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Research Article

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Set a bi-objective redundancy allocation model to optimize the reliability and cost of the Series-parallel systems using NSGA II problem

M. R. Shahriari *†

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Abstract

With the huge global and wide range of attention placed upon quality, promoting and optimize the reliability of the products during the design process has turned out to be a high priority. In this study, the researcher have adopted one of the existing models in the reliability science and propose a bi-objective model for redundancy allocation in the series-parallel systems in accordance with the redundancy policy- given that failure rate depends on the number of the active elements. The objective behind the proposed model is to maximize the reliability and to minimize the total cost of the system. Internal connection cost, which is the most common parameter in electronic systems, put in this model in order to simulate the real-world conditions. As the proposed model is an NP-Hard one(for getting results), the researcher adopted a Non-dominated Sorting Genetic Algorithm (NSGA II) after optimizing its operators rate by using Response Surface Methodology (RSM).

Keywords : Reliability; Series; Parallel System; Redundancy Allocation Problem; Non-dominated Sorting Genetic Algorithm; Response Surface Methodology.

1 Introduction

THe main goal of the reliability science is to improve and optimize the reliability of the systems. During the initial stages of the design, the redundancy allocation directly enhances the system reliability. Fyffe and his colleagues first introduced redundancy Allocation Problem (RAP) in 1986. RAP is a combinatorial optimization problem, which focuses on determining an optimal assignment for the components in a system, and is widely used in system designing in a variety of practical circumstances, especially in electrical, computer and industrial engineering (Ramirez-Marquez and Coit, 2004) [12]. RAP involves the simultaneous selection of the components and configuration of system-level design, which can collectively meet all design constraints in order to optimize some objective functions such as system cost and/or reliability. RAP can be categorized into RAP with and without component mixing (Coit, 2001 [4], Coit and Smith, 1995 [2]). Over the recent years, researchers have developed reliability models, especially in series-parallel and redundancy problems. The historical development of the redundancy problem in a series-parallel problem is being summarized in Table 1. This section describes briefly the literature and concept of the redundancy allocation problem in series-parallel systems. The proposed model will be presented in the next section. Section 3 re-

*Corresponding author. Shahriari.mr@gmail.com
†Faculty of Management, South Tehran Branch, Islamic Azad University, Tehran, Iran.

views the concept and results of the Response Surface Methodology for optimizing the operators rate of Non-dominated Sorting Genetic Algorithm (NSGA II). In Section 4, the test results of applying non-dominated sorting genetic algorithm to solve the proposed model are presented and finally Section 5 discusses the conclusions.

2 Problem formulation

The purpose of the proposed model is to maximize the reliability and to minimize the total cost of a series-parallel system considering different redundancy strategies without component mixing (RAPCM) in each subsystem and the failure rate of active elements in a subsystem, which are dependent on the number of the active elements. The selected strategies for each subsystem are active and cold standby. To formulate the problem, first the assumptions are defined in Section *2.1*, and then the parameters are presented in section *2.2*.
Eventually, the proposed model is presented in section *2.3*.

Authors	Decision/Subject	Objective in study	
		single	multi
Fyffe et al 1968	RAP and Computational Algorithm(Dynamic Programimnig)	•	
coint and Smith,1996	Relaibility Optimaization of Series- Parallel System Uinsing GA	•	
Liang and Smith,2004	An Ant Colony Optimization Algorithm for RAP	•	
Yun and Kim,2004	Multi- Level Redundancy Optimization in series Systems		•
Liang and Wu, 2005	A Variable Neighborhood Descent Algorithm (VNA)for RAP	•	
coint and konak,2006	Multiple Weighted Objective Heuristic for the RAP		•
Liang and Chen, 2007	A VNA for RAP IN series- parallel system	•	
Wamg et al., 2009	Multi- objective Approach to RAP in Parallel series Systems		•
Khalili Damghani, Amiri, 2012	Solving binary- state Multi- objective for RAP		•
Chambari et al.,2012	A bi- objective model to opimize reliability and cost of system		•

Table 1: History of the development of the redundancy problem in a series-parallel problem

2.1 Assumptions

- The system consists of some series subsystem which redundant elements in these subsystems are parallel.
- Only one kind of element can be assigned to each subsystem.
- Each subsystem can only choose on active or cold standby strategy in redundancy allocation.
- For each redundancy strategy, the failure rate of elements depends on active elements.
- Elements of system and subsystem have intact or failed state.
- For redundant elements, internal connection cost will be used.
- Costs and weights of the elements are definite.
- The reliability of the elements is deterministic and definite.
- None of the elements use preventive maintenance strategy.
- The failure of the elements is independent.
- Inactive elements do not harm the system.
- The failure of the switch only happens in response to a failure.

2.2 Parameters

i subsystems index $(i = 1, 2, ..., s)$

j redundant element index $(j = 1, 2, ..., m_i)$

K_i The kind of the element assigned to i^{th} subsystem index $K_i \in \{1, 2, ..., m_i\}$

k $(k_1, k_2, ..., k_s)$

n_i Number of the elements in i^{th} subsystem $n_i \in \{1, 2, ..., n_{\max,i}\}$

n $(n_1, n_2, ..., n_s)$

st_i Redundancy strategy chosen for i^{th} subsystem $st_i \in \{A\} or \{S\}$

st $\{st_1, st_2, ..., st_s\}$

s Number of subsystems

m_i number of the kind of the elements for assigning in i^{th} subsystem

$n_{\max,i}$ Upper limit for n_i

t Mission time of the system

$r_{ij}(t)$ Reliability of j^{th} part assigned to j^{th} subsystem

$c_{ij}(t)$ Cost of j^{th} element assigned to i^{th} subsystem

$w_{ij}(t)$ Weight of j^{th} element assigned to i^{th} subsystem

C Upper limit of the cost of the system

W Upper limit of the weight of the system

ρ_i switching success probability in failure detection

$F^{(j)}_{i,k_i}(t)$ Probability function distribution for j^{th} failure in i^{th} subsystem with assigning element k_i to subsystem in time t

A Set of subsystems with active strategy

S Set of subsystems with cold stand by strategy

λ_j Failure rate of the j^{th} kind of the element

λ_{i,n_i} Failure rate of the elements in i^{th} subsystem when ni elements are active

ρ_{ij} Internal connection cost of j^{th} element to i^{th} subsystem

$R(t; st, k, n)$ Reliability of the system in time t with assigning vector k of all kind of elements and vector n of number of the elements

2.3 Proposed model

$$\max Z_1 = R(t; st, k, n) \tag{2.1}$$

$$\min Z_2 = \sum_{i=1}^{s}\sum_{j=1}^{mi}(c_{ij}(n_i + e^{\rho_{ik_i}n_i})) \tag{2.2}$$

$$s.t$$
$$\sum_{i=1}^{s}\sum_{j=1}^{mi}(w_{ij}n_i) \leq W \tag{2.3}$$

$$n_i \leq n_{\max,i} \forall i \in s$$
$$n_i, k_i \ln t, st_i \in \{0, 1\} \tag{2.4}$$

2.3.1 First objective

Total reliability of the system is evaluated by Equation (2.1) and for the cold standby and active subsystem and total reliability of the system are calculated by Equation (2.5), Equation (2.6), and Equation (2.7) respectively (Sharifi et al., 2010) [13], considered the effects of the component failure on remaining components failure rates. They proved that the failure rate of each element in ith subsystem with ki kind and ni active element, which is described in Equation (2.8). They also proposed that the best value for δ is 0.5.

$$R(t; S, k, n) = \Pi_{i\in S} \left(r_{i,k_i}(t) + \sum_{j=1}^{n_i-1} \int_t^0 \rho_i(u) F^{(j)}_{i,k_i}(u) r_{i,k_i}(t-u)du \right) \tag{2.5}$$

$$R(t; A, k, n) = \left(\Pi_{j=1}^{m}\lambda_{i,n_i}\right) \sum_{i=k}^{m}\{\tfrac{n_i!}{i(k-1)!}\{\tfrac{1}{\prod_{\theta=k,\theta\neq i}^{m}(\theta\lambda_\theta - i\lambda_{i,n_i})}\}\tfrac{e^{-i\lambda_{i,n_i}t}}{\lambda_i}\} \tag{2.6}$$

$$\lambda_{i,n_i} = \frac{n_i - (\delta(n_i-1))}{n_i} * \lambda_{k_i} \tag{2.7}$$

$$R(t; st, k, n) = \prod_{i\in A}\left(\prod_{j=1}^{m}\lambda_{i,n_i}\right) \sum_{i=k}^{m}\{\tfrac{n_i!}{i(k-1)!}\{\tfrac{1}{\prod_{\theta=k,\theta\neq i}^{m}(\theta\lambda_\theta - i\lambda_{i,n_i})}\}\tfrac{e^{-i\lambda_{i,n_i}t}}{\lambda_i}\} * \prod_{i\in S} \left(r_{i,k_i}(t) + \sum_{j=1}^{n_i-1} \int_t^0 \rho_i(u) F^{(j)}_{i,k_i}(u) r_{i,k_i}(t-u)du \right) \tag{2.8}$$

2.3.2 Second objective

Total cost of the system consists of the assignment of the redundant elements and the internal connection of elements. Internal connection cost has an exponential nature because of the limited space in electronic systems as surcharging each redundant element to each subsystem incurs progressive costs. This constraint is shown in Equation (2.2).

2.3.3 Constraints

The researchers took into account the total weight of the system and the total component that was assigned to each subsystem. There is an upper bound that is being presented in Equation (2.3), and Equation (2.4). Finally, the objective of the model is to define the best strategy, number and kind of redundant component assigned to each subsystem considering the constraints.

3 Optimizing the operators rates

Non-dominated Sorting Genetic Algorithm (NSGA II) has been adopted for solving the problem and getting test results upon optimizing the operators rate of the algorithm using Response Surface Methodology (RSM). Response Surface Methodology (RSM) explores the relationships between several explanatory variables and one or more response variables (Box and Wilson, 1951) [1]. The main philosophy behind RSM is to use a sequence of designed experiments to obtain an optimal response. The non-linear programing model and the best solution of applying RSM for optimizing the operators of the NSGA II are presented in Equations (3.9) to (3.12) and Table 2.

$$\begin{aligned} \min MID/DIV &= -9.77404 + 0.0862565* \\ & npop + 9.29748 * pc - 74.7459 * pm \\ & -0.156791 * npop * pc - 1.19643* \\ & npop * pm + 101.616 * pc * pm \\ & +0.000919053 * npop\hat{\ }2 + 23.6708* \\ & pc\hat{\ }2 + 548.207 * pm\hat{\ }2; \end{aligned} \tag{3.9}$$

$$\begin{aligned} & S.T. \\ & 100 \leq npop \leq 200 \end{aligned} \tag{3.10}$$

$$0.4 \leq pc \leq 0.8 \tag{3.11}$$

$$0.1 \leq pm \leq 0.3 \tag{3.12}$$

npop is an integer variable

4 Numerical results

Non-dominated Sorting Genetic Algorithm II (NSGA II) is one of the most widely-utilized and applicable multi-objective optimization algorithms.

To evaluate the model, the researchers put to test the very example that, for the first time, Coit (2003) [5] used in his study.

Algorithm	NSGA II		
Operator	npop	Pc	Pm
Best value	70	0.8	0.2

Table 2: The best solution for operators rate of the NSGA II.

Consider a series-parallel system with 14 series subsystems. Each subsystem can assign up to six elements from three to four different kinds of elements. Inter-failure time for all of the assignable elements follows negative exponential distribution with deterministic parameter. Cost, weight and exponential distribution parameter of the elements are presented in Table 3. Each subsystem can choose one of the redundancy strategies: active or cold standby. In the subsystems with cold standby strategy, the switch reliability is 0.99. The objective is to increase the reliability of the system in time 100 under cost ($C = 130, Max$) and weight ($W = 170, Max$) constraint. To find the best solutions for the model, the algorithm was implemented 5 times and the best feasible solution in these steps was recorded, we repeated these steps 5 times and finally, the averages of each multi-objective algorithm indicator were concluded, the results are shown in Table 4.

	choice1 (j=1)			choice2 (j=2)			choice3 (j=3)			choice4 (j=4)		
i	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}	λ_{ij}	C_{ij}	W_{ij}
1	0.00532	1	3	0.000726	1	4	0.00499	2	2	0.00818	2	5
2	0.00818	2	8	0.000619	1	10	0.00431	1	9	*		
3	0.0133	2	7	0.011	3	5	0.0124	1	6	0.00466	4	4
4	0.00741	3	5	0.0124	4	6	0.00683	5	4	*		
5	0.00619	2	4	0.00431	2	3	0.00818	3	5	*		
6	0.00436	3	5	0.00567	3	4	0.00268	2	5	0.000408	2	4
7	0.0105	4	7	0.00466	4	8	0.00394	5	9	*		
8	0.015	3	4	0.00105	5	7	0.0105	6	6	*		
9	0.00268	2	8	0.000101	3	9	0.000408	4	7	0.000943	3	8
10	0.0141	4	6	0.00683	4	5	0.00105	5	6	*		
11	0.00394	3	5	0.00355	4	6	0.00314	5	6	*		
12	0.00236	2	4	0.00769	3	5	0.0133	4	6	0.011	5	7
13	0.00215	2	5	0.00436	3	5	0.00665	2	6	*		
14	0.011	4	6	0.00834	4	7	0.00355	5	6	0.00436	6	9

Table 3: Values of the parameters.

5 Conclusion

In this research, an integer nonlinear programming model for the redundancy allocation problem, without component mixing, was presented

test no. / Indicators	1	2	3	4	5	Average
MID	288600	311120	301550	286030	287090	294878
Diversity	9844.9	41316	35160	7273.9	13664	21451.76
NPS	35	27	38	100	74	54.8
Spacing	189.19	460.14	1434.7	59.2	23.12	474.886
Time	564.7251	135.5238	492.2808	1901246	684.9804	413.5269
MID/Diversity	29.31	7.53	8.58	39.32	21.01	21.15

Table 4: The results of solving the model using NSGA II.

with regard to the dependence of the component failure rates work and the interconnection cost of the system. In other words, the purpose of this paper was to allocate components and the redundancy strategy to any subsystem without allocating the component mixing to any subsystems in order to increase system reliability and to reduce the total cost of the system under certain physical restrictions. Then, the results of solving the proposed model using a famous example and a multi-objective algorithm were presented. Presenting multi-objective models, considering new restrictions such as volume and factors like weight, possible cost, and more than two active and cold standby strategies for each subsystem are recommended for further research into the development of the future models.

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Mohammadreza Shahriari is an Assistant professor at the Department of Management in Islamic Azad University UAE branch , Dubai, UAE. he received his Master and PhD. in Industrial Management from Islamic Azad University Science and Research Branch, Tehran. he currently is chancellor of the UAE Branch of Islamic Azad University. His research interests includes decision making techniques and supply chain management issues.