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Research Article

Malmquist Productivity Index for Two-stage Structures and its Applications in Bank Branches

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Abstract
This study uses the data envelopment analysis (DEA) to assess the progress/regression of decision-making units (DMUs) having a two-stage structure. The progress/regression of these DMU can be assessed in the first stage, the second stage, and the whole system. In the first stage, the progress/regression of bank branches in collecting resources and in the second stage their progress/regression in allocating the resources as well as gaining profit are calculated, and the combination of both types of data is ultimately analyzed. The progress/regression is calculated using two separate indices: the The Malmquist indexand the Meta-Malmquist index. This study applied the proposed models to 20 branches of a commercial bank with two-stage structure. The obtained results were analyzed using GAMS.
Keywords : Data Envelopment Analysis (DEA); Two-stage; The Malmquist productivity index.

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1 Introduction

Todays world is facing rapid developments in commercial activities. This situation requires a performance assessment system in every organization. DEA is a technique for assessing the performance of a set of homogenous DMUs. This assessment includes the calculation of efficiency, rank, progress and regression and so on. Efficiency scores, derived from DEA models, can assess the performance of DMUs relying on some inputs and outputs. Assessment scores derived from standard DEA models range from zero to 1. The primary results of the models classify DMUs to efficient and inefficient units. DMUs with the efficiency score of 1 are considered as efficient and those with efficiency scores of zero are considered as inefficient [1, 2]. The progress/regression of a DMU was first discussed by adhering to the idea of dividing present efficiency to past efficiency. Later, Malmquist (19530 [8] introduced an index due to existence of many problems in early technique. This

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index was the base of Quiz et al (1982) work for building productivity index [4]. The Malmquist index is the most important index for assessing and identifying the extent of the progress of a DMU in a given period and the status of this growth with respect to society growth. This index presents a measure for the progress or regression of a DMU by comparing the growth (downfall) of the DMU with the growth (downfall) of society. Currently, the attentions of researchers analyzing DMUs performance have been attracted to the measurement and analysis of productivity change. Relying on discussions on the calculation of this index using DEA technique, Chen (2003) [5, 8] introduced non-radial The Malmquist index where the opinion of decision makers on the priority of items is taken into account. The advantage of this index is that it can remove the likelihood of inefficiency through non-zero variables. In addition, it is possible to obtain the progress or regression of DMUs in T periods using the Meta-Malmquist productivity index. In the process of calculating Meta-Malmquist productivity index, a DMU is studied in a given period with respect to a society built during T periods of time [6, 11]. Approaching the main target of a bank, its branches divide their tasks to two stages. In other words, any unit can perform two activities: 1) collecting resources and deposits, and 2) allocating resources and gaining profit. However, not all units are able to perform both these activities at higher levels of quality because different factors including geographical position and culture cause them to exhibit good performance in some activities and unacceptable performance in other activities [7, 10]. Therefore, identification of branch performance (progress or regression) in each activity is of high importance, and this is what the present study focuses on. Malmquist productivity and Meta-Malmquist productivity are the most important techniques for calculating the progress/regression of a DMU [9, 12]. This study expands both indices to DMUs with two-stage structure. In DMUs with network structure, each unit has inputs and outputs. The network is considered as a two-stage network positioned in series. Therefore, the outputs of the first stage are considered as the inputs of the second stage. [13, 14] This paper is structured as follows. Section 2 briefly discusses DEA models, the Malmquist index, and the Meta-The Malmquist index. Section 3 introduces the input, intermediate, and output indices of 20 branches of a commercial bank in Tehran and generalizes the Malmquist productivity index and the Meta-Malmquist productivity index to the network structure. Section 4 analyzes the results obtained from the employed models, and Section 5 concludes the paper.

2 Progress and Regression

Assume n DMU_s as $\{(X_j, Y_j) = 1, ..., n\}$ where the j^{th} DMU uses $Xj = (x_{1j}, , x_{mj})$ input vector to generate $Y_j = (y_{1j}, , y_{mj})$ output vector where:

$$X_j \geq 0, X_j \neq 0, Y_j \geq 0, Y_j \neq 0$$

CCR model proposed by Charnes, Cooper and Rhoads (1978) for assessing $DMU_p$is shown as follows [3]:

$$
\begin{aligned}
\theta = \max \quad & \sum_{r=1}^{s} u_r y_{rp} \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ip} = 1, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, j = 1, ..., n, \\
& AU \leq 0, \\
& BV \leq 0, \\
& u_r, v_i \geq 0, r = 1, ..., s, i = 1, ..., m,
\end{aligned} \tag{2.1}
$$

The third and fourth groups of constraints are weight control constraints for the inputs and outputs respectively, and are determined by the manager. However, in some Standard DEA models, the managers opinion is not considered. In real world situations, however, inputs and outputs do not have the same value. Applying weight control is a solution for considering this difference and solving this problem. After solving the following model in four states, the extent of progress and regression is derived via the Malmquist productivity index:

$$
\begin{aligned}
\theta^m(X_p^\tau, Y_p^\tau) = \max \quad & \sum_{r=1}^{s} u_r y_{rp}^\tau \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ip}^\tau = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^t - \sum_{i=1}^{m} v_i x_{ij}^t \leq 0, j = 1, ..., n, \\
& AU \leq 0, \\
& BV \leq 0, \\
& u_r, v_i \geq 0, r = 1, ..., s, i = 1, ..., m,
\end{aligned} \tag{2.2}
$$

where $(X_p^\tau, Y_p^\tau$ shows the coordinates of DMU_p at time τ and ι, $\tau \in t, t+1$. In this way, the progress and regression of DMU_p can be calculated from the following relation:

$$
MPI_p = \frac{\theta_p^{t+1}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^t(X_p^t, Y_p^t)} [\frac{\theta_p^t(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{t+1}(X_p^{t+1}, Y_p^{t+1})} \times \frac{\theta_p^t(X_p^t, Y_p^t)}{\theta_p^{t+1}(X_p^t, Y_p^t)}]^{1/2}, \tag{2.3}
$$

where $MPI_p > 1$ means that DMU_p has progressed. The higher the value of DMU_p, the more would be its progress. Conversely, if $MPI_p < 1$, then DMU_p undergoes regression. Similarly, the smaller the values of DMU_p, the more sever will be its regression. Finally, if $MPI_p = 1$, then DMU_p would demonstrate neither progress nor regression.

Meta-Malmquist productivity index is another productivity index for calculating the progress/regression of DMUs in T periods of time [4]. The proposed model for this index is as follows:

$$\begin{aligned}
\theta_p^m(X_p^\tau, Y_p^\tau) = \max \quad & \sum_{r=1}^{s} u_r y_{rp}^\tau \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ip}^\tau = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^t - \sum_{i=1}^{m} v_i x_{ij}^t \leq 0, j = 1, ..., n, t = 1, ..., T, \\
& AU \leq 0, \\
& BV \leq 0, \\
& u_r, v_i \geq 0, r = 1, ..., s, i = 1, ..., m,
\end{aligned} \tag{2.4}$$

To calculate the progress/regression of DMU_p, the above model is solved T times in order to calculate the following indices:

$$MetaMPI_p = \frac{\theta_p^m(X_p^{t+1}, Y_p^{t+1})}{\theta_p^m(X_p^t, Y_p^t)}, \quad t \in \{1, ..., T-1\} \tag{2.5}$$

If $MetaMPI_p > 1$, then MDU_p shall progress. Similar to the Malmquist productivity index, the higher the index, the more its progress. If $MetaMPI_p < 1$, then MDU_p experiences regression. Again, the smaller the index, the more sever its regression. $MetaMPI_p = 1$ indicates that MDU_p experiences neither progress nor regression.

3 An Applied Example

The collected data in this study included those obtained from 20 branches of a commercial bank with a two-stage structure. The data were collected in 2011 during two different periods. Table 1 shows inputs and outputs

Table 1: Inputs and outputs

Inputs	Intermediate products	Outputs
Gained profit Received interest	Resources	Employees' scores Paid profit

The following model is adopted for calculating the progress/regression of with a two-stage structure:

$$
\begin{aligned}
\theta^m(X_p^\tau, Y_p^\tau) = \max \quad & \sum_{r=1}^{s} u_r y_{rp}^\tau \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ip}^\tau = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^l - \sum_{d=1}^{D} W_d Z_{dj}^l \leq 0, j = 1, ..., n, \\
& \sum_{d=1}^{D} W_d Z_{dj}^l - \sum_{i=1}^{m} V_i X_{ij}^l \leq 0, j = 1, ..., n, \\
& AU \leq 0, \\
& BV \leq 0, \\
& CW \leq 0, \\
& u_r, v_i, w_d \geq 0, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D,
\end{aligned} \tag{3.6}
$$

where $l, \tau \in t, t+1$.
Note that the fourth, fifth, and sixth groups of the constraints serve as constraints for applying the managers control on weights of output, input and intermediate products respectively. After solving the above model in four states, the extent of progress/regression of the first, second and whole system is derived for the studied DMU from the following relation (similar to relation 3):

$$
MPI_p Stage1 = \frac{\theta_p^{S1t+1}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{S1t}(X_p^t, Y_p^t)} [\frac{\theta_p^{S1t}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{S1t+1}(X_p^{t+1}, Y_p^{t+1})} \times \frac{\theta_p^{S1t}(X_p^t, Y_p^t)}{\theta_p^{S1t+1}(X_p^t, Y_p^t)}]^{1/2},
$$

$$
MPI_p Stage2 = \frac{\theta_p^{S2t+1}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{S2t}(X_p^t, Y_p^t)} [\frac{\theta_p^{S2t}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{S2t+1}(X_p^{t+1}, Y_p^{t+1})} \times \frac{\theta_p^{S2t}(X_p^t, Y_p^t)}{\theta_p^{S2t+1}(X_p^t, Y_p^t)}]^{1/2}, \tag{3.7}
$$

$$
MPI_p Overall = \frac{\theta_p^{Ot+1}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{Ot}(X_p^t, Y_p^t)} [\frac{\theta_p^{Ot}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{Ot+1}(X_p^{t+1}, Y_p^{t+1})} \times \frac{\theta_p^{Ot}(X_p^t, Y_p^t)}{\theta_p^{Ot+1}(X_p^t, Y_p^t)}]^{1/2}.
$$

where θ^{S1}, θ^{S2} and θ^0 are the efficiencies of the first stage, the second stage, and the whole system respectively.
The following model is adopted for calculating the progress/regression of DMU_p with

two-stage structure by Meta-Malmquist productivity index:

$$\begin{aligned}
\theta_p^m(X_p^\tau, Y_p^\tau) = \max \quad & \sum_{r=1}^{s} u_r y_{rp}^\tau \\
s.t. \quad & \sum_{i=1}^{m} v_i x_{ip}^\tau = 1, \\
& \sum_{r=1}^{s} u_r y_{rj}^t - \sum_{d=1}^{D} W_d Z_{dj}^t \leq 0, j = 1, ..., n,\ t = 1, ..., T, \\
& \sum_{d=1}^{D} W_d Z_{dj}^t - \sum_{i=1}^{m} V_i X_{ij}^t \leq 0, j = 1, ..., n,\ t = 1, ..., T, \\
& u_r, v_i, w_d \geq 0, r = 1, ..., s, i = 1, ..., m, d = 1, ..., D,
\end{aligned} \tag{3.8}$$

where $\tau \in 1, ..., n$

After running the model for T times (3.8), the progress/regression of DMU_p is computed from the following relations (similar to relation 5):

$$\begin{aligned}
MetaMPI_pStage1 &= \frac{\theta_p^{S1m}(X_p^{t+1}, Y_p^{t+1})'}{\theta_p^{S1m}(X_p^t, Y_p^t)} \\
MetaMPI_pStage2 &= \frac{\theta_p^{S2m}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{S2m}(X_p^t, Y_p^t)}, \\
MetaMPI_pOverall &= \frac{\theta_p^{Om}(X_p^{t+1}, Y_p^{t+1})}{\theta_p^{Om}(X_p^t, Y_p^t)},
\end{aligned} \tag{3.9}$$

where: $\tau \in 1, ..., T-1$

4 Result Analysis

Table 2 lists the progress/regression of each branch in the first and second stages as well as in the whole system based on the Malmquist productivity index after solving four problems generated by model 6.

According to table 2, in the first stage, 10 branches (branches no. 1, 2, 3, 5, 6, 7, 9, 10, 14 and 15) showed regression and 10 branches showed progress. In the second stage, 8 branches (branches no. 4, 11, 12, 13, 16, 17, 18 and 19) show regression and 12 branches show progress. In the whole system, only 3 branches (branches 3, 9 and 14) show regression and 17 branches show progress. As can be seen, a branch with a regression within the whole system shows at least one instance of regression in one of the stages and its progress score will not be necessarily high in the next stage. Assessment of the whole system reveals that branch no. 4 has the highest progress because it shows a progress in the first stage and a very weak regression in the second stage. Therefore, it was introduced as the best branch. On the other side, the assessment of the whole system introduces branch no. 9 as the weakest efficiency among 20 branches. This branch has a strong regression in the first stage and a slight progress in the second stage and finally it was introduced as the

Table 2: progress/regression of both stages as well as whole system derived from Malmquist productivity index

	Stage 1	Stage 2	Whole system
DMU 1	0.261	13.520	3.535
DMU 2	0.894	4.927	4.405
DMU 3	0.122	6.386	0.781
DMU 4	6.009	0.848	5.097
DMU 5	0.099	21.124	2.095
DMU 6	0.559	6.792	3.797
DMU 7	0.283	8.706	2.465
DMU 8	1.484	1.194	1.772
DMU 9	0.090	1.668	0.150
DMU 10	0.386	2.770	1.069
DMU 11	2.233	0.737	1.645
DMU 12	4.686	0.237	1.109
DMU 13	5.024	0.230	1.153
DMU 14	0.428	2.248	0.962
DMU 15	0.800	1.935	1.547
DMU 16	3.615	0.555	2.006
DMU 17	9.038	0.337	3.041
DMU 18	9.841	0.376	3.704
DMU 19	6.996	0.338	2.367
DMU 20	1.105	1.417	1.567

weakest branch. The diagrams of DMUs progress/regression in the first and second stages as well as the whole system, derived from Malmquist productivity index, are displayed in the following figures.

Diagram 1: progress/regression in both stages and the whole system derived from Malmquist productivity index

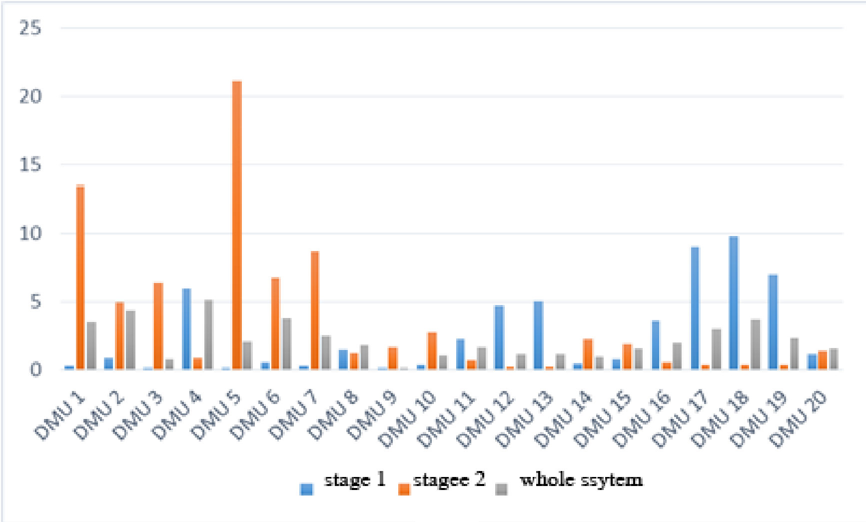


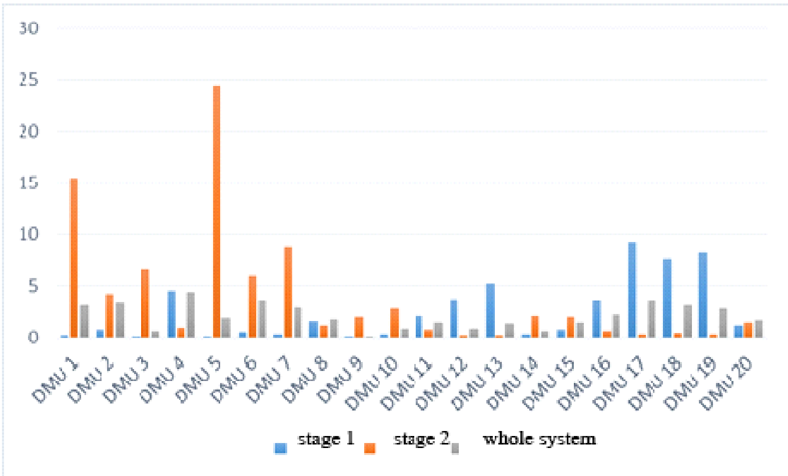
Table 3 lists the progress/regression of each branch in the first and second stages as well as the whole system based on Meta-Malmquist productivity index after solving model (3.8) for T times. Since only data of two periods were available, model (3.8) is solved only twice for each branch.

Table 3: progress/regression in both stages and the whole system derived from Meta-Malmquist productivity index

	Stage 1	Stage 2	The whole system
DMU 1	0.197	15.479	3.056
DMU 2	0.786	4.249	3.339
DMU 3	0.101	6.617	0.670
DMU 4	4.548	0.966	4.393
DMU 5	0.078	24.488	1.916
DMU 6	0.581	6.005	3.489
DMU 7	0.325	8.809	2.864
DMU 8	1.543	1.144	1.765
DMU 9	0.085	1.929	0.163
DMU 10	0.316	2.802	0.886
DMU 11	2.045	0.726	1.484
DMU 12	3.747	0.217	0.813
DMU 13	5.217	0.265	1.380
DMU 14	0.331	2.059	0.682
DMU 15	0.767	1.926	1.477
DMU 16	3.592	0.608	2.182
DMU 17	9.253	0.382	3.537
DMU 18	7.657	0.400	3.062
DMU 19	8.339	0.337	2.814
DMU 20	1.143	1.416	1.619

According to Table 3, in the first stage, 10 branches (branches no. 1, 2, 3, 5, 6, 7, 9, 10, 14 and 15) show regression and 10 branches show progress. In the second stage, however, 8 branches (branches no. 4, 11, 12, 13, 16, 17, 18 and 19) show regression and 12 branches show progress. Regarding the progress/recession of the whole system, 5 branches (branches no. 3, 9, 10, 12 and 14) show regression and 15 branches show progress. It is apparent that Malmquist and Meta-Malmquist indices give the same results, with different values, for the progress/regression in the first and second stages. The results of the whole system are similar to Malmquist productivity index to some extent. Generally, it can be concluded that Meta-Malmquist productivity index is a tighter index than Malmquist productivity index. The diagrams of DMUs progress/regression in the first and second stages as well as the whole system, derived from meta-Malmquist productivity index, are displayed in the following.

Diagram 2: progress/recession in both stages and the whole system derived from Meta-Malmquist productivity index



5 Conclusion

Progress/regression techniques can be used to assess the status of a unit along with its subsets as compared with its past status as well as other units. The Malmquist productivity index and the Meta-Malmquist productivity index are the most important progress/regression techniques used for DMU_s. This paper calculated the progress/regression of homogeneous DMU_s with two-stage structures using multiplicative models with weight limitation. The results revealed that in both techniques, the total progress of branches was higher in stage 2 than in stage 1. In addition, all the branches with a regression in the first stage showed a progress in the second stage. According to the tables and diagrams, illustrated in the analysis section, both indices introduced branch no. 4 as the branch with the highest progress in the whole system.

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