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Research Article

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A Study on Intuitionistic Fuzzy and Normal Fuzzy M-Subgroup, M-Homomorphism and Isomorphism

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Abstract

In this paper, we introduce some properties of an intuitionistic normal fuzzy m-subgroup of m- group with m-homomorphism and isomorphism. We study the image, the pre-image and the inverse mapping of the intuitionistic normal fuzzy m-subgroups.

Keywords : Intuitionistic Fuzzy Sets; M-Groups; Intuitionistic Fuzzy M-Subgroups; Intuitionistic Normal Fuzzy M-Subgroups; M-Homomorphism.

1 Introduction

In 1971 Rosenfeld. A [8] introduced the concept of fuzzy subgroups. In 1981 Wu [10] studied the normal fuzzy subgroups. Gu. Wx et al [3] further studied in 1994 the fuzzy groups theory and gave some new concepts such as fuzzy m-subgroups, normal fuzzy m-subgroups. Several mathematicians have followed them in investigating the fuzzy m-subgroups in [5, 6, 9].The intuitionistic fuzzy set idea was first published by Atanassov [1, 2] as a generalization of the fuzzy sets notion. The basic concepts of intuitionistic fuzzy subgroups are in [4, 7]. In this paper, we introduce some properties of an intuitionistic normal fuzzy m-subgroups of m-groups with m-homomorphism and isomorphism and we study the image, pre-image and other properties in this subject.

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2 Preliminaries

Definition 2.1 *[3] Let G be a group,M be a set, if*

(i) $mx \in G \quad \forall x \in G,\ x \in M$.

(ii) $m(xy) = (mx)y = x(my) \quad \forall x,y \in G,\ x \in M$.

Then m is said to be a left operator of G, M is said to be a left operator set of G. G is said to be a group with operators. We use phrase "G is an M-group" in stead of a group with operators. If a subgroup of M-group G is also M-group, then it is said to be an M subgroup of G.

Definition 2.2 *[1] An intuitionistic fuzzy subse* μ *in a set* X *is defined as an object of the form* $\mu = \{< x, \delta_\mu(x), \lambda_\mu(x) >; x \in X\}$, *where* $\delta_\mu : X \to [0,1]$ *and* $\lambda_\mu : X \to [0,1]$ *define the degree of membership and the degree of non- membership of the element* $x \in X$ *respectively and for every* $x \in X$ *satisfying* $0 \leq \delta_\mu(x) + \lambda_\mu(x) \leq 1$. *All the intuitionistic fuzzy sets on* X *are written as* $IFS(X)$*for short.*

Definition 2.3 *[11]Let X, Y be a non empty classical sets, $\Phi : X \to Y$ be a mapping and $\mu = \{y \in Y, \delta_\mu(y), \lambda_\mu(y)\}$ be an intuitionistic fuzzy set on Y ($\mu \in IFS(Y)$ $\Psi_\Phi^{-1} : IFS(Y) \to IFS(X)$ is the inverse mapping induced by Φ, the pre- image $\Psi_\Phi^{-1}(\mu) = \{x \in X;\ \Psi_\Phi^{-1}(\delta_\mu)(x),\ \Psi_\Phi^{-1}(\lambda_\mu)(x)\}$. Where $\Psi_\Phi^{-1}(\delta_\mu)$, $\Psi_\Phi^{-1}(\lambda_\mu)$ obey the classical extension principle of Zadeh. L. A.*

Definition 2.4 *[11]Let X, Y be a non empty classical sets, $\Phi : X \to Y$ be a mapping and $\mu = \{y \in Y, \delta_\mu(y), \lambda_\mu(y)\}$ be an intuitionistic fuzzy set on Y ($\mu \in IFS(Y)$ $\Psi_\Phi : IFS(Y) \to IFS(X)$ is the inverse mapping induced by Φ, the image $\Psi_\Phi(\mu)$ of $\mu$is an intuitionistic fuzzy set on Y, and define $\Psi_\Phi(\mu) = \{y \in Y;\ \Psi_\Phi(\delta_\mu)(y),\ \Psi_\Phi(\lambda_\mu)(y)\}$ Where*

$$\Psi_\Phi(\delta_\mu)(y) = \begin{cases} Sup\{\delta_\mu(x); \Phi(x) = y, x \in X\}; \\ \Phi^{-1}(y) \neq \phi, \\ \\ 0; \quad \Phi^{-1}(y) = \phi \end{cases}$$

$$\Psi_\Phi(\lambda_\mu)(y) = \begin{cases} Inf\{\lambda_\mu(x); \Phi(x) = y, x \in X\}; \\ \Phi^{-1}(y) \neq \phi \\ \\ 0; \quad \Phi^{-1}(y) = \phi \end{cases}$$

Definition 2.5 *Let G be an M-group and μbe an intuitionistic fuzzy group of $\delta_\mu(mx) \geq \delta_\mu(x)$ and $\lambda_\mu(mx) \leq \lambda_\mu(x)$ for all $x \in G$and $m \in M$ then μ is said to be an intuitionistic fuzzy subgroup with operator of G. We use the phrase μ is an intuitionistic fuzzy M -subgroup of G.All the intuitionistic fuzzy M-subgroups on $G$are written as $IFMS(G)$for short.*

Example 2.1 *Let H be M -subgroup of an M -group G and let μ be an intuitionistic fuzzy set in G defined by.*

$$\delta_\mu(x) = \begin{cases} 0.8 & ; x \in H, \\ \\ 0 & ; otherwise \end{cases}$$

$$\lambda_\mu(x) = \begin{cases} 0.4 & ; x \in H, \\ \\ 0.6 & ; otherwise \end{cases}$$

For all $x \in G$. Then it is easy to verify that μ is an intuitionistic fuzzy M-subgroup of

Proposition 2.1 *If μ is an intuitionistic fuzzy M -subgroup of an M-group G, then for any $x, y \in G$ and $m \in M$*

1-δ_μ $(m(xy)) \geq min\{\delta_\mu(mx)$, $\delta_\mu(my)\}$ and λ_μ $(m(xy)) \leq max$ $\{\lambda_\mu$ (mx) , λ_μ (my) $\}$ 2- δ_μ $(mx^{-1}) \leq \delta_\mu(x)$ and λ_μ $(mx^{-1}) \leq \lambda_\mu$ (x).

Definition 2.6 *Let G be m-group, μ be an intuitionistic fuzzy m-subgroup of G, then μ is called intuitionistic normal fuzzy m-subgroup if $\delta_\mu(m(xyx^{-1})) \geq \delta_\mu(my)$ and λ_μ $(m(xyx^{-1})) \leq \lambda_\mu m(xy)$for all $x, y \in G$ and $m \in M$. All the intuitionistic fuzzy M -subgroups on $G$are written as $INFMS(G)$for short.*

Definition 2.7 *[5]Let $G1$ onto $G2$ be two m-groups, Ψ be a homomorphism from $G1$ onto $G2$. If $\Phi(mx) = m\ \Phi(x)$ for all $x \in G1$ and $m \in M$, then Ψ is called m- homomorphism.*

3 M-Homomorphim and isomorphism for intuitionistic fuzzy m-subgroups

Theorem 3.1 *Let $G1, G2$be m-groups, $\Phi : G1 \longrightarrow G2$ be m-homomorphic mapping. If $\mu \in IFMS(G1)$, $\gamma \in IFMS(G2)$. Then $\Psi_\Phi(\mu) \in IFMS(G2)$ and $\Psi_\Phi^{-1}(\gamma) \in IFMS(G1)$.*

Theorem 3.2 *Let $G1, G2$be m-groups, $\Phi : G1 \longrightarrow G2$ be m-homomorphic mapping. If μ be intuitionistic fuzzy m-subgroup of $G1$. Define for any $x \in G1$,then $\mu^{-1} \in IFMS(G1)$ and $\mu^{-1}; \delta_{\mu^{-1}}(x) = \delta_\mu(x^{-1}), \lambda_{\mu^{-1}}(x) = \lambda_\mu(x^{-1})$ and $\Psi_\Phi(\mu^{-1}) = (\Psi_\Phi(\mu))^{-1}$.*

Theorem 3.3 *Let $G1, G2$be m–groups, $\Phi : G1 \longrightarrow G2$ be m-homomorphic surjective mapping. $\mu \in IFMS(G1)$ then $\Psi_\Phi(\mu) \in IFMS(G2)$.*

Proof. By Theorem 3.1, clearly we have $\Psi_\Phi(\mu) \in IFMS(G2)$.We need to prove the normality fuzzy for $\Psi_\Phi(\mu)$, for any $y1, y2 \in G2, m \in M$ by the extension principle, $\Phi : G1 \longrightarrow G2$ is m-homomorphism surjective mapping. This means that $\Phi(G1) = G2$, $\Phi^{-1}(my1) \neq \phi$ and $\Phi^{-1}(my2) \neq \phi$, $\Phi^{-1}(m\ (y1y2y1^{-1})) \neq \phi$ and we have

$$\Psi_\Phi(\delta_\mu)(m(y1y2y1^{-1})) = \sup_{z \in \Phi^{-1}(m(y1y2y1^{-1}))} \delta_\mu(z)$$

$\Psi_\Phi(\delta_\mu)(my2) = \sup_{z\in\Phi^{-1}(my2)} \delta_\mu(z)$ For all $mx2 \in \Phi^{-1}(my2))$ and for all $mx1 \in \Phi^{-1}(my1)$, then$(mx1)^{-1} \in \Phi^{-1}((my1)^{-1}$)since$\mu \in IFMS(G)$. We get $\delta_\mu \in (m(x1x2x1^{-1}))$ $\geq \delta_\mu$ $(mx2)$, as Φ is m-homomorphism then $\Phi(m(x1x2x1^{-1})) = m(\Phi(x1)\Phi(x2)\Phi(x1^{-1}) = m(\Phi(x1)\Phi(x2)(\Phi(x1))^{-1} = m(y1y2y1^{-1})$. Consequently m(x1x2 x1^{-1}) $\in \Phi^{-1}(m(y1y2y1^{-1}))$, therefore $\sup_{z\in\Phi(m\ (y1y2y1^{-1}))} \delta_\mu(z) \geq \sup_{mx1\in\Phi^{-1}(my1).mx2\in\Phi^{-1}(my2)} \delta_\mu(m(x1x2x1^{-1}))$ $\geq \sup_{mx2\in\Phi^{-1}(my2)} \delta_\mu(mx2)$

This means that $\Psi_\Phi(\delta_\mu)(m\ (y1y2y1^{-1}) \geq \Psi_\Phi(\delta_\mu)(m\ y2)$ for all $y1, y2 \in G2, m \in M$. On the other hand, similarly $y1, y2 \in G2, m \in M\Phi^{-1}(my1) \neq \phi$ and $\Phi^{-1}(my2) \neq \phi$, $\Phi^{-1}(m(y1y2y1^{-1})) \neq \phi$and $mx2 \in \Phi^{-1}(my2)$, $mx1 \in \Phi^{-1}(my1)$then $(mx1)^{-1} \in \Phi^{-1}((my1)^{-1})$and $\lambda_\mu(m(x1x2x1^{-1})) \leq \lambda_\mu(mx2)$,thus $\inf_{z\ \in\ \Phi^{-1}(m\ (y1y2y1^{-1}))} \lambda_\mu(z) \leq \inf_{mx1\in\Phi^{-1}(my1).mx2\in\Phi^{-1}(my2)} \lambda_\mu(m(x1x2x1^{-1}))$ $\leq \inf_{mx2\in\Phi^{-1}(my2)} \lambda_\mu(mx2)$

This means that $\Psi_\Phi(\lambda_\mu)(m(y1y2y1^{-1})) \in \Psi_\Phi(\lambda_\mu)(my2)$ for all $y1, y2 \in G2, m \in M$.Hence$\Psi_\Phi(\mu) \in INFMS(G2)$.

Theorem 3.4 *: Let $G1, G2$ be m-groups, $\Phi : G1 \longrightarrow G2$be m-homomorphism mapping. If $\gamma \in INFMS(G2)$, then $\Psi_\Phi^{-1}(\gamma) \in INFMS(G1)$.*

Proof. By Theorem3.1 $\Psi_\Phi^{-1}(\gamma) \in IFMS(G1)$, thus we need to prove the normality fuzzy. Since $\gamma \in INFMS(G2)$ for any $x, y \in G1, m \in M$ from the extension principle, we obtain $\Psi_\Phi^{-1}(\delta_\gamma)(m(xyx^{-1})) = (\delta_\gamma)(\Phi(m(xyx^{-1}))) = \delta_\gamma(m(\Phi(x).\Phi(y).\Phi(x^{-1})) = \delta_\gamma(m(\Phi(x).\Phi(y).\Phi(x)^{-1}) \geq \delta_\gamma(m\Phi(y)) = \Psi_\Phi\ (\delta_\gamma)(my)$.

Similarly we get $\Psi_\Phi^{-1}(\lambda_\gamma)(m(xyx^{-1})) = (\lambda_\gamma)(\Phi(m(xyx^{-1}))) = \lambda_\gamma(m(\Phi(x)\Phi(y)\Phi(x^{-1}))$=$\lambda_\gamma$(m(Φ(x)Φ(y)Φ(x)$^{-1}$) $\leq$ λ_γ(mΦ(y))=$\Psi_\Phi(\lambda_\gamma)$(my), therefore$\Psi_\Phi^{-1}(\gamma) \in INFMS(G1)$.

Theorem 3.5 *: Let $G1, G2$ be m-groups, $\Phi : G1 \longrightarrow G2$be m-homomorphism mapping. If $\mu \in INFMS(G2)$, then $\mu^{-1} \in INFMS(G1)$ and $\Psi_\Phi(\mu^{-1})$ =$(\Psi_\Phi(\mu))^{-1}$.*

Proof.

Let μ be intuitionistic fuzzy m-subgroup of $G1$, then $\mu^{-1} = \{< x \in G1; \delta_{\mu^{-1}}(mx), \lambda_{\mu^{-1}}(mx), m \in M >\}$where $\delta_{\mu^{-1}}(mx) = \delta_\mu(mx^{-1})$ and $\lambda_{\mu^{-1}}(mx) = \lambda_\mu(mx^{-1})$ since$\mu \in INFMS(G1)$ and by Theorem3.1. We know$\mu^{-1} \in IFMS(G1)$, for any $x, y \in G1, m \in M$ we have $\delta_{\mu^{-1}}(m(xyx^{-1})) = \delta_\mu\ (m(xyx^{-1})^{-1}) \geq \delta_\mu(m(xyx^{-1})) \geq \delta_\mu(my) = \delta_{\mu^{-1}}(my^{-1}) \geq \delta_{\mu^{-1}}(my)$ and $\lambda_{\mu^{-1}}(m(xyx^{-1})) = \lambda_\mu(m(xyx^{-1})^{-1}) \leq \lambda_\mu(m(xyx^{-1})) \leq \lambda_\mu(my) = \lambda_{\mu^{-1}}(my^{-1}) \leq \lambda_{\mu^{-1}}(my)$.Then μ is intuitionistic normal fuzzy m-subgroup, consequently we get $\mu^{-1} \in INFMS(G1)$ by Theorem 3.3 we have $\Psi_\Phi\ (\mu) \in INFMS(G2)$, thus $\Psi_\Phi(\mu^{-1}) \in INFMS(G2)$ and$\Psi_\Phi\ (\mu) \in IFMS(G2), \Psi_\Phi(\mu^{-1}) \in IFMS(G2)$ utilizing Theorem 3.1 we $\Psi_\Phi(\mu^{-1})) = (\Psi_\Phi(\mu))^{-1}$.

Corollary 3.1 *: Let $G1, G2$ be m-groups, $\Phi : G1 \longrightarrow G2$be m-homomorphism mapping. If $\gamma \in INFMS(G2)$, then $(\Psi_\Phi^{-1}(\gamma))^{-1} = \Psi_\Phi^{-1}(\gamma^{-1})$*

Theorem 3.6 *: Let $G1, G2$ be m-groups, $\Phi : G1 \longrightarrow G2$be an isomorphic mapping. If $\mu \in INFMS(G1)$, then $\Psi_\Phi^{-1}\ (\Psi_\Phi\ (\mu)) = \mu$,*

Proof. Let $x \in G1, m \in M$ and$\Phi(mx) = my$ as Φ is an isomorphic mapping $\Psi^{-1}(my) = \{mx\}$,applying the extension principle we obtain$\Psi_\Phi^{-1}\ (\Psi_\Phi\ (\delta_\mu))(mx) = \Psi_\Phi(\delta_\mu)(\Phi(mx)) = \Psi_\Phi(\delta_\mu)(\Phi(my) = \sup_{mx\in\Phi^{-1}(my)} \delta_\mu(mx) = \delta_\mu(mx)$
$\Psi_\Phi^{-1}\ (\Psi_\Phi\ (\lambda_\mu))(mx) = \Psi_\Phi(\lambda_\mu)(\Phi(mx)) = \Psi_\Phi(\lambda_\mu)(\Phi(my) = \inf_{mx\in\Phi^{-1}(my)} \lambda_\mu(mx) = \lambda_\mu(mx)$
Hence $\Psi_\Phi^{-1}\ (\Psi_\Phi\ (\mu)) = \mu$

Corollary 3.2 *: Let $G1, G2$ be m-groups. 1- If $\Phi : G1 \longrightarrow G2$be an isomorphic mapping and $\gamma \in INFMS(G1)$ Ψ_Φ ($\Psi_\Phi^{-1}(\gamma)) = \gamma$.*

2- If $\Phi : G1 \longrightarrow G2$be an automorphism mapping and $\mu \in INFMS(G1)$,then $\Psi_\Phi(\mu) = \mu$ iff $\Psi_\Phi^{-1}(\mu) = \mu$

4 Conclusion

Further work is in progress in order to develop the intuitionistic anti L-normal fuzzy m-subgroups and its applications and properties.

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