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# A new multi-mode and multi-product hub covering problem: A priority M/M/c queue approach

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**Abstract**

One main group of a transportation network is a discrete hub covering problem that seeks to minimize the total transportation cost. This paper presents a multi-product and multi-mode hub covering model, in which the transportation time depends on travelling mode between each pair of hubs. Indeed, the nature of products is considered different and hub capacity constraint is also applied. Due to the transport volume and related traffic, a new priority M/M/c queuing system is considered, in which products with high priority are selected for service ahead of those with low priority. The objectives of this model minimize the total transportation cost and total time. Besides, because of the computational complexity, a multi-objective parallel simulated annealing (MOPSA) algorithm is proposed and some computational experiments are provided to illustrate the efficiency of the presented model and proposed MOPSA algorithm. The performance of this algorithm is compared with two well-known multi-objective evolutionary algorithms, namely non-dominated sorting genetic algorithm (NSGA-II) and Pareto archive evolution strategy (PAES).

*Keywords* : Multi-objective hub covering problem; Priority queuing model; Multi modes; Parallel simulated annealing.

## 1 Introduction

A Hub network has been generally applied in telecommunication and transportation areas. Hubs are special facilities transferred commodities (e.g., goods or passengers) between origin and destination nodes and used to decrease the number of transportations. The function of a hub facility is to collect, switch, sort and transfer commodity, and it decreases the number of links in the network. Therefore, hubs reduce costs and enhance the efficiency of the network. O'Kelly [12] introduced the first mathematical formulation in the HLP. O'Kelly [13] also developed the first quadratic mathematical formulation for HLP and presented a p-hub median problem, in which the hub nodes are completely linked together where every non-hub node is linked to a single hub node. Wagner [15] presented a new model formulation for single and multiple allocation hub covering problem. Ernst et al. [6] applied an integer programming formulation based on the radius of hubs concept for single and multiple allocation hub covering problem.

Costa et al.[5] presented a multi-objective hub location problem so that the first objective minimizes the total travelling cost, whereas the second

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one minimizes the maximum service time between each pair of nodes. In this problem, each non-hub node is allocated to one hub and the number of hub nodes is predefined and denoted by Tavakkoli-Moghaddam et al. [14] applied a new multi-objective capacitated hub location model with regarding a set of capacities for each potential hub, in which only one of them can be chosen. The authors considered balancing requirements in this model and proposed a multi objective imperialist competitive algorithm (MOICA) to solve the model. Ghodratnama et al. [7] presented a novel mathematical model for a new p-hub location-allocation problem with different types of vehicles and capacity. They also proposed simulated annealing (SA), genetic algorithm (GA) and particle swarm optimization (PSO) to solve the presented model.

Marianove and Sera [9] introduced a novel formulation of hub location problem in airline networks behaving as an M/D/c queue system. They proposed a probabilistic constraint that the probability of having more than a specified number of air planes in a queue should be limited. The authors solved the model by using of the tabu search algorithm. Ishfaq and Sox [8] proposed an integrated hub operation queuing model and hub location-allocation model and studied the on the effect of limited hub resources on the design of inter modal logistics networks under service time requirements. Mohammadi et al. [10] presented a capacitated single allocation p-hub covering problem behaving as M/M/c queues. They developed meta-heuristic algorithm based on the imperialist competitive algorithm (ICA) to solve the model.

In HLPs generally supposed that one hub type and one type of transportation mode are existed whereas, in the real world, we have a choice between different types of transportation systems. Alumur et al. [1] designed a new multi-modal hub location network, in which different transportation modes between hubs and different types of service time between the origindestination nodes are considered. Then, an efficient heuristic algorithm is developed for solving the presented model. Mohammadi et al. [11] developed a new stochastic multi-objective multi-mode hub covering problem, in which each transportation mode has a risk factor and transportation time is influenced by the risk.

The interested reader is referred to [2, 3, 16], for surveys on HLPs. In the real world, some restrictions such as capacity constraint and time limitation in hub nodes cause to have a queue in these nodes. Moreover, in reality, types of products which are transferred between nodes are different; so, transferring cost and waiting time for different kinds of products are not the same. In other word, the priority of products is dissimilar and products with high priority should be selected for service ahead of those with low priority. Thus, the priority queue model is proposed to address the waiting time of products in the hub nodes.

Another feature of designing hub problem that researchers have recently considered is the choice of transportation mode. Transportation time and cost also can be depended on the transportation mode (e.g., road, intermodal rail, air and waterways) so paying more attention to select the proper mode causes to get close to the real situations. In this paper, we present the capacitated multi-modal multi-product p-hub covering problem under queuing approach. The major differences of this paper with the other studies are as follows:

- Designing a new multi-modal multi-product hub covering problem;
- Considering a single allocation strategy for each product so that each non-hub product at each node can be allocated to exactly one hub;
- Incorporating a priority M/M/c queue model to consider the waiting time in the hub nodes;
- Describing a set of available capacity levels for each potential hub so that only one of them can be chosen;
- Proposing a multi-objective model, in which the total cost and the maximum time of transportation between each pair of node should be minimized;
- Developing a multi-objective parallel simulated annealing (MOPSA) algorithm for solving this problem.

The remainder of this paper is organized as follows. Section 2 describes a mathematical formulation for multi-objective multi-modal hub location problem. Section 3 introduces the proposed

MOPSA to solve the problem. Section 4 presents the computational results. Finally conclusions are discussed in Section 5.

## 2 Problem formulation

In HLPs, there is a set of $n$ nodes that some of them can be selected as hubs and $p$ hubs are located and the remaining non-hub nodes are allocated to these located hubs. We first present notations and then the model presents. According to the notations, we present a mathematical formulation for the single allocation multi-modal hub location network design problem. The aim of this problem is to locate hub nodes and allocate non-hub nodes to these located hubs.

Indices:

| | |
|---|---|
| $n = \{1, 2, ..., N\}$ | Set of nodes |
| $c = \{1, 2, ..., C\}$ | Set of products [set of priorities] |
| $m = \{1, 2, ..., M\}$ | Set of modes |

Parameters:

| | |
|---|---|
| $w^c_{i,j}$ | Flow to be sent from node $i$ to node $j$ for product $c$ $(i, j \in N)$. |
| $C^c_{i,k}$ | Cost of sending product c from node $i$ to node $k$ $(i, j \in N)$. |
| $f^q_k$ | Fixed cost of opening hub $k$ with capacity level $q$. |
| $t^c_{i,k}$ | Transportation time of sending product $c$ from node $i$ to hub $k$ $(i, j \in N)$. |
| $O^c_i = \sum_j w^c_{i,j}$ | Total flow originating at node $i$ for product $c$. |
| $D^c_i = \sum_j w^c_{j,i}$ | Total flow destined for node $i$ for product $c$. |
| $P$ | Number of hubs that should be established. |
| $\mu_k$ | Processing rate of products at hub $k$. |
| $\varsigma_k$ | Number of service providers at hub $k$ |
| $R_k$ | Coverage radius at hub $k$ |
| $C^k_m$ | Modal connectivity cost of serving mode $m$ at hub $k$. |

Variables:

| | |
|---|---|
| $x^{c,m}_{i,j,k,l}$ | 1 if product $c$ travels from node $i$ to $j$ through hub pair $(k, l)$ using mode $m$, otherwise 0. |
| $s^m_k$ | 1 if hub $k$ is served by mode $m$; 0, otherwise. |
| $z^q_k$ | 1 if node $k$ is a hub with capacity level $q$; 0, otherwise. |
| $WT_k$ | Waiting time at node $k$ |
| $T$ | The maximum allowed time between each pair of node |

Objective functions:

$$min \quad \sum_{i,j,k,l,m,c} w^c_{i,j}(C^c_{i,k} + \alpha^m C^{c,m}_{k,l} + C^c_{l,j})x^{c,m}_{i,j,k,l} + \sum_{k,q} f^q_k z^q_k + \sum_{k,m} C^m_k s^m_k \quad (1)$$

$$min \quad T \quad (2)$$

Constraints:

$$x^{mc}_{ijkl} \leq \sum_q z^q_k \quad \forall i, j, k, l, m, c \quad (3)$$

$$x^{mc}_{ijkl} \leq \sum_q z^q_l \quad \forall i, j, k, l, m, c \quad (4)$$

$$\sum_q z^q_k \leq l \quad \forall k \quad (5)$$

$$\sum_{k,q} z^q_k = l \quad (6)$$

$$\sum_{k,l,m} x^{mc}_{ijkl} = l \quad \forall i, j, c \quad (7)$$

$$x^{mc}_{ijkl} \leq s^m_k \quad \forall i, j, k, l, m, c \quad (8)$$

$$x^{mc}_{ijkl} \leq s^m_l \quad \forall i, j, k, l, m, c \quad (9)$$

$$\sum_{l,j,c,m} C^c_{i,k} x^{mc}_{ijkl} \leq R_k \quad \forall i, k \quad (10)$$

$$\lambda^c_k = \sum_{i,j,l,m} w^c_{i,j} x^{mc}_{ijkl} \quad \forall k, c \quad (11)$$

$$WT^c_k = \frac{\sum_m s^m_k}{\varsigma_k!(\varsigma_k\mu_k - \lambda^c_k)(\frac{\lambda^c_k}{\mu_k})\sum_{j=0}^{\varsigma_k - 1}\frac{(\frac{\lambda^c_k}{\mu_k})^j}{j!} + \varsigma_k\mu_k} \times \frac{1}{(1 - \frac{\sum_{i=1}^{c-1}\lambda^c_k}{\varsigma_k\mu_k}) \times (1 - \frac{\sum_{i=1}^{c}\lambda^c_k}{\varsigma_k\mu_k})} \quad \forall k, c \quad (12)$$

$$(t^c_{ik} + WT^c_k + \alpha^m t^{cm}_{kl} + WT^c_l + t^c_{ij})x^{mc}_{ijkl} \leq T \quad \forall i, j, k, l, m, c \quad (13)$$

$$\sum_{i,j,l,c,m} w^c_{ij} x^{mc}_{ijkl} \leq \sum_q cap^q_k z^q_k \quad \forall k \quad (14)$$

$$x^{mc}_{ijkl}, z^q_k, s^m_k \in \{0, 1\} \quad \forall i, j, k, l, m, c \quad (15)$$

The first objective function of this model minimizes the sum of transportation costs and fixed costs of locating hubs while the second one minimizes the maximum time including transportation time and waiting time between each pair of node. Constraints (3) and (4) enforce that every hub pair assignment for an origindestination pair is restricted to open hubs. Constraint (5) ensures that each hub can choose only one capacity level. Constraint (6) shows that $p$ hub node is selected. Constraint (7) makes sure that, on a hub link, only one mode can be selected. Constraints (8) and (9) assure that if transportation of product c from node $i$ to node $j$ through a hub pair $(k,l)$ by using of mode $m$ is occurring, hubs $k$ and $l$ should have been served by mode $m$. Constraint (10) assures that each node $i$ can only be allocated to hub $k$ if the unit cost of $C_{ik}^{c}$ not exceed radius $R_k$ . Equation (11) calculates the arrival rate of each product at each hub. Equation (12) also calculates the waiting time at each hub. Constraint (13) ensures that the total transportation time and the waiting time in hub nodes should be less than the maximum allowed time which will be determined by the second objective function. Constraint (14) shows the capacity constraints which limit the amount of flows processed by each hub. Finally, Constraint (15) shows the type of variables.

## 3 Proposed algorithm

One special kind of multi-objective simulated annealing algorithm called multi-objective parallel SA (MOPSA) is proposed to solve the model in which more than one solution is used for searching in the solution space to obtain Pareto optimal solutions. The proposed MOPSA is able to search the solution space broadly. The performance of this algorithm is compared with two well-known meta-heuristic algorithms such as NSGA-II and PAES and the better performance of proposed MOPSA has been presented for validating this algorithm in HLPs.

### *3.1* Proposed MOPSA

Parallel simulated annealing algorithm starts with random initial solutions. In this algorithm for HLPs, the continues solution encoding (CSE) can consist of a matrix where $n$ is a number of nodes and $p$ is a number of hubs that should be located in the network. This matrix includes two parts; the first part is related to allocation phase and the second one is related to location phase. The allocation part includes real random values which are placed in bits 1 to $n$; while the location part includes random integer values limited to 1 to $n$ interval placed in bits $n+1$ to $n+p$. Furthermore, it should be considered that the value of the $(n+p)$th bit must be equal to $n$. Then, the random values of the first part are sorted increasingly so that the place of the biggest one includes number 1, 2 for the next big one, and so on. After that, the sorted bits of the second part are considered as the place of hubs in first part and the other bits are allocated to the nearest right signed bit which becomes hub. Fig. 1 illustrates the sample of CSE with 4 hubs and 10 nodes in which the colorful bits are hubs and the remained gray ones are nodes which are allocated to their nearest colorful bit. For instance, in Fig. 1, nodes 1, 2, 6 and 7 are hubs and nodes 8, 10, 4, 9, 3 and 5 are respectively allocated to hubs 1, 2, 2, 6, 7 and 7 [11].

The proposed algorithm uses three steps for creating neighborhood for the initial solution and achievement to the better solutions in the each iteration. These three steps include: mutation, assimilation and crossover. We used the non-dominance strategy and crowding distance metric to choose the *nPop* better solutions among the all created solutions for applying in the next iterations initial solutions. While two corresponding solutions are being compared to choose for next iteration initial solutions, three different cases may occur:

- If one solution dominates another one, it will be chosen.
- If two solutions cannot dominate each other, a solution with higher crowding distance metric will be selected for the next iteration.
- If the old solution dominates the new one, the probable acceptance function of simulated annealing will be accepted as stated bellows.

This probability is detected by the Boltzmann function, $P = e^{-\frac{\Delta f}{kT}}$ where $\Delta$ is a different value of objective functions between the current and

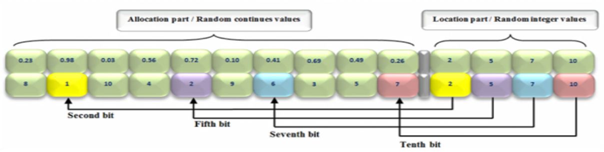


**Figure 1:** Continuous solution encoding for the HLP.

the new solutions, $k$ is a constant and $T$ is a current temperature. Moreover, the number of function calls (NFC) is considered as stopping criteria. The pseudo code of the proposed MOPSA is shown below:

## 3.2 NSGAII and PAES assumption

The NSGA-II assumptions and the value of parameters are as follows:

- The initial solution is randomly generated.
- Crossover operator is applied random selected solutions using one of these operators: one-point crossover, two-point crossover and uniform crossover.
- Mutation operator is exerted on random selected solution using one of these operators: inversion, swap and reversion.
- The crossover and mutation ratios are set to 0.75 and 0.3, respectively.
- Number of the initial population is set to 200 and 300 for small and large-sized problems, respectively.
- The NFCs stopping criteria was set on 4000 and 12000 for small and large size problems, respectively.

The PAES assumptions and the value of parameters are as follows:

- The size of archive is equal to 200.
- One of revolution operators, namely inversion, swap and reversion, are selected randomly.
- The NFCs stopping criteria was set on 4000 and 12000 for small and large size problems, respectively.

## 3.3 Comparison metrics

To illustrate the performance of the proposed MOPSA, four comparison metrics are used as follows [11]:

**Quality Metric (QM)** This metric is measured by putting together the non-dominated solutions found by the algorithms and calculating the ratios between non-dominated solutions of each algorithm. An algorithm with higher value of the QM has better performance.

**Mean Ideal Distance (MID)** The closeness between Pareto solutions and ideal point is detected using MID index. The MID index is calculated by:

$$MID = \frac{\sum_i^n \sqrt{\left(\frac{f_{1i}-f_1^{best}}{f_{1,total}^{max}-f_{1,total}^{min}}\right)^2 + \left(\frac{f_{2i}-f_2^{best}}{f_{2,total}^{max}-f_{2,total}^{min}}\right)^2}}{n} \tag{3.1}$$

where $n$ is the number of non-dominated solutions and $f_{i,total}^{max}$ and $f_{i,total}^{min}$ are the maximum and minimum values of each fitness functions of all non-dominated solutions obtained by the algorithm. According to this definition, the algorithm with a lower value of the MID has a better performance.

**Diversification Metric (DM)** This metric shows the spread of the Pareto solutions set and is measured by Eq. (3.2). Base on the definition, the algorithm with a higher value of the DM has a better performance.

$$MD = \sqrt{\left(\frac{max f_{1i} - min f_{1i}}{f_{1,total}^{p,max} - f_{1,total}^{p,min}}\right)^2 + \left(\frac{max f_{2i} - min f_{2i}}{f_{2,total}^{p,max} - f_{2,total}^{p,min}}\right)^2} \tag{3.2}$$

**Spacing Metric (SM)** This metric measures the uniformity of the spread of the non-dominated set solutions. This metric is obtained according to Eq. (3.3). The algorithm with a lower value of the SM has a better performance.

$$SM = \frac{\sum_{i=1}^{n-1} |\bar{d} - d_i|}{(n-1)\bar{d}} \tag{3.3}$$

where $d_i$ is the Euclidean distance between consecutive solutions in the obtained non-dominated set of solutions and $\overline{d}$ is the average of these distances.

### *3.4* Data generation

The required data for the presented problem consists of a number of nodes, number of hubs, transportation cost, transportation time, fixed costs, capacity of hubs and flows. The value and distribution of the above input parameters are as Table 1. Some special numbers of hubs are considered for each number of nodes. Also, each problem instance is shown as Number of nodes # number of hubs (e.g., 30#8 means 30 nodes and 8 hubs).

## 4 Computational results

The proposed MOPSA is used to number of test problems and its performance compared with NSGA-II and PAES. Tables 2 and 3 show the above four comparison metrics for small-sized problems. Tables 4 and 5 list the above four comparison metrics for large-sized problems. All tables show that the proposed MOPSA outperforms the NSGA-II and PAES in all test problems. The results of comparisons are as follows:

- Proposed MOPSA has more contribution of obtaining Pareto optimal solutions with higher qualities in comparison with both NSGA-II and PAES.
- Proposed MOPSA provides non-dominated solutions that have less average values of the spacing metric, so the MOPSA are more uniformly distributed in comparison with both NSGA-II and PAES in proposed model.
- The average values of the diversification metric in the proposed MOPSA are greater than NSGA-II and PAES in the most of the test problems;
- The values of MID in the proposed MOPSA are smaller than those of NSGA-II and PAES.

Therefore, the proposed algorithm has a better performance rather than two well-known algorithms (NSGAII & PAES) for solving the presented novel model.

**Algorithm 1** Pseudo code of constructive algorithm

```
NFC ← 0
Set the parameters of PSA (nPop, nMutate, pCrossover,
Create initial solution ← nPop
Terminate ← false
while Terminate=false do
  mutate each initial solution ← nMutate
  find the best solution (imperialist) ← non-dominated s
  update NFC
  assimilate mutated solutions toward imperialist ← β
  merge whole new created solution
  apply crossover ← pCrossover
  update NFC
  find better new solutions ← nPop
  update NFC
  if new solution dominates old solution then
    accept the new solution
  else if no one dominate the other one then
    calculate crowding distance - CD of each solution
    if CD of new solution > CD of old solution then
      accept the new solution
    end if
  else
    apply probable acceptance function ← P = e^(-Δf/kT)
    create random value r
    if (r < P) then
      accept new solution
    else
      accept old solution
    end if
  end if
  update NFC
  T ← α × T
  if NFC= predefined value then
    terminate=true
```

**Table 1:** Value and distribution of input parameters

| | Parameters | | | | | |
|---|---|---|---|---|---|---|
| Value and | $n$ | $w_{ij}^c$ | $C_{ij}^c$ | $f_k^q$ | $cap_k^q$ | $t_{kl}^m$ |
| distribution | 30#8 | $p \sim (20)$ | $U \sim (1,10)$ | $U \sim (1000,2000)$ | $U \sim (100,1000)$ | $U \sim (10,20)$ |

**Table 2:** Comparison metrics for small-sized problems

| | Quality metric(QM) | | | Spacing metric(SM) | | |
|---|---|---|---|---|---|---|
| Problem No | PAES | NSGA-II | MOHCG | PAES | NSGA-II | MOHCG |
| 10#3 | 0 | 0.115 | 0.695 | 0.545 | 0.657 | 0.741 |
| 10#4 | 0 | 0.254 | 0.846 | 0.472 | 0.644 | 0.878 |
| 15#3 | 0 | 0.201 | 0.802 | 0.708 | 0.71 | 0.920 |
| 15#4 | 0 | 0.103 | 0.807 | 0.625 | 0.738 | 0.923 |
| 15#5 | 0 | 0.100 | 1 | 0.012 | 0.571 | 0.903 |
| 20#3 | 0 | 0 | 1 | 1.084 | 0.491 | 0.901 |
| 20#4 | 0 | 0.172 | 0.827 | 1.304 | 0.75 | 0.970 |
| 20#5 | 0.019 | 0.208 | 0.433 | 0.827 | 0.583 | 0.892 |
| 20#6 | 0 | 0.105 | 0.638 | 1.131 | 0.803 | 0.954 |
| 25#3 | 0.20 | 0.200 | 0.597 | 1.026 | 0.101 | 1.245 |
| 25#4 | 0 | 0.243 | 0.737 | 0.927 | 0.311 | 0.851 |
| 25#5 | 0 | 0.175 | 0.815 | 1.151 | 0.955 | 1.238 |
| 25#6 | 0 | 0 | 1 | 0.927 | 0.864 | 1.305 |
| 30#3 | 0 | 0 | 1 | 1.105 | 0.731 | 1.176 |
| 30#4 | 0.246 | 0.109 | 0.595 | 1.002 | 1.08 | 1.103 |
| 30#5 | 0 | 0 | 1 | 0.863 | 0.934 | 0.998 |
| 30#6 | 0.431 | 0.207 | 0.750 | 0.846 | 0.560 | 0.799 |
| 30#7 | 0.095 | 0.138 | 0.765 | 1.020 | 0.959 | 1.094 |
| 30#8 | 0 | 0 | 1 | 0.327 | 0.296 | 0.745 |

**Table 3:** Comparison metrics for small-sized problems

| | Diversity Metric (DM) | | | Mean Ideal Distance (MID) | | |
|---|---|---|---|---|---|---|
| Problem No | PAES | NSGA-II | MOHCG | PAES | NSGA-II | MOHCG |
| 10#3 | 0.930 | 1.159 | 1.702 | 0.695 | 0.653 | 0.573 |
| 10#4 | 1.048 | 1.275 | 1.839 | 0.587 | 0.614 | 0.546 |
| 15#3 | 0.346 | 0.831 | 1.566 | 0.762 | 0.838 | 0.431 |
| 15#4 | 1.431 | 0.502 | 0.957 | 0.851 | 0.699 | 0.329 |
| 15#5 | 1.118 | 0.423 | 0.940 | 0.541 | 0.396 | 0.361 |
| 20#3 | 1.270 | 1.323 | 1.087 | 0.783 | 0.716 | 0.537 |
| 20#4 | 1.279 | 0.861 | 0.903 | 0.851 | 0.548 | 0.342 |
| 20#5 | 1.150 | 0.931 | 1.319 | 0.993 | 0.529 | 0.518 |
| 20#6 | 1.031 | 1.100 | 1.158 | 0.743 | 0.685 | 0.623 |
| 25#3 | 0.538 | 1.391 | 1.456 | 0.535 | 0.681 | 0.258 |
| 25#4 | 0.947 | 0.995 | 1.288 | 0.783 | 0.653 | 0.710 |
| 25#5 | 0.739 | 1.148 | 1.309 | 0.693 | 0.555 | 0.521 |
| 25#6 | 0.957 | 0.527 | 1.463 | 0.582 | 0.474 | 0.263 |
| 30#3 | 1.043 | 0.513 | 1.224 | 0.634 | 0.684 | 0.633 |
| 30#4 | 0.929 | 1.119 | 1.188 | 0.574 | 0.737 | 0.459 |
| 30#5 | 0.813 | 1.146 | 1.354 | 0.723 | 0.800 | 0.513 |
| 30#6 | 1.224 | 0.643 | 1.130 | 0.409 | 0.286 | 0.357 |
| 30#7 | 1.236 | 0.943 | 0.905 | 0.638 | 0.458 | 0.281 |
| 30#8 | 0.232 | 0.589 | 1.263 | 0.753 | 0.588 | 0.513 |

**Table 4:** Comparison metrics for large-sized problems

| | Quality metric(QM) | | | Spacing metric(SM) | | |
|---|---|---|---|---|---|---|
| Problem No | PAES | NSGA-II | MOHCG | PAES | NSGA-II | MOHCG |
| 100#3 | 0 | 0 | 1 | 1.24 | 0.471 | 0.661 |
| 100#4 | 0 | 0.43 | 0.57 | 0.51 | 1.052 | 1.370 |
| 100#5 | 0 | 0 | 1 | 0.20 | 0.011 | 1.181 |
| 100#6 | 0.34 | 0 | 0.654 | 0.35 | 0.501 | 0.581 |
| 100#7 | 0 | 0 | 1 | 0.80 | 0.662 | 1.053 |
| 100#8 | 0 | 0 | 1 | 0.29 | 1.713 | 0.553 |
| 100#9 | 0 | 0 | 1 | 1.47 | 0.901 | 0.454 |
| 100#10 | 0 | 0 | 1 | 0.85 | 1.042 | 0.971 |
| 100#11 | 0 | 0 | 1 | 1.00 | 0.709 | 0.594 |
| 100#12 | 0 | 0 | 1 | 0.07 | 1.072 | 0.731 |
| 100#13 | 0 | 0 | 1 | 0.97 | 0.653 | 0.401 |
| 100#14 | 0.08 | 0.347 | 0.5697 | 0.36 | 1.236 | 1.059 |
| 100#15 | 0 | 0 | 1 | 0.85 | 0.920 | 1.368 |
| 100#16 | 0 | 0 | 1 | 1.05 | 0.631 | 1.123 |
| 100#17 | 0 | 0 | 1 | 0.894 | 1.138 | 0.775 |
| 100#18 | 0 | 0 | 1 | 1.11 | 0.981 | 1.043 |

**Table 5:** Comparison metrics for large-sized problems

| | Diversity Metric (DM) | | | Mean Ideal Distance (MID) | | |
|---|---|---|---|---|---|---|
| Problem No | PAES | NSGA-II | MOHCG | PAES | NSGA-II | MOHCG |
| 100#3 | 0.65 | 0.200 | 0.661 | 0.74 | 1.251 | 0.239 |
| 100#4 | 0.55 | 1.042 | 1.084 | 0.45 | 0.551 | 0.428 |
| 100#5 | 0.43 | 1.041 | 1.172 | 0.80 | 0.824 | 0.109 |
| 100#6 | 1.30 | 0.871 | 0.845 | 0.73 | 0.758 | 0.503 |
| 100#7 | 1.05 | 0.430 | 1.020 | 0.31 | 0.371 | 0.125 |
| 100#8 | 0.25 | 1.154 | 0.367 | 0.68 | 0.598 | 0.249 |
| 100#9 | 1.11 | 0.191 | 0.552 | 0.40 | 0.883 | 0.031 |
| 100#10 | 0.86 | 1.136 | 0.961 | 0.65 | 0.590 | 0.265 |
| 100#11 | 1.15 | 0.291 | 0.679 | 0.47 | 0.8251 | 0.281 |
| 100#12 | 0.34 | 1.050 | 1.188 | 0.58 | 0.207 | 0.350 |
| 100#13 | 0.96 | 0.578 | 0.52 | 0.56 | 0.863 | 0.181 |
| 100#14 | 0.73 | 1.041 | 0.905 | 0.63 | 0.335 | 0.215 |
| 100#15 | 0.56 | 1.204 | 1.120 | 0.68 | 0.741 | 0.479 |
| 100#16 | 0.65 | 0.803 | 1.153 | 0.96 | 0.689 | 0.397 |
| 100#17 | 1.05 | 0.711 | 0.901 | 0.76 | 0.821 | 0.179 |
| 100#18 | 1.19 | 0.573 | 0.951 | 0.82 | 0.571 | 0.272 |

## 5 Conclusion

This paper studied the capacitated multi-mode p-hub covering problem, in which the mode of transportation was a part of the decision-making process and for each potential hub, a set of capacity level and modes was available . The travelling time depended on the transportation mode. The presented model followed the single allocation principles for each product that could receive and send flow only through one hub. Since the priority of products was different, the authors extended the M/M/c queue model to priority M/M/c one, in which the product with high priority was selected for service ahead of that with low priority. The objectives of the proposed model minimized total transportation cost and maximum time (transportation time and waiting time). For solving this problem, a multi-objective parallel simulated annealing (MOPSA) was proposed, whose performance was compared with NSGA-II and PAES algorithms. The results showed that the proposed MOPSA provided non-dominated solutions with less average values of the spacing metric. Its Pareto-optimal solutions considerably had higher qualities in comparison with both NSGA-II and PAES algorithms.

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