Vibration Analysis of a Multi-disk, Bearing and Mass Unbalance Rotor Using Assumed Modes Method

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Abstract: This paper aims to presents a simple and efficient approach for modelling and solving the equations of a rotor with any number of disks, bearings and mass unbalances using the assumed modes method. This model consists of a continuous shaft, arbitrary number of mass unbalances in any axial location and phase angle, and any number of rigid disks and bearings. This arrangement is extensively used in diverse applications. In this study, final governing differential equations are not derived because the assumed modes method is directly inserted into solving process. Some examples in both cases of free and forced vibration have been dealt with. The results show the accuracy of this approach and its prediction of vibration behaviour of the rotor in a complex combination of shaft, disk and bearing. This study also shows the results are as accurate as the results of the most popular approach, i.e. the Finite Element Method.

Keywords: Assumed Modes Method, Rotor-Bearing, Vibration Analysis

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1 INTRODUCTION

The most common sources of vibration in machinery are related to the inertia of their moving parts of which one of the most important parts is rotor. Rotor is usually a flexible shaft with a number of disks mounted on it and has some bearing supports. The vibration of the rotor bearings increases due to bad installation, high rotating speed, poor lubrication, unbalances, etc. Recognizing the roots of these vibrations can help engineers to return the operating conditions of the rotor to safe mode. This is important because rotor is usually the most expensive part of large machines.

Since some types of rotor faults are diagnosable using common vibration based condition monitoring methods and some others are not, then engineers need to use the simple models of rotors so that they can predict the behaviour of rotor under a group of its faults. Many relevant studies have been accomplished on rotor dynamics in recent years, where reference is covering a complete review in this regard [1].

Rotor modelling can be divided in two categories: a) simple theories with discrete shafts, b) complicated theories with continuous shaft. Among simple theories, Jeffcott model is very common for clamp-clamp rotors. Also, Stodola-Green model is used for clamp-free rotors. Furthermore, a combination of these models is sometimes used to study the gyroscopic effects and rotating inertia. Some of complicated theories such as Rayleigh [2], [3], Timoshenko [4], [5], [6] and Euler-Bernoulli beam [7] theories have been also used.

The first analysis of a spinning shaft was done by Rankin in 1869. In 1919, Jeffcott introduced some models for rotor with massless and elastic shaft, rigid disc and unbalance forces [8]. A modal analysis of continuous rotor-bearing system was studied by Lee et al., [9] examining the effects of rotary inertia and gyroscopic moment. They investigated the effects of asymmetry boundary conditions on dynamic characteristics of the system [9]. Jun et al., [4] analysed the free bending vibration of a rotating shaft composed of multi-step segments with each segment having a uniform circular cross-section using the Timoshenko beam model.

Shabaneh investigated the dynamic analysis of a rotating disc-shaft system with linear elastic bearings at the ends mounted on viscoelastic suspensions [5]. Stability and steady state response of symmetric rotors using the finite element method have been investigated by Oncescu et al., [10]. Athanasios [3] analysed a rotor-bearing system consisting a continuous Rayleigh's shaft and finite fluid film bearings. A novel wavelet-based finite element method was used for the analysis of rotor-bearing systems by Xiang et al., [11]. They have considered the effects of translational and rotary inertia, the gyroscopic moments, the transverse shear deformations, and the internal viscous and hysteretic damping using the Rayleigh-Timoshenko element. Khanlo et al., [7] modelled the rotor-bearing system as a continuous shaft with a rigid disc in its midsection with Coriolis and centrifugal effects included. They extracted the governing partial differential equations of motion based on the Euler–Bernoulli beam theory.

The assumed modes method was used to discrete partial differential equations and the resulting equations were solved numerically [7]. One can conclude that the most of the previous works have been done based on complicated theories, simulated lateral vibrations of rotor using the finite element method. The finite element method in this area does not have the required flexibility for changing the position of each member and boundary conditions, because in each of these cases a new problem must be produced. Furthermore, finding mode shapes, sometimes, is very difficult because of spatial condition on rotor-bearing system. The aim of this research is to make a user friendly model for complicate rotor using the Rayleigh's theory which is suitable for condition monitoring studies. Also in this paper, the assumed modes method was inserted into solving process after extracting of energy terms, while the governing differential equations were not derived. This approach whose software was written in MATLAB allows the user to change all inputs and increase the accuracy by including any number of modes easily. The resulting equation will be solved using the Newmark's method and Crank-Nicholson coefficients.

2 ROTOR WITH MULTI-DISK, BEARING AND MASS UNBALANCE

A complete model of rotor-bearing system with arbitrary conditions is shown in Fig. 1.

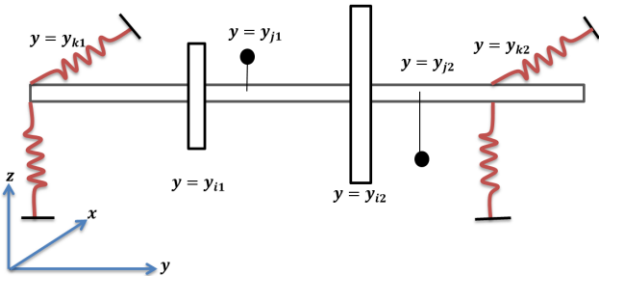

Fig. 1 A general model of the rotor-bearing system with arbitrary conditions.

In this model, the number of discs and their axial locations, the number and position of the bearings, the number of unbalance masses with different radius, magnitude, phase angle and their axial locations are

completely arbitrary. In this model, the shaft is continuous and each bearing is modelled as two springs in horizontal and vertical directions.

3 DERIVATION OF ENERGY TERMS

Consider a model of a rotor with an arbitrary number of discs, unbalance masses and bearings with arbitrary locations mounted on the continuous flexible shaft as shown in Fig. 1. All energy terms of each of these parts are derived individually as follows:

3.1. Kinetic energy of a disc

It is assumed that the disk is rigid. Fig. 2 shows the reference frame of a disc mounted on a rotating shaft. $R_0(XYZ)$ is an inertial frame and $R(x, y, z)$ is attached to the disc. The reference frame fixed to the disc is related to the inertial reference frame through a set of three angles of θ_x, θ_y and θ_z. In order to find the orientation of the disc, it is rotated around the Z axis by amount of θ_z, then by amount of θ_x around the new axis which is x_1. Finally it is rotated by amount of θ_y around the new y axis.

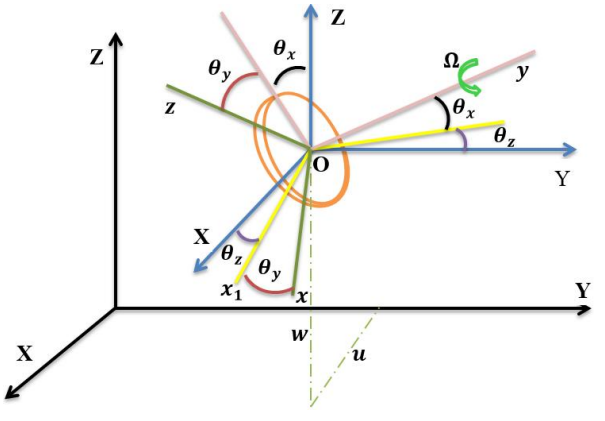

Fig. 2 Rotating frames of a disc on a rotating flexible shaft

Let u and w denote the coordinates of O in R, the coordinate along Y being constant and also the inertia tensor components are I_{dx}, I_{dy} and I_{dz}. So the kinetic energy of the disc can be written as in Eq. (1). If the disc is symmetric i.e. $I_{dx} = I_{dz}$, the angles of θ_x and θ_z are small, and the angular velocity is constant, i.e. $\dot{\theta}_y = \Omega$, the Eq. (1) can be written as in Eq. (2). The last term in this relation represents the gyroscopic (Coriolis) effect [12]. So for a rotor with many discs located at different axial distances from the left end of the rotor, kinetic energy of each disc can be written according to the distance from the left end of the rotor as in Eq. (3). It is worth noting that in this relations, θ_x and θ_z are related to u and w as $\theta_x = dw/dy$ and $\theta_z = -du/dy$.

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{1}{2}\left(I_{dx}\omega_x^2 + I_{dy}\omega_y^2 + I_{dz}\omega_z^2\right) \tag{1}$$

$$T_D = \frac{1}{2} M_D \left(\dot{u}^2 + \dot{w}^2\right) + \frac{I_{dx}}{2}\left(\dot{\theta}_x^2 + \dot{\theta}_z^2\right) + I_{dy}\Omega\dot{\theta}_z\theta_x \tag{2}$$

$$T_{D_n} = \frac{1}{2} M_{D_n} \left(\dot{u}^2 + \dot{w}^2\right) + \frac{I_{dx_n}}{2}\left(\dot{\theta}_x^2 + \dot{\theta}_z^2\right) + I_{dy_n}\Omega\dot{\theta}_z\theta_x\Big|_{y = y_n} \tag{3}$$

3.2. Kinetic energy of the shaft

The kinetic energy of the shaft can be written by extending the kinetic energy of a disk in longitudinal direction. Fig. 2 shows the reference frame of a disc mounted on flexible shaft. Then, it is possible to have [12].

$$T_S = \int_0^L \frac{\rho A}{2}\left(\dot{u}^2 + \dot{w}^2\right)dy + \int_0^L \frac{\rho I}{2}\left(\dot{\theta}_x^2 + \dot{\theta}_z^2\right)dy + 2\int_0^L \rho I\Omega\dot{\theta}_z\theta_x dy \tag{4}$$

3.3. Kinetic energy of the unbalance mass

The kinetic energy of unbalance mass is shown as T_U. The mass remains in a plane perpendicular to the y axis and its coordinate along the y axis is a constant or zero depending on the origin of the reference frame. The displacements of unbalance mass in the x and z directions are $u + d_U \sin\Omega t$ and $w + d_U \cos\Omega t$ respectively with the constant in the y direction. So the kinetic energy of the unbalance mass is given as Eq. (5). The term $m_U\Omega^2 d_U^2/2$ is a constant and can be omitted because the derivatives of kinetic energy remain in final equations, this constant term, thus, does not appear in next equations. Also, the unbalance mass m_U is so smaller than the mass of the rotor [13]. Therefore, for a rotor-bearing system with multi unbalance masses that each of them has a phase angle of φ_{U_n} from horizontal line and located at distance of $y = y_U$ from the left end of the rotor, the kinetic energy is obtained as Eq. (6).

$$T_U = \frac{1}{2} m_U \left(\dot{u}^2 + \dot{w}^2 + \Omega^2 d_U^2\right.$$

$$+2\Omega d_U \dot{u}\cos\Omega t - 2d_U \Omega \dot{w}\sin\Omega t\Big) \quad (5)$$

$$T_{U_n} \cong m_{U_n}\Omega d_{U_n}\left(\dot{u}\cos(\Omega t+\varphi_{U_n}) - \dot{w}\sin(\Omega t+\varphi_{U_n})\right)\Big|_{y=y_j} \quad (6)$$

3.4. Strain energy of the shaft

The displacements of the shaft in the *x*, *y* and *z* directions are $u_x = u$, $u_y = -Z\theta_x + X\theta_z$ and $u_z = w$. So, the longitudinal strain in the *y* direction will be as Eq. (7). Using Hook's law $\sigma_{yy} = E\varepsilon_{yy}$ and ignoring the non-linear terms, the strain energy can be written as Eq. (8). Furthermore, the third term in this equation can be neglected due to the symmetry of the cross section. Also, $I_x = I_z = I$, $I_x = \int_A Z^2 dA$, $I_x = \int_A X^2 dA$, (due to symmetry) and $A = \int_S ds$ is the area of the cross section. So, the strain energy of the shaft will be obtained as Eq. (9) [12].

$$\varepsilon_{yy} = -Z\frac{d\theta_x}{dy} + X\frac{d\theta_z}{dy} + \frac{1}{2}\theta_x^2 + \frac{1}{2}\theta_z^2 \quad (7)$$

$$U_S = \frac{E}{2}\int_0^L\iint_A\left[Z^2\left(\frac{d\theta_x}{dy}\right)^2 + X^2\left(\frac{d\theta_z}{dy}\right)^2 - 2ZX\left(\frac{d\theta_x}{dy}\right)\left(\frac{d\theta_z}{dy}\right)\right]dAdy \quad (8)$$

$$U_S = \frac{EI}{2}\int_0^L\iint_A\left[\left(\frac{d\theta_x}{dy}\right)^2 + \left(\frac{d\theta_z}{dy}\right)^2\right]dAdy \quad (9)$$

3.5. Strain and damped energy of the bearings

If any of the bearings is modelled as a linear spring and damper with constant coefficients, the strain energy of each of the bearings located in $y = y_{B_n}$ of the left end of the rotor will be obtained from (10). Also, assuming the cross damping coefficients are zero, the wasted energy of each bearings will be obtained as Eq. (11).

$$U_{B_n} = \frac{1}{2}K_{u_n}u^2 + \frac{1}{2}K_{w_n}w^2\Big|_{y=y_{B_n}} \quad (10)$$

$$D_{B_n} = \frac{1}{2}C_{u_n}\dot{u}^2 + \frac{1}{2}C_{w_n}\dot{w}^2\Big|_{y=y_{B_n}} \quad (11)$$

4 THE GENERAL EQUATIONS OF THE ROTOR

Substituting the energy terms into Lagrange's equations or Hamilton's principle, some mathematical operations lead to governing differential equations of the rotor. Derivation of these equations is not important because there is no exact solution for them. Then, it is better to use the assumed modes method as approximate in this stage. Based on this method we can assume that rotor responses at any point and time, i.e. u and w, are a linear combinations of some assumed modes instead of real ones as in Eq. (12), [15].

$$\begin{aligned} u(y,t) &= \sum_{i=1}^{n}\phi_i(y)\eta_i(t) \\ w(y,t) &= \sum_{i=1}^{n}\psi_i(y)\xi_i(t) \end{aligned} \quad (12)$$

In these relations $\phi_i(y)$ and $\psi_i(y)$ are known trial functions that satisfy the geometric boundary conditions, as in Eq. (13), and $\eta_i(t)$ and $\xi_i(t)$ are unknown functions of time and n is the number of trial functions.

$$\phi_i(y) = \psi_i(y) = \begin{cases} 1 & i = 1 \\ \left(\frac{2}{L}\left(y - \frac{L}{2}\right)\right)^{i-1} & i \geq 2 \end{cases} \quad (13)$$

Substituting the above responses in the energy relations, approximated matrices of mass, stiffness and damping of the system are derived and the following system of equations is obtained [14].

$$[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = [F] \quad (14)$$

In this relation, vector $\{q\}$ is a $2n \times 1$ vector defined by equation (15). Also, the damping matrix which includes both gyroscopic and damping effects is as equation (16). In the following equations, $[c]$ is the damping matrix of the system, $[G^d]$ is the gyroscopic matrix related to the disc, and $[G^s]$ is the gyroscopic matrix related to the shaft [14].

$$\{q\} = \begin{Bmatrix} \{\eta_i(t)\} \\ \{\xi_i(t)\} \end{Bmatrix} \quad (15)$$

$$[D] = [c] + [G] = \begin{bmatrix} [c_{11}] & [c_{12}] + [G^d + G^s] \\ [c_{21}] - [G^d + G^s] & [c_{22}] \end{bmatrix} \quad (16)$$

In the next section, derivation of each of the matrices in equation (14) based on the assumed modes method is given in details.

5 DERIVATION OF THE MASS AND STIFFNESS MATRICES

According to the assumed modes method, u and w could be given a set of trial functions as in equations (12). In that relation, the functions $\phi_i(y)$ and $\psi_i(y)$ are known trial functions that can be a set of assumed mode shapes, polynomials or even some eigen functions. Rayleigh's quotient is expressed as relation (17). In this relation $\pi_{\max}$ and $T^*_{\max}$ denote, respectively, the maximum strain energy and reference kinetic energy of the system. Minimization of Rayleigh's quotient results in the relation (18) in which $\lambda_i^{(2n)}$ are the eigenvalues. The roots of this eigenvalue problem represent the natural frequencies and each of $\{q^{(i)}\}$ will be the vectors of Ritz coefficients [15].

$$R(a) = \omega^2 = \frac{\pi_{\max}}{T^*_{\max}} \tag{17}$$

$$[[K] - \lambda_i^{(2n)}[M]]\{q^{(i)}\} = \vec{0} \qquad i = 1,...,2n \tag{18}$$

Where

$$\begin{aligned} m_{ij} &= \int_0^L \rho A \phi_i \phi_j dy + \int_0^L \rho A \psi_i \psi_j dy + \\ &\int_0^L \rho I \frac{d\psi_i}{dy}\frac{d\psi_j}{dy} dy + \int_0^L \rho I \frac{d\phi_i}{dy}\frac{d\phi_j}{dy} + \\ &M_{D_n}\phi_i\phi_j\Big|_{y=y_{D_n}} + M_{D_n}\psi_i\psi_j\Big|_{y=y_{D_n}} \\ &+ I_{dx_n}\phi_i\phi_j\Big|_{y=y_{D_n}} + I_{dx_n}\psi_i\psi_j\Big|_{y=y_{D_n}} \end{aligned} \tag{19}$$

$$\begin{aligned} k_{ij} &= \int_A \int_0^L EI \frac{d^2\psi_i}{dy^2}\frac{d^2\psi_j}{dy^2} dA dy + \\ &\int_A \int_0^L EI \frac{d^2\phi_i}{dy^2}\frac{d^2\phi_j}{dy^2} dA dy + \\ &\left(K_{u_n}\phi_i\phi_j + K_{w_n}\psi_i\psi_j\right)\Big|_{y=y_{B_n}} \end{aligned} \tag{20}$$

1. Kinetic energy of a disc

The third term in equations (3) and (4) represents the gyroscopic effects and the relation (11) represents the damping effects on the system. Thus, the elements of each of the gyroscopic Eq. (16) and damping matrices will be obtained as in equations (21) to (23).

$$G_{ij}^s = 2\Omega \int_0^L \rho I \frac{d\phi_i}{dy}\frac{d\psi_j}{dy} dy \tag{21}$$

$$G_{ij}^d = \Omega I_{dy} \frac{d\phi_i}{dy}\frac{d\psi_j}{dy}\Big|_{y=y_{D_n}} \tag{22}$$

$$c_{ij} = \left(C_{u_n}\phi_i\phi_j + C_{w_n}\psi_i\psi_j\right)\Big|_{y=y_{B_n}} \tag{23}$$

2. Vector of force

According to equation (6), the energy from the unbalance mass could be expressed as a kinetic energy term. Applying Hamilton's principle on equation (6) leads to the following two equations:

$$\int_{t1}^{t2} \frac{\partial T_{u_n}}{\partial u}\delta u dt = \int_{t1}^{t2} \left(m_{U_n}\Omega^2 d_{U_n}\sin(\Omega t + \varphi_{U_n})\right)\delta u dt \tag{24}$$

$$\int_{t1}^{t2} \frac{\partial T_{wn}}{\partial w}\delta w dt = \int_{t1}^{t2} \left(m_{U_n}\Omega^2 d_{U_n}\cos(\Omega t + \varphi_{U_n})\right)\delta w dt \tag{25}$$

Therefore, the integrands of these two relations are the external forces due to unbalance mass in u and w directions respectively. The unbalance mass may be located at any distance of the shaft, so the matrix of external force would be affected by its location. Thus, the external force exerted on the system is obtained as the equation (26) which $\{F_u\}$ and $\{F_w\}$ will be as the relations (27) and (28).

$$\{F\} = \begin{Bmatrix} \{F_u\} \\ \{F_w\} \end{Bmatrix} \tag{26}$$

$$\{F_u\} = \begin{Bmatrix} \phi_1(y_{U_n}) \\ \phi_2(y_{U_n}) \\ \vdots \\ \phi_n(y_{U_n}) \end{Bmatrix} m_{U_n}\Omega^2 d_{U_n}\sin(\Omega t + \varphi_{U_n}) \tag{26}$$

$$\{F_w\} = \begin{Bmatrix} \psi_1(y_{U_n}) \\ \psi_2(y_{U_n}) \\ \vdots \\ \psi_n(y_{U_n}) \end{Bmatrix} m_{U_n}\Omega^2 d_{U_n}\cos(\Omega t + \varphi_{U_n}) \tag{27}$$

6 VALIDATION OF THE MODEL AND SOLUTION METHOD

Newmark's method and Crank-Nicholson's coefficients were used to solve the resulting equations. To verify the modelling and method of solution, some examples are solved in this section and the accuracy of the results in this study has been compared with those obtained from other references.

1. Example 1

Consider a rotor without disc and neglect the effects of the bearing on both sides. The length of the shaft is 1 m and its diameter is 50 mm. The material properties are: E= 2.058e11 N/m^2, $\nu = 0.29$ and $\rho = 7800\,kg/m^3$. It is assumed that the rotating speed is zero for ignoring the effects of gyroscopic forces. Natural frequencies of this rotor in reference [4] were obtained based on two Euler-Bernoulli and Timoshenko beam theories. The results are shown in Table 1.

Table 1 Natural frequencies comparison of simple rotor (Hz)

Mode No.	Timoshenko Theory [4]	Euler-Bernoulli Theory [4]	Present Research
0	100.6	100.9	100.8
1	399.6	403.4	402.2
2	888.1	907.7	901.5

2. Example 2

Consider a simple rotor consisting of a flexible massless shaft with a massive disc at its center mounted on bearings with stiffness $K = 1\times10^{10}\,N/m$ at both ends. The properties of this rotor are summarized in Table 2. In this example, the impacts of rotor speed and gyroscopic effect on the natural frequencies and critical speeds of the rotor were investigated. The variations of rotor speed were from zero to 2000 rpm.

Table 2 Mass and geometrical properties [16]

Length of shaft	Diameter of shaft	Poisson's ratio	Mass
1.2 m	0.04 m	0.3	120.072 Kg
Young's modulus	Density	Diametral inertia	Polar inertia
2.1×10^{11} N/m^2	7800 Kg/m^3	3.6932 Kg-m^2	7.3544 Kg-m^2

Table 3 Eigen frequencies of the rotor at 2000 rpm

Mode No.	Finite Element Method [16]	Present Study	Error Percent
1	12.121	11.615	4.1
2	12.123	11.617	4.1
3	20.607	21.293	3.3
4	85.538	84.841	0.8

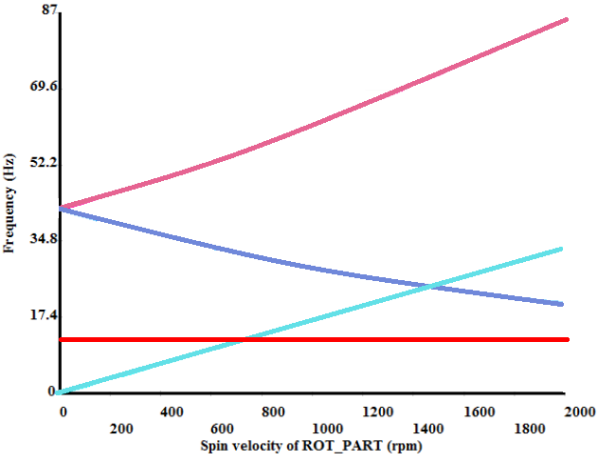

Fig. 3 Campbell's diagram of the rotor using Ansys [16]

Figs. 3 and 4 and Table 3 show the changes of eigen frequencies in Campbell's diagram. It is clear that Campbell's diagram drawn in this study fits very well with diagrams drawn by finite element method using Ansys software [16]. Rotor critical speeds are also compared in Table 4 with [16].

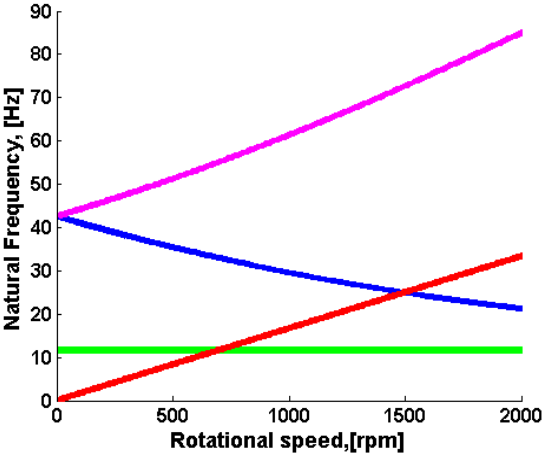

Fig. 4 Campbell's diagram of the rotor in the present study

Table 4 Critical speed of simple rotor

Mode No.	Finite Element Method [16]	Present Study	Whirl	Error Percent
1	727.29	696.93	Backward	4.1
2	727.33	696.99	Forward	4.1
3	1467.23	1496.09	Backward	1.9

3. Example 3

To investigate the forced response of the system, the frequency response of the rotor in Example 2 was considered. In this example it is assumed that the

mounted disc on the rotor has an eccentricity of e=0.001 m. The flexible bearings have stiffness value of 4 KN/mm and damping value of 2000 $N-s/m$. To determine the response of the rotor for this unbalance loading, a harmonic analysis was carried out within the speed range from 0 rpm to 1000 rpm. Frequency response graph at the location of the disc was plotted in Figs. 5 and 6. Moreover, the maximum displacement of the rotor, rotor critical speed, and relevant comparisons are given in Table 5.

Table 5 Maximum response of simple rotor

Response of Rotor	Present Study	Ref. [16]	Error Percent
critical speed (rpm)	697.19	695.52	0.2
Maximum displacement of the rotor (m)	0.3014	0.3038	0.8

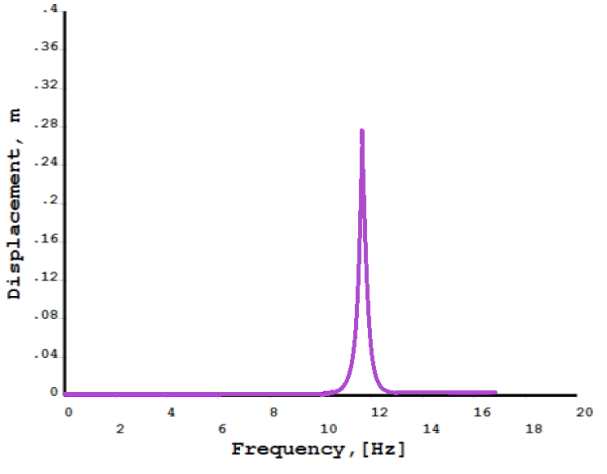

Fig. 5 Maximum response of the simple rotor in Ref. [16]

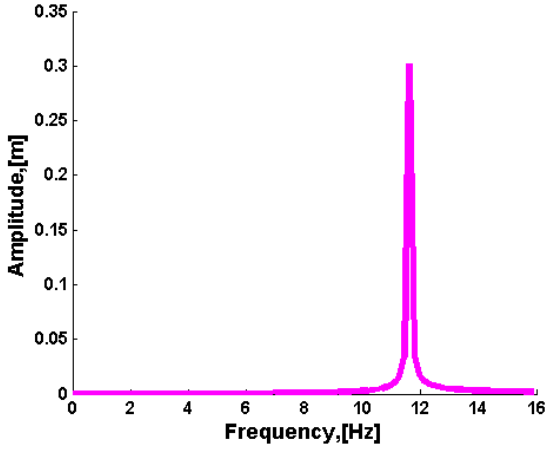

Fig. 6 Maximum response of the simple rotor in the present study

Table 7 Details of the discs [17]

Disc No.	Outside diameter (m)	Thickness (mm)	Mass unbalance (kg-m)	Position from the left end of rotor (m)
1	0.24	0.005	0	0.2
2	0.4	0.005	0.2	0.5
3	0.4	0.006	0	1

4. Example 4

Consider a complex rotor as shown in Fig. 7. The rotor consists of a 100 mm diameter uniform steel shaft with length of 1.3 m. It is supported on two identical orthotropic bearings at its ends and carries three rigid discs at different locations within its span. The location, geometry and other properties of the discs are given in Table 6. Density of the shaft material is 7800 kg/m^3 and Young's modulus is 200 GPa. The stiffness and damping coefficients of the two identical bearings at each end are: $K_{yy}=7\times10^7 N/m$, $K_{zz}=5\times10^7 N/m$, $C_{yy}=5\times10^7 Ns/m$, $C_{zz}=5\times10^7 N/m$.

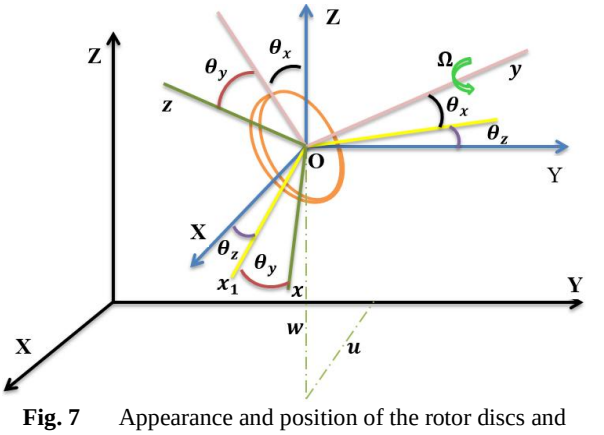

Fig. 7 Appearance and position of the rotor discs and bearings in Example 4 [17]

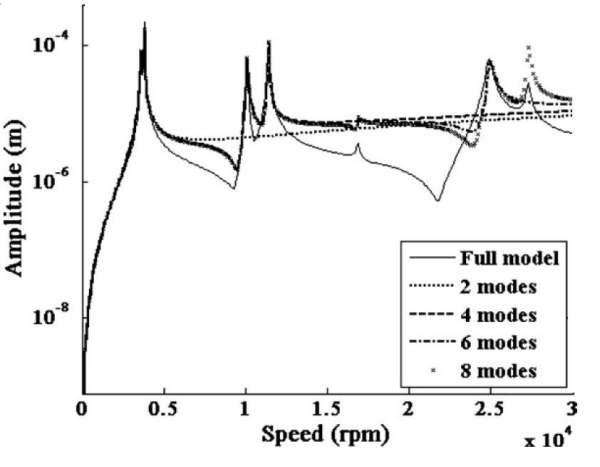

Fig. 8 Frequency response of the rotor of example 4 using the finite element method [17] for point 6

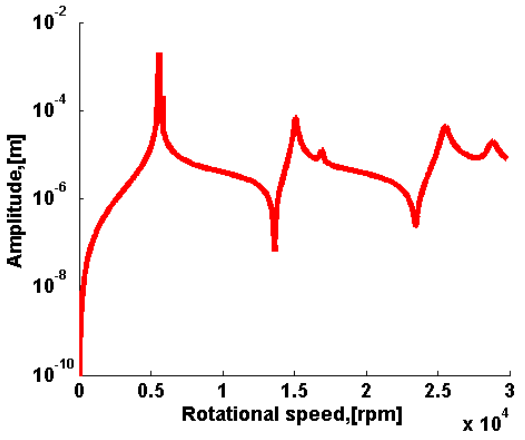

Fig. 9 Frequency response of the rotor of example 4 in the present study for point 6

Frequency response functions at the location of the second disc are shown in Figs. 8 and 9. The result is quite clear that the number of peaks, rotational speed of each peak and amplitude of the peaks are in good agreement with the full model of [17]. Thus, the methods used in this study were capable to model complicated rotor with multiple discs and different conditions, which is comparable to hard and time consuming methods such as finite element.

7 CONCLUSION

In this study the modelling of a rotor-bearing system with desired conditions using the Rayleigh's theory and approximate analytical method of assumed modes was investigated. Various models of the rotor were considered and the results were compared with those of other reliable references. The present work showed that the method used in this research is more efficient for determining the natural frequencies of the rotor, studying of the gyroscopic effects and forced response of the complex rotors. The examples verified the high accuracy along with greater ease and less time consuming of the present method. Thus, the procedure developed in this study can be used for complicated models of rotors apart from the difficulties in selecting mode shapes, higher volume of equations and consuming a lot of time to achieve the desired results.

8 APPENDIX OR NOMENCLATURE

List of Symbols

A	Area of shaft (m^2)
A_m	Amplitude of vibration (m)
C_u	Bearing damping in direction u (N.s/m)
C_w	Bearing damping in direction w (N.s/m)
d_U	Radius of unbalances (m)
E	Young's modulus (GPa)
f	Frequency (Hz)
K_u	Bearing stiffness in direction u (N.s/m)
K_w	Bearing stiffness in direction w (N.s/m)
L	Length of shaft (m)
M_D	Mass of disc(Kg)
m_U	Mass of unbalance (Kg)
Ra	Railey's quotient
I	Moment of the shaft (m^4)
I_{dx}	Moment of inertia of the disc about the axis perpendicular to the shaft ($Kg.m^2$)
I_{dy}	Polar Moment of disc about shaft axis ($Kg.m^2$)
t	Time (s)
u	Displacement in the horizontal direction (m)
w	Displacement in the vertical direction (m)

Greek Symptoms

ρ	Density (Kg/m^3)
Ω	Angular velocity (rad/s)
ω	Natural frequency (rad/s)
φ_U	Phase of unbalance mass (rad)
σ_{yy}	Stress in direction of y axis (N/m^2)
ε_{yy}	Strain in direction of y axis

Subtitles

B	Related to bearing
D	Related to disc
n	Counter of disc, bearing and unbalance mass
U	Related to unbalance mass

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