Available online at http://ijim.srbiau.ac.ir
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 4, No. 2, Year 2012 Article ID IJIM-00207, 11 pages
Research Article

Estimated Returns to Scale with Interval Data in Parallel Manufacturing Systems with Shared Resources

S. Kordrostami[a]*, A. Amirteimoori[b], O. Azmayandeh[a], Z. Bakhoda[a]
(a) *Department of Applied Mathematics,Lahijan Branch,Islamic Azad University, Lahijan , Iran*
(b) *Department of Applied Mathematics, Rasht Branch,Islamic Azad University, Rasht, Iran*

Abstract
Many models in DEA have been proposed to estimate returns to scale. Determining the nature of returns to scale has considerable of importance in the theory of production. Knowing the fact that returns to scale is a constant, ascending or descending decision making unit, proper actions can be performed to develop the decision making units. In this study, the efficiency of parallel production systems with shared resources is evaluated such that the data is inexact and interval, so the function of these systems is interval too. Then a model is proposed to estimate returns to scale of interval data on these systems; when the data is inexact, the nature of returns to scale of these units is inexact too, so returns to scale is estimated as multiple in best and worst conditions.
Keywords : Returns to scale; Parallel systems; Interval data; Efficiency.

---

1 Introduction

Determining the nature of returns to scale of decision making units in the production theory is of high importance; Knowing the fact that returns to scale is a constant, ascending or descending decision making unit, proper actions can be carried out. One of the research topics with functional value is to estimate the nature of returns to scale of decision making units when the input and output data is inexact and interval. Kao (2006) believed that when the input and output data is inexact and interval, the efficiency should be interval too, so we should define an interval efficiency for decision-making units with inexact data. In the real world there are systems which are composed of independent production units. These systems use input data to produce output. Kao (2008) evaluated the performance of the production systems which are composed of parallel production units. Kordrostami et al (2010) considered production systems which are composed of parallel subunits such that each subunit uses given inputs and part of shared resources to produce the final output.

*Corresponding author. Email address: sohrabkrostami@gmail.com

But when the input and output data is inexact, we can propose models to evaluate the performance of these systems in the most optimistic and pessimistic cases. In this paper, we evaluate the efficiency of parallel production systems with shared resources when the data is inexact and interval. Also, we estimate returns to scale of these systems. Moreover, we believe that when the input and output data is inexact, returns to scale are inexact too, so returns to scale is estimated as multiple in best and worst conditions. The structure of this paper is as follows: In the next section, we provide the related models. In the third section, the models to evaluate the efficiency of the parallel production systems with shared resources and the estimation of returns to scale in these systems are provided. In order to analyze the proposed models, we provide a practical example in the fourth Section, and finally in the last section the conclusion is drawn.

2 The related models

In this section we briefly explain the models used in this paper.

2.1 Parallel model with shared resources

Consider a parallel production system as shown in Figure 1. Assume that we have n decision making units, and each unit has T parallel subunits where each subunit uses its own input and the shared resources. In particular, $t-th$ subunit of $DMUP$ uses its own input$X_p^{(t)}$ and part of the shared resource of $X_p^{(s)}$. Assume that $X_p^{(s,t)}$ is part of the shared input of $X_p^{(t)}$ which is assigned to the $t-th$ subunit.
The process is shown in Figure 1. $Y_p^{(t)}$ is the produced output of the $t-th$ subunit. Clearly, we have:$X_p^{(s)} = \sum_{t=1}^{T} X_p^{(s,t)}$.

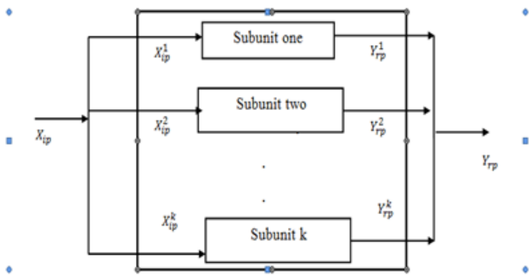

Figure 1. The parallel production system, where a DMUp has k production units.

The production possibility set (PPS) of the $t-th$ subunit under the assumption of variable returns to scale is as follows:

$$T_v^{(t)} = \{(X^{(t)}, Y^{(t)}, X^{(s,t)}) : \sum_{j=1}^{n} \lambda_j X_j^{(t)} \le X^{(t)}, \sum_{j=1}^{n} \lambda_j Y_j^{(t)} \ge Y^{(t)}, \sum_{j=1}^{n} \lambda_j X_j^{(s,t)} \le X^{(s,t)}\},$$

$$\sum_{j=1}^{n} \lambda_j = 1, \lambda_j \ge 0.$$

To calculate the technical efficiency of DMUP, we are to solve the following programming problem:

$$
\begin{array}{ll}
\min \sum_{t=1}^{T} w_t \frac{\theta_t+\acute{\theta}_t}{2} & \\
\quad s.t. & \\
\sum_{j=1}^{n} \lambda_j X_j^{(t)} \leq \theta_t X_p^{(t)}, & t=1,...,T \\
\sum_{j=1}^{n} \lambda_j X_j^{(s,t)} \leq \acute{\theta}_t X_p^{(s,t)}, & t=1,...,T \\
\sum_{j=1}^{n} \lambda_j Y_j^{(t)} \geq Y_p^{(t)}, & t=1,...,T, \\
\sum_{j=1}^{n} \lambda_j = 1, & \lambda_j \geq 0, j=1,...,n, \\
\theta_t \leq 1, \quad \acute{\theta}_t \leq 1. &
\end{array}
\tag{2.1}
$$

The objective function of the above model is a weighted set of $E_p^{(t)} = \frac{\theta_t+\acute{\theta}_t}{2}, (t=1,...,T)$. w_ts are the assumed coefficients by a specific user and we have: $\sum_{t=1}^{T} w_t = 1$. In this model,$E_p^{(t)}$is the bonus of the efficiency of t-th subunit which is equal to the average of$\theta_t$and$\acute{\theta}_t$or$E_p^{(t)} = \frac{\theta_t+\acute{\theta}_t}{2}$.Also $E_p$is the bonus of the efficiency of the whole system i.e.$E_p = \sum_{t=1}^{T} w_t \frac{\theta_t+\acute{\theta}_t}{2} = \sum_{t=1}^{T} w_t E_p^{(t)}$. Moreover, we can show the feasibly and boundedness of the linear programming problem (2.1).

2.2 The estimation of the returns to scale in DEA

In this section, we examine a model proposed by Khodabakhshi, et al (2010). They developed a model to estimate returns to scale in data envelopment analysis as follows. They tried to use the proposed model to make a non-efficient$(\xi x_o, \xi y_o)$to determine returns to scale (x_o, y_o).There, if the non-efficiency $(\xi x_o, \xi y_o)$ increases, ξin one level, returns to scale will increase, and if it decreases, ξin one level, returns to scale will decrease as well. If$(\xi x_o, \xi y_o)$ is never non-efficient, returns to scale are constant. The presented model is as follows:

$$
\begin{array}{l}
\max 1s^- + 1s^+ \\
s.t. \\
\sum_{j=1}^{n} \lambda_j x_j + s^- = \xi x_o, \\
\sum_{j=1}^{n} \lambda_j y_j - s^+ = \xi y_o, \\
\sum_{j=1}^{n} \lambda_j = 1 \\
\lambda_j \geq 0, j=1,...,n, \qquad s^-, s^+ \geq 0
\end{array}
\tag{2.2}
$$

The following theorem provides returns to scale with model (2.2).

Theorem 2.1. *The assumption DMU_o with the input output mix (x_o, y_o) are efficient. The following conditions provide returns to scale of DMU_o with model (2.2).*
A-The optimal amount of objective function is greater than zero and $\xi^* > 1$ if and only if DMU_o has increasing returns to scale.
B - The optimal amount of objective function is greater than zero and $\xi^* < 1$ if and only if DMU_o has decreasing returns to scale.
C - The optimal amount of objective function is zero and this is possible if and only if DMU_o has a constant returns to scale.

3 Proposed Models

3.1 The Parallel Model with Shared Resources on Interval Data

Consider a parallel production system as shown in Figure 1. Suppose we have $nDMUs$, and each DMU has T parallel subunits, so that each subunit has its own inputs and all subunits use a shared resource. In particular, the $t-th$ subunit of DMU_P uses its own inputs $X_p^{(t)}$ and a part of the shared inputs $X_p^{(s)}$. Suppose $X_p^{(s,t)}$ is a part of the shared inputs $X_p^{(s)}$ dedicated to subunit t. The output produced by subunit t is $Y_p^{(t)}$ clearly, we have $X_p^{(s)} = \sum_{t=1}^{T} X_p^{(s,t)}$ .Also assume that the input and outputs are known to lie within bounded intervals, i.e. $X_p^{(t)} \in [^L X_p^{(t)}, ^U X_p^{(t)}]$, $X_p^{(s,t)} \in [^L X_p^{(s,t)}, ^U X_p^{(s,t)}]$ and$Y_p^{(t)} \in [^L Y_p^{(t)}, ^U Y_p^{(t)}]$.
Also, the provided parallel model with shared sources in section 2-2, i.e. model (1) is considered as the basis of the work.

Definition 3.1. *Suppose $[^L X_j^{(t)}, ^U X_j^{(t)}]$, $[^L X_j^{(s,t)}, ^U X_j^{(s,t)}]$ and $[^L Y_j^{(t)}, ^U Y_j^{(t)}]$ are the own input, shared input and output of DMU_j, respectively. The best situation of DMU_p is defined as follows:*

$$X_p^{(t)} =^L X_p^{(t)}, \qquad X_p^{(s,t)} =^L X_p^{(s,t)}, \qquad Y_p^{(t)} =^U Y_p^{(t)}$$

$$X_j^{(t)} =^U X_j^{(t)}, \qquad X_j^{(s,t)} =^U X_j^{(s,t)}, \qquad Y_j^{(t)} =^L Y_j^{(t)}, \qquad j \neq p, \qquad j = 1, ..., n$$

Definition 3.2. *Suppose $[^L X_j^{(t)}, ^U X_j^{(t)}]$, $[^L X_j^{(s,t)}, ^U X_j^{(s,t)}]$ and $[^L Y_j^{(t)}, ^U Y_j^{(t)}]$ are the own input, shared input and output of DMU_j , respectively. The best situation of DMU_p is defined as follows:*

$$X_p^{(t)} =^U X_p^{(t)}, \qquad X_p^{(s,t)} =^U X_p^{(s,t)}, \qquad Y_p^{(t)} =^L Y_p^{(t)}$$

$$X_j^{(t)} =^L X_j^{(t)}, \qquad X_j^{(s,t)} =^L X_j^{(s,t)}, \qquad Y_j^{(t)} =^U Y_j^{(t)}, \qquad j \neq p, \qquad j = 1, ..., n$$

Definition 3.3. *Consider DMU_p. The lower bound of efficiency score of DMU_p is cal-*

culated by using the following model:

$$\theta^L = \min \sum_{t=1}^{T} w_t \frac{\theta_t + \acute{\theta_t}}{2}$$

$$s.t.$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^L X_j^{(t)} + \lambda_p^U X_p^{(t)} + s^{-(t)} = \theta_t^U X_p^t, t = 1, ..., T$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^L X_j^{(s,t)} + \lambda_p^U X_p^{(s,t)} + s^{\acute{-}(t)} = \acute{\theta_t}^U X_p^{(s,t)}, t = 1, \ldots, T$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^U Y_j^{(t)} + \lambda_p^L Y_p^{(t)} - s^{+(t)} =^L Y_p^{(t)},\ t = 1, \ldots, T \qquad (3.3)$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\theta_t \leq 1, \qquad t = 1, ..., T$$

$$\acute{\theta_t} \leq 1, \qquad t = 1, ..., T$$

$$\lambda_j \geq 0, s^{-(t)}, s^{\acute{-}(t)}, s^{+(t)} \geq 0 \qquad j = 1, ..., n$$

Also, the upper bound of the efficiency score can be calculated by using the following model:

$$\theta^U = \min \sum_{t=1}^{T} w_t \frac{\theta_t + \acute{\theta_t}}{2}$$

$$s.t.$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^U X_j^{(t)} + \lambda_p^L X_p^{(t)} + s^{-(t)} = \theta_t^L X_p^t, \qquad t = 1, ..., T$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^U X_j^{(s,t)} + \lambda_p^L X_p^{(s,t)} + s^{\acute{-}(t)} = \acute{\theta_t}^L X_p^{(s,t)}, \qquad t = 1, \ldots, T$$

$$\sum_{j=1,j\neq p}^{n} \lambda_j^L Y_j^{(t)} + \lambda_p^U Y_p^{(t)} - s^{+(t)} =^U Y_p^{(t)},\ t = 1, \ldots, T \qquad (3.4)$$

$$\sum_{j=1}^{n} \lambda_j = 1,$$

$$\theta_t \leq 1, \qquad t = 1, ..., T$$

$$\acute{\theta_t} \leq 1, \qquad t = 1, ..., T$$

$$\lambda_j \geq 0, s^{-(t)}, s^{\acute{-}(t)}, s^{+(t)} \geq 0 \qquad j = 1, ..., n$$

θ^L *together with* θ^U *constitute the efficiency interval as* $[\theta^L, \theta^U]$ *that covers all possible efficiency scores for the whole system. Also, in model(3.3)* ${}^U E_p^t = \frac{{}^U\theta_t + {}^U\acute{\theta_t}}{2}$ *is the upper efficiency score of the subunit* t*, and in model (3.4)* ${}^L E_p^t = \frac{{}^L\theta_t + {}^L\acute{\theta_t}}{2}$ *is the lower efficiency score of the subunit* t*.*

3.2 The Estimation of Returns to Scale with Interval Data

In parallel production systems with shared resources as shown in the previous section, when the data in the parallel production systems with shared resources is interval, the efficiency is interval too. So when the data is interval, a return to scale is estimated as multiple in best and worst conditions. The models to estimate returns to scale in best and worst conditions are defined as follows:

$$\begin{aligned}
&\max \quad 1s^{-(t)} + 1s^{-\acute{}(t)} + 1s^{+(t)} \\
&s.t. \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^L X_j^{(t)} + \lambda_p^U X_p^{(t)} + s^{-(t)} = \xi^U X_p^t, \qquad t = 1, ..., T \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^L X_j^{(s,t)} + \lambda_p^U X_p^{(s,t)} + s^{-\acute{}(t)} = \xi^U X_p^{(s,t)}, t = 1, \dots, T \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^U Y_j^{(t)} + \lambda_p^L Y_p^{(t)} - s^{+(t)} = \xi^L Y_p^{(t)}, \ t = 1, \dots, T \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1, \\
&\lambda_j \geq 0, s^{-(t)}, s^{-\acute{}(t)}, s^{+(t)} \geq 0 \qquad j = 1, ..., n
\end{aligned} \tag{3.5}$$

$$\begin{aligned}
&\max 1s^{-(t)} + 1s^{-\acute{}(t)} + 1s^{+(t)} \\
&s.t. \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^U X_j^{(t)} + \lambda_p^L X_p^{(t)} + s^{-(t)} = \xi^L X_p^t, \qquad t = 1, ..., T \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^U X_j^{(s,t)} + \lambda_p^L X_p^{(s,t)} + s^{-\acute{}(t)} = \xi^L X_p^{(s,t)}, t = 1, \dots, T \\
&\textstyle\sum_{j=1, j\neq p}^{n} \lambda_j^L Y_j^{(t)} + \lambda_p^U Y_p^{(t)} - s^{+(t)} = \xi^U Y_p^{(t)}, \ t = 1, \dots, T \\
&\textstyle\sum_{j=1}^{n} \lambda_j = 1, \quad \lambda_j \geq 0, s^{-(t)}, s^{-\acute{}(t)}, s^{+(t)} \geq 0 \qquad j = 1, ..., n.
\end{aligned} \tag{3.6}$$

Definition 3.4. *Suppose DMU_P with input-output $(X_p, X_p^{(s)}, Y_p)$ is efficient. Therefore, we have:*
(i) The optimal value of the objective functions of models (3.5) and (3.6) are greater than zero and $\xi^* > 1$ if and only if DMU_P has increasing returns to scale (IRS).
(ii) The optimal value of the objective functions of models (3.5) and (3.6) are greater than zero and $\xi^* < 1$ if and only if DMU_P has decreasing returns to scale (DRS).
(iii) The optimal value of the objective functions of models (3.5) and (3.6) are zero if and only if DMU_P has constant returns to scale (CRS).

4 The Applied Study on Iran's Bank

In this section, we chose 20 areas of one of Iran's Banks to describe returns to scale and the efficiency of parallel production units with shared resources using interval data.

Here each area is composed of three branches and each branch uses the network expenses as shared input and branch resources as the owned input to produce payment facilities (loans) as output. In Table (1),(2) we see the input and output data.

Table 1. The input-output data of 20 areas of Iran's Bank.

	Owned Input Branch Resource			Shared Input Network Expenses		
Area	Branch1	Branch2	Branch3	Branch1	Branch2	Branch3
1	[5007, 9613]	[175768, 179671]	[87243, 87243]	[10378, 11440]	[19021, 20463]	[9745, 10021]
2	[2926, 5961]	[119532, 121117]	[9945, 2120]	[16065, 16463]	[21547, 22590]	[10464, 11440]
3	[8732, 17752]	[123521, 129375]	[47575, 50013]	[14641, 15442]	[15980, 16442]	[8322, 8427]
4	[945, 1966]	[114678, 118050]	[19292, 19753]	[15231, 15440]	[9846, 10738]	[10497, 11816]
5	[8487, 17521]	[44647, 45212]	[3428, 3911]	[8652, 8745]	[10105, 10365]	[12026, 12426]
6	[13759, 27359]	[76514, 77698]	[13929, 15657]	[13976, 14746]	[14298, 14652]	[9798, 9907]
7	[587, 1205]	[78987, 80378]	[27827, 29005]	[12440, 13943]	[9725, 9907]	[10162, 10365]
8	[4646, 9559]	[42456, 44396]	[9070, 9983]	[5462, 5647]	[9147, 9295]	[5190, 5283]
9	[1554, 3427]	[34972, 35112]	[412036, 413902]	[4922, 5087]	[4361, 4491]	[5731, 5856]
10	[17528, 36297]	[25034, 25548]	[8638, 10229]	[3327, 3497]	[1902, 1934]	[8437, 8745]
11	[2444, 4955]	[15326, 16060]	[500, 937]	[1864, 1932]	[3521, 3604]	[7026, 7326]
12	[7303, 14178]	[23374, 24286]	[16148, 21353]	[3763, 3963]	[5542, 5647]	[8304, 8326]
13	[9852, 19742]	[45130, 45221]	[17163, 17290]	[10427, 11684]	[11012, 11135]	[6176, 6525]
14	[4540, 9312]	[45190, 45577]	[17918, 17964]	[8214, 8326]	[8217, 8316]	[8028, 8158]
15	[3039, 6304]	[27643, 28122]	[51582, 55136]	[2572, 2672]	[4280, 4491]	[6860, 6920]
16	[6585, 13453]	[61741, 62179]	[20975, 23992]	[2847, 3006]	[4601, 4652]	[5627, 5864]
17	[4209, 8603]	[25637, 26453]	[41960, 43103]	[3523, 3641]	[5428, 5647]	[8098, 8378]
18	[1015, 2037]	[22193, 22465]	[18641, 19354]	[5142, 5283]	[4637, 4795]	[6254, 6307]
19	[5800, 11875]	[17310, 17834]	[19500, 19569]	[2795, 2932]	[3410, 3497]	[8023, 8100]
20	[1445, 2922]	[159937, 163443]	[31700, 32061]	[10456, 11816]	[9412, 10020]	[9450, 9640]

Table 2. The interval output data for the 20 area of Iran's Bank.

	output payment facilities(loans)		
Area	Branch1	Branch2	Branch3
1	[2696995, 3126798]	[116798, 129817]	[109053, 109053]
2	[430377, 440355]	[71324, 72003]	[70976, 71860]
3	[1027546, 1061260]	[141728, 146599]	[141735, 147744]
4	[1145235, 1213541]	[68819, 70610]	[70345, 71194]
5	[390902, 395241]	[54523, 55579]	[55179, 56027]
6	[988115, 1087392]	[39538, 40518]	[3850, 40456]
7	[144906, 165818]	[38976, 40176]	[39773, 40796]
8	[408163, 416416]	[10379, 11637]	[13112, 13515]
9	[335070, 410427]	[47964, 48393]	[48423, 48998]
10	[700842, 768593]	[17862, 18173]	[16647, 18086]
11	[641680, 696338]	[9576, 9640]	[9834, 9989]
12	[453170, 481943]	[6689, 6777]	[7056, 7241]
13	[553167, 574989]	[14826, 15640]	[14310, 15404]
14	[309670, 342598]	[46361, 47301]	[45782, 46939]
15	[286149, 317186]	[11773, 12008]	[12085, 12208]
16	[321435, 347848]	[39474, 40974]	[41970, 42429]
17	[618105, 835839]	[13213, 14494]	[14311, 14544]
18	[248125, 320974]	[12170, 12871]	[15642, 16969]
19	[640890, 679916]	[6092, 6193]	[6631, 6710]
20	[119948, 120208]	[38978, 39877]	[37927, 38791]

We applied the parallel models (3.3) and (3.4) for these data, and we calculated the lower and upper bounds of efficiency for these 20 areas and their subordinated branches. With the use of models (3.5) and (3.6) and definition 4 we estimate returns to scale of the areas in both best and worst conditions. We assume weights as: $w_1 = 0.5, w_2 = 0.3, w_3 = 0.2$. The results of the efficiency measurement and returns to scale estimation are presented in Table 3.

Table 3. The results of the returns to scale estimation and the efficiency interval for 20 areas of Iran's Bank.

Area	E^1	E^2	E^3	Efficiency in the worst situation	Efficiency in the best situation	RTS in the worst situation	RTS in the best situation
1	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
2	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
3	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
4	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
5	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
6	[0.2129, 1.0000]	[0.5216, 1.0000]	[0.8669, 1.0000]	0.4363	1.0000	DRS	CRS
7	[0.7560, 1.0000]	[0.7510, 1.0000]	[0.7206, 1.0000]	0.7474	1.0000	IRS	CRS
8	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	IRS	IRS
9	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
10	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
11	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
12	[0.3214, 0.5959]	[0.6273, 0.6687]	[0.4336, 0.4701]	0.4356	0.5926	DRS	DRS
13	[0.1969, 0.6339]	[0.5002, 0.8054]	[0.6448, 0.8119]	0.3775	0.7210	IRS	IRS
14	[0.5394, 1.0000]	[0.8989, 1.0000]	[0.8171, 1.0000]	0.7028	1.0000	IRS	CRS
15	[0.5815, 1.0000]	[0.7361, 1.0000]	[0.5186, 1.0000]	0.6153	1.0000	IRS	IRS
16	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	CRS	CRS
17	[0.4645, 1.0000]	[0.7002, 1.0000]	[0.4447, 1.0000]	0.5315	1.0000	DRS	CRS
18	[1.0000, 1.0000]	[1.0000, 1.0000]	[1.0000, 1.0000]	1.0000	1.0000	IRS	CRS
19	[0.4338, 1.0000]	[0.9337, 1.0000]	[0.4508, 1.0000]	0.5872	1.0000	DRS	CRS
20	[0.5788, 1.0000]	[0.6064, 1.0000]	[0.7192, 1.0000]	0.5967	1.0000	IRS	IRS

Because the efficiency of the whole system is equal to the weighted sum of efficiencies of the subunits of the system, then the efficiency of each area is the weighted sum of the efficiencies of the subordinated branches. Columns 1, 2 and 3 in Table 2 show the efficiency of the branches 1, 2 and 3 in each area, respectively. In fact, in these 3 columns, the efficiencies of branches are reported in the best and worst situations. But, in columns 4 and 5, the efficiency of each area is presented in the best and worst situations, also in the last two columns of Table 2, returns to scale of areas are given in the best and worst conditions. According to the table of results, it can be seen that areas 1, 2, 3, 4, 5, 8, 9, 10, 11, 16 and 18 are efficient; namely, the efficiencies of these areas are equal to 1 in each two situations. But it can be seen that areas 6, 7, 14, 15, 19 and 20 are efficient in the upper bound and inefficient in the lower bound. Also, it can be observed that the returns to scale are not precise, and they are estimated as multiple. In areas 6, 7, 14, 17, 18 and 19 it is shown that the returns to scale are not estimated precisely. For example, area 6 has decreasing returns to scale in the worst situation and it has constant returns to scale in the best situation.

5 Conclusion

In this paper, models for calculating efficiency and estimating returns to scale in parallel production systems with shared resources were presented on imprecise data where the inputs and outputs were interval. With the use of these models, the efficiency of subunits and the whole system was obtained as interval and inexact. Then, the data was applied on 20 areas of Iran's Banks. In this paper, returns to scale were not exact and were estimated as multiple.

References

[1] A. Charnes, A. Cooper, WW. Rhodes, E. Measuring Efficiency of Decision Making Units, European Journal of Operational Research 1 (1978) 429-444.

[2] WD. Cook, M. Kress, LM. Seiford, On the use of ordinal data in data envelopment analysis, Journal of the Operational Research Society 44 (1993) 133-140.

[3] WW. Cooper, KS. Park, G. Yu, IDEA and AR-IDEA: Models for dealing with imprecise data in DEA, Management Science 45 (1999) 597-607.

[4] D. Primont, Efficiency Measures for Multiplan Firms, Operations Research Letters 3 (1984) 257-260.

[5] C. Kao, PL. Chang, SN. Hwang, Data Envelopment Analysis in Measuring the Efficiency of Forest Management, Journal of Environmental Management 38(1993) 73-83.

[6] C. Kao, the Efficiency Measurement for Parallel Production Systems, European Journal of Operational Research 4 (2008) 56-66.

[7] C. Kao, Interval efficiency measures in data envelopment analysis with imprecise data, European Journal of Operational Research 174 (2006) 1087-1099 .

[8] C. Kao, Measuring the Efficiency of Forest District with Multiple Working Circles, Journal of Operational Research Society 49 (1998) 583-590.

[9] C. Kao, PL. Chang, SN. Hwang, Data Envelopment Analysis in Measuring the Efficiency of Forest Management, Journal of Environmental Management 38(1993) 73-83.

[10] M. Khodabakhshi, Y. Gholami, H. Kheirollahi, An additive model approach for estimating returns to scale in imprecise data envelopment analysis, Applied Mathematical Modelling 34 (2010) 1247-1257.