have indicated that HS method is a more suitable method for solving the problem of path planning for robot. Because, this method is not depended on initial guess and start point in iteration process and general optimal solution is more appropriate than SQP by using this method [17]. Chettibi and Lemoine have planned a point to point path using SQP. For solving the problem of optimization, objective function of minimum transmission time and electromechanical constraints are used.

For approximating path profile, they have used Spline functions and the results of path planning are presented in two theoretical and laboratory states [18]. Zanotto et al. have investigated the problem of planning point to point path considering multi-criteria objective function and laboratory validation. They have considered minimum transmission time and jerk value of joints as the objective function in optimization problem. Experimental results are also presented to compare with theoretical simulation [19]. In [20], hybrid optimization method is presented for planning optimal path based on kinematic constraints. Optimization problem is introduced by using complex optimization method resulted by the description of path and continued to considering kinematic constraints of manipulator. Ramabalan et al. have investigated planning a path for robot considering multi-criteria objective function and the process of evolutionary optimization. They have used genetic algorithm and multi-objective differential evolutionary algorithm (MODE) for planning the path. The results of simulation have indicated that MODE method is more suitable than genetic algorithm and has a faster convergence speed [21].

Different methods are proposed to solve the trajectory planning problem of a mobile manipulator in closed loop form. Kelly and Nagy have addressed a method to find optimal paths by formulating an optimal control problem and solving the non-linear programming problem via the Lagrange method [22]. A linear interpolation-based methodology, which is the basis for many numerical methods, is presented in [23] to design trajectory tracking control algorithms of mobile robot. Also some simulation and experimental results are presented and discussed and it is mentioned that the proposed methodology is simple and can be applied to the design of a large class of control systems.

Furuno et al. [24] have solved the trajectory planning problem of a mobile manipulator. A zero moment point criterion is used as a stability index. The problem is formulated as an optimal control problem and a hierarchical gradient method is used to solve it. The trajectory-tracking problem to reference mobile robot is addressed based on dynamic extension approach in [25]. The exact feedback linearization on the kinematic error model of mobile robot is realized and the non-linear system is transferred to two reduced-order linear time invariant systems that can be controlled easily. Xu et al. [26] have investigated motion planning for a mobile manipulator with redundant degrees of freedom to track a desired trajectory. Alibakhshi and Daniali have addressed an efficient method for singularity-free path planning and obstacle avoidance of parallel manipulator based on neural networks. A modified interpolating polynomial is used to plan a trajectory for a spherical parallel manipulator and an artificial neural network is implemented to solve forward kinematics of the manipulator to estimate the distance between gripper and singularity or obstacle in Euler coordinate [27].

This paper includes the following aspects of innovations. An optimal path is planned for the manipulator by considering the most complete non-linear dynamics, large deformation model, Timoshenko beam model, and joint flexibility. While, in previous papers that have used direct methods, the effects of the flexibility are not considered in manipulators and joints at all. In addition, by the use of optimal control method, the simplification of non-linear equations is avoidable due to the excessive complexity of the problem solution. In most papers using direct methods, the minimum transfer time, minimum energy consumption and minimum jerk are considered as the objective functions. However, in this paper, the minimum transfer time and the maximum load carrying capacity of the manipulators are studied by the most complete constraints existing in the problems such as kinematic and dynamic constraints, end accuracy constraint and stability constraint.

In this paper, harmony search method is used to determine the load carrying capacity of the manipulators that has no restrictions to non-linear dynamics of the system. While the methods like optimal control have some difficulties when dealing with such issues. So, here, the most complete process is used to design an optimal path using HS method. The proposed optimization method has the following advantages: (1) a global optimal solution is possible, (2) it is easy to program and to implement efficiently, (3) it ensures that the resulting optimized trajectory is smoother, faster, and nonsingular, (4) it can also be extended to get optimized trajectories for other types of robots, (5) it considers both kinematic and dynamic aspects of the robot, (6) it considers the payload constraint, and (7) it is computationally superior and faster.

The paper is organized as follows: In section 2, dynamic modeling of flexible manipulator with N-flexible links and joints under large deformation is studied. In section 3, the problem of planning point to point path by direct method is presented. In this section, the problem of planning the path is presented to determine maximum load carrying capacity and minimum transmission time in the form of parametric optimization problems and related solutions. The results of simulation for two-link flexible manipulator are presented in section 4 and then conclusions are described at the end.

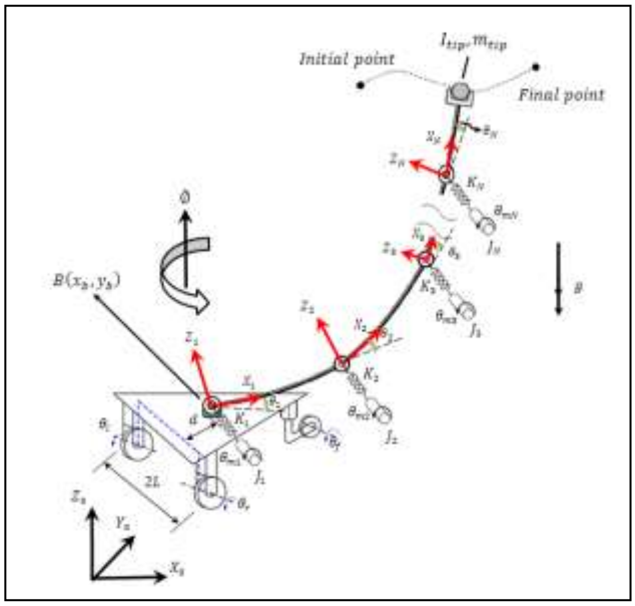

Fig. 1 N-flexible links and N-flexible joints manipulator with mobile base

2 DYNAMIC MODELING OF MOBILE FLEXIBLE MANIPULATOR WITH ELASTIC JOINTS UNDER LARGE DEFORMATION

Consider N-links mobile manipulator with n-degrees of freedom $\left(n = n_{base} + n_{arm} \quad , \quad n > N\right)$ that must move from primary position X^{ini} to final position X^{fin} in end effector space (Fig. 1). For investigating base kinematics, position vector of the center of gravity of the plate $B = \left(x_b, y_b\right)$, rotation angle of base relative to symmetry axis (ϕ) and rotation angles of deriving wheels of mobile base $\left(\theta_r, \theta_l\right)$ are considered as the physical coordinates. Flexible links are considered as Timoshenko beam in which the effects of shear and rotational inertia are investigated. The motions of links are described by rotation angles of the link θ_i , angle of motors θ_{mi} , flexible displacement $w_i(x_i, t)$, and rotation due to such flexible displacement as β_i . Kinetic energy of system is consisted of kinetic energy of the base, kinetic energy of the links, kinetic energy of the motors, and kinetic energy of the tip mass. Therefore, for total kinetic energy of system we have:

$$KE = \frac{1}{2}\sum_{i=1}^{N}\left\{\int \rho_i \left(\dot{d}_i^T(x_i,t)\cdot \dot{d}_i(x_i,t)\right)dv_i\right\} + \frac{1}{2}m_{tip}\left(\dot{d}_{tip}^T\cdot \dot{d}_{tip}\right) + \frac{1}{2}I_{tip}\left(\sum_{i=1}^{N}\dot{\theta}_i\right)^2 + \frac{1}{2}\sum_{i=1}^{N}J_i\dot{\theta}_{mi}^2 + \frac{1}{2}m_{base}\left(\dot{x}_b^2+\dot{y}_b^2\right) + \frac{1}{2}I_{base}\dot{\phi}^2 + \frac{1}{2}J_r\dot{\theta}_r^2 + \frac{1}{2}J_l\dot{\theta}_l^2 \quad (1)$$

$d_i(x_i,t)$, is the position of a small element on i[th] flexible link and d_{tip}, is the position vector of tip mass relative to absolute coordinate $\{X_0 Y_0 Z_0\}$ defined as follows:

$$\begin{aligned} &\vec{d}_i(x_i,t) = x_b\vec{I} + y_b\vec{J} + \sum_{j=0}^{i-1}\vec{P}_j(l_j,t) + \vec{P}_i(x_i,t) \\ &\vec{P}_i(x_i,t) = \begin{pmatrix} x_i\cos(\theta_{i-1}+\theta_i)\cos\phi - \\ w_i(x_i,t)\sin(\theta_{i-1}+\theta_i)\cos\phi \end{pmatrix}\vec{I} + \\ &\left(x_i\cos(\theta_{i-1}+\theta_i)\sin\phi - w_i(x_i,t)\sin(\theta_{i-1}+\theta_i)\sin\phi\right)\vec{J} + \\ &\left(x_i\sin(\theta_{i-1}+\theta_i) + w_i(x_i,t)\cos(\theta_{i-1}+\theta_i)\right)\vec{K} \\ &i=1,2,\ldots,N \quad , \quad \vec{P}_0 = 0 \ \ and \ \ \theta_0 = 0 \end{aligned} \quad (2)$$

where I, J, K are the unit vectors along the X_0, Y_0 and Z_0 axis, respectively. Considering Timoshenko's beam model, displacement field in large deformation model is as follows [28].

$$u_x = -Z\sin\beta(x) \ , \ u_y = 0 \ , \ u_z = w(x) + Z\cos\beta(x) \quad (3)$$

Where, Z is the distance to neutral axis of beam. Non-zero elements of Green's strain tensor in large deformation model may be written as follows [28].

$$\begin{aligned} E_{xx} &= \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - Z\left(\frac{\partial \beta}{\partial x}\right)\left[\cos\beta + \left(\frac{\partial w}{\partial x}\right)\sin\beta\right] = E_0 + ZK_b \\ E_{xz} &= \frac{1}{2}\left[-\sin\beta + \left(\frac{\partial w}{\partial x}\right)\cos\beta\right] = \Gamma \end{aligned} \quad (4)$$

Potential energy of the system includes strain energy, gravitational potential energy and potential energy due to flexible joints [28].

$$\begin{aligned} PE &= \frac{1}{2}\sum_{i=1}^{N}\int_0^{l_i}\left(E_iA_iE_{i,0}^2 + E_iI_iK_{i,b}^2 + kG_iA_i\Gamma_i^2\right)dx_i + \\ &\sum_{i=1}^{N}\rho_iA_ig\left(\sum_{j=0}^{i-1}M_j(l_j,t) + \int_0^{l_i}M_i(x_i,t)dx_i\right) + \\ &m_{tip}g\sum_{j=1}^{N}M_j(l_j,t) + \frac{1}{2}\sum_{i=1}^{N}K_i(\theta_i-\theta_{mi})^2 \\ M_i(x_i,t) &= x_i\sin(\theta_{i-1}+\theta_i) + w_i(x_i,t)\cos(\theta_{i-1}+\theta_i) \quad , \\ &M_0 = 0 \ \ and \ \ \theta_0 = 0 \end{aligned} \quad (5)$$

For having non-slip motion in wheels and movement along symmetry axis, mobile base is regarded as non- holonom system. Non-holonom constraints of the base are expressed as follows [1].

$$\underbrace{\begin{bmatrix} -\sin\phi & \cos\phi & -d & 0 & 0 \\ \cos\phi & \sin\phi & -L & -r & 0 \\ \cos\phi & \sin\phi & L & 0 & -r \end{bmatrix}}_{A}\begin{bmatrix}\dot{x}_b \\ \dot{y}_b \\ \dot{\phi} \\ \dot{\theta}_l \\ \dot{\theta}_r\end{bmatrix} = [0] \quad (6)$$

Using extended Hamilton method and considering vectors (7), equations of motion for mobile manipulator are presented as follows.

$$\begin{aligned} q_{mb} &= \left[x_b \quad y_b \quad \phi \quad \theta_r \quad \theta_l \quad q_1 \quad q_2 \quad \ldots \quad q_i \quad \ldots \quad q_N\right]^T \\ q_i &= \left[\theta_i \quad w_{i,1} \quad \ldots \quad w_{i,n_i+1} \quad \beta_{i,1} \quad \ldots \quad \beta_{i,n_i+1}\right]^T \qquad i=1,2,\ldots,N \\ q_{fj} &= \left[\theta_{m1},\theta_{m2},\ldots,\theta_{mN}\right]^T \\ q_r &= \left[\theta_1,\theta_2,\ldots,\theta_N\right]^T \end{aligned} \quad (7)$$

$$\begin{bmatrix} M_{mb} & 0 \\ 0 & J_{fj}\end{bmatrix}\begin{bmatrix}\ddot{q}_{mb} \\ \ddot{q}_{fj}\end{bmatrix} + \begin{bmatrix} K_{mb} & K_m \\ 0 & -K_{fj}\end{bmatrix}\begin{bmatrix} q_{mb} \\ q_r - q_{fj}\end{bmatrix} + \begin{bmatrix} C_{mb}(q_{mb},\dot{q}_{mb}) \\ 0\end{bmatrix} = \begin{bmatrix}\tau_{mb} + A^T\lambda \\ \tau\end{bmatrix} \quad (8)$$

Where

$$\begin{aligned} &for \ \ i = 1:1:2\sum_{r=1}^{N}n_r + 3N + 5 \\ &for \ \ \ j = 1:1:N \\ &if \ \ q_{mb}(i) = q_j(1) \qquad\qquad (K_m)_{ij} = K_j \\ &else \qquad\qquad\qquad\qquad (K_m)_{ij} = 0 \end{aligned} \quad (9)$$

Where, $[M]$ is inertia matrix and $C(q,\dot{q})$ is the vector indicating centrifugal effects and Coriolis. τ_{mb} is the vector of external forces imposed on mobile base and τ is torque vector imposed on motors. $J_{fj} = diag\ [J_1, J_2, \cdots, J_N]$ is diagonal matrix modeling the inertia of motors and $K_{fj} = diag\ [K_1, K_2, \cdots, K_N]$ is diagonal matrix indicating torsional stiffness in elastic joints. λ is Lagrange coefficient and k is shear correction factor.

3 EXPRESSING THE PROBLEM OF PLANNING POINT TO POINT PATH BY DIRECT METHOD

The problem of solving optimal path by direct method includes presenting a model for the path of joints, in which all dynamic and kinematic constraints are dealt with by the path proposed and objective function is also minimized (maximized) for it. In general, polynomial, Spline and B-Spline functions are suitable functions for path profile. Cubic Spline profile is used as the profile of elastic joints path. By changing the variable $(0 \le \zeta \le 1)$ $\xi = \frac{t}{T}$, general form of cubic Spline profile with two free points ζ_b, ζ_a will be as follows [15, 17].

$$\tilde{q}_j(\zeta)=\begin{cases} a_0+a_1\zeta+a_2\zeta^2+a_3\zeta^3 & 0\le\zeta\le\zeta_a \\ a_4+a_5(\zeta-\zeta_a)+a_6(\zeta-\zeta_a)^2+a_7(\zeta-\zeta_a) & \zeta_a\le\zeta\le\zeta_b \\ a_8+a_9(\zeta-\zeta_b)+a_{10}(\zeta-\zeta_b)^2+a_{11}(\zeta-\zeta_b)^3 & \zeta_b\le\zeta\le 1 \end{cases}$$

$$a_0=q_j^{ini}\ ,\quad a_1=a_2=0$$

$$a_{11}=\frac{q_j^{fin}-q_j^{ini}}{D}\ ,$$

$$D=\left\{\begin{array}{l}\left(\frac{(1-\zeta_b)(\zeta_b-\zeta_a)+(1-\zeta_b)^2}{\zeta_b}\right)\times \\ \left(\zeta_a{}^2+3\zeta_a(\zeta_b-\zeta_a)+2(\zeta_b-\zeta_a)^2\right)- \\ (1-\zeta_b)(\zeta_b-\zeta_a)^2+(1-\zeta_b)^3 \end{array}\right\} \tag{10}$$

$$a_3=\frac{a_{11}\left[(1-\zeta_b)(\zeta_b-\zeta_a)+(1-\zeta_b)^2\right]}{\zeta_a{}^2+\zeta_a(\zeta_b-\zeta_a)}$$

$$a_4=a_0+a_3\zeta_a{}^2,\qquad a_5=3a_3\zeta_a{}^2\ ,\qquad a_6=3a_3\zeta_a\ ,$$

$$a_7=\frac{a_{10}-a_6}{3(\zeta_b-\zeta_a)}\quad ,\quad a_8=-a_{11}(1-\zeta_b)^3+q_j^{fin}$$

$$a_9=3a_{11}(1-\zeta_b)^2,\qquad a_{10}=-3a_{11}(1-\zeta_b)$$

ζ_b,ζ_a are free points between beginning and end points of motion (Fig. 2).

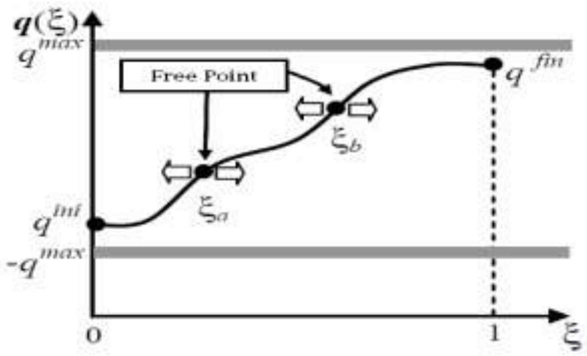

Fig. 2 Proposed path profile with two free points

The purpose of path planning problem is to determine these points in optimal manner by considering a special objective function and kinematic and dynamic constraints. Therefore, by proposing the profile of joints path, the problem of planning the path is converted to the problem of optimization by the purpose of determining free points of the profile proposed in an optimal manner. Then, the problem of optimization by the use of meta-heuristic optimization methods will be solved. Maximum load carrying capacity and minimum transmission time in mobile manipulator are considered as the objective functions of optimization problem and kinematic constraints of velocity and acceleration of joints, dynamic constraint of torque, constraint of end point accuracy and constraint of dynamic stability of motion as the constraints.

3.1 Planning the path by maximum load carrying capacity

The purpose of path planning in this case is to calculate a path for the end effector of mobile manipulator along which the robot would be able to carry maximum dynamic load and mentioned constraints are satisfied. In addition, optimal path must be planned such that it is not faced with singularity configuration of manipulator. So that, corresponding optimization problem for this state is presented as follows.

$$f_{obj}=\max\left(m_{tip},I_{tip}\right)$$

$$subject\ to\ \begin{cases} \left|q_{fj,i}(t)\right|\le q_{fj,i}^{\max} \\ \left|\dot{q}_{fj,i}(t)\right|\le M_{vi} \\ \left|\ddot{q}_{fj,i}(t)\right|\le M_{ai} \\ \left|\tau_i(t)\right|\le\tau_i^{\max} \\ \left|q_{r,i}^{fin}-q_{r,i}(T)\right|\le\varepsilon \\ \det(J(q))\ne 0 \end{cases}\qquad i=1,2,\ldots,N \tag{11}$$

3.2 Planning the path by minimum transmission time

In this case, minimum transmission time between beginning and end points of movement in end effector is considered as the objective function in optimization problem. For extracting optimization problem, it is necessary to present kinematic and dynamic constraints in the form of time constraints. Therefore, if $\tilde{q}_{fj}(t)$ is selected as a candidate for the path profile of elastic joints, each kinematic and dynamic constraint given in previous section is posed as bands from transmission time by changing $\xi=\frac{t}{T}$ variable. The constraints governing on velocity and acceleration of joints may be stated as follows applying derivation chain rule [15].

$$T\ge T_v\qquad T_v=\max_{i=1,2,\ldots,N}\left[\max_{\zeta\in[0,1]}\frac{\left|\tilde{q}'_{fj,i}(\zeta)\right|}{M_{vi}}\right]\quad ,\quad \tilde{q}'_{fj,i}(\zeta)=\frac{d\tilde{q}_{fj,i}(\zeta)}{d\zeta}$$

$$T\ge T_A\ \ T_A=\max_{i=1,2,\ldots,N}\left[\max_{\zeta\in[0,1]}\frac{\left|\tilde{q}''_{fj,i}(\zeta)\right|}{M_{ai}}\right]^{1/2}\quad ,\quad \tilde{q}''_{fj,i}(\zeta)=\frac{d^2\tilde{q}_{fj,i}(\zeta)}{d\zeta^2} \tag{12}$$

Dynamic constraints governing the torque of motors may be converted as constraints by two bands based on transmission time such that, for these constraints, we have $T\in[T_L,T_R]$. By investigating obtained time bands, we can finally obtain end band $\left[T_{lower},T_{upper}\right]$ for the time of path transfer. Optimal transmission time for each path profile $\tilde{q}_{fj}(\xi)$, will be obtained by minimizing the objective function based on transmission time in this interval. By changing the variable ξ, differential equation of motion for i^{th} elastic joint of N- links manipulator will be as follows [15].

$$\bar{\tau}_i(\zeta)=\frac{1}{T^2}.\bar{h}_i(\zeta)+\overline{Q}_{i(\zeta)}\qquad \zeta\in[0,1]\quad ,\quad i=1,2,\cdots,N$$

$$h_i(\zeta)=\sum_{j=1}^{N}J_{fj,ij}\tilde{q}''_{fj,j}(\zeta)\qquad \bar{h}_i(\zeta)=\frac{h_i(\zeta)}{\tau_i^{\max}} \tag{13}$$

$$\overline{Q}_i(\zeta)=\frac{Q_i(\tilde{q}(\zeta))}{\tau_i^{\max}}\qquad \bar{\tau}_i(\zeta)=\frac{\tau_i(\zeta)}{\tau_i^{\max}}$$

Therefore, dynamic constraints will be converted to following [15].

$$-1\le \frac{1}{T^2}.\bar{h}_i(\zeta)+\bar{Q}_{i(\zeta)}\le 1 \quad -b_i(\zeta)\le \frac{1}{T^2}.\bar{h}_i(\zeta)\le a_i(\zeta) \qquad (14)$$
$$a_i(\zeta)=1-\bar{Q}_{i(\zeta)} \quad , \quad b_i(\zeta)=1+\bar{Q}_{i(\zeta)}$$

So that, for each $\xi\in[0,1]$, time bands T related to torque constraint are presented in Table 1. In general, the problem of planning optimal path by objective function of minimum transmission time and constraints is presented as following in the form of optimization problem.

$$f_{obj}=\min(T)$$
$$subject\ \ to\ \begin{cases} T\ge T_v \qquad T_v=\max\limits_{i=1,2,\ldots,N}\left[\max\limits_{\zeta\in[0,1]}\dfrac{\left|\tilde{q}'_{fj,i}(\zeta)\right|}{M_{vi}}\right] \\ T\ge T_A \qquad T_A=\max\limits_{i=1,2,\ldots,N}\left[\max\limits_{\zeta\in[0,1]}\dfrac{\left|\tilde{q}''_{fj,i}(\zeta)\right|}{M_{ai}}\right]^{1/2} \\ T_L\le T\le T_R \qquad\qquad Table(1) \\ \left|\tilde{q}_{r,i}^{fin}-\tilde{q}_{r,i}(T)\right|\le\varepsilon \\ \det(J(q))\ne 0 \end{cases} \qquad (15)$$

3.3 Solving the problem of optimization by meta-heuristic optimization methods

Based on optimization problems obtained (Eqs. 11 and 15) and non-linear dynamics of mobile manipulator, the methods should be applied for solving these problems which have higher convergence speed and are appropriate to large optimization problems. Therefore, HS and SA meta-heuristic methods as possessing these capabilities are used. The flowchart of optimal path planning using HS and SA methods is presented in Fig. 3.

4 SIMULATIONS

Generalized coordinates of two-link flexible manipulator with elastic joints and mobile base are considered as the vector $q=[x_b \quad y_b \quad \phi \quad \theta_1 \quad \theta_2 \quad \theta_{m1} \quad \theta_{m2} \quad w_1 \quad \beta_1 \quad w_2 \quad \beta_2]$. w_1, w_2 are the variables of transverse vibrations of the first and second links, respectively. β_1, β_2 indicate the rotation due to transverse vibrations of first and second links, respectively. By using finite element method and dividing the links to various elements, flexibility variables of i[th] element of first arm and flexibility variables of j[th] element of second arm are presented as follows.

$$\begin{aligned} &w_{1i}=[N_1(x) \quad N_2(x)]\{\tilde{w}_{1i}(t)\},\ \beta_{1i}=[N_1(x) \quad N_2(x)]\{\tilde{\beta}_{1i}(t)\} \\ &w_{2j}=[N_1(x) \quad N_2(x)]\{\tilde{w}_{2j}(t)\},\ \beta_{2j}=[N_1(x) \quad N_2(x)]\{\tilde{\beta}_{2j}(t)\} \\ &i=1,2,\ldots,n_1,\ j=1,2,\ldots,n_2 \qquad [N_1(x) \quad N_2(x)]=\left[\left(1-\frac{x}{l}\right) \quad \frac{x}{l}\right] \end{aligned} \qquad (16)$$

where, n_1, n_2 are nods considered in element grid of the first and second arm, respectively. Therefore, closed form of dynamic equations for two-link flexible manipulator with mobile base and elastic joints is as follows.

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} & m_{15} & m_{16} & m_{17} & m_{18} & m_{19} & m_{110} & m_{111} \\ & m_{22} & m_{23} & m_{24} & m_{25} & m_{26} & m_{27} & m_{28} & m_{29} & m_{210} & m_{211} \\ & & m_{33} & m_{34} & m_{35} & m_{36} & m_{37} & m_{38} & m_{39} & m_{310} & m_{311} \\ & & & m_{44} & m_{45} & m_{46} & m_{47} & m_{48} & m_{49} & m_{410} & m_{411} \\ & & & & m_{55} & m_{56} & m_{57} & m_{58} & m_{59} & m_{510} & m_{511} \\ & & & & & m_{66} & m_{67} & m_{68} & m_{69} & m_{610} & m_{611} \\ & & & & & & m_{77} & m_{78} & m_{79} & m_{710} & m_{711} \\ & & & sym & & & & m_{88} & m_{89} & m_{810} & m_{811} \\ & & & & & & & & m_{99} & m_{910} & m_{911} \\ & & & & & & & & & m_{1010} & m_{1011} \\ & & & & & & & & & & m_{1111} \end{bmatrix} \begin{bmatrix} \ddot{x}_b \\ \ddot{y}_b \\ \ddot{\phi} \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_{m1} \\ \ddot{\theta}_{m2} \\ \ddot{\tilde{w}}_{1i} \\ \ddot{\tilde{\beta}}_{1i} \\ \ddot{\tilde{w}}_{2j} \\ \ddot{\tilde{\beta}}_{2j} \end{bmatrix} +$$
$$\begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ 0 \\ 0 \\ C_{8i} \\ 0 \\ C_{10j} \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q_4 \\ Q_5 \\ K_1(\theta_{m1}-\theta_1) \\ K_2(\theta_{m2}-\theta_2) \\ Q_{8i} \\ Q_{9i} \\ Q_{10j} \\ Q_{11j} \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ 0 \\ 0 \\ \tau_1 \\ \tau_2 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{aligned} &i=1,2,\ldots,n_1 \\ &j=1,2,\ldots,n_2 \end{aligned} \qquad (17)$$

Where, the vector $[Q]$ indicates the effects of gravitation and potential energy of system. Position of end effector is indicated through the following relation.

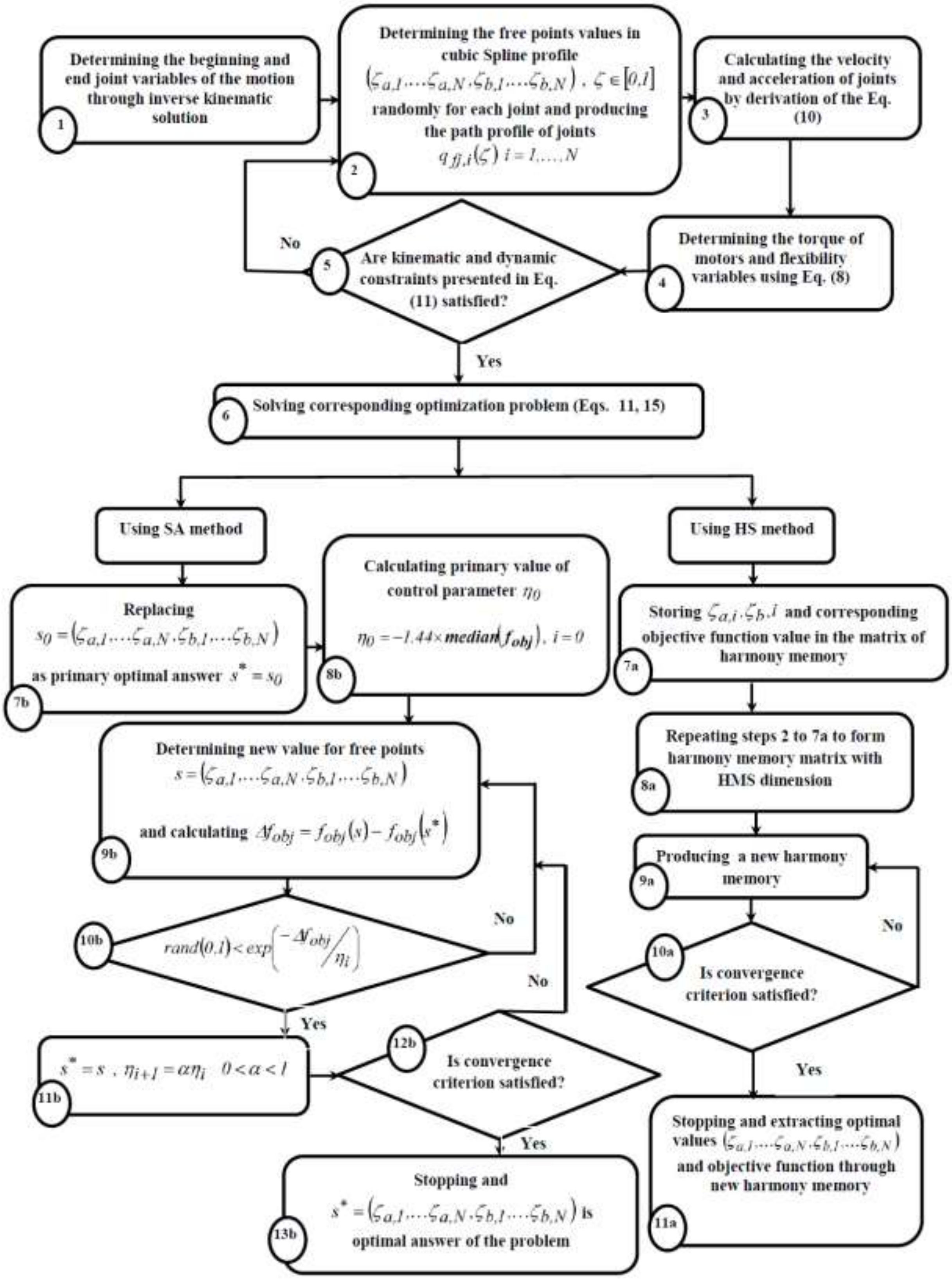

Fig. 3 Flowchart of the planning an optimal path using meta-heuristic methods

$$X_e = (x_b + l_1\cos\theta_1\cos\phi + l_2\cos(\theta_1+\theta_2)\cos\phi)\vec{I} + (y_b + l_1\cos\theta_1\sin\phi + l_2\cos(\theta_1+\theta_2)\sin\phi)\vec{J} + (l_1\sin\theta_1 + l_2\sin(\theta_1+\theta_2))\vec{K} \quad (18)$$

In order to calculate maximum load carrying capacity and minimum transmission time of planning optimal path for two-link mobile manipulator, the corresponding problem of optimization is presented in Table 2. In order to determine the accuracy of the results, simulation of two-link planar mobile manipulator presented in [2] is conducted. In [2], the path planning is done based on optimal control method. Parameters of this manipulator are indicated in Table 3. For planning the path, the total motion time is considered as $T = 1.5$ sec and the beginning and end positions of joints are considered as follows.

$$\theta_1(t=0) = \theta_{m1}(t=0) = 1.5\ rad\ , \theta_2(t=0) = \theta_{m2}(t=0) = 2\ rad \\ \theta_1(t=1.5) = \theta_{m1}(t=1.5) = -1\ rad\ , \\ \theta_2(t=1.5) = \theta_{m2}(t=1.5) = 1\ rad \quad (19)$$

The velocity of end effector in the beginning and end of the path is zero. The base also begins the movement from the point $x_1 = 0.6\ , y_1 = 0.8\ (m)$ and the end is the point $x_1 = 1.4\ , y_1 = 1.2\ (m)$. The results relating to joint variables, torque of joints and the path passed by end by considering minimum torque as the objective function and model of rigid links are presented in Figures 5-8. The figures indicate that, because of flexibility in links and joints, the obtained results have oscillatory behaviour. By the help of Figure 5, it is known that the angular position of the first arm gets oscillatory behaviour at the middle of motion time that is more visible than other times. The torque of first arm is also bigger at the middle of motion time. The obtained path is not smooth and there are some fluctuations due to the flexibility in links and joints. The shapes of angular displacement curves of links and torque of links are similar, such that for changing angular displacement of first arm between positive and negative values, the torque of first arm also changes its direction and for the angle of second arm to have a symmetrical behaviour, the torque of second arm also behaves symmetrically

The results are very consistent to the results reported in [2]. Due to the flexibility in the arms, implementing optimal control method is not so simple. But, the proposed method will get optimal answer sooner and without simplification of the dynamic equations.

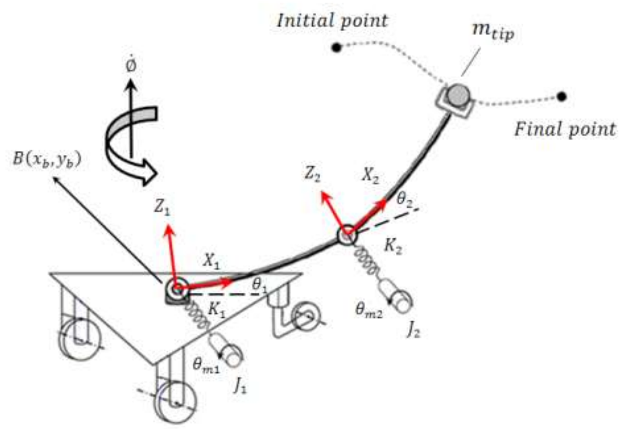

Fig. 4 Flexible two-link manipulator with flexible joints and mobile base

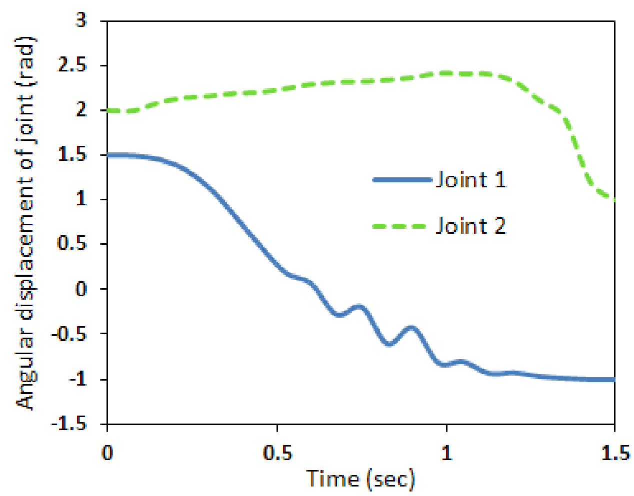

Fig. 5 Angular displacement of first and second motors

Table 1 Bounds on T due to dynamic constraints for a given path profile

$\bar{h}_i(\zeta)\geq 0$			$\bar{h}_i(\zeta)<0$		
$a_i(\zeta)<0$	$a_i(\zeta)\geq 0$		$b_i(\zeta)<0$	$b_i(\zeta)\geq 0$	
	$b_i(\zeta)<0$	$b_i(\zeta)\geq 0$		$a_i(\zeta)<0$	$a_i(\zeta)\geq 0$
Ø	$T_{L,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{a_i}}$ $T_{R,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{-b_i}}$	$T_{L,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{a_i}}$ $T_{R,i}=+\infty$	Ø	$T_{L,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{a_i}}$ $T_{R,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{-b_i}}$	$T_{L,i}=\sqrt{\frac{\bar{h}_i(\zeta)}{-b_i}}$ $T_{R,i}=+\infty$

Table 2 Optimization problems for optimal path planning of mobile two-link manipulator

Planning the path by maximum load carrying capacity	Planning the path by minimum transmission time
$f_{obj}=max(m_{tip})$ $design\ \ variables=(\zeta_{a1},\zeta_{b1},\zeta_{a2},\zeta_{b2})$ $\theta_{mi}=\tilde{q}_i$ *from Eq.(10)* $\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}J_1 & 0\\ 0 & J_2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_{m1}\\ \ddot{\theta}_{m2}\end{bmatrix}+\begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix}\begin{bmatrix}\theta_{m1}-\theta_1\\ \theta_{m2}-\theta_2\end{bmatrix}$ *from Eq.(17)* $subject\ to\ \begin{cases}\lvert\theta m_i(t)\rvert\leq\theta_{mi}^{max}\\ \lvert\dot{\theta}_{mi}(t)\rvert\leq M_{vi}\\ \lvert\ddot{\theta}_{mi}(t)\rvert\leq M_{ai}\\ \lvert\tau_i(t)\rvert\leq\tau_i^{max} \quad i=1,2\\ \lvert X_e(T)-X^{fin}\rvert\leq\varepsilon\\ det(J(q))\neq 0 \quad from\ Eq.(20)\end{cases}$	$f_{obj}=min(T)$ $design\ \ variables=(\zeta_{a1},\zeta_{b1},\zeta_{a2},\zeta_{b2})$ $\theta_{mi}=\tilde{q}_i$ *from Eq.(10)* $\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix}J_1 & 0\\ 0 & J_2\end{bmatrix}\begin{bmatrix}\ddot{\theta}_{m1}\\ \ddot{\theta}_{m2}\end{bmatrix}+\begin{bmatrix}K_1 & 0\\ 0 & K_2\end{bmatrix}\begin{bmatrix}\theta_{m1}-\theta_1\\ \theta_{m2}-\theta_2\end{bmatrix}$ *from Eq.(17)* $subject\ to\ \begin{cases}T\geq T_V \quad T_V=\max\limits_{i=1,2}\left[\max\limits_{\zeta\in[0,1]}\frac{\lvert\tilde{q}'_{fj,i}(\zeta)\rvert}{M_{vi}}\right]\\ T\geq T_A \quad T_A=\max\limits_{i=1,2}\left[\max\limits_{\zeta\in[0,1]}\frac{\lvert\tilde{q}''_{fj,i}(\zeta)\rvert}{M_{ai}}\right]^{1/2}\\ T_L\leq T\leq T_R \quad from\ Eq.(13,14)\ and\ Table(1)\\ \lvert X_e(T)-X^{fin}\rvert\leq\varepsilon\\ det(J(q))\neq 0 \quad from\ Eq.(20)\end{cases}$

Table 3 Parameters of the planar two flexible links and flexible joints manipulator [2]

Parameter (unit)	Value	Parameter (unit)	Value	Parameter (unit)	Value
Length of links (m)	$l_1=l_2=0.7$	Mass of links (kg)	$m_1=m_2=6$	Mass of wheels (kg)	$m_{wheel}=0.3$
Spring constant (N/m)	$k_1=k_2=2000$	Moment of inertia (kg/m^2)	$J_1=J_2=2$	Radius of wheels (m)	$r=0.3$
Max. no load speed of actuators (rad/sec)	$w_{s1}=w_{s2}=3$	Actuator stall torque $(N.m)$	$\tau_{s1}=\tau_{s2}=70$	Mass of base (kg)	$m_{base}=2$

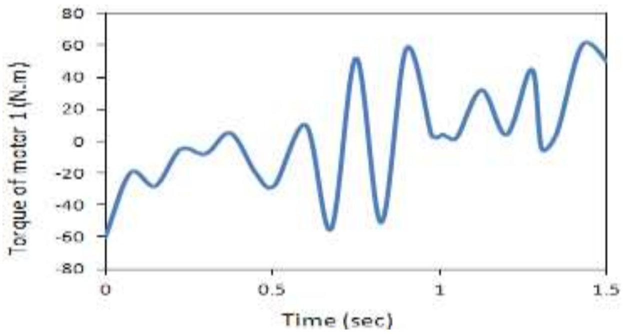

Fig. 6 Applied torque for the first motor

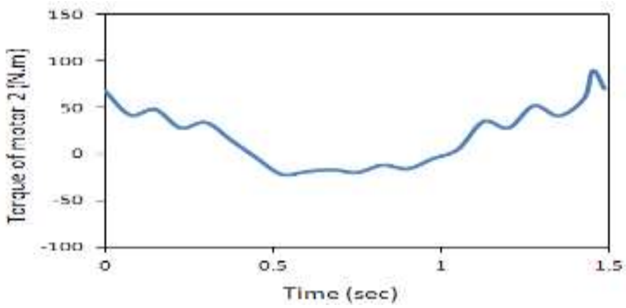

Fig. 7 Applied torque for the second motor

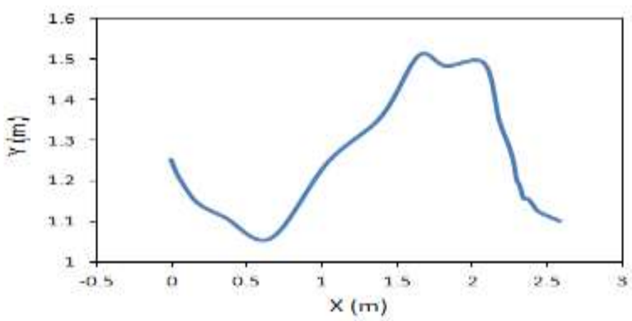

Fig. 8 End effector trajectory for planar two link manipulator

4.1 Results of path planning by maximum load carrying capacity

In this part, the problem of planning optimal path for two-link mobile manipulator by elastic joints is considered in point to point movement in three dimensional spaces (Fig. 4) by considering maximum load carrying capacity as the objective function. The primary position of the end effector is $x_1 = 1.6$, $y_1 = 0$, $z_1 = 0$ (m) and the final position is equal to $x_2 = 2.1$, $y_2 = 0.5$, $z_2 = 0.3$ (m) at $T = 2$ sec. The base also begins a direct path from the point $x_1 = 0$, $y_1 = 0$ (m) and continues the movement with velocity $v_b = 0.2$ m/s along a line that has 68° angle relative to horizon line. For kinematic constraints, maximum velocity and acceleration are considered equal to $M_v = 3\,rad/s$, $M_a = 10\,rad/s^2$, maximum torque is considered as $\tau_{\max} = \pm 230\ N.m$ and the value of standard deviation from end point is considered as $\varepsilon = 1cm$. Parameters of two-link flexible manipulator with elastic joints and mobile base are presented in Table 4. The results for two states of links with small and large deformations are presented. Jacobean matrix for two-link mobile manipulator is based on Eq. 20. Figures 9 and 10 indicate the angular displacement of motors for both models during the time. Angular position of links by HS and SA methods are represented in Figures 11and 12. In Figures 13 and 14, torque imposed on links is presented. Figure 15 also indicates the path paved by end effector. The results relating to carrying load capacity are presented in Table 5 considering the kinematic and dynamic constraints. Due to flexibility of the links and joints, angular positions of the links have oscillatory behaviour differing from angular position of motors.

$$J(q) = \begin{bmatrix} 1 & 0 & -l_1 \cos\theta_1 \sin\varphi - l_2 \cos(\theta_1+\theta_2)\sin\varphi & -l_1 \sin\theta_1 \cos\varphi - l_2 \sin(\theta_1+\theta_2)\cos\varphi & -l_2 \sin(\theta_1+\theta_2)\cos\varphi \\ 0 & 1 & l_1 \cos\theta_1 \cos\varphi + l_2 \cos(\theta_1+\theta_2)\cos\varphi & -l_1 \sin\theta_1 \sin\varphi - l_2 \sin(\theta_1+\theta_2)\sin\varphi & -l_2 \sin(\theta_1+\theta_2)\sin\varphi \\ 0 & 0 & 0 & l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix} \quad (20)$$

The amplitude indicated in figures of links angular displacement, torque of motors and the path passed by end effector resulted based on the model of large deformation is more than the figures reached by small deformation model. Given this point, it is expected that the torque of motors achieves the saturation limit in large deformation model early and finally, load carrying capacity of manipulator is reduced. The results presented in Table 5 confirm this and dynamic allowable load by manipulator under large deformation is less than the load resulted from small deformation model by $0.4\,kg$. The path resulted from large deformation represents more severe fluctuations than small deformation. Therefore, the combination of large deformation of links and elastic joints develop severe oscillatory behaviour and indicate the importance of dynamic modeling by elastic joints. The obtained results by HS and SA methods are consistent in all figures.

Therefore, the existence of controller in the system is essential to reduce the effects of flexibility and increase the load capacity of flexible manipulator. The obtained results indicate that the developed method has no problem with non-linear dynamics of the system and the path planning is possible by the most complete dynamic equations. While, most previous methods have basic challenges when dealing with non-linear equations and there is no choice but to simplify the equations. Moreover, in the proposed method, optimal path is planned by considering the most complete constraints dominating the problem. Load carrying capacity for manipulator based on small deformation is equal to $m_{tip} = 1kg$. Since, this load may lead to reversal of manipulator during movement, dynamic stability of manipulator is using Moment-Height Stability (MHS) method. Figure 16 indicates dynamic stability limit of manipulator. Due to positive sign of this parameter during motion, the manipulator will be able to carry this load without reversal. For details on the formulation of stability constraint the reader is referred to Moosavian [29].

Table 4 Parameters of the two flexible links and flexible joints manipulator with mobile base

Parameter	Value (unit)
Length of links	$l_1 = 0.4\ m\ ,\ \ l_2 = 1.6\ m$
Density of links	$\rho_1 = \rho_2 = 3000\ {kg}/{m^3}$
Young's modulus of material	$E_1 = E_2 = 0.3\times10^{11}\ N/m^2$
Spring constant	$K_1 = K_2 = 600\ N/m$
Cross section area	$A_1 = A_2 = 2.5\times10^{-3}\ m^2$
Moment of inertia	$I_1 = I_2 = 5.2\times10^{-7}\ m^4$
Shear modulus of material	$G_1 = G_2 = 16\times10^6\ N/m^2$
Motor's moment of inertia	$J_1 = J_2 = 0.5\ kgm^2$
Mass of base	$m_{base} = 65\ kg$
Base's moment of inertia	$I_{base} = 0.297\ m^4$

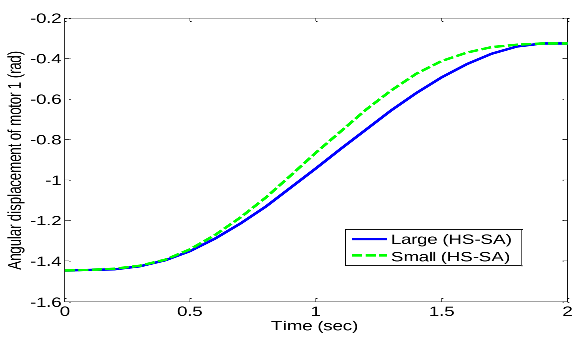

Fig. 9 Angular displacement of the first motor

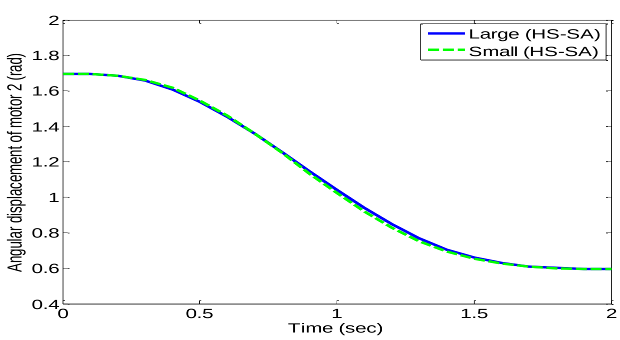

Fig. 10 Angular displacement of the second motor

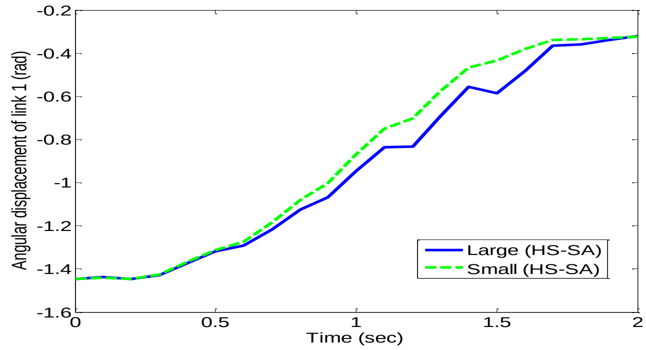

Fig. 11 Angular displacement of the first link

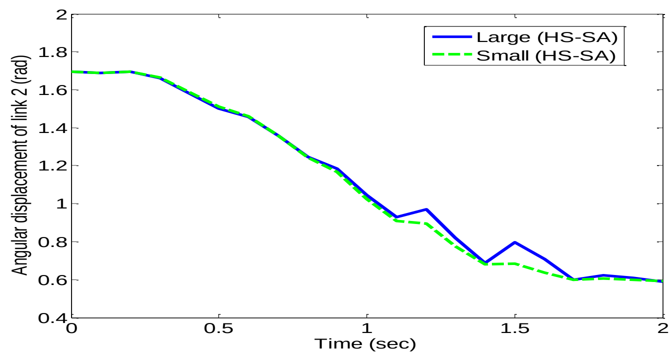

Fig. 12 Angular displacement of the second link

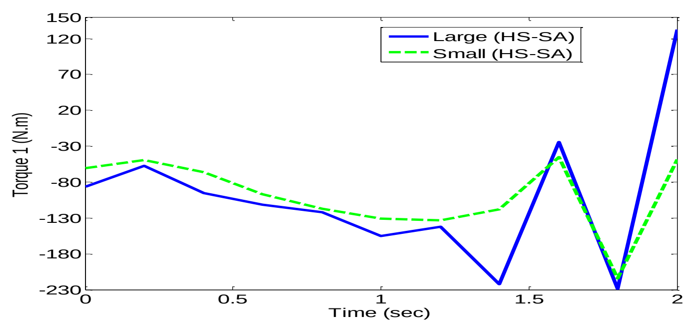

Fig. 13 Applied torque of the first motor

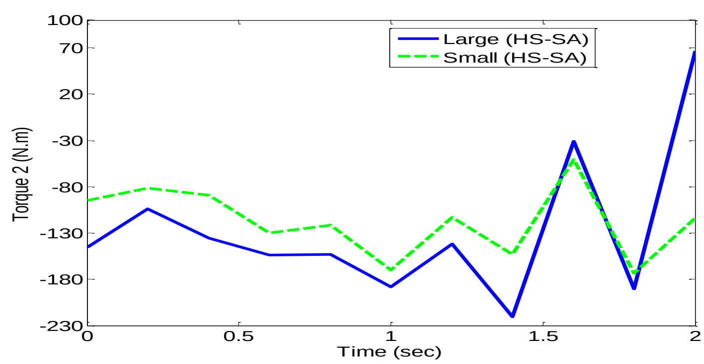

Fig. 14 Applied torque of the second motor

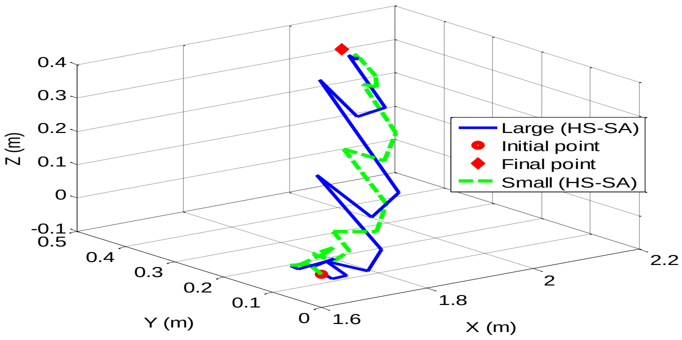

Fig. 15 End effector path based on small and large deformation models

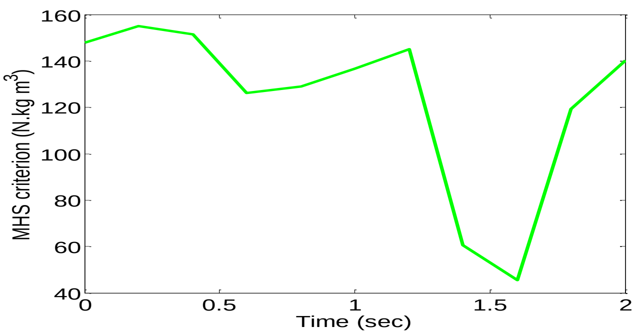

Fig. 16 MHS measure related to the stability region

To investigate the effect of flexibility in joints, the path paved by end effector is planned for different values of torsional stiffness in Figure 17. By studying these figures, it is clear that, by the increase of torsional stiffness, the behaviour of system approaches to a system with rigid joints and load carrying capacity of robot is also enhanced, i.e. the fluctuations of end effector is reduced and motors approach to their saturation limit by carrying more loads. It is known that by the reduction of flexibility effect in joints, the obtained path becomes flatter. Therefore, by not considering the flexibility in joints, the answers obtained for the problem of planning the path have not suitable accuracy and they always suffer errors.

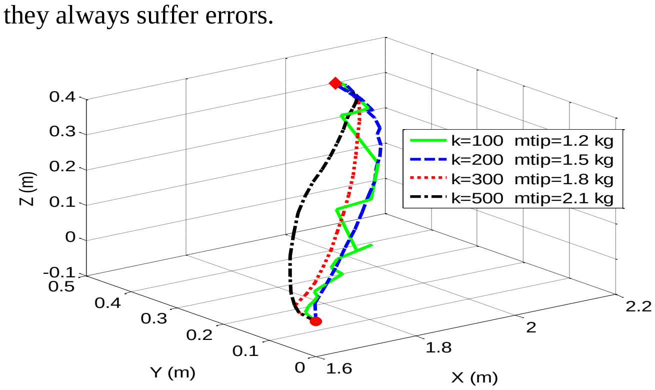

Fig. 17 End effector trajectory based on maximum load with different values of torsional spring constant

4.2 Results of planning optimal path by minimum transmission time

In this part, the problem of planning optimal path is investigated considering minimum transmission time as the objective function. For the results presented in this part, it is known that $k_1 = k_2 = 100 N/m$, $m_{tip} = 0.3 kg$. Primary position of end effector is $x_1 = 1.2$, $y_1 = 0$, $z_1 = 0$ (m) and its final position is $x_2 = 2$, $y_2 = 0.5$, $z_2 = 0.8$ (m). Maximum torque in this state is equal to $\tau_{max} = \pm 250 \ N.m$.

Angular displacement of the links and motors is indicated in Figures 18 and 20 considering the models of small and large deformations of the links. The torque of motors for both models is indicated in Figures 19 and 21. Figure 22 represents optimal path planned for both models. The results relating to minimum transmission time are indicated in Table 6 considering kinematic and dynamic constraints. Figures 18 and 20 indicate that angular position of links and joints differ by the presence of elastic joints and angular position of links has an oscillatory motion around angular position of motors.

Moreover, the value of deviation in large deformation model is higher. In other words, when the motor rotates by a certain value, the links move by different value. Besides, due to flexibility in links, the position of end effector of manipulator deviates more than in rigid state. So that, for accurate motion of end effector due to rotation of motors, presence of controller is necessary in the system. Angular displacement of the first and second arm will get more distance from each other at the end of the path in both models. By analyzing the torque applied to the arms, it is clear that the torque amplitude is more at the end of the path and the effect of flexibility is more in the joints. By adding the flexibility of the arm, the diversion will be more apparent in large deformation model.

By investigating the Figure 21, it is known that the torque of first and second motors approaches its saturation limit in $t = [4{,}5]\ sec$ in interval. Figure 21 has also oscillatory behaviour in this interval by bigger ranges and the path paved is rougher at the middle of path. Transmission time between beginning and end points of movement in large deformation is more than that of small deformation. In general, by investigating the results, it can be said that flexibility in joints has a significant effect on path planning problem and for achieving accurate answer, using a perfect dynamic model of system is of high importance.

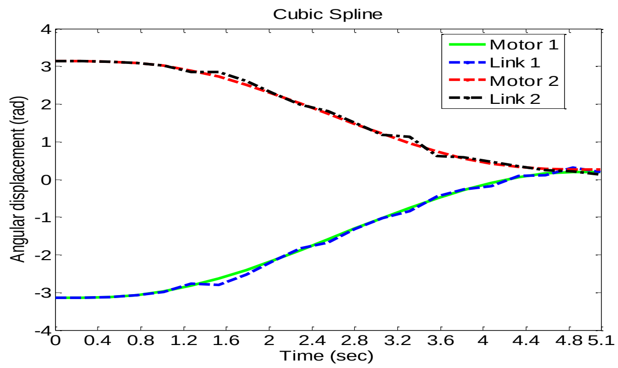

Fig. 18 Angular position of the first and second motors and links for small deformation model

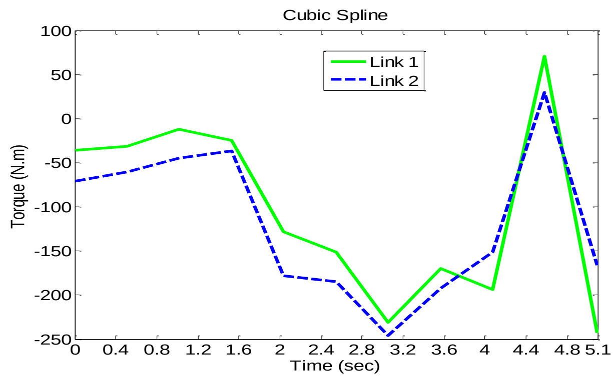

Fig. 19 Applied torques of the first and second links for small deformation model

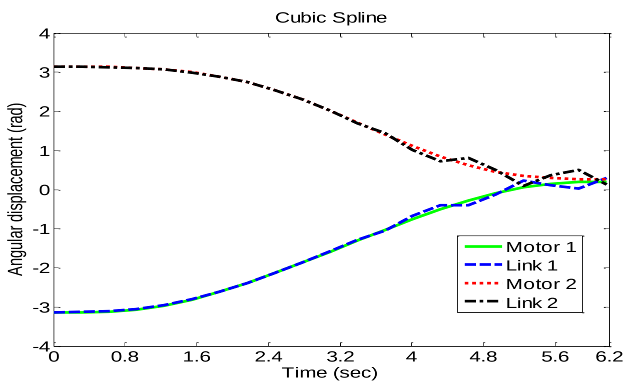

Fig. 20 Angular position of the first and second motors and links for large deformation model

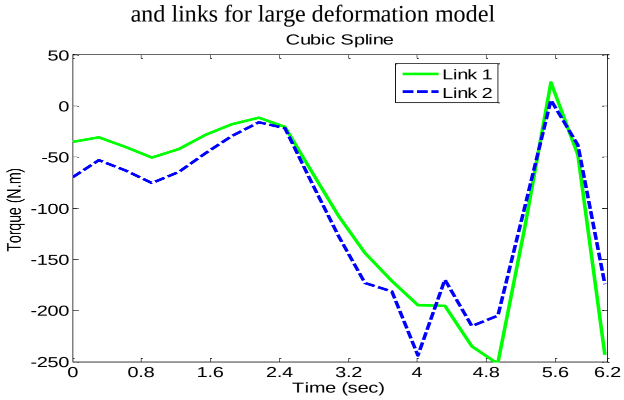

Fig. 21 Applied torques of the first and second links for large deformation model

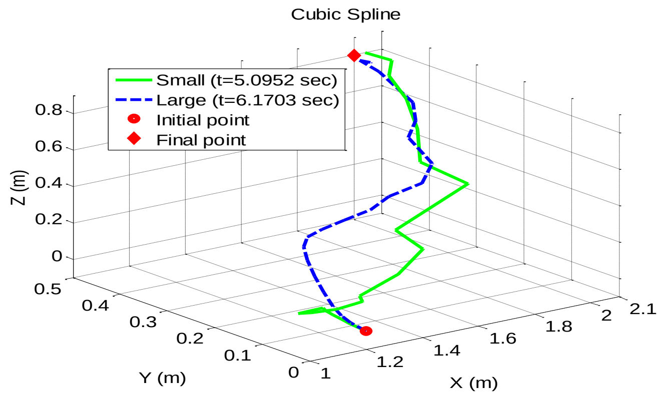

Fig. 22 End effector trajectory based on minimum transmission time for both small and large deformation models

In this case, for studying the effect of flexibility in joints, end effector path for different values of torsional stiffness is presented in Figure 23. By reviewing the figure, it is clear that by the increase of torsional stiffness value, the flexibility of joints will be reduced as well as transmission time of manipulator. In addition, by the reduction of flexibility effects in joints, the obtained path will be flatter, so that, the importance of the effects of flexibility in joints for dynamic modeling of manipulator to get a more real solution becomes more evident.

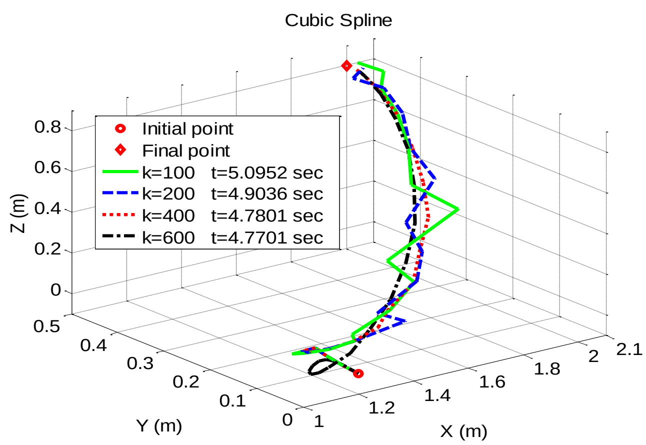

Fig. 23 End effector trajectory based on minimum transmission time with different values of torsional spring constant

Table 5 Results of optimal path planning for mobile flexible link and joint manipulator based on maximum load

Model	ξ_{a1}	ξ_{b1}	ξ_{a2}	ξ_{b2}	maximum load (kg)
Small	0.228	0.740	0.175	0.628	1
Large	0.196	0.915	0.138	0.680	0.6

Table 6 Results of optimal path planning for mobile flexible link and joint manipulator based on minimum Transmission time

Model	ξ_{a1}	ξ_{b1}	ξ_{a2}	ξ_{b2}	Minimum Transmission Time (sec)
Small	0.171	0.913	0.22	0.805	5.095
Large	0.162	0.917	0.387	0.796	6.170

5 CONCLUSION

In this paper, planning an optimal path for flexible mobile manipulator under point to point case is investigated. A perfect dynamic model is presented considering large deformation of the links, flexibility of joints and non-holonomic constraints of base for mobile manipulator. Optimal path of manipulator is planned based on direct method and also HS and SA optimization methods. Maximum load carrying capacity and minimum transmission time are considered as two criteria for determining the efficiency of mobile manipulators.
In addition to kinematic and dynamic constraints, the constraint of closeness of path to end point and the constraint of dynamic stability are considered by using MHS method. In planning an optimal path, it is found that HS and SA methods has no problem with non- linear dynamics of system and, by considering the most perfect dynamic model for system, these methods would solve the problem by suitable accuracy. The results indicates that the flexibility in joints have significant effect on planning the path for end effector.
Neglecting this factor in planning the path would lead to errors in the response of system relative to real state. In addition, the results obtained through HS and SA methods are consistent to each other. For investigating the effect of flexibility of joints, the optimal path is obtained by different values of torsional stiffness. The results indicate that, by the increase of torsional stiffness value, the response of system approaches to the state of rigid joints and effect of flexibility in joints is disappeared. Moreover, by the increase of torsional stiffness value, elastic joints of the path obtained will be flatter.
A comparison is also made between the results obtained from small and large deformation models. Fluctuation range in obtained figures for angular displacement of links and end effector path is bigger for large deformation model and flexibility effect of joints is more visible in this model. Comparing the results obtained to experimental results, implementing the proposed method in industrial robots and planning an optimal path in closed loop mode can foster the future researches about this paper.

6 NOMENCLATURE

(n_{base}, n_{arm})	Degree of freedom for the base and arm
(m_{tip}, I_{tip})	Concentrated gravity and moment inertia of end manipulator
η	Control parameter in SA method
Γ	Shear strain
$\tilde{w}_{1i}(t), \tilde{w}_{2i}(t)$	Nodal values of transverse displacement in links
$\tilde{\beta}_{1i}(t), \tilde{\beta}_{2i}(t)$	Nodal values of shear angle in links
K_b	Curvature developed in element of beam due to large deformation
E_0	Axial strain
g	Acceleration of gravity
$J(q)$	Jacobean matrix
HMS	size of harmony memory
$rand(0,1)$	Random number between zero and one

REFERENCES

[1] Korayem, M. H., Rahimi, H.N, Nikoobin, A., "Analysis of four wheeled flexible joint robotic arms with application on optimal motion design", New Advances in intelligent Decision Technologies, Vol. 199, 2009, pp. 107–116.
[2] Korayem, M. H., Rahimi Nohooji, H., Nikoobin A., "Path planning of mobile elastic robotic arms by indirect approach of optimal control", Journal of Advanced Robotic Systems, Vol. 8, No. 1, 2011, pp. 10-20.
[3] Korayem, M. H., Rahimi Nohooji, H., Nikoobin, A., "Optimal motion generating of non-holonomic manipulators with elastic revolute joints in generalized point-to-point task", Journal of Advanced Design and Manufacturing Technology ,Vol. 3, No. 2, 2010, pp.1-9.
[4] Korayem, M. H., Rahimi Nohooji, H., "Trajectory optimization of flexible mobile manipulators using open-loop optimal control method", Springer-Verlag Berlin Heidelberg, Vol. 5314, No. 1, 2008, pp. 54–63.
[5] Rahimi, H. N., Korayem, M. H., Nikoobin, A., "Optimal motion planning of manipulators with elastic links and joints in generalized point-to-point task", Design Engineering Technical Conferences and Computers and Information in Engineering Conference, ASME, 2009, pp. 1167-1174.
[6] Korayem, M. H., Nikoobin, A., "Formulation and numerical solution of robot manipulators in point-topoint motion with maximum load carrying capacity", Scientia Iranica, Trans. B, Vol. 6, 2009, pp. 101-109.
[7] Korayem, M. H., Nikoobin, A., Azimirad, V., "Trajectory optimization of flexible link manipulators in point-to-point motion", Robotica, Vol. 27, 2009, pp. 825-840.
[8] Korayem, M. H., Rahimi, H. N., Nikoobin, A., "Mathematical modeling and trajectory planning of mobile manipulators with flexible links and joints", Applied Mathematical Modeling, Vol. 36, 2012, pp. 3229-3244.
[9] Gasparetto, A., Boscariol, P., Lanzutti, A., Vidoni, R., "Trajectory planning in robotics", Mathematics in Computer Science, Vol. 6, No. 3, 2012, pp. 269–279.

[10] Chettibi, T., Lehtihet, H. E., Haddad, M., “Minimum cost trajectory planning for industrial robots”, European Journal of Mechnics A /Solids, Vol. 23, 2004, pp.703–715.

[11] Haddad, M., Chettibi, T., Hanchi, S., Lehtihet, H. S., “A new approach for minimum time motion planning problem of wheeled mobile robots”, 16th IFAC World Congress, Prague, 2005.

[12] Haddad, M., Hanchi, S. Lehtihet, H. E., “Sub-optimal motion planner of a mobile manipulator with end-effector’s specified path”, IFTOMM World Congress, 2007, pp.1-6.

[13] Saravanan, R., Ramabalan, S., “Evolutionary minimum cost trajectory planning for industrial robots”, Journal of Intelligent and Robotic Systems, Vol. 52, No. 1, 2008, pp. 45–77.

[14] Saravanan, R., Ramabalan, S., Balamurugan, C., “Evolutionary optimal trajectory planning for industrial robot with payload constraints”, Journal of Advanced Manufacturing Technology, Vol. 38, No. 11, 2008, pp. 1213–1226.

[15] Tangpattanakul, P., Artrit, P., “Minimum-time trajectory of robot manipulator using harmony search algorithm”, 6th International Conference on Electrical Engineering/Electronics, Computer, Telecommunication, Pattaya, Chonburi, 2009, pp. 354 - 357.

[16] Devendra, P. G., Kumar, M., “Optimization techniques applied to multiple manipulators for path planning and torque minimization”, Engineering Applications of Artificial Intelligence, Vol. 15, 2002, pp. 241–252.

[17] Tangpattanakul, P., Meesomboon, A., Artrit, P., “Optimal trajectory of robot manipulator using harmony search algorithms”, Recent Advances in Harmony Search Algorithm, Edited by Zong Woo Geem, Springer, 2010, pp. 23–36.

[18] Chettibi, T., Lemoine, P., “Generation of point to point trajectories for robotic manipulators under electro-mechanical constraints”, International Review of Mechanical Engineering (IREME), Vol. 1, No. 2, 2007, pp. 131-143.

[19] Zanotto, V., Gasparetto, A., Lanzutti, A., Boscariol, P., Vidoni, R., “Experimental validation of minimum time-jerk algorithms for industrial robots”, Journal of Intelligent and Robotic Systems, Vol. 64, No. 2, 2011, pp. 197–219.

[20] Dijk, V., de Wouw, V., Nijmeijer, N., Pancras, H., “Path-constrained motion planning for robotics based on kinematic constraints”, In Proceedings of IDETC/CIE ASME, 2007, pp. 1-10.

[21] Ramabalan, S., Saravanan, R., Balamurugan, C., “Multi-objective dynamic optimal trajectory planning of robot manipulators in the presence of obstacles”, Journal of Advanced Manufacturing Technology, Vol. 41, No. 5, 2009, pp.580–594.

[22] Kelly, A., Nagy, B., “Reactive non-holonomic trajectory generation via parametric optimal control”, Journal of Robotics Research, Vol. 22, No. 7, 2003, pp. 583-601.

[23] Scaglia, G., Rosales, A., Quintero, L., Mut, V., Agarwal, R., “A linear-interpolation-based controller design for trajectory tracking of mobile robots”, Control Engineering Practice, Vol. 18, 2010, pp.318–329.

[24] Furuno, S., Yamamoto, M., Mohri, A., “Trajectory planning of mobile manipulator with stability considerations” IEEE International Conference on Robotics & Automation, Taipei, Taiwan, 2003, pp. 3403-3408.

[25] Sun, S., “Designing approach on trajectory-tracking control of mobile robot”, Robotic and Computer-Integrated Manufacturing, Vol. 21, No. 1, 2005, pp.81-85.

[26] Xu, D., Hu, H., Calderon, C. A. A., Tan, M., “Motion planning for a mobile manipulator with redundant DOFs”, Proceedings of International Conference on Intelligent Computing (ICIC'05), China, 2005, pp. 23-26.

[27] Alibakhshi, R., Mohammadi Daniali, H. R., “Trajectory optimization of spherical parallel robots using artificial neural network”, Journal of Advanced Design and Manufacturing Technology, Vol. 7, No. 1, 2014, pp. 91-98.

[28] Sharma, C., “Displacement / Mixed finite element formulation for beam and frame problems”, MSc. Dissertation, European School of Advanced Studies in Reduction of Seismic, October, 2007.

[29] Moosavian, S. A. A., Alipour, K., “Stability evaluation of mobile robotic systems using moment– height measure”, The IEEE International Conference on Robotics, Automation and Mechatronics, Bangkok, 2006, pp. 1–6.