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Research Article

Determining Malmquist Productivity Index in DEA and DEA-R based on Value Efficiency

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Abstract

Malmquist Productivity Index (MPI) is a numeric index that is of great importance in measuring productivity and its changes. In recent years, tools like DEA have been utilized for determining MPI. In the present paper, some models are recommended for calculating MPI when there are just ratio data available. Then, using DEA and DEA-R, some models are proposed under the constant returns to scale (CRS) technology and based on value efficiency (VE) in order to calculate MPI when there is just a ratio of the output to the input data (and vice versa). Finally, in an applied study on 30 welfare service companies under CRS technology, the progress and/or regression of companies are determined in DEA and DEA-R.

Keywords : Malmquist; Value Efficiency; DEA; DEA-R.

1 Introduction

DEA is a nonparametric method to evaluate efficiency, utilized for calculating the relative efficiency and evaluating the performance of a set of DMUs. This method considers a frontier function around the input and output components. This frontier not only is the most efficient units, but also provides an analysis for inefficient units. Farrell (1957) for the first time determined the efficiency by using a non-parametric method [1]. Data Envelopment Analysis (DEA) was the subject of Rhodes study. The results of his initial research in cooperation with Cooper and Charnes were published in 1978 [2]. By introducing the CCR model, Charnes et al. (1978), in fact, extended Farrells idea to multiple inputs and multiple outputs. DEA models are divided into two basic groups: input-oriented and output-oriented models. In input-oriented models, inputs decrease by keeping the status quo for outputs and in output-oriented models, outputs increase by keeping the status quo for inputs. Returns to scale is a concept that expresses the ratio of inputs to outputs. This ratio can be constant or variable, i.e., it can be increasing or decreasing. Banker et al. (1984) introduced models of variable returns to scale [3].

In 1953, Stan Malmquist, a Swedish economist, introduced Malmquist index as an indicator of living standards [4]. Productivity is one of the concepts in studying performance over time. Productivity index is based on pairwise comparison, which generally refers to the efficiency of an organization over two different periods. To calculate efficiency, DEA and Malmquist Index are utilized. This index made it possible to divide total productivity into two major components of the change in allocative technological efficiency and

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technical efficiency. Then, Caves et al. (1982) utilized this index for the first time in the production theory [5]. Färe et al. (1994) used DEA for calculating Malmquist index [6]. Balf et al. (2010) divided this index into two factors of change in the efficiency and technology [7].

Maudos et al. (1999) performed, for the first time, the total factor productivity measurement [8]. Then, Tulkens (1995) paid attention to the non-parametric efficiency in this context [9]. Chen (2003) made use of non-radial Malmquist productivity index for the Chinese industry [10]. Chen and Ali (2004) calculated Malmquist productivity index by using DEA [11]. Jesus et al. (2005) calculated global Malmquist productivity index with a new viewpoint [12]. Since the evaluation of decision-making units using a common set of weights (CSW) is of great importance, Kao (2010) proposed Malmquist productivity index based on the CSW and conducted an applied research on the data of the Taiwan forests [13]. Wang and Lan (2011) suggested a new method to compute Malmquist productivity index [14].

Along with the dramatic growth of DEA and the focus on the input and output data, the topic of ratio data was introduced. With the integration of DEA and Ratio analysis, Despic et al. (2007) suggested the ratio-based DEA (DEA-R) [15]. Wei et al. (2011) showed the false inefficiency in 21 Taiwan medical centers using DEA-R models [16]. Afterwards, Wei et al. (2011) studied the problems of the CCR model in DEA and the advantages of DEA-R over the previous model [17]. In addition, Wei et al. (2011) computed the efficiency and super-efficiency by developing input-oriented DEA-R models under constant returns to scale technology [18]. Once DEA-R was introduced for ratio data, from a different point of view Liu et al. (2011) proposed DEA models without explicit inputs and studied 15 research institutes in China [19]. Mozaffari et al. (2012) studied the relationship between DEA and DEA-R models [20]. Also, Mozaffari et al. (2013) compared cost and revenue efficiency in DEA and DEA-R [21].

Many real problems in organizations can be modeled into a multi-objective program to achieve Pareto-optimal solutions. Pareto-optimal solutions in an organization can be determined with regard to preferences of the decision maker (DM). Korhonen (1986) solved a multiple criteria problem using interactive methods [22]. In addition, Joro et al. (1998) compared DEA and multi-objective linear programming [23]. At this time, Hamle et al. (1999) suggested a new method in data envelopment analysis by using value efficiency (VE) [24]. Korhonen and Hamle (2000) also raised the subjects of VE with weights restrictions and later Korhonen et al. (2002) extended the topic so that value efficiency came to the fore [25, 26]. In this regard, Korhonen et al. (2005) and Soleimani-damaneh et al. (2014) studied VE and paid special attention to its applications [27, 28]. Hamle and Korhonen (2013) proposed a new model for benchmarking in heterogeneous units using value efficiency analysis [29]. Hamle et al. (2014) conducted value efficiency analysis of branches of a bank and proposed FDH models based on VE [30]. Korhonen et al. meticulously studied the relationship between DEA and VE [31].

In the second section of the article, the basic concepts of MPI and value efficiency are briefly reviewed. In the third section, the MPI model is provided based on VE in DEA. In the fourth section, first the calculation of MPI in DEA-R is presented, and then the model and relevant theorems are provided. Computing model of MPI based on the VE in DEA-R will also be proposed. In the fifth section, an example is provided in two subsections: numerical example and applied study. Finally, some conclusions are stated.

2 Basic concepts

In this section, first a brief review of the efficiency index in DEA is provided. Then, the concept of value efficiency is defined.

2.1 The productivity index in DEA

Numerical indicators play an important role in measuring productivity and its changes. In this respect, numerous indicators have also been proposed, each with specific characteristics, among which Malmquist Productivity Index (MPI) is the most prominent. Malmquist productivity index has been introduced as an index that measures the changes in total factor productivity with the separation of its components. The significant feature of MPI is that, unlike other major indicators used to measure total factor productivity, there is no need with this index to have the production cost and product cost data, which are sometimes difficult or impossible to access. It requires no behavioral assumptions such as profit maximization or cost minimization. Meanwhile, the attractive feature of this index is to decompose the impact of the change in technical efficiency and the impact of the change in technological change. The mathematical model of Malmquist productivity index is defined based on the distance function in which the change in total factor productivity between two points of the data is measured by calculating the ratio of the distance of each data point relative to a common technology. Distance Function has many applications in the field of economics, including the fact that it is possible to utilize it to measure and analyze the efficiency and productivity. The distance function can be defined and analyzed based on two points: one of them is based on inputs or production factors, as the input-oriented distance function of production that focuses on the minimum use of production factors and the other is based on the output, as the output-oriented distance function that focuses on the maximum production or output. The point that should be emphasized is that in the measurement of Malmquist productivity changes index the feature of returns to scale in production is of great significance. Grifell-Tatje. and Lovell (1995) showed that in case that it is assumed production technology has variable returns to scale, Malmquist productivity index may not properly measure the changes in total factor productivity, so it is important in calculation of distance functions and measurement of Malmquist index the assumption of constant returns to scale to be applied [28].

Suppose the jth decision-making unit produces outputs $Y_j^t = (y_{1j}^t, ..., y_{sj}^t)$ with the consumption of inputs $X_j^t = (x_{1j}^t, ..., x_{mj}^t)$ at time t. Also, suppose the decision-making unit oth produces outputs $Y_o^{t+1} = (y_{1o}^{t+1}, ..., y_{so}^{t+1})$ with the consumption of outputs $X_o^{t+1} = (x_{1o}^{t+1}, ..., x_{mo}^{t+1})$ at time $t+1$. Known as output-oriented DEA model under CRS technology, model (2.1) evaluates DMUo at time $t+1$ and other DMUs at time t.

$$\begin{array}{lll} \varphi^* = & Max \quad \varphi & \\ s.t & \sum_{j=1}^{n} \lambda_j x_{ij}^t + s_i^- = x_{io}^{t+1}, & \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t - s_r^+ = \varphi y_{ro}^{t+1}, & \forall r \\ & s_i^- \geq 0, \quad s_r^+ \geq 0, & \forall i, \forall r, \\ & \lambda_j \geq 0 \quad \forall i & \end{array} \tag{2.1}$$

The value of the objective function in model (2.1) for decision-making unit O at time t and for other units at time $t+1$ is represented by the following symbol:

$$(\varphi^*)^{-1} = D_o^t(X_o^{t+1}, Y_O^{t+1}) \tag{2.2}$$

Therefore, Malmquist productivity index in DEA is calculated by the following equation:

$$MD_o = \left(\frac{D_o^t(X_o^{t+1}, Y_O^{t+1}) D_o^{t+1}(X_o^{t+1}, Y_O^{t+1})}{D_o^t(X_o^t, Y_O^t) D_o^{t+1}(X_o^t, Y_O^t)}\right)^{1/2} \tag{2.3}$$

In linear programming model (2.1) the value of φ^* can be calculated in the first phase and then in the second phase the maximum value of slack variables (s_i^-, s_r^+) can be calculated. Units in model (2.1) are called efficient if $\varphi^* = 1$ in the first phase and all optimal solutions of slack variables be zero, i.e. $(s_i^{-*}, s_r^{+*}) = (o, o)$, in the second phase.

2.2 Value Efficiency (VE)

In this section, initial definitions of cone and the cone of feasible directions are provided and then the value efficiency model is defined and presented.

Definition 2.1 *A nonempty set defined in an n-dimensional Euclidean space R^n is called a cone with vertex x, if $x + y \in G_X \Rightarrow x + \lambda y \in G_X$ for all R^n. The cone with the origin as vertex is denoted by G. Note that vertex. A singleton $\{x\}$ is also a cone with vertex x.*

Definition 2.2 *Let X is a nonempty polytope in R^n and let $x \in X$. The cone $D(x)$ in R^n is called the cone of feasible directions of X at x, if $D(x) = \{d | x + \lambda d \in X, for \quad all \lambda \in (0, \delta) for \quad \delta > 0\}$. Each $d \in D(x)$, $\quad d \neq 0$, is called a feasible direction. The cone $G_X = \{y | y = x + d, d \in D(x)\}$ is called the tangent cone of X at x and the cone $W_X = \{s | s = y + z, z \in R^n\}$ the augmented tangent cone of X at x.*

In general, the output-oriented technical efficiency of the decision-making units in DEA is equal to the ratio of the output of the unit under analysis to the output obtained by stretching the radial line which passes through the origin and cuts the DEA frontier. However, in computing the technical efficiency in DEA units called MPS do not play any role, though it is the manager that determines MPS and uses it in the analysis. Of course, VE enjoys problems such as using the approximate function of VE. But due to the use of manager views for the introduction of MPS and taking into account the managers idea in the analysis of the efficiency and also measuring the distance difference with MPSs, VE can be used following DEA models. VE scale can be easily calculated and it is required to display MPS as non-negative linear combination (with the condition of constant returns to scale) of units that are on DEA efficiency frontier. Khorhoneh proposed the output-oriented DEA model for calculating value efficiency (VE) under CRS as follows:

$$\begin{array}{ll} Max & \sigma \\ s.t & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{rj} - \sigma y_{ro} - s_r^+ = y_{ro}, \forall r \\ & s_i^- \geq 0, \quad s_r^+ \geq 0, \qquad \forall i, \forall r, \\ & \lambda_j : \begin{cases} \geq 0 & if \lambda_j^* = 0 \\ = free & if \lambda_j^* > 0 \end{cases} \end{array} \tag{2.4}$$

In model (2.4) λ_j^* is calculated from the equation $(X^*, Y^*) = (\sum_{j \in MPS} \lambda_j^* x_{ij}, \sum_{j \in MPS} \lambda_j^* y_{rj})$. In addition, (X^*, Y^*) is located on the CRS frontier.

Definition 2.3 *DMU0 is called value efficiency if in model (2.4) $\sigma^* = 0$ and in all optimal solutions the slack variables are zero, i.e. $(s_i^{-*}, s_r^{+*}) = (o, o)$.*

3 MPI based on VE in DEA

To calculate the output-oriented Malmquist Productivity Index (MPI) under CRS technology the following model is proposed based on the efficiency value:

$$\begin{array}{rl} \alpha^* = & Max \quad \alpha \\ s.t & \sum_{j=1}^{n} \lambda_j x_{ij}^t + s_i = x_{io}^{t+1}, \quad \forall i \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t - \alpha y_{ro}^{t+1} - s_r = y_{ro}^{t+1}, \forall r \\ & s_i \geq 0, \quad s_r \geq 0, \qquad \forall i, \forall r, \\ & \lambda_j : \begin{cases} \geq 0 & if \lambda_j^* = 0 \\ = free & if \lambda_j^* > 0 \end{cases} \end{array} \tag{3.5}$$

The value of objective function (3.5) for decision-making unit o at time $t+1$ and in case that other units are at time t is represented by the symbol below:

$$(1 + \alpha^*)^{-1} = E_o^t(X_o^{t+1}, Y_O^{t+1}) \tag{3.6}$$

The efficiency index in DEA is calculated based on the value efficiency from the following equation:

$$ME_o = \left(\frac{E_o^t(X_o^{t+1}, Y_O^{t+1}) E_o^{t+1}(X_o^{t+1}, Y_O^{t+1})}{E_o^t(X_o^t, Y_O^t) E_o^{t+1}(X_o^t, Y_O^t)}\right)^{1/2} \tag{3.7}$$

In general, in DEA the scale efficiency is calculated from basic input- and/or output-oriented DEA models without the taking into account the manager's idea in the analysis. But in models proposed Joro et al. [27] the value efficiency is calculated based on the MPS and the manager's idea. The difference model (2.1) and (3.5) is only in determining the variable symbol of λ_j.

4 Computing MPI for ratio data

In this section, first computing models of MPI in DEA-R and then computing models of MPI based on VE in DEA-R are suggested.

4.1 MPI in DEA-R

Suppose j^{th} decision-making unit with the consumption of inputs, $X_j^t = (x_{1j}^t, ..., x_{mj}^t)$, at time t produces outputs, $Y_j^t = (y_{1j}^t, ..., y_{sj}^t)$, at time t. Suppose also that the ratio of output to input are defined at two successive times t and $t+1$.

$$\begin{array}{ll} Max & \theta \\ s.t & \sum_{j=1}^{n} \mu_j (\frac{y_{rj}^t}{x_{ij}^t}) - \theta (\frac{y_{ro}^{t+1}}{x_{io}^{t+1}}) - s_{ir} \\ & = (\frac{y_{ro}^{t+1}}{x_{io}^{t+1}}) \quad \forall i, \forall r, \\ & \sum_{j=1}^{n} \mu_j = 1, \\ & s_{ir} \geq 0, \qquad \forall i, \forall r, \\ & \mu_j \geq 0, \qquad \forall j. \end{array} \tag{4.8}$$

Model (4.8) is a linear programming problem introduced as the output-oriented DEA-R for the evaluation of DMUo at time $t+1$ when the other units are at time $t+1$ [15, 16]. Using optimal model (4.8), we have:

$$(\theta^*)^{-1} = H_o^t(X_o^{t+1}, Y_O^{t+1}) \tag{4.9}$$

Malmquist Productivity Index (MPI) in DEA-R is obtained.by equation (4.10)

$$MH_o = \left(\frac{H_o^t(X_o^{t+1}, Y_O^{t+1})H_o^{t+1}(X_o^{t+1}, Y_O^{t+1})}{H_o^t(X_o^t, Y_O^t)H_o^{t+1}(X_o^t, Y_O^t)}\right)^{1/2} \tag{4.10}$$

In this section, theorems that states the relationship among the proposed models in the DEA and DEA-R in order to calculate Malmquist Productivity Index are provided.

Theorem 4.1 *In models (2.1) and (4.8) in case of one input and s output there is* $H_o^t(X_o^{t+1}, Y_O^{t+1}) = D_o^t(X_o^{t+1}, Y_O^{t+1})$ *(the scale efficiency obtained from models (2.1) and (4.8) in case of one input and s output are equal.)*

Proof. Consider the output-oriented multiplier model under constant returns to scale technology for evaluating DMU_o in DEA as following.

$$\begin{array}{ll} Min & \sum_{i=1}^m v_i x_{io} \\ s.t & \sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \le 0 \quad \forall j, \\ & \sum_{r=1}^s u_r y_{ro} = 1, \\ & v_i \ge 0 \quad u_r \ge 0, \quad \forall i, \forall r. \end{array} \tag{4.11}$$

In addition, consider the output-oriented model in multiplier form with constant returns to scale technology for analyzing DEA-R as follows [17].

$$\begin{array}{ll} Min & \theta \\ s.t & \sum_{i=1}^m \sum_{r=1}^s w_{ir} \left(\frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{ro}}{x_{io}}}\right) \\ & \sum_{i=1}^m \sum_{r=1}^s w_{ir} = 1, \\ & s_{ir} \ge 0, \quad \forall i, \forall r. \end{array} \tag{4.12}$$

Without any change in the generality of the argument, assume there is one input and s outputs. So, considering the value of the objective function as θ, we have $v_1 x_{1o} = \theta$, then $v_1 = \frac{\theta}{x_{1o}}$. Also, with defining $u_r = \frac{1}{y_{rp}} w_{1r}$ we have $\sum_{r=1}^s u_r y_{rp} = \sum_{r=1}^s \frac{1}{y_{rp}} w_{1r}(y_{rp}) = 1$. Furthermore, with the placement of v_1 and u_r in the provision $\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \le 0$ we have:

$$\sum_{r=1}^s \frac{1}{y_{ro}} w_{1r} y_{rj} - \frac{\theta}{x_{1o}} x_{ij} \le 0$$

$$\Longrightarrow \sum_{r=1}^s \frac{1}{y_{ro}} w_{1r} y_{rj} \le \frac{\theta}{x_{1o}} x_{ij}$$

$$\Longrightarrow \sum_{r=1}^s w_{ir} \frac{\frac{y_{rj}}{x_{ij}}}{\frac{y_{ro}}{x_{io}}} \le \theta$$

If we consider DMU_o at time $t+1$ and $DMU_j \quad j \ne 0$ at time t, model (4.11) is the dual model (2.1) and also model (4.12) is the dual model (4.8) in case of one input and s outputs. So the optimal values of models (2.1) and (4.8) are equal.

Theorem 4.2 *In models 2 and 9 in case of m inputs and s outputs, we have:*

$$H_o^t(X_o^{t+1}, Y_O^{t+1}) \le D_o^t(X_o^{t+1}, Y_O^{t+1})$$

Proof. For evaluating, the DMU_o output-oriented DEA multiplier model under CRS technology taken from the idea of Despic et al. [15] is presented as below:

$$\begin{array}{l} \varphi^*{}_{DEA} = Min_{(u,v)} \quad Max_j \quad \frac{\sum_{r=1}^s u_r(\frac{y_{rj}}{y_{ro}})}{\sum_{i=1}^m v_i(\frac{x_{ij}}{x_{io}})} \\ \sum_{r=1}^s u_r = 1, \sum_{i=1}^m v_i = 1 \\ u_r \ge 0, v_i \ge 0. \quad i = 1,..,m, \quad r = 1,..,s. \end{array} \tag{4.13}$$

Harmonic efficiency models for evaluating DMU_o based on the idea of Wei et al. [16] is presented as follows.

$$\begin{array}{l} \varphi^*_{H-DEA-R} = \\ Min_{(u,v)} Max_j (\sum_{r=1}^s u_r(\frac{y_{rj}}{y_{ro}})).(\sum_{i=1}^m v_i(\frac{x_{ij}}{x_{io}})) \\ \sum_{r=1}^s u_r = 1, \sum_{i=1}^m v_i = 1 \\ u_r \ge 0, v_i \ge 0. \quad i = 1,..,m, \quad r = 1,..,s. \end{array} \tag{4.14}$$

In models (4.13) and (4.14) definitions of are considered and applied. By multiplying model (4.13) in the expression $\frac{\sum_{i=1}^m v_i(1/X'_{ij})}{\sum_{i=1}^m v_i(1/X'_{ij})}$ we have: $Min_{(u,v)} \quad Max_j \quad \frac{\sum_{r=1}^s u_r(Y'_{rj})}{\sum_{i=1}^m v_i(X'_{ij})} \times \frac{\sum_{i=1}^m v_i(1/X'_{ij})}{\sum_{i=1}^m v_i(1/X'_{ij})}$; therefore, using the equation $\sum_{i=1}^m v_i(X'_{ij}) \times \sum_{i=1}^m v_i(1/X'_{ij}) \ge 1$ the relationship between models (4.13) and (4.14) can be obtained as follows.

$$\varphi^*_{DEA} = Min_{(u,v)} Max_j \frac{\sum_{r=1}^{s} u_r(Y'_{rj})}{\sum_{i=1}^{m} v_i(X'_{ij})} \times \frac{\sum_{i=1}^{m} v_i(1/X'_{ij})}{\sum_{i=1}^{m} v_i(1/X'_{ij})} \leq Min_{(u,v)} Max_j \sum_{r=1}^{s} u_r(Y'_{rj}). \sum_{i=1}^{m} v_i(1/X'_{ij})$$

$$\varphi^*_{DEA} \leq \varphi^*_{H-DEA-R} \Longrightarrow \frac{1}{\varphi^*_{DEA}} \geq \frac{1}{\varphi^*_{H-DEA-R}}$$

This means:
Now, considering DMU_o at time $t+1$ and DMU_j $j \neq 0$ at time t and model (4.13) is equal to the **dual model (2.1)** and model (4.14) is equal to dual model (4.8), so

$$H^t_o(X^{t+1}_o, Y^{t+1}_O) \leq D^t_o(X^{t+1}_o, Y^{t+1}_O)$$

4.2 MPI based on the VE in DEA-R

In this section, first the output-oriented DEA-R models based on the value efficiency in CRS technology is recommended to calculate MPI. The related theorems to the proposed models are then provided. Therefore, considering DMU_o at time $t+1$ and DMU_j such that $j \neq 0$ at time t, the value efficiency models in the output-oriented DEA-R are proposed as following:

$$\begin{aligned} Max \quad & \beta \\ & \sum_{j=1}^{n} \mu_j(\frac{y^t_{rj}}{x^t_{ij}}) - \beta(\frac{y^{t+1}_{ro}}{x^{t+1}_{io}}) - sir = (\frac{y^{t+1}_{ro}}{x^{t+1}_{io}}) \\ s.t \\ & \mu_j : \begin{cases} \geq 0 & if \quad \mu^*_j = 0 \\ = free & if \quad \mu^*_j > 0 \end{cases} \\ & \sum_{j=1}^{n} \mu_j = 1, \quad s_{ir} \geq 0, \end{aligned} \tag{4.15}$$

In model (4.15) the optimal value is calculated from the equation

$$\sum_{j \in MPS} \mu^*_j = 1, \quad (X^*, Y^*) = (\sum_{j \in MPS} \mu^*_j x_{ij}, \sum_{j \in MPS} \mu^*_j y_{rj}).$$

The productivity index in DEA-R based on the value efficiency is calculated from the following equation.

$$(1+\beta^*)^{-1} = R^t_o(X^{t+1}_o, Y^{t+1}_O) \tag{4.16}$$

$$MR_o = (\frac{R^t_o(X^{t+1}_o, Y^{t+1}_O) R^{t+1}_o(X^{t+1}_o, Y^{t+1}_O)}{R^t_o(X^t_o, Y^t_O) R^{t+1}_o(X^t_o, Y^t_O)})^{1/2} \tag{4.17}$$

In GAMS program constraints of model (4.15) with the separation of free variables in symbols and nonnegative variables is as follows.

```
Equations
    Objective, Con1(i,r), Con2;
Objective..   z = e = Teta;
Con1(i,r)..
    Sum(j, (y1(j,r)/x1(j,i)) * Lambda(j))
     + Sum(p, (y1p(p,r)/x1p(p,i)) * mu(p))
     - Teta * (y2o(r)/x2o(i)) - s(i,r)
     = e = y2o(r)/x2o(i);
Con2..
    Sum(j, Lambda(j))
     + Sum(p, mu(p)) = e = 1;
    FileResults/Results.txt/;
    Modele_Model/All/;
```

The reason of proposing model (4.15) based on the ideas of Korhonen is presented as follows [27].

Lemma 4.1 *Let* $\Lambda = \{\mu = (\mu_1, ..., \mu_n) | \sum_{j=1}^{n} \mu_j = 1, \quad \mu_j \geq 0 \quad j = 1, ..., n\}$ *be a nonempty polytope and* $\mu^o \in \Lambda$ *an arbitrary point. Then* $G_{\mu^o} = \Lambda^O$, $\Lambda^O = \{(\mu_1, ..., \mu_n) | \sum_{j=1}^{n} \mu_j = 1, \mu_j \geq 0 \; if \; \mu^o_j = 0, and \; otherwise \; \mu^o_j \; is \; free, \; j = 1, ..., n\}$

Proof. Clearly the tangent cone of an affine set $\Lambda_a = \{(\mu_1, ..., \mu_n) | \sum_{j=1}^{n} \mu_j = 1\}$ at μ^o is Λ_a itself. Moreover, the tangent cone of the closed half-space $H_j = \{\mu = (\mu_1, ..., \mu_n) | \quad \mu_j \geq 0 \quad j = 1, ..., n\}$ at μ^o is R^n, if $\mu^o_j > 0$ and H_j if $j = 1, ..., n$. Because Λ is the intersection of Λ_a and the half-spaces H_j, $j = 1, ..., n$, the tangent cone of Λ at μ^o is the intersection of their tangent cones, respectively, i.e., set Λ^o.

Lemma 4.2 *Let* $T_R = \{\frac{y}{x} | \sum_{j=1}^{n} \mu_j (\frac{y_{rj}}{x_{ij}}) = \frac{y}{x}, \mu \in \Lambda\}$ *where,* $\Lambda = \{(\mu_1, ..., \mu_n) | \sum_{j=1}^{n} \mu_j = 1, \ \mu_j \geq 0 \ j = 1, ..., n\}$*, be a linear transformation of a nonempty polytope* Λ *and* $(\frac{y_o}{x_o}) \in T_R$ *an arbitrary point. Let* $\mu^0 \in \Lambda$ *be any point such that* $\sum_{j=1}^{n} \mu_j^o (\frac{y_{rj}}{x_{ij}}) = \frac{y_o}{x_o}$*. Then the tangent cone of* T_R *at* $(\frac{y_o}{x_o})$ *is* $G_{(\frac{y_o}{x_o}) = \sum_{j=1}^{n} \mu_j^o (\frac{y_{rj}}{x_{ij}}) = \{\frac{y}{x} | \frac{y}{x} = \sum_{j=1}^{n} \mu_j (\frac{y_{rj}}{x_{ij}}), \mu \in G_{\mu^o}\}}$.

Proof. Any $\mu \in G_{\mu^o}, \frac{y}{x} \neq \frac{y_o}{x_o}$, defines a feasible direction $(\frac{y}{x} - \frac{y_o}{x_o})$ for T_R at $(\frac{y_o}{x_o})$, which must be generated by a feasible direction $(\mu - \mu^o)$ for Λ at μ^o. Thus $G_{\frac{y_o}{x_o}} \subset (\sum_{j=1}^{n} \mu_j (\frac{y_{rj}}{x_{ij}})) G_{\mu^o}$. Any $\mu \in G_{\mu^o}, (\mu \neq \mu^o)$ defines a feasible direction $(\mu - \mu^o)$ for Λ at μ^o, which defines a feasible direction $(\frac{y}{x} - \frac{y_o}{x_o})$ for T_R at $(\frac{y_o}{x_o})$. Thus $G_{\frac{y_o}{x_o}} \supset (\sum_{j=1}^{n} \mu_j (\frac{y_{rj}}{x_{ij}})) G_{\mu^o}$.

Theorem 4.3 W_{u^*} *is the largest cone with the property* $W_{u^*} \subset V = \{u | v(u) \leq v(u^*),\ for\ any\ v \in E(u^*)\}$.

Proof. See [27] Let $u^* = [\frac{y^*}{x^*}] \in T_R$ be the DMs most preferred solution. Then $u^* \in T_R$, an arbitrary point in the input/output space, is value inefficient with respect to any strictly increasing pseudoconcave value function $v(u)$, $u = [\frac{y}{x}]$ with a maximum at point u^*, if the optimum value Z^* of the following problem is strictly positive:

$$\begin{aligned} &Max \quad Z = \theta \\ &s.t \quad \sum_{j=1}^{n} \mu_j (\tfrac{y_{rj}}{x_{ij}}) - \theta(\tfrac{y_{ro}}{x_{io}}) - s_{ir} \\ &\qquad = (\tfrac{y_{ro}}{x_{io}}) \quad \forall i, \forall r, \\ &\qquad \sum_{j=1}^{n} \mu_j = 1, \\ &\qquad s_{ir} \geq 0, \qquad \forall i, \forall r, \end{aligned} \tag{4.18}$$

Where $\mu^* \in \Lambda$ correspond to the most preferred solutions, $\frac{y}{x} = \sum_{j=1}^{n} \mu_j^* (\frac{y_{rj}}{x_{ij}})$.

Proof. By Lemmas 4.1 and 4.2 the tangent cone of T_R at u^* is the set where $T_R = \{\frac{y}{x} | u = \sum_{j=1}^{n} \mu_j (\frac{y_{ri}}{x_{ij}}) = \frac{y}{x}, \mu \in \Lambda\}$, where the tangent cone of Λ at μ^* is $G_{\mu^*} = \{(\mu_1, ..., \mu_n) | \sum_{j=1}^{n} \mu_j = 1, \mu_j \geq 0 if \mu_j^* = 0,\ and\ otherwise\ \mu_j^*\ is\ free,\ j = 1, ..., n\}$. The augmented tangent cone W_{u^*} of T at u^* is the set $\{\frac{y}{x} | \frac{y}{x} = \sum_{j=1}^{n} \mu_j (\frac{y_{rj}}{x_{ij}}) + d_{xy}, d_{xy} \leq 0 \quad, \mu \in \Lambda\}$. Therefore (8) hasa solution with $\theta \geq 0$ only if $[\frac{y}{x}] \in W_{u^*}$. Now let (Z^*, θ^s, μ^s) be a solution of (4.8). With $\varepsilon > 0, Z^8 > 0$ only if either $\theta^s > 0$ or $\theta^s = 0$ and $(s_{ir}) \neq 0$. In either case, $[\frac{y^s}{x^s}] \in W_{u^*}$ and $\frac{y^s}{x^s} = \sum_{j=1}^{n} \mu^s j (\frac{y_{rj}}{x_{ij}}) \geq \frac{y}{x}$ and $\frac{y^s}{x^s} \neq \frac{y}{x}$. Thus $v(\frac{y}{x}) \leq v(\frac{y^s}{x^s}) \leq v(\frac{y^*}{x^*})$, and by Theorem 4.1, (y,x) is value inefficient.

Definition 4.1 *The (weighted) value efficiency score for point* $u^o = [\frac{y_o}{x_o}]$ *is defined as Where* θ^s *is the value of* θ *at the optimal solution of problem (4.18).*

Therefore, model (4.15) is of great significance in calculating MPI when the ratio data of the output to input is available.

5 Numerical Example

In this section, first five decision-making units are considered in the numerical example for calculating MPI in DEA and DEA-R. Then an applied study on 30 welfare companies is provided in order to compare Malmquist productivity index in DEA and DEA-R.

5.1 Numerical Example

In this section, five decision-making units with two outputs (o1 and o2) and one input (I1) are considered in calculating Malmquist productivity index in DEA and DEA-R and the efficiency is treated based on the value efficiency. Figure 1 shows the CRS production possibility set of five decision-making units at two times of t and $t+1$ with two outputs and one input. All units at time t and $t+1$ are on the efficiency frontier and are compared using Malmquist efficiency index in the table below. Dashed line shows the frontier of five decision-making units (D1, A1, B1, C1, E1) at time t and solid line displays the frontier of five decision-making units (D2, A2, B2, C2, E2) at time $t+1$. In Table 2, the second up to fifth

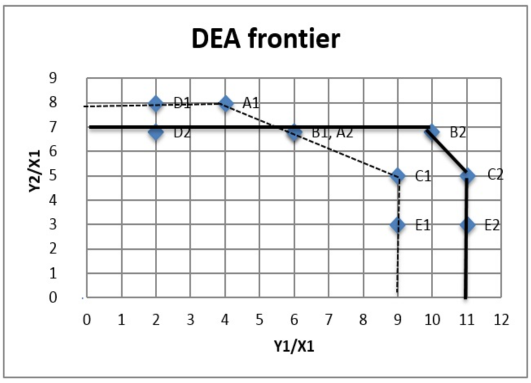

Figure 1: The production possibility set at time t and $t+1$.

columns show the scale efficiencies of model (2.1) and (4.8) and the sixth column displays the value

Table 1: Input and Output data at time t and $t+1$

	DMU	I1	O1	O2	DMU	I1	O1	O2
A	A1	1	4	8	A2	1	6	6.8
B	B1	1	6	6.8	B2	1	10	6.8
C	C1	1	9	5	C2	1	11	5
D	D1	1	2	8	D2	1	2	6.8
E	E1	1	9	3	E2	1	11	3

Table 2: The value efficiency and the value of MPI at times t and $t+1$

DMU	D11	D22	D21	D12	MD=MH	ALL DMU
A	1	1	1.1765	1	0.921943	-
B	1	1	1	1.2308	1.109414	+
C	1	1	0.8548	1.2222	1.195746	+
D	1	1	1.1765	0.85	0.849989	-
E	1	1	0.8182	1.2222	1.222198	+

Table 3: The value of MPI based on the scale efficiency at times t and $t+1$ (units B and C are MPS)

DMU	D11	D22	D21	D12	ME=MR	DMU3, DMU4
A	1	0.7097	0.6129	1	1.076075	+
D	0.8846	0.4194	0.4677	0.7692	0.883033	-
E	0.8077	0.9194	0.7742	0.9231	1.164998	+

of MPI in DEA and DEA-R, i.e. models (2.3) and (4.10). According to Theorem 4.1, since in the example there are multiple inputs and one output, it is observed that the efficiency and value of MPI obtained by equations (2.3) and (4.10) are exactly equal. Overall, units B, C and E show progress at sequential times t and $t+1$, while units D and A show regression at same times. In Table 3, with considering units B and C as MPS by the manager, Malmquist productivity index was compared based on VE in DEA and DEA-R. Unit A is compared with MPS units, i.e. B and C. Therefore, the second output of unit A1 which is 8 compared to the second output of unit B1 which is 6.8 showed an increase, and similarly at time $t+1$ the second output of unit A2 which is 6.8 is constant compared to the second output of unit B2 which is 6.8. Generally, the results of models of unit A showed a progress considering MPS of B and C. Of course, the mount of progress in unit A is less than unit E, as shown in Table 3 below. Similarly, with MPI value equal to 0.883033 unit D had a regression at two consecutive times.

5.2 Applied Study

In this section, 30 welfare companies, which provide retirees with utilities such as seasonal outings, meals and holding special celebrations are to be analyzed. In Table 4, input and output data related the second quarter of 2014 and 2015 are presented. The government has a plan to evaluate the productivity index of all companies during two consecutive periods. The problem is that many companies do not provide real data of their current liability, current cost and asset as well as total assets for the government.

The welfare companies do not require a large investment for providing services and just the management of welfare services to retirees is important because the cost of the ticket, food and services are pre-paid. Therefore, welfare companies can provide services with low capital. The input and output data of 30 welfare companies in the second quarter of 2014 and 2015 are presented in Table 4 and 5, respectively. The first input I1 is related to the current liability and the

Table 4: Input and output data related to the second quarter of 2014

DMU	I1	I2	O1	O2
1	1200	35800	58991	8549600
2	8540	85210	48555	8549700
3	9870	23320	23869	1278900
4	4571	75400	89700	2255140
5	3500	35100	69800	6980000
6	2530	45000	48510	8627500
7	3148	95780	71450	3879000
8	2385	74500	24851	6987000
9	3569	65100	35888	5846000
10	3458	35880	78910	3489000
11	2358	35570	35840	4589000
12	3148	78440	23150	3275000
13	2215	96800	65700	1857000
14	3158	78469	98710	4866700
15	7548	36900	85477	3798000
16	3258	23660	72782	2147000
17	1478	45890	36910	3586000
18	1748	33700	57651	3333000
19	5489	72400	32826	2187000
20	3125	39800	78541	1222000
21	9854	41220	14780	9685400
22	3125	36900	35890	6597700
23	8974	50000	21587	2147800
24	3125	35600	93887	5477800
25	2548	47000	31253	3255000
26	3125	37800	56100	2100000
27	1233	81200	7418	8899000
28	1478	51390	25867	4158000
29	1369	36900	87651	9833000
30	2544	35000	93900	8800000

second input I2 is related to the cost of the company. Current liabilities include bank overdrafts, taxes and other obligations that are reasonably expected to be. The first output O1 is related to current assets the and second output O2 is related to total assets in US Dollars. Current assets consisted of cash, temporary investments, and prepaid expenses.

However, from the viewpoint of government welfare companies are required to provide financial guarantees and have sufficient experience in providing related services. As observed in this study, the companies under study first refused to provide the related input and output data defined in the previous sections as they tried to show the best of their companies. This means that they only provided the ratio data, i.e. the ratio of output data to input data; though, the government obtained the necessary data with inspection and using backup data. Therefore, in order to calculate the Malmquist productivity index, we consider two viewpoints.

A) Input and output data are available

At the end of 2014 and 2015, the government is able to collect data by making quarterly backups and using online data, although problems such as the malfunction of data transmission systems or the change of evaluation criteria for companies (inputs and outputs) still exist in this regard.

B) A ratio of output data to input data is available

This is the case when the government requires the input and output data from companies, but they just provide a ratio of output data to input data. In this case, the ratio data are as follows:

The ratio $\frac{O1}{I1}$ is the ratio of companies current assets to the current liability, which is defined as quick ratio.

The ratio $\frac{O1}{I2}$ is the ratio of current assets of a company to its expenses and indicates the extent

Table 5: Input and Output Data of the Second Quarter of 2015

DMU	I1	I2	O1	O2
1	1200	35800	58991	8549600
2	8540	85210	48555	8549700
3	9870	23320	23869	1278900
4	4571	75400	89700	2255140
5	3500	35100	69800	6980000
6	2530	45000	48510	8627500
7	3148	95780	71450	3879000
8	2385	74500	24851	6987000
9	3569	65100	35888	5846000
10	3458	35880	78910	3489000
11	2358	35570	35840	4589000
12	3148	78440	23150	3275000
13	2215	96800	65700	1857000
14	3158	78469	98710	4866700
15	7548	36900	85477	3798000
16	3258	23660	72782	2147000
17	1478	45890	36910	3586000
18	1748	33700	57651	3333000
19	5489	72400	32826	2187000
20	3125	39800	78541	1222000
21	9854	41220	14780	9685400
22	3125	36900	35890	6597700
23	8974	50000	21587	2147800
24	3125	35600	93887	5477800
25	2548	47000	31253	3255000
26	3125	37800	56100	2100000
27	1233	81200	7418	8899000
28	1478	51390	25867	4158000
29	1369	36900	87651	9833000
30	2544	35000	93900	8800000

to which a company can continue to operate.
In Table 6, the efficiency scores of model (2.2) and model (4.9) are presented in the second to fifth columns and in the sixth to ninth columns, respectively. According to Theorem 4.2, as the applied study is based on two inputs and two outputs, it is observed that the efficiency scores of model (2.2) are greater than or equal to those of model (10).
The managers determined a combination of units 29 and 30 as the MPS. Accordingly, Malmquist productivity index is presented based on DEA and DEA-R in Table 7. Considering the extensive experience of companies 29 and 30 in providing welfare services, the manager insists on calculating their efficiencies and then Malmquist productivity index. When the input and output data are available, Malmquist productivity index can be calculated by models (2.3) and (3.7). However, if only the ratio of output data to input data is available, Malmquist productivity index can be calculated by models (4.10) and (4.17). Using management viewpoints, computing technical efficiency and Malmquist productivity index according to management viewpoints, and applying the managers views are of great importance as a tool in discussing value efficiency analysis and DEA [27].
In Table 7, a combination of units 29 and 30 is the MPS. In addition, the values obtained by the four models (2.3), (4.10), (3.7), and (4.17) are represented by DEA, DEA-R, VDEA and VDEA-R, respectively, in the second to fifth columns of Table 7.
Considering Table 7, ME and MR represent the productivity index in DEA and DEA-R with the value efficiency structure. Companies (1, 2, 4, 5, 6, 9, 10, 11, 13, 18, 20, 22, 24, 27, 28) in DEA and DEA-R based on the value efficiency have shown regression in two periods, but companies

Table 6: The scale efficiencies of DEA and DEA-R in 2014 and 2015 (solutions of Model 2 and 9)

DMU	D11	D22	D21	D12	H11	H22	H21	H12
1	0.9916	0.0142	0.7383	0.0956	0.9912	0.0100	0.7383	0.0956
2	0.3765	0.0744	0.1251	0.277	0.3765	0.0564	0.1068	0.277
3	0.351	0.3495	0.0421	1.7746	0.351	0.2443	0.0294	1.7746
4	0.4614	0.0461	0.2572	0.1973	0.4402	0.0308	0.1883	0.1973
5	0.7739	0.1144	0.2884	0.3152	0.7739	0.105	0.22	0.3152
6	0.7195	0.0623	0.3534	0.1158	0.7195	0.0456	0.3534	0.1105
7	0.3545	0.2137	0.2814	0.559	0.3545	0.2065	0.2133	0.559
8	0.4077	0.1573	0.3036	0.4127	0.4074	0.1514	0.3036	0.4127
9	0.337	0.0861	0.1697	0.2299	0.337	0.0814	0.1697	0.2299
10	0.7782	0.0772	0.3275	0.2457	0.7499	0.0507	0.2247	0.2457
11	0.4841	0.1569	0.2017	0.514	0.4841	0.1025	0.2017	0.514
12	0.1567	0.3359	0.1078	0.4504	0.1567	0.3359	0.1078	0.4504
13	0.4633	0.1667	0.3498	0.2166	0.4633	0.1297	0.2767	0.2166
14	0.5236	0.5465	0.3961	0.6906	0.5167	0.4093	0.2954	0.688
15	0.7787	0.2539	0.1846	1.3753	0.7787	0.1763	0.1202	1.3753
16	1	0.3374	0.3439	2.7384	1	0.24	0.2265	2.7384
17	0.39	1	0.3089	1.6931	0.39	1	0.2544	1.6931
18	0.6835	0.0641	0.4273	1.457	0.6562	0.0641	0.3144	1.457
19	0.1678	0.2616	0.0809	0.3262	0.1629	0.2059	0.058	0.3259
20	0.726	0.1279	0.343	0.4966	0.6941	0.0953	0.2443	0.4966
21	0.8818	0.2014	0.1698	1.1364	0.8818	0.1395	0.1184	1.1364
22	0.671	0.0752	0.2426	0.4504	0.671	0.0503	0.2219	0.4504
23	0.1674	0.0969	0.0386	0.3119	0.1674	0.0791	0.0282	0.3119
24	0.9507	0.3325	0.4218	0.5134	0.9235	0.3039	0.294	0.5134
25	0.266	0.6169	0.1595	2.7865	0.266	0.4024	0.1348	2.7865
26	0.541	0.1528	0.2483	1.5683	0.5174	0.1133	0.175	1.5683
27	1	0.1814	0.7479	0.4846	1	0.1716	0.7479	0.4846
28	0.3913	0.1388	0.2915	0.3002	0.391	0.0981	0.2915	0.2871
29	1	1	0.8046	2.5355	1	1	0.7458	2.5355
30	1	1	0.4933	25.4196	1	1	0.3772	25.4196

8 and 14 have progressed in VDEA and regressed in VDEA-R.

DEA and DEA-R models based on the value efficiency are models (3.7) and (4.17), respectively, and their results are shown in Table 7. Companies 1 and 6 enjoy the lowest values of output-oriented Malmquist productivity index and show the highest regression. Companies 3 and 30 enjoy the highest values of output-oriented Malmquist productivity index and show the highest progress. Similarly, in DEA-R based on the vale efficiency, there are Max and Min values in Malmquist productivity index for the aforementioned companies and they present similar behaviors in DEA and DEA-R models.

Companies 3, 7, 12, 14, 15, 16, 17, 19, 21, 23, 25, 29 and 30 have increasing MPI index in DEA and DEA-R. If based on VE the MPI index is considered in DEA and DEA-R, the difference of columns are 1.042678 and 0.804642 Unit 8 have a progress in VDEA, i.e. it shows progress comparing with MPSs; though, it shows regression in the normal sate. Unit 12 in VDEA and VDEA-R have less progress in comparison to the normal state, i.e. the selecting MPS for unit 12 is not appropriate. However, compared to the normal state unit 16 shows more progress in VDEA and VDEA-R, i.e. selecting MPS for 16 units is suitable. Overall, the highest progress in VDEA is related to units 3 and 25, respectively and the highest regression in VDEA is related to unit 1 and 6, respectively. Similar behavior was noted for VDEA-R.

In general, the following strategy is recommended for applied study: companies 1 and 6 should revisit their inputs and outputs and/or replace them with other companies. In addition, companies 3 and 30 that enjoy the highest MPI values are based on DEA and DEA-R have very good or appropriate progress during 2014 and 2015. Sim-

Table 7: MPI values in DEA, DEA-R, VDEA, and VDEA-R in 2014 and 2015.

DMU	MD (DEA)	MH(DEA-R)	ME(VDEA)	MR(VDEA-R)	MPI
1	0.043061	0.036144	0.056821	0.039538	-
2	0.661478	0.62332	0.679656	0.64045	-
3	6.478571	6.481638	7.16385	7.403469	+
4	0.276847	0.270763	0.278493	0.422834	-
5	0.401944	0.440894	0.390627	0.42848	-
6	0.168442	0.140772	0.188621	0.12781	-
7	1.094304	1.235556	1.253748	1.567933	+
8	0.724203	0.710753	1.042678	0.804642	-/+
9	0.588322	0.572039	0.624904	0.568972	-
10	0.27281	0.271896	0.25934	0.290557	-
11	0.908809	0.734553	0.886416	0.439771	-
12	2.99268	2.99268	1.126882	1.861228	+
13	0.472015	0.468127	0.511831	0.671516	-
14	1.348982	1.358284	1.336193	0.956768	+/-
15	1.55858	1.609487	1.594438	1.839579	+
16	1.639098	1.703413	1.683363	2.455505	+
17	3.748867	4.130955	1.51968	2.683071	+
18	0.565488	0.672821	0.453161	0.673722	-
19	2.507209	2.664991	2.301096	2.942039	+
20	0.505037	0.528296	0.454005	0.840846	-
21	1.23635	1.23223	1.410553	1.405852	+
22	0.456143	0.390071	0.390443	0.292927	-
23	2.16271	2.286092	2.084681	2.203612	+
24	0.652452	0.758056	0.2752	0.465366	-
25	6.365254	5.592072	5.692803	4.415272	+
26	1.33564	1.400868	1.341855	1.930466	+
27	0.342838	0.333448	0.983526	0.559972	-
28	0.604402	0.4971	0.956087	0.446643	-
29	1.775178	1.843829	MPS	MPS	
30	7.178419	8.209156	MPS	MPS	

ilarly, units 29 and 30 are considered as MPS, units 3 and 25 as units with the highest progress and units 1 and 6 as units with the greatest regressions in both VDEA and VDEA-R.

6 Conclusion

In DEA when data are ratio the scale efficiencies and then MPI cannot easily be identified. With using DEA-R it is possible to find MPI for ratio data beside problems such as (input-oriented) false-inefficiency using ε as a non-Archimedean number which leads to weight restrictions in DEA. The relationship between MPI in DEA and DEA-R and classification of units in this evaluation is very important. In this paper, in addition to calculating MPI in DEA and DEA-R the discussion of VE analysis has also been used. With introducing units as MPS, VE applies some criteria according to the management viewpoints that have important roles in calculating technical efficiency and MPI. Although the efficiency index of all companies in the applied study using DEA and DEA-R models has been determined with two views of real data and ratio data, the use of VE and VDEA and VDEA-R models allows the manager to compare progress and regression when introducing MPS. As advantages of the proposed research, we can first mention the presentation of models that determine MPI with ratio data and without considering management viewpoints and then the models that easily determine MPI with applying management viewpoints and using MPS. For future research, computing MPI using non-radial DEA-R and DEA models and also utilizing ZW methods are recommended.

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