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Research Article

Another Method for Defuzzification Based On Characterization of Fuzzy Numbers

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Abstract

Here we consider approaches to the ranking of fuzzy numbers based upon the idea of associating with a fuzzy number a scalar value, its signal/noise ratios, where the signal and the noise are defined as the middle-point and the spread of each γ-cut of a fuzzy number, respectively. We use the value of a as the weight of the signal/noise ratio of each γ-cut of a fuzzy number to calculate the ranking index of each fuzzy number. The proposed method can rank any kinds of fuzzy numbers with different kinds of membership functions.

Keywords : Ranking; Fuzzy number; Defuzzification; Signal/noise ratios.

1 Introduction

In many applications, ranking of fuzzy numbers is an important component of the decision process. In addition to a fuzzy environment, ranking is a very important decision making procedure. Since Jain [2, 3] employed the concept of maximizing set to order the fuzzy numbers in 1976(1978), many authors have investigated various ranking methods. Some of these ranking methods have been compared and reviewed by Bortolan and Degani [4], and more recently by Chen and Hwang [5]. Other contributions in this field include: an index for ordering fuzzy numbers defined by Choobineh and Li [6], ranking alternatives using fuzzy numbers studied by Dias [7], automatic ranking of fuzzy numbers using artificial neural networks proposed by Requena et al. [8], ranking fuzzy values with satisfaction function investigated by Lee et al. [9], ranking and defuzzification methods based on area compensation presented by Fortemps and Roubens [10], and ranking alternatives with fuzzy weights using maximizing set and minimizing set given by Raj and Kumar [11]. However, some of these methods are computationally complex and difficult to implement, and others are counterintuitive and not discriminating. Furthermore, many of them produce different ranking outcomes for the same problem. In 1988, Lee and Li [12], proposed a comparison of fuzzy numbers by considering the mean and dispersion (standard deviation) based on the uniform and the proportional probability distributions. Having reviewed the previous methods, this article proposes a method to use the concept of median value, so as to find the order of fuzzy numbers. This method can distinguish the alternatives clearly. The main purpose of this article is that, the median value can be used as a crisp approximation of a fuzzy number. Therefore, by the means of this defuzzification, this article aims to present a new method for ranking of fuzzy numbers. In addition to its ranking features, this method removes the ambi-

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guities resulted from the comparison of previous ranking. In Section 2, we recall some fundamental results on fuzzy numbers. In Section 3, a crisp approximation of a fuzzy number is obtained. In this Section some theorems and remarks are proposed and illustrated. Proposed method for ranking fuzzy numbers is in this section.

2 Basic Definitions and Notations

The basic definitions of a fuzzy number are given in [12, 13, 14, 15, 16, 17, 18, 19] as follows:

Definition 2.1 *A fuzzy number A is a mapping* $\mu_A(x) : \Re \rightarrow [0,1]$ *with the following properties:*

1. μ_A *is an upper semi-continuous function on* $\Re$,
2. $\mu_A(x) = 0$ *outside of some interval* $[a_1, b_2] \subset \Re$.
3. *There are real numbers* a_2, b_1 *such that* $a_1 \leq a_2 \leq b_1 \leq b_2$ *and*
   - 3.1 $\mu_A(x)$ *is a monotonic increasing function on* $[a_1,\ a_2]$,
   - 3.2 $\mu_A(x)$ *is a monotonic decreasing function on* $[b_1,\ b_2]$,
   - 3.3 $\mu_A(x) = 1$ *for all x in* $[a_2, b_1]$.

The set of all fuzzy numbers is denoted by F.

Definition 2.2 *Let* $\Re$ *be the set of all real numbers. We assume a fuzzy number A that can be expressed for all* $x \in \Re$ *in the form*

$$A(x) = \begin{cases} g(x) = \left(\frac{x-a}{b-a}\right)^n & \text{when } x \in [a,b), \\ 1, & \text{when } x \in [b,c], \\ h(x) = \left(\frac{d-x}{d-c}\right)^n, & \text{when } x \in (c,d], \\ 0 & \text{otherwise}. \end{cases} \tag{2.1}$$

where a, b, c, d *are real numbers such that* $a < b \leq c < d$ *and g is a real valued function that is increasing and right continuous and h is a real valued function that is decreasing and left continuous. A fuzzy number A with shape function g and h, where* $n > 0$*, will be denoted by* $A = \langle a,b,c,d \rangle_n$. *If* $n = 1$*, we simply write* $A = \langle a,b,c,d \rangle$*, which is known as a trapezoidal fuzzy number. If* $n \neq 1$*, a fuzzy number* $A^* = \langle a,b,c,d \rangle_n$ *is a concentration of A. If* $0 < n < 1$*, then* A^* *is a dilation of A. Concentration of A by* $n = 2$ *is often interpreted as the linguistic hedge "very". Dilation of A by* $n = 0.5$ *is often interpreted as the linguistic hedge "more or less". More about linguistic hedges can be found in [21].*

Another important notion connected with fuzzy number A is an cardinality of a fuzzy number A.

Definition 2.3 *[1]. Cardinality of a fuzzy number A described by (2.1) is the value of the integral*

$$cardA = \int_a^b A(x)dx = \int_0^1 (b_\alpha - a_\alpha)d\alpha. \tag{2.2}$$

If $A = \langle a,b,c,d \rangle_n$ *then*

$$cardA = \frac{b-a}{n+1} + (c-d) + \frac{d-c}{n+1}. \tag{2.3}$$

In this paper we will always refer to fuzzy number A described by (2.1).

3 A novel method for ranking fuzzy numbers

In this section, we present a new method for ranking fuzzy numbers. The proposed method integrates many concepts, such as the approximate area measure [19], the belief feature [12] and the signal/noise ratio [13]. Assume that a decision maker wants to determine the ranking order of m fuzzy numbers $A_1, A_2, \cdots,$ and A_m. The kth γ-cut $A_i^{\gamma_k}$ of fuzzy number A_i is defined as follows:

$$A_i^{\gamma_k} = \{x | f_{A_i}(x) \geq \gamma_k, x \in X\},\ \gamma_k = \frac{k}{n},\ k \in \{0,1,\cdots,n\},\ n \in N \tag{3.4}$$

where n denotes the number of γ-cuts.
The minimal value $l_{i,k}$ and the maximal value $r_{i,k}$ of the kth γ-cut of the fuzzy number A_i are defined as follows:

$$l_{i,k} = \inf_{x\in X}\{x | f_{A_i}(x) \geq \gamma_k\}. \tag{3.5}$$

$$r_{i,k} = \sup_{x\in X}\{x | f_{A_i}(x) \geq \gamma_k\}. \tag{3.6}$$

respectively. The maximal barrier U and the minimal barrier L of the m fuzzy numbers $A_1, A_2, \cdots,$ and A_m are defined as follows:

$$U = \max_{\forall i}\{x|x \in A_i^{\gamma},\ 0 \leq \gamma \leq h_{A_i},\ 1 = 1, 2, \cdots, m\}, \quad (3.7)$$

$$L = \min_{\forall i}\{x|x \in A_i^{\gamma},\ 0 \leq \gamma \leq h_{A_i},\ 1 = 1, 2, \cdots, m\}. \quad (3.8)$$

where A_i^{γ} denotes the γ-cut of the fuzzy number A_i and h_{A_i} denotes the height of A_i defined as follows:

$$h_{A_i} = \sup_{x \in X} f_{A_i}(x). \quad (3.9)$$

The signal/noise ratio $\eta_{i,k}$ of the kth γ-cut of the fuzzy number A_i used in the proposed method is defined as follows:

$$\eta_{i,k} = \frac{m_{i,k} - L}{\delta_{i,k} + c}, \quad (3.10)$$

where $m_{i,k}$ and $d_{i,k}$ denote the middle-point and the spread of $A_i^{\gamma_k}$, respectively, defined as follows:

$$m_{i,k} = \frac{r_{i,k} + l_{i,k}}{2}, \quad (3.11)$$

$$\delta_{i,k} = r_{i,k} - l_{i,k}. \quad (3.12)$$

L denotes the minimal barrier of the m fuzzy numbers $A_1, A_2, \cdots, A_m$ defined by Eq. (3.8), c is a parameter, and $c > 0$. The parameter $c > 0$ is used to avoid the case that if the fuzzy number A_i is the crisp value "0", the signal/noise ratio will be indeterminate. From Eq. (3.10), we can find that the larger the value of c, the smaller the influence of $\delta_{i,k}$ on the signal/noise ratio $\eta_{i,k}$. Therefore, we think that the influence of $\delta_{i,k}$ on $\eta_{i,k}$ should be smaller than the influence of $m_{i,k}$ on $\eta_{i,k}$. The value of c should be greater than the value of $R - L$ in order to avoid the special case that if we want to obtain the ranking order of two equal crisp values A_1 and A_2, the values of $R-L$ and $\delta_{i,k}$ of the kth γ-cut of the fuzzy number A_1 and A_2 will be all zero and the signal/noise ratio will be indeterminate or undefined, where $\gamma_k \in [0, 1]$. In the following, we present a new approach for comparing fuzzy numbers based on the distance method. The method not only considers the signal/noise ratio of a fuzzy number, but also considers the minimum crisp value of fuzzy numbers. The proposed method for ranking fuzzy numbers $A_1, A_2, \cdots, A_m$ is now presented as follows:

Use the point $(RI(A_j), 0)$ to calculate the ranking value $sn/r(A_j) = D(RI(A_j), x_{min})$ of the fuzzy numbers A_j, where A_j , where $1 \leq j \leq m$, as follows:

$$D(RI(A_j), x_{min}) = \|RI(A_j) - x_{min}\| \quad (3.13)$$

From formula (3.13), we can see that $sn/r(A_j) = D(RI(A_j), x_{min})$ can be considered as the Euclidean distance between the point $(RI(A_j), 0)$ and the point $(x_{min}, 0)$. We can see that the larger the value of $sn/r(A_j)$, the better the ranking of A_j, where $1 \leq j \leq m$. When ranking n fuzzy numbers $A_1, A_2, \cdots, A_m$, the minimum crisp value $x_{\min}$ is defined as:

$$x_{\min} = min\{x|x \in Domain(A_1, A_2, \cdots, A_m)\}. \quad (3.14)$$

The index $RI(A_j)$ of fuzzy numbers A_i is calculated as $RI(A_j) = \frac{h_{A_i} \sum_{k=1}^{n} \gamma_k \times \eta_{i,k}}{\sum_{k=1}^{n} \gamma_k}$, where $\gamma = h_{A_i} \times \frac{k}{n}$, $k \in \{1, 2, \cdots, n\}$, $n \in N$, and n denotes the number of γ-cuts.

3.1 Using The Proposed Ranking Method In Selecting Army Equip System

From experimental results, the proposed method with some advantages: (a) without normalizing process, (b) fit all kind of ranking fuzzy number, (c) correct Kerre's concept. Therefore we can apply median value of fuzzy ranking method in practical examples. In the following, the algorithm of selecting equip systems is proposed, and then adopted to ranking a army example.

3.1.1 An algorithm for selecting equip system

We summarize the algorithm for evaluating equip system as below:

Step 1: Construct a hierarchical structure model for equip system.

Step 2: Build a fuzzy performance matrix $\tilde{A}$. We compute the performance score of the sub factor, which is represented by triangular fuzzy numbers based on expert's ratings, average all the scores corresponding to its criteria. Then, build a fuzzy performance matrix $\tilde{A}$.

Step 3: Build a fuzzy weighting matrix $\tilde{W}$. According to the attributes of the equip systems, experts give the weight for each criterion by fuzzy numbers, and then form a fuzzy weighting matrix $\tilde{W}$.

Table 1: Linguistic values for the ratings

Linguistic value	TFNs
Very Poor(VP)	(0,0,0.16)
Poor	(0,0.16,0.33)
Slightly(SP)	(0.16,0.33,0.5)
Fair(F)	(0.33,0.5,0.66)
Slightly good(SG)	(0.5,0.66.0.83)
Good(G)	(0.66,0.83,1)
Very good(VG)	(0.83,1,1)

Table 2: Basic performance data for five types of main battle Tanks.

Item	Type				
	Tank A	Tank B	Tank C	Tank D	Tank E
Armament	120 mm gun 15.2 mm MG 12.7 mm MG	120 mm gun 15.2 mm MG	120 mm gun 15.2 mm MG	105 mm gun 15.2 mm MG	120mm gun 7.62mm MG 12.7 mm MG
Ammunition	40 1000 11400	Up to 50 4000	42 4750	40 4	44 1500 10000
Smoke grenade discharges	2×6	2×5	2×8	None	2×9
Power to weight ratio(hp/t)	26.2	19.2	27.2	19.0	27.5
Max. road speed(km/h)	67	56	72	60	71
Max. range(km)	480	450	550	300	550
Fording(m)	1.21	1.07	1.0	1.2	1.23
Gradient	60	60	60	55	60
Trench	2.74	2.43	3.00	2.51	2.92
Armor protection	Good	Excellent	Good	Fair	Excellent
Acclimatization	Good	Fair	Good	Fair	Good
Communication	Fair	Fair	Fair	Poor	Fair
Scout	Medium	Medium	Medium	Medium	Good

Table 3: Linguistic values of the importance weights

Linguistic value	TFNs
Very low(VL)	(0,0,0.167)
Low(L)	(0,0.167,0.333)
Slightly	(0.167,0.333,0.5)
Medium(M)	(0.333,0.5,0.667)
Slightly high(SH)	(0.5,0.667,0.833)
High(H)	(0.667,0.833,1)
Very High(VH)	(0.833,1,1)

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix and fuzzy weighting matrix $\tilde{W}$, then get fuzzy aggregative evaluation matrix $\tilde{R}$. (i.e. $\tilde{R} = \tilde{A} \otimes \tilde{W}^t$).
Step 5: Determinate the best alternative. After step 4, we can get the fuzzy aggregative performance for each alternative, and then rank fuzzy numbers by median value of fuzzy numbers.

Table 4: The importance weights of linguistic criteria and its mean

Criteria	Experts			Mean of TFNs
	D_1	D_2	D_3	
Attack ($\tilde{W}_1$)	VH	H	H	(0.72,0.89,1)
Mobility ($\tilde{W}_2$)	VH	H	VH	(0.78,0.94,1)
Self-defence ($\tilde{W}_3$)	M	VH	SH	(0.56,0.72,0.83)
Communication and command ($\tilde{W}_4$)	M	M	M	(0.33,0.5,0.67)

Table 5: Basic performance data for five types of main battle Tanks

Criteria	Type				
	Tank A	Tank B	Tank C	Tank D	Tank E
Attack					
Armament	G	SG	SG	F	SG
Ammunition	VG	SG	SG	F	G
Smoke grenade dischargers	G	SP	VG	VP	VG
Mean	(0.7,0.8,1)	(0.3,0.5,0.7)	(0.6,0.7,0.8)	(0.2,0.3,0.5)	(0.6,0.8,0.9)
Mobility					
Power to weight ratio	G	F	G	F	G
Max. road speed	G	F	VG	SG	VG
Max. range	G	SG	VG	P	VG
Fording/Gradient Trench	G	SG	SG	F	G
Mean	(0.6,0.8,1)	(0.4,0.5,0.7)	(0.7,0.8,0.9)	(0.2,0.4,0.6)	(0.7,0.9,1)
Self-defence					
Armor protection	SG	G	F	F	G
Acclimatization	SG	F	SG	F	G
Mean	(0.5,0.6,0.8)	(0.5,0.6,0.8)	(0.4,0.5,0.7)	(0.3,0.5,0.6)	(0.5,0.7,0.9)
Communication and command					
Communication	G	G	G	F	G
Scout	SG	SG	SG	SG	G
Mean	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.5,0.7,0.9)	(0.4,0.5,0.7)	(0.6,0.8,1)

3.1.2 The selecting of best main battle tank

In [22], the authors have constructed a practical example for evaluating the best main battle tank, and they selected $x_1 = M_1A_1$ (USA), $x_2 = Challenger2$ (UK), $x_3 = Leopard2$ (Germany) as alternatives. In [22], the experts opinion were described by linguistic terms, which can be repressed in triangular fuzzy numbers. The fuzzy Delphi method is adopted to adjust the fuzzy rating of each expert to achieve the consensus condition. The evaluating criteria of main battle tank are $a_1 : attackcapability$, $a_2 : mobilitycapability$, $a_3 : self-defencecapability$ and $a_4 : communicationandcontrolcapability$. In this example, we adopted the hierarchical structure constructed in [22] for selection of five main battle tanks, and the step-by-step illustrations based on Sec. *3.1.1*s algorithm are described bellow:

Step 1: Construct a hierarchical structure model for equip system.

Step 2: Build a fuzzy performance matrix $\tilde{A}$. The basic performance data for five types of main battle tanks are summarized in Table 1. Then based on the linguistic values in Table 2, the fuzzy preference of five tanks toward four criteria are collected and shown in Table 3.

Step 3: Build a fuzzy weighting matrix $\tilde{W}$.

The aggregative fuzzy weights of four criteria, according to the linguistic values of importance in Table 2, are shown in Table 4.

Step 4: Aggregate evaluation. To multiple fuzzy performance matrix $\tilde{A}$ and fuzzy weighting matrix $\tilde{W}$, then get fuzzy aggregative evaluation matrix $\tilde{R} = \tilde{A} \otimes \tilde{W}^t$. Therefore, from Table 4 and 5, we have

$$\tilde{R} = \begin{bmatrix} (0.38, 0.61, 0.82) \\ (0.27, 0.48, 0.69) \\ (0.36, 0.58, 0.77) \\ (0.18, 0.34, 0.55) \\ (0.40, 0.64, 0.84) \end{bmatrix}.$$

Step 5: Determinate the best alternative. According to Eq. 3.14, we can get the signal value of fuzzy numbers of Tanks A-E, which are equal to 0.234, 0.423, 0.236, 0.323 and 0.289, respectively. Therefore, we find that the ordering of median value is $Tank\ A < Tank\ C < Tank\ F < Tank\ D < Tank\ B$. So, the best type of main battle Tank is Tank F.

4 Conclusion

In this paper, we have presented a new approach for ranking of fuzzy numbers. First, we present a new method for ranking fuzzy numbers based on the γ-cuts, the belief features and the signal/noise ratios of fuzzy numbers. The proposed method calculates the signal/noise ratio of each γ-cut of a fuzzy number to evaluate the quantity and the quality of a fuzzy number, where the signal and the noise are defined as the middle-point and the spread of each γ-cut of a fuzzy number, respectively. We use the value of a as the weight of the signal/noise ratio of each γ-cut of a fuzzy number to calculate the ranking index of each fuzzy number. The proposed fuzzy ranking method can rank any kinds of fuzzy numbers with different kinds of membership functions.

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