Multiple Moving Cracks in a Non-Homogeneous Orthotropic Plane

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Abstract

In this paper, a theoretical study of the behavior of multiple moving cracks in a non-homogeneous orthotropic plane under anti-plate deformation is presented. Material properties of the functionally graded (FG) orthotropic plane are assumed to vary exponentially in the *y*-direction. First, the distributed dislocation method is used to perform stress analysis, and the Galilean transformation is used to express the wave equations in terms of the coordinates attached to the moving crack. Then, the solution of the moving screw dislocation in the non-homogeneous orthotropic plane is obtained using the Fourier transform and shows that the stress components have the familiar Cauchy singularity at the location of dislocation. The solution is employed to derive integral equations for a non-homogeneous orthotropic plane weakened by multiple moving cracks. Numerical calculations are performed to show the effects of material properties and the cracks propagating velocity on the dynamic stress intensity factors of crack tips.

Keywords: Dislocation Method, Non-Homogeneous Material, Multiple Cracks, Dynamic Stress Intensity Factor.

1. Introduction

Recently, functionally graded materials (FGMs) have widely been introduced and applied in the environments with extremely high temperature. The major advantages of the graded material, especially in elevated temperature environments, stem from the tailoring capability to produce a general variety of its thermomechanical properties in the spatial domain. The knowledge of crack growth and propagation in functionally graded materials is important in designing components of FGMs and improving its fracture toughness. From the fracture mechanics viewpoint, a crack in FGMs may exhibit the complex behavior due to the variety of the mechanical properties of the material. The increasing attention to the study of crack problems in functionally graded materials in the last decade has led to a lot of significant work. Especially, the influence of the crack moving speed on the stress intensity factors was a popular subject in classical elastodynamics. Problems of crack propagation at constant speed can be classified into three classes depending on the boundary conditions [1]. The first class is the steady-state crack growth. Here, the crack tip moves at constant speed for all the time and the mechanical fields are invariant with respect to an observer moving with the crack tip. The prototype problem in this category is the two-dimensional Yoffe problem of a crack of fixed length propagating in a body subjected to uniform far field tensile loading [2]. The second class of problems is the self-similar crack growth subject to time-independent loading. In this case, the crack tip has been moving at constant speed since some initial instant, and certain mechanical fields are invariant with respect to an observer moving steadily away from the process being observed. The third category of problems corresponds to crack in a body initially at rest and subjected to time-independent loading.

Wang and Meguid [3] obtained the stress field around a moving finite crack that propagates in a non-homogeneous interfacial layer between two dissimilar elastic half-planes under anti-plane loading conditions. The plane strain problem for determining the dynamic stress intensity factor in orthotropic medium when a moving Griffith crack is situated at the interface of two dissimilar half spaces was considered by Das et al. [4]. Asymptotic expansion of out of plane displacement fields for a crack propagating with a constant velocity at an angle to the property gradient was obtained by Chalivendra et al. [5]. Jiang and Wang [6] studied the dynamic plane behavior of a Yoffe type crack propagating in a functionally graded interlayer bonded to dissimilar half planes. The dynamic stress intensity factor and strain energy density for moving crack in an infinite strip of functionally graded material subjected to antiplane shear was determined by Bi et al. [7]. Li [8] solved the dynamic problem of an impermeable crack of length 2a propagating in a piezoelectric strip. The problem for the crack propagating at constant speed in a functionally graded piezoelectric ceramic strip under combined anti-plane shear and in-plane electrical loadings was studied by Kwon [9]. Ma et al. [10] investigated the theoretical analysis of the dynamic plane behavior of a Yoffe type crack [11] propagating in a functionally graded orthotropic medium.

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The elastic stiffness constants and mass density of materials are assumed to vary exponentially perpendicular to the direction of the crack propagation. Numerical examples were given to show the effects of the material properties, the thickness of the functionally graded orthotropic strip and the speed of the crack propagation upon the dynamic fracture behavior. Das [12], Considered the interaction between three moving collinear Griffith cracks under anti-plane shear stress situated at the interface of an elastic layer overlying a different half plane. The problem of a Griffith crack of constant length propagating at a uniform speed in a non-homogeneous plane under uniform load is investigated by Singh et al. [13]. The finite crack with constant length (Yoffe-type crack) propagating in a functionally graded strip with spatially varying elastic properties between two dissimilar homogeneous layers under in-plane loading was studied by Cheng et al. [14].
Yan [15] investigated the problem of a propagating finite crack in functionally graded piezoelectric materials. The solution procedures devised in all above studies are neither capable of handling multiple cracks nor arbitrary arrangement. In the most of studies, FGMs are assumed to be isotropic. However, because of the nature of the techniques used in processing, the graded materials are seldom isotropic. For example, FGMs processed by using a plasma spray technique have generally a lamella structure, whereas processing by electron beam physical vapor deposition would lead to a highly columnar structure. Thus, the orthotropic properties should be considered in studying the mechanics of FGMs. However, due to the problem complexity, up to now, only a few researchers considered the crack problem for functionally graded orthotropic materials.
The primary objective of this study is to provide a theoretical analysis of multiple moving cracks with arbitrary arrangements propagating in a functionally graded orthotropic plane under anti-plane traction. The complex Fourier transform is employed to obtain transformed displacement and stress fields. The inversion of transforming displacement and stress fields is carried out by changing the contour of integration.
The dislocation solutions are then used to formulate integral equations for a plane weakened by several cracks.
The integral equations are of Cauchy singular types which are solved numerically for the dislocation density on the crack.
To confirm the validity of formulations, numerical values of dynamic stress intensity factors for a crack are compared with the results in the literature. Several examples of cracks are solved to study the effects speed of crack on the stress intensity factor of cracks to illustrate the applicability of the procedure.

2. Description of the Problem and Governing Equations

Consider a functionally graded orthotropic plane with moving screw dislocation along x-axis, as shown in Fig. (1). The X- and Y-axes are in the direction of principal material orthotropic.
The distributed dislocation technique is an efficient means for treating multiple moving cracks. However, determining stress fields due to a single dislocation in the region has been a major obstacle to the utilization of this method. We now take up this task for a functionally graded orthotropic plane containing a moving screw dislocation. Under the assumption of anti-plane deformation, the only nonzero displacement component is the out of plane component W(X, Y, t). Consequently, the constitutive equations are given by Eq. (1) [7].

$$\sigma_{zx}(X,Y,t) = \mu_X(Y)\frac{\partial W}{\partial X}$$
$$\sigma_{zy}(X,Y,t) = \mu_y(Y)\frac{\partial W}{\partial Y} \qquad \text{Eq. (1)}$$

where $\mu_y(Y) = \mu_{0X}e^{2\zeta Y}$ and $\mu_y(Y) = \mu_{0Y}e^{2\zeta Y}$ are the shear modulus. Utilizing Eq. (1) in the absence of body forces, the governing equation of dynamic anti-plane deformation in terms of displacement may be written as Eq. (2).

$$\frac{\partial^2 W}{\partial X^2} + \frac{1}{f^2}\frac{\partial^2 W}{\partial Y^2} + \frac{2\zeta}{f^2}\frac{\partial W}{\partial Y} = \frac{1}{C_X{}^2}\frac{\partial^2 W}{\partial t^2} \qquad \text{Eq. (2)}$$

Where $p(y) = p_0 e^{2\zeta Y}$ is the material mass density. Also $C_X = \sqrt{\frac{\mu_{0X}}{p_0}}$ is the characteristic elastic shear wave velocity for the material in the *x* direction. The crack is assumed to be open at one end and closed at the other, to have a constant velocity V, and to maintain its length constant within the plane. Localized crack-tip plasticity and three-dimensional effects are neglected in this formulation. For the current problem of a crack propagating at constant velocity V along the X–direction, it is convenient to employ the Galilean transformation such as Eq. (3) [7].

$$X = x + Vt,\ Y=y,\ \frac{\partial}{\partial t} = -V\frac{\partial}{\partial x} \qquad \text{Eq. (3)}$$

Where (x, y) is the translating coordinate system attached to the propagating crack. Therefore, Eq. (1). becomes independent of time and can be converted into:

$$a^2\frac{\partial^2 w}{\partial x^2} + \frac{1}{f^2}\frac{\partial^2 w}{\partial y^2} + \frac{2\zeta}{f^2}\frac{\partial w}{\partial y} = 0 \qquad \text{Eq. (4)}$$

Where w(x, y) = W(X, Y, t) and a = $\sqrt{(\frac{1-V^2}{C_x^2})}$. Let a Volterra type screw dislocation with Bergers vector b_z be situated at the origin of the coordinate system with the dislocation line x > 0 .

The conditions representing the screw dislocation are:

$$W(x, 0^+) - w(x, 0^-) = b_2 H(x)$$
$$\sigma_{zy}(x, 0^+) = \sigma_{zy}(x, 0^-) \qquad \text{Eq. (5)}$$

Here, H(x) is the Heaviside step-function. The first Eq. (5). shows the multivaluedness of displacement while the second implies the continuity of traction along the dislocation line.

It is worth mentioning that the above conditions for screw dislocation were utilized by several investigators, e. g., Weertman and Weertman [16]. To obtain a solution of the differential Eq. (4). subject to the conditions Eq. (5). , the complex Fourier transform is defined as follows:

$$f^*(\lambda) = \int_{-\infty}^{+\infty} e^{i\lambda x} f(x)dx$$
$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} e^{-i\lambda x} f^*(\lambda)d\lambda \qquad \text{Eq. (6)}$$

In the above equation $i=\sqrt{-1}$, Applying Fourier transform Eq. (6). to Eq. (4). leads to a second order ordinary differential equation for $w^*(\lambda, y)$. The solution to this equation readily known:

$$w^*(\lambda, y) = A(\lambda)e^{(-\zeta-\sqrt{\zeta^2+f^2a^2\lambda^2})y} + B(\lambda)e^{(-\zeta-\sqrt{\zeta^2+f^2a^2\lambda^2})y} \qquad \text{Eq. (7)}$$

where A(λ) and B(λ) are unknown. Application of conditions Eq. (5). to Eq. (7). gives the unknown coefficients. Therefore, the expressions for transformed component of displacement field become:

$$w^*(\lambda, y) = \frac{b_z}{2}(\pi\delta(\lambda)+i/\lambda)e^{(-\zeta-\sqrt{\zeta^2+f^2a^2\lambda^2})y} \qquad \text{Eq. (8)}$$

where б(λ) is the Dirac delta function. The displacement component in view of Eq. (6). and Eq. (8). leads to:

$$w(x, y) = \frac{b_z}{4}e^{-2\zeta y} + \frac{ib_z e^{-\zeta y}}{4\pi}\int_{-\infty}^{+\infty}\frac{e^{-y\sqrt{\zeta^2+f^2a^2\lambda^2}-i\lambda}}{\lambda}d\lambda \qquad \text{Eq. (9)}$$

It is elementary to show that Eq. (9). satisfy the first condition Eq. (5). The associated stress components by virtue of Eq. (1). and Eq. (9)., are given by:

$$\sigma_{zx}(x, y) = \frac{\mu_{0x} b_z e^{\zeta y}}{4\pi}\int_{-\infty}^{+\infty} e^{-y\sqrt{\zeta^2+f^2a^2\lambda^2}-i\lambda x}d\lambda \qquad \text{Eq. (10)}$$

$$\sigma_{zy}(x, y) = \frac{-\zeta\mu_{0y}b_z}{2} - \frac{i\mu_{0y}b_z e^{y\zeta}}{4\pi}\int_{-\infty}^{+\infty}\left(\frac{\zeta+\sqrt{\zeta^2+f^2a^2\lambda^2}}{\lambda}\right) e^{-y\sqrt{\zeta^2+f^2a^2x^2}-i\lambda x}d\lambda \qquad \text{Eq. (11)}$$

The integrals in Eq. (10). and Eq. (11). can be evaluated with the contour integration. For the sake of brevity, the details of manipulation are not given here. The final results are Eq. (12):

$$\sigma_{zx}(x, y) = \frac{\mu_{0x} y\zeta b_z e^{y\zeta}}{2\pi r}\, k_1\left(\frac{r\zeta}{af}\right)$$

$$\sigma_{zy}(x, y) = \frac{\mu_{0y} x\zeta b_z e^{y\zeta}}{2\pi r}\Bigg\{k_1\left(\frac{r\zeta}{af}\right) + \frac{r\zeta}{af}\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-\left(\frac{r\zeta}{af}\right)u}}{\left(\frac{f^2a^2y^2}{r^2-u^2}\right)}du - 2\int_1^{+\infty}\frac{u\sqrt{u^2-1}e^{-\left(\frac{r\zeta}{fa}\right)u}}{\left(\frac{f^2a^2y^2}{r^2-u^2}\right)^2}du - y\zeta\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-\left(\frac{r\zeta}{fa}\right)u}}{u\left(\frac{f^2a^2y^2}{r^2-u^2}\right)}du - \frac{afy}{r}\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-\left(\frac{r\zeta}{fa}\right)u}}{u^2\left(\frac{f^2a^2y^2}{r^2-u^2}\right)}du - \frac{2afy}{r}\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-\left(\frac{r\zeta}{fa}\right)u}}{\left(\frac{f^2a^2y^2}{r^2-u^2}\right)^2}du\Bigg\} \qquad \text{Eq. (12)}$$

Where $k_1(\ldots)$ is the modified Bessel function of the second kind and $r=\sqrt{x^2+f^2a^2y^2}$. From Eq. (12)., it is obvious that stress components are Cauchy singular at dislocation position which is a well-known feature of stress fields due to volterra dislocation.

3. Formulation for Moving Cracks

The dislocation solutions obtained in Section 2 are utilized to analyze functionally graded orthotropic plane weakened by N arbitrary moving straight cracks. The distributed dislocation technique is an efficient means to carry out this task, see for instance [17]. The moving cracks configuration may be described in parametric form as Eq. (13):

$$X_i = x_{0i} + l_i s$$
$$Y_i = y_{0i} \qquad i=1,2,\ldots,N \qquad -1 \le s \le 1 \qquad \text{Eq. (13)}$$

We consider local coordinate systems moving on the face of ith crack. The anti-plane traction on the face of the ith crack in terms of stress components in Cartesian coordinates becomes Eq. (14):

$$\sigma_{nz}(x_i, y_i) = \tau_0 \qquad \text{Eq. (14)}$$

Suppose dislocations with unknown density B_{zj} is distributed on the infinitesimal segment dl_i located at the face of the jth crack where the parameter $-1 \le p \le 1$ and prime denotes differentiation with respect to the relevant argument. The traction on the face of ith crack due to the presence of distribution of dislocations on the face of all N moving cracks yields Eq. (15):

$$\sigma_{nz}\big(x(s), y(s)\big) = \sum_{j=1}^{N}\int_{-1}^{1} k_{ij}(s,p)l_j B_{zj}(p)dp$$
$$i=1,2,\ldots,N. \qquad \text{Eq. (15)}$$

Where from Eq. (12)., the kernel of integral equation is:

$$k_{ij}\left(x_i(s),y_i(s),x_j(p),y_j(p)\right) = \frac{\mu_{0y}\left(x_i(s)-x_j(p)\right)\xi e^{\left(y_i(S)-y_j(p)\right)\xi}}{2\pi r_{ij}}\{k_1\left(\frac{r_{ij}\xi}{af}\right) + \frac{r_{ij}\xi}{af}\int_1^{+\infty}\frac{\sqrt{u^2-le^{-\left(\frac{r_{ij}\xi}{af}\right)u}}}{\frac{(f^2a^2(y_i(S)-y_j(P))^2}{r_{ij}^2}-U^2}du - 2\int_1^{+\infty}\frac{u\sqrt{u^2-le^{-\left(\frac{r_{ij}\xi}{fa}\right)u}}}{\left(\frac{(f^2a^2(y_i(S)-y_j(P))^2}{(r_{ij}^2-u^2)}\right)^2}du - \left(y_i(S)-y_j(P)\right)\xi - \int_1^{+\infty}\frac{\sqrt{u^2-le^{-\left(\frac{r_{ij}\xi}{fa}\right)u}}}{\left(\frac{u(f^2a^2(y_i(S)-y_j(P))^2}{(r_{ij}^2-u^2)}\right)}du - \left(\frac{af\left(y_i(S)-y_j(P)\right)}{(r_{ij})}\right)\int_1^{+\infty}\frac{\sqrt{u^2-le^{-\left(\frac{r_{ij}\xi}{fa}\right)u}}}{\left(\frac{u^2(f^2a^2(y_i(S)-y_j(P))^2}{(r_{ij}^2-u^2)}\right)}du + \left(\frac{2af\left(y_i(S)-y_j(P)\right)}{(r_{ij})}\right)\int_1^{+\infty}\frac{\sqrt{u^2-le^{-\left(\frac{r_{ij}\xi}{fa}\right)u}}}{\left(\frac{(f^2a^2(y_i(S)-y_j(P))^2}{(r_{ij}^2-u^2)}\right)^2}du\}$$

Eq. (16)

We substitute Eq. (14) and (16) into Eq. (15), becomes:

$$\sum_{j=1}^{N}\int_{-1}^{1}[\frac{\mu_{0y}(x_i(s)-x_j(p))\zeta e^{(y_i(s)-y_j(p))\zeta}}{2\pi r_{ij}}\{k_1(r_{ij}\zeta/\alpha f) + \frac{r_{ij}\zeta}{\alpha f}\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-(r_{ij}\zeta/\alpha f)u}}{(f^2\alpha^2(y_i(s)-y_j(p))^2/r_{ij}^2-u^2)}du - 2\int_1^{+\infty}\frac{u\sqrt{u^2-1}e^{-(r_{ij}\zeta/f\alpha)u}}{(f^2\alpha^2(y_i(s)-y_j(p))^2/r_{ij}^2-u^2)^2}du - (y_i(s)-y_j(p))\zeta\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-(r_{ij}\zeta/f\alpha)u}}{u(f^2\alpha^2(y_i(s)-y_j(p))^2/r_{ij}^2-u^2)}du - \frac{\alpha f(y_i(s)-y_j(p))}{r_{ij}}\int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-(r_{ij}\zeta/f\alpha)u}}{u^2(f^2\alpha^2(y_i(s)-y_j(p))^2/r_{ij}^2-u^2)}du + \frac{2\alpha f(y_i(s)-y_j(p))}{r_{ij}}\times \int_1^{+\infty}\frac{\sqrt{u^2-1}e^{-(r_{ij}\zeta/f\alpha)u}}{(f^2\alpha^2(y_i(s)-y_j(p))^2/r_{ij}^2-u^2)^2}du\}]B_{zj}(p)l_j\,dp=\tau_0$$

Eq. (17)

Since the singularity of stress fields for dislocation is of Cauchy type then the Eq. (17). is Cauchy singular equations for unknown dislocation densities. Employing the definition of the dislocation density function, the equation for the crack opening displacement across the jth crack is

$W_j^-(S)-W_j^+(S)=\int_{-1}^{s}l_jB_{zj}(p)dp$, j=1,2,3,.,N Eq. (18)

The displacement field is single-valued for the faces of cracks. Consequently, the dislocation density functions are subject to the following closure requirements

$l_j\int_{-1}^{1}B_{zj}(p)dp=0,$ j=1,2,3,..,N Eq. (19)

The Cauchy singular integral Eq. (17) and Eq. (19) are solved simultaneously. To determine dislocation density functions this task is taken up by the methodology developed by Erdogan et al. [18]. The stress fields in the neighborhood of crack tips behave like $1/\sqrt{r}$ where r is the distance from the crack tip. Therefore, the dislocation densities are taken as

$$B_{zj}(p)=\frac{g_{zj}(p)}{\sqrt{1-p^2}},$$
$$-1\le p\le 1 \quad j=1,2,3,\ldots,N$$

Eq. (20)

Substituting Eq. (20) into Eq. (17) and Eq. (19) and discretizating of the domain, $-1\le p\le 1$, by m+1 segments, we arrive at the following system of Nx2m algebraic equations:

$$\begin{bmatrix} A_{11} & A_{12} & \Lambda & A_{1N}\\ A_{21} & A_{22} & \Lambda & A_{2N}\\ M & M & O & M\\ A_{N1} & A_{N2} & \Lambda & A_{NN}\end{bmatrix}\begin{bmatrix} g_{z1}(p_n)\\ g_{z2}(p_n)\\ M\\ g_{zN}(p_n)\end{bmatrix}=\begin{bmatrix} q_1(s_r)\\ q_2(s_r)\\ M\\ q_N(s_r)\end{bmatrix},$$

Eq. (21)

Where the collocation points are:

$$\begin{cases} s_r=\cos(\frac{\pi r}{m}) & r=1,2,\ldots,m-1\\ p_n=\cos(\frac{\pi(2n-1)}{2m}) & n=1,2,\ldots,m\end{cases}$$

Eq. (22)

The components of matrix in Eq. (21) are:

$$A_{ij}=\frac{\pi}{m}\begin{bmatrix} k_{ij}(s_1,p_1) & k_{ij}(s_1,p_2) & \Lambda & k_{ij}(s_1,p_m)\\ k_{ij}(s_2,p_1) & k_{ij}(s_2,p_2) & \Lambda & k_{ij}(s_2,p_m)\\ M & M & O & M\\ k_{ij}(s_{m-1},p_1) & k_{ij}(s_{m-1},p_2) & \Lambda & k_{ij}(s_{m-1},p_m)\\ \delta_{ij}l_i & \delta_{ij}l_i & \Lambda & \delta_{ij}l_i\end{bmatrix},$$

Eq. (23)

In Eq. (23.), δ_{ij} in the last row of A_{ij} designates the Kronecker delta. The components of vectors in Eq. (21). Are Eq. 24.:

$g_{zj}(P_n)$=[$g_{zj}(P_1)$ $g_{zj}(P_2)$ Λ $g_{zj}(P_m)$]^T,
$q_j(S_r)$ = [$б_{yz}(x_i(S_j(S_1),y_j(S_1))$ $б_{yz}(x_j(S_2),y_j(S_2))$… $б_{yz}(x_j(S_{m-1}),y_j(S_{m-1}))0]^T$ Eq. (24)

Where superscript T stands for the transpose of a vector. The stress intensity factors at the tip of ith crack in terms of the crack opening displacement can be determined as follows

$$k_{Li}=\frac{\sqrt{2}}{4}\mu(y_{Li})\lim_{r_{L_i}\to 0}\frac{W_i^-(s)-W_i^+(s)}{\sqrt{r_{L_i}}},$$
$$k_{Ri}=\frac{\sqrt{2}}{4}\mu(y_{Ri})\lim_{r_{R_i}\to 0}\frac{W_i^-(s)-W_i^+(s)}{\sqrt{r_{R_i}}}$$

Eq. (25)

where L and R designate, the left and right tips of a crack, respectively. The geometry of a crack implies

$$r_{Li}=[(x_i(s)-x_i(-1))^2+(y_i(s)-y_i(-1)^2]^{1/2},$$
$$r_{Ri}=[(x_i(s)-x_i(1))^2+(y_i(s)-y_i(1)^2]^{1/2},$$

Eq. (26)

In order to take the limits for $r_{Li}\to 0$ and $r_{Ri}\to 0$, we should let, in Eq. (26)., the parameter $s\to -1$ and $s\to 1$, respectively.

The substitution of Eq. (20). into E. (18)., and the resultant equations and Eq. (26). into Eq. (25). in conjunction with the Taylor series expansion of functions $x_i(s)$ and $y_i(s)$ around the points $S=\pm 1$ yield:

$$k_{Li}=\frac{\mu(y_{Li})f}{2}\left(\left(x_i'(-1)\right)^2+\left(y_i'(-1)\right)^2\right)^{\frac{1}{4}}g_i(-1),$$
$$k_{Ri}=\frac{\mu(y_{Ri})f}{2}\left(\left(x_i'(1)\right)^2+\left(y_i'(1)\right)^2\right)^{\frac{1}{4}}g_i(1),$$

i=1,2,3,…N. Eq. (27).

The solutions of Eq. (21). are plugged into Eq. (27). thereby the dynamic stress intensity factors are obtained.

4. Results and Discussion

In this section, attention will be focused on the effect of the speed of crack propagation and material properties upon the dynamic stress intensity factors. Several examples are solved to demonstrate the applicability of the distributed dislocation technique. The analysis developed in the preceding section allows the consideration of a functionally graded orthotropic plane with any number of moving straight cracks. The stress distribution around the moving crack tip, are far more complicated than for the case of a stationary crack.

All of the field variables have fields intensity factors and these intensity factors are all dependent on the crack moving velocity. In order to investigate the effects of the materials properties gradient and the crack moving velocity on the stress intensity factors, we now furnish some numerical works to demonstrate the applicability of the applied method. In all examples, the plane is under anti-plane shear deformation with magnitude τ_0 .

First, consider the case where example deals the moving crack is propagating parallel to the x-axis with constant velocity V in the positive X – direction. The moving crack situated at l=1 with different ratios of moduli and FGM constant. In these examples, the effects of material properties and dimensionless crack speed V/C on the dynamic stress intensity factors are investigated. The problem is symmetric with respect to the y-axis. As it may be observed k/k_0, is increased by growing V/C. The values of the normalized stress intensity factors for single crack versus the dimensionless crack speed V/C, are given in Fig. (2). The trend of variation remain the same by changing the FGM constant and the ratio of moduli of elasticity of the non-homogeneous orthotropic plane.

In the second example, we consider two collinear moving cracks with length 2l which are placed on the x-axis. The ratio of the moduli of elasticity of the functionally graded plane is chosen f=1.0,1.2. The graphical representation of the normalized stress intensity factors of crack tips k/k_0, against the V/C, is depicted in Fig. (3). In this figure 2a is the distance between two crack tips. As expected, the value of k/k_0, increases with increasing crack moving velocities. Conversely, with increasing ratios of moduli, the values of the k/k_0 decreases.

In the third example, the non-homogeneous orthotropic plane contains two parallel identical moving cracks with lengths 2l which are also parallel with the *x*-axis. Fig. (4)., shows the variation of normalized stress intensity factors with crack speed V/C, for various values of f. The stress intensity factors of the first crack which located on the vertical distance 2a with x-axis, is higher than the second one, mainly because it is closer to the load.

Consider now the case where the two equal-length cracks are parallel with x-axis are shown in Fig. (5). The length of cracks remain fixed while the crack velocity are changing. The dimensionless stress intensity factors verses the dimensionless crack velocity, for value of FGM constant ξ=1.0, is depicted in Fig. (5). As it might be observed the maximum stress intensity factor for the crack tips occur when the crack velocity is increased.

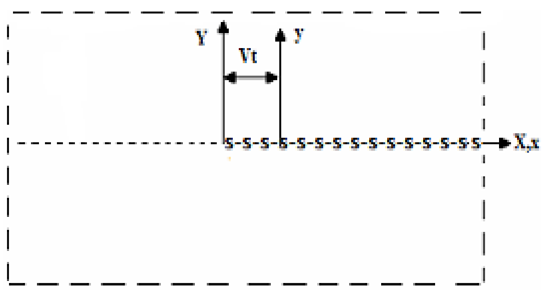

Fig. 1. Schematic view of a non-homogeneous orthotropic plane with a screw dislocation.

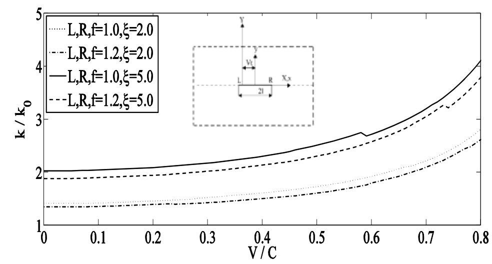

Fig. 2. Normalized stress intensity factors of crack tips versus the dimensionless crack speed.

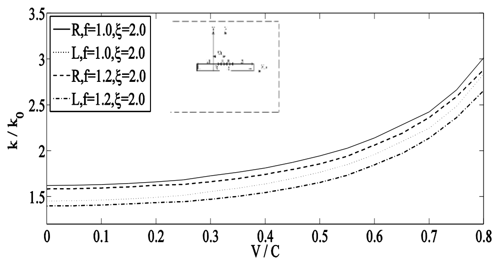

Fig. 3. Normalized stress intensity factor versus the dimensionless crack velocity for different ratio of the moduli.

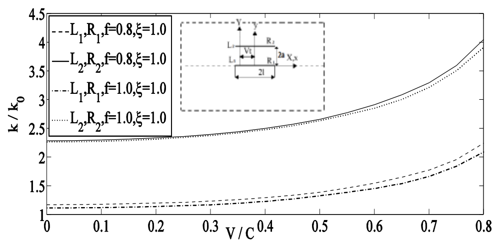

Fig. 4. Variation of Normalized stress intensity factors with V/C.

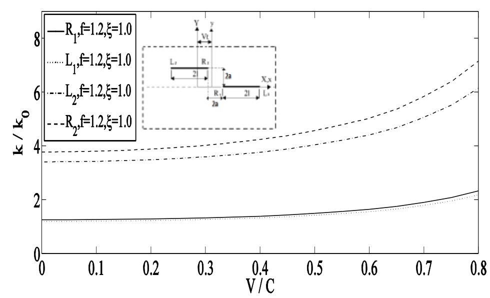

Fig. 5. Normalized stress intensity factors of crack tips versus the dimensionless crack speed.

8. Conclusion

1.The anti-plane stress analysis of a non-homogeneous orthotropic plane weakened by several moving cracks is carried out in this article.
2.The dislocation solutions are used to construct integral equations for a functionally graded orthotropic plane weakened by multiple moving straight cracks.
3.The effect of crack velocity, cracks and the ratio of the module of elasticity of the functionally graded orthotropic plane on the stress intensity factor was studied.
4.To show the applicability of the procedure more examples are solved wherein the interaction between moving cracks is investigated.

References

[1] L. B. Freund and D.. Fract. Mech., Camb. Univers. Press., (1998)
[2] E. H. Yoffe , Philos. Mag. 42, (1951), 739.
[3] X. D. Wang and S. A. Meguid, Elast. prop. Int. J. Fract. Mech. 69, (1995), 87.
[4] S. Das and B. Patra, Eng. Fract. Mech. 54, (1996), 523.
[5] V. B. Chalivendra and A. Shukla, Mater. J. Elast. 69, (2002), 99.
[6] L. Y. Jiang and X. D. Wang, Int. J. Fract., 114, (2002), 225.
[7] X. S. Bi, J. Cheng and X. L. Chen, Theor. Appl. Fract. Mech., 39, (2003), 89.
[8] X. F. Li, Arch. Appl. Mech., 72, (2003), 745.
[9] S. M. Kwon, Acta Mech., 167, (2004), 73.
[10] L. Ma, L. Z. Wu and L. C. Guo, Int. J. Fracture., 136, (2005), 187.
[11] E. H. Yoffe, Philos. Mag., 42, (1951), 739.
[12] S. Das, shear Int. J. Solids Struct., 43, (2006), 7880.
[13] B.M. Singh, J. Rokne, J. Vrbik, J., R.S. Dhaliwal, Eur. J. Mech., 25, (2006), 867.
[14] Z. Cheng, Z. Zhong, Int. J. Mech. Sci., 49, (2007), 1038.
[15] Z. Yan and L.Y. Jiang, Int. J. Solids Struct., 46, (2009), 1362.
[16] J. Weertman and J. R. Weertman, Elementary Dislocation Theory. Oxford University Press, New York, (1992).
[17] J. Weertman, Dislocation Based Fracture Mechanics, World Scientific Publishing Co., Singapore (1996).
[18] F. Erdogan, G. D. Gupta and T. S. Cook, Numerical Solution of Singular Integral Equations, Method of Analysis and Solution of Crack Problems, Edited by G.C. Sih, Noordhoof, Leyden, Holland, (1973).