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Research Article

Defuzzification Method for Solving Fuzzy Linear Programming Problems

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Abstract

Several authors have proposed different methods to find solution to fully fuzzy linear programming (FFLP) problems. However, it seems all the existing methods are based on the assumption that all fuzzy coefficients and fuzzy variables are non-negative fuzzy numbers. Nevertheless, a new method is proposed herein to solve FFLP problems with arbitrary fuzzy coefficients and arbitrary fuzzy variables, where there are no restrictions on the elements used in the FFLP problems. By utilizing the radius of gyration function (ROG), it is demonstrated that fuzzy solution obtained by solving FFLP problems, produce exact fuzzy optimal solutions for those problems. The introduced in this article is very easly understood and simple to apply to fully fuzzy linear systems occurring in real life situations.

Keywords : Fully fuzzy linear programming; Exact fuzzy optimal solution; Ranking function; Radius of gyration.

1 Introduction

Linear programming (LP) is one of the most frequently applied operations research techniques. In real world situations, a linear programming model involves many parameters with values assigned by experts. However, frequently neither experts nor decision makers the precise know the values of those parameters. Therefore, it is useful to consider the experts, knowledge regarding the parameters as fuzzy data [1], such as the first formulation of fuzzy linear programming (FLP) proposed by Saneifard et al. [12]. Afterwards, many authors have considered various kinds of FLP problems and have proposed several approaches for solving these problems [2, 3, 4, 5, 7, 8, 9]. Fuzzy set theory, for instance, has been applied to many disciplines such as control theory, management science, mathematical modeling, and industrial applications. The concept of fuzzy linear programming (FLP) on a general level was first proposed by Saneifard et al [11]. Later on, other authors considered various types of FLP problems and proposed several approaches to solve them. The fuzzy linear programming problem in which all parameters and variables are represented by fuzzy numbers is known as fully fuzzy linear programing problem (FFLP) problems. Wang et al. [9] proposed a method to find the

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solution for a fully fuzzified linear programming problem by changing the objective function into a multi-objective LP problem. Allahviranloo et al [1] offered a method based on ranking functions for solving FFLP problems. Murakami et al. [10] suggested a method for solving LP problems with vagueness in constraints by using ranking functions. The main disadvantage of the solutions obtained by existing methods is they do not satisfy the constraints with precision. In varios financial applications, linear systems of equations play a major role. Yager [19] proposed a fuzzy linear programming approach for finding the exact solution of, fully fuzzy linear system of equations; however, it is applicable only if all the elements of the coefficient matrix are nonnegative fuzzy numbers. Later, Chen et al. [6] proposed a new method to attain fuzzy optimal solutions to FFLP problems with equality constraints; however, these can only be applied if the elements of the coefficient matrix are symmetrical fuzzy numbers and the obtained solutions are approximate but not exact. Saneifard [12] proposed a new effect of radius of gyration with network FFLP. Saneifard [15] later proposed a new method for defuzzification by weighted distance. The primary focus in this paper is to introduce a new method for solving FFLP problems using ranking functions, and illustrated the proposed method with an example in section 5.

The paper is organized as follows: Section 2 reviews some basic definitions and arithmetics between two trapezoidal fuzzy numbers. In Section 3, Ranking function radius of gyration for solving FFLP problems is presented. In Section 4, the general form of FFLP problems with equality constraints is presented. Section 5 proposes a new method for solving FFLP problems with equality constraints having arbitraris fuzzy coefficients and unrestricted fuzzy variables.

2 Preliminaries

In this section some basic definitions of fuzzy set theory are reviewed.

Definition 2.1 *The Characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set fall within a specified range i.e. $\mu_{\tilde{A}} : X \longrightarrow [0,1]$.*

The assigned value indicate the membership grade of the element in the set A.

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$is called a fuzzy set.

Definition 2.2 *A fuzzy number $\widetilde{A}$ is said to be an unrestricted fuzzy number if the domain of its membership function is a set of real numbers.*

The set of unrestricted fuzzy numbers can be represented by $F(R)$.

Definition 2.3 *A fuzzy number $\tilde{A}$ is said to be non-negative fuzzy number if and only if $\mu_{\tilde{A}}(x) = 0 \quad \forall x < 0$.*

The set of non-negative fuzzy numbers may be represented by $F(R^+)$.

Definition 2.4 *A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by , where $a \leqslant b \leqslant c \leqslant d$*

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a \leqslant x < b \\ 1 & b \leqslant x \leqslant c \\ \frac{d-x}{d-c} & c < x \leqslant d. \end{cases}$$

Definition 2.5 *A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a non-negative (or non-positive) trapezoidal fuzzy number i.e., $\tilde{A} \geqslant 0\,(\tilde{A} \leqslant 0)$ if and only if $a \geqslant 0\,(c \leqslant 0)$.*

The set of non-negative trapezoidal fuzzy numbers can be represented by $TF(R^+)$.

Definition 2.6 *Two trapezoidal fuzzy number $\tilde{A}_1 = (a, b, c, d)$ and $\tilde{A}_2 = (e, f, g, h)$ are said to be equal i.e., $\tilde{A}_1 = \tilde{A}_2 =$ if and only if $a = e$, $b = f$, $c = g$, $d = h$.*

Definition 2.7 *A fuzzy number $\tilde{A}$ is called N-zero fuzzy number ,denoted by $\tilde{A} \equiv 0$ if its membership function $\mu_{\tilde{A}}(x)$ satisfies $\mu_{\tilde{A}}(0^-) = \mu_{\tilde{A}}(0^+) = \mu_{\tilde{A}}(=) \neq 0$.*

Henceforth a trapezoidal fuzzy number $\tilde{A} = (a, b, c, d) \equiv 0$ if and only if: $0 < 0 < d$.

Remark 2.1 *A trapezoidal fuzzy number that is not positive, not negative and not zero is a N-zero trapezoidal fuzzy number.*

Remark 2.2 *A trapezoidal fuzzy number is a triangular fuzzy number if* $b = c$ *and a N-zero triangular fuzzy number if* $0 < 0 < d; b = c$.

Definition 2.8 *Let* $\tilde{A}_1 = (a, b, c, d)$ *and* $\tilde{A}_2 = (e, f, g, h)$ *be two non-negative trapezoidal fuzzy number then*

i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a, b, c, d) \oplus (e, f, g, h) = (a + e, b + f, c + g, d + h)$,

ii) $-\tilde{A}_1 = -(a, b, c, d) = (-d, -c, -b, -a)$,

iii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a, b, c, d) \ominus (e, f, g, h) = (a - h, b - g, c - f, d - e)$,

iv) $\tilde{A}_1 \otimes \tilde{A}_2 = (a, b, c, d) \otimes (e, f, g, h) \cong (ae, bf, cg, dh)$,

Remark 2.3 *Let* $\tilde{A}_1 = (a, b, c, d)$ *and* $\tilde{A}_2 = (x, y, z, w)$ *denote two unrestricted trapezoidal fuzzy numbers. Then*

$$\tilde{A}_1 \otimes \tilde{A}_2 = \begin{cases} (\min(ax, dx), \min(by, cy), \max(bz, cz), \max(aw, dw)) \\ if (a, b, c, d) \geq 0, that is, if a \geq 0 \\ (\min(aw, dw), \min(by, cy), \max(bz, cz), \max(dw, ax)) \\ if (a, b, c, d) \equiv 0, that is, if a < 0 \leq b \leq c < d \\ (\min(aw, dx), \min(bz, cy), \max(by, cz), \max(dw, ax)) \\ if (a, b, c, d) \equiv 0, that is, if a < b < 0 < c < d \\ (\min(aw, dx), cz, bz, \max(dw, ax)) \\ if (a, b, c, d) \equiv 0, that is, if a < b \leqslant c < 0 < d \end{cases}$$

Remark 2.4 *The minimum and maximum are represented by min and max respectively, where*

$$\min(a, b) = \left(\frac{a+b}{2}\right) - \left|\frac{a-b}{2}\right|,$$
$$\max(a, b) = \left(\frac{a+b}{2}\right) - \left|\frac{a-b}{2}\right|.$$

Definition 2.9 *The moment of inertia of the area A with respect to the x axis and the moment of inertia of the area A with respect to the y axis are defined respectively as:*

$$\begin{cases} I_x = \int y^2 dA, \\ I_y = \int x^2 dA, \end{cases}$$

The moment of inertia of the trapezoidal fuzzy number can be obtained as follows :

$$\begin{cases} I_x = (I_x)_1 + (I_x)_2 + (I_x)_3, \\ I_y = (I_y)_1 + (I_y)_2 + (I_y)_3, \end{cases}$$

where

$$\begin{cases} (I_x)_1 = \frac{(b-a)}{12}, \\ (I_x)_2 = \frac{(c-b)}{3}, \\ (I_x)_3 = \frac{(d-c)}{12}. \end{cases}$$

$$\begin{cases} (I_y)_1 = \frac{(b-a)^3}{4} + \frac{(b-a)^2}{2}a^2 + \frac{2(b-a)^2}{3}a, \\ (I_y)_2 = \frac{(c-b)^3}{3} + (c-b)b^2 + (c-b)^2 b, \\ (I_y)_3 = \frac{(d-c)^3}{12} + \frac{(d-c)}{2}c^2 + \frac{(d-c)^2}{3}c. \end{cases}$$

Therefore, the (ROG) point of a trapezoidal fuzzy number (a, b, c, d) can be calculated as:

$$r_x = \sqrt{\frac{(I_x)_1 + (I_x)_2 + (I_x)_3}{((c-b)+(d-a))/2}}$$

$$r_y = \sqrt{\frac{(I_y)_1 + (I_y)_2 + (I_y)_3}{((c-b)+(d-a))/2}}$$

Definition 2.10 *For n fuzzy numbers $A_1, A_2, ..., A_n$ the minimum crisp value $\tau_{\min}$ is defined as:*

$$\tau_{\min} = \min\{x | x \in Domain(A_1, A_2, ..., A_n)\}$$

3 Ranking function

A ranking function is a function $RR : F(R) \longrightarrow R$, where F(R) is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists.
Let $\tilde{A}_1, \tilde{A}_1 \in F$ be two arbitrary fuzzy numbers. Then efine the ranking of $\tilde{A}_1$ and $\tilde{A}_1$ by RR on F as follows:

1) $RR(\tilde{A}_1) > RR(\tilde{A}_2) \Longleftrightarrow \tilde{A}_1 > \tilde{A}_2$,

2) $RR(\tilde{A}_1) < RR(\tilde{A}_2) \Longleftrightarrow \tilde{A}_1 < \tilde{A}_2$,

3) $RR(\tilde{A}_1) = RR(\tilde{A}_2) \Longleftrightarrow \tilde{A}_1 \approx \tilde{A}_2$.

For any arbitrary fuzzy number $\tilde{A}$, the ranking function proposed by Saneifard [15] is as follows:

$$RR\tilde{A} = \sqrt{(r_x^A - \tau_{\min})^2 + (r_y^A - 0)^2}$$

4 Fully fuzzy linear programming problem

Consider the following fully fuzzy linear programming problems with m fuzzy equality constraints and n fuzzy variables that may be formulated as follows:

$$Max(Min)\tilde{z} = \tilde{c}_1 \otimes \tilde{x}_1 \oplus \cdots \tilde{c}_n \otimes \tilde{x}_n$$

$$Subjectto \tilde{a}_{11} \otimes \tilde{x}_1 \oplus \cdots \tilde{a}_{1n} \otimes \tilde{x}_n = \tilde{b}_1$$

$$\vdots$$

$$\tilde{a}_{m1} \otimes \tilde{x}_1 \oplus \cdots \tilde{a}_{mn} \otimes \tilde{x}_n = \tilde{b}_m$$

$$\tilde{x} \in F(R).$$

The matrix form of the above equation is:

$$Max(Min)\tilde{z} = \tilde{c}^T \otimes \tilde{X},$$

$$Subjectto \tilde{A} \otimes \tilde{X} = \tilde{b},$$

Where $\tilde{c}^T = [\tilde{c}_j]_{1\times n}, \tilde{X} = [\tilde{x}_j]_{n\times 1}, \tilde{A} = [\tilde{a}_{ij}]_{m\times 1}, \tilde{b} = [\tilde{b}_i]_{m\times 1} and \tilde{A}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$.

Definition 4.1 *Any $\tilde{X} = [\tilde{x}_j]_{n\times 1}$ which satisfies the set of constraints of FFLP is called a feasible solution.*

Let be the set of all feasible solutions of FFLP. We say that $\tilde{x}^*$ is optimal feasible solution for FFLP if for all $x \in S$:

$$RR(\tilde{c}^T \otimes \tilde{x}^*) \geq RR(\tilde{c}^T \otimes \tilde{x})$$

(in case of maximization problem),

$$RR(\tilde{c}^T \otimes \tilde{x}^*) \leq RR(\tilde{c}^T \otimes \tilde{x})$$

(in case of minimization problem).

5 Proposed method

In this section a new method is proposed to solve a fully fuzzy linear programming problem with no restrictions on the parameters The steps of the proposed method are as follows:

Step 1:

Substituting $\tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_i]_{m\times 1}, \tilde{X} = [\tilde{x}_j]_{n\times 1} and \tilde{c}^T = [\tilde{c}_j]_{1\times n} where \tilde{a}_{ij}, \tilde{c}_j, \tilde{x}_j, \tilde{b}_i \in F(R)$.

The FFLP problem (P_1) can be written as:

$$Maximize(orminimize)(\sum \tilde{c}_j \otimes \tilde{x}_j),$$

$$Subjectto \sum \tilde{a}_{ij} \otimes \tilde{x}_j = \tilde{b}_i, \ \ \forall i = 1, 2, \cdots, m$$
$$where, \tilde{x}_j \in F(R).$$

Step 2:

Substituting $\tilde{c}_j, \tilde{a}_{ij}, \tilde{b}_i$ and $\tilde{x}_j$ are represented by unrestricted trapezoidal fuzzy numbers, $(\tilde{p}_j, \tilde{q}_j, \tilde{r}_j, \tilde{s}_j)$, $(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij})$, $(\tilde{b}_i, \tilde{g}_i, \tilde{h}_i, \tilde{k}_i)$, $(\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j)$ respectively the FFLP problem, obtained in step 1, can be written as:

$$Max(min)(\sum_{j=1}^{n}(\tilde{p}_j, \tilde{q}_j, \tilde{r}_j, \tilde{s}_j) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j)),$$

Subject to

$$\sum_{j=1}^{n}(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) = (\tilde{b}_i, \tilde{g}_i, \tilde{h}_i, \tilde{k}_i),$$

$$\forall i = 1, 2, \cdots, m$$

where, $(\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) \in TF(R)$.

Step 3:

Assuming

$$(\tilde{p}_j, \tilde{q}_j, \tilde{r}_j, \tilde{s}_j) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) = (\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{o}_{ij}),$$

$$(\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) = (\tilde{f}_{ij}, \tilde{p}_{ij}, \tilde{g}_{ij}, \tilde{r}_{ij}),$$

where $(\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) \in TF(R)$.

The FFLP problems obtained in step 2, may be written as:

$$Max(orMin)(\sum_{j=1}^{n}(\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{o}_{ij}), \ \ \forall i = 1, \cdots, m$$

$$Subjectto \sum_{j=1}^{n}(\tilde{f}_{ij}, \tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij}) = (\tilde{b}_j, \tilde{g}_j, \tilde{h}_j, \tilde{k}_j),$$

where $(\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j) \in TF(R)$. And

$$(\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{o}_{ij}) = (\tilde{p}_j, \tilde{q}_j, \tilde{r}_j, \tilde{s}_j) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j)$$

$$(\tilde{f}_{ij}, \tilde{p}_{ij}, \tilde{q}_{ij}, \tilde{r}_{ij}) = (\tilde{a}_{ij}, \tilde{b}_{ij}, \tilde{c}_{ij}, \tilde{d}_{ij}) \otimes (\tilde{x}_j, \tilde{y}_j, \tilde{z}_j, \tilde{w}_j).$$

As $\tilde{a}_{ij}, \tilde{x}_j$ and $\tilde{c}_j$ are orbitrary fuzzy numbers, the above product can be defined in four subcases.

Step 4:

Using ranking function radius of gyration, the fuzzy objective function of the FFLP problem obtained in Step 3, can be converted into the crisp objective function and using Definition 2.6 the fuzzy linear programming problem, can be converted into the following crisp non-linear programming problem:

$$Max(Min)PR(\sum_{j=1}^{n}(\tilde{l}_{ij}, \tilde{m}_{ij}, \tilde{n}_{ij}, \tilde{o}_{ij})$$

$$Subjectto \sum_{j=1}^{n}(\tilde{f}_{ij} \ \ \forall i = 1, 2, \cdots, m$$

$$\sum_{j=1}^{n}(\tilde{p}_{ij} \ \ \forall i = 1, 2, \cdots, m$$

$$\sum_{j=1}^{n}(\tilde{q}_{ij} \ \ \forall i = 1, 2, \cdots, m$$

$$\sum_{j=1}^{n}(\tilde{r}_{ij} \ \ \forall i = 1, 2, \cdots, m$$

$$y_j - x_j \geq 0, \ \ z_j - y_j \geq 0, \ \ w_j - z_j \geq 0$$

where $x_j, y_j, z_j, w_j \in R \ \ \forall i = 1, 2, \cdots, m, \ \ j = 1, 2, \ldots, n$.

Step 5:

Solve the crisp non-linear programming problem, obtained in step 4 to find the optimal solution $x_j, y_j, z - j$ and w_j.

Step 6:

Put the values of $x_j, y_j, z - j$ and w_j, obtained in step 5, in $\tilde{x}_j = (x_j, y_j, z - j, w_j)$ to find the exact fuzzy optimal solution$\tilde{x}_j$.

Case 2:

If $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \equiv 0, (p_j, q_j, r_j, s_j) \equiv 0$ that is $a_{ij} < 0 \leq b_{ij} \leqslant c_{ij} < d_{ij}, p_j < 0 < q_j \leq r_j < s_j$.

$$(l_{ij}, m_{ij}, n_{ij}, o_{ij}) = ((\frac{p_j w_j - q_j x_j}{2}) - |\frac{p_j w_j - q_j x_j}{2}|, (\frac{q_j + r_j}{2})y_i - (\frac{r_j - q_j}{2})|y_j|, (\frac{q_j + r_j}{2})z_j + (\frac{r_j - q_j}{2})|z_j|, (\frac{s_j w_j + pjx_j}{2}) + |\frac{s_j wj - p_j w_j}{2}|).$$

$$(f_{ij}, p_{ij}, q_{ij}, r_{ij}) = ((\frac{a_{ij} w_j - d_{ij} x_j}{2}) - |\frac{a_{ij} w_j - d_{ij} x_j}{2}|, (\frac{b_{ij} + c_{ij}}{2})y_i - (\frac{c_{ij} - b_{ij}}{2})|y_j|, (\frac{b_{ij} + c_{ij}}{2})z_j + (\frac{c_{ij} - b_{ij}}{2})|z_j|, (\frac{d_{ij} w_j + aijx_j}{2}) + |\frac{d_{ij} wj - a_{ij} x_j}{2}|).$$

Case 1:

If $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \geq 0$, $(p_j, q_j, r_j, s_j) \geq 0$ that is $\tilde{a}_{ij} \geq 0, \tilde{p}_j \geq 0$

$$(l_{ij}, m_{ij}, n_{ij}, o_{ij}) = ((\frac{p_j + s_j}{2})x_j - (\frac{s_j - p_j}{2})|x_j|, (\frac{q_j - r_j}{2})y_i - (\frac{r_j - q_j}{2})|y_j|, (\frac{q_j + r_j}{2})z_j + (\frac{r_j - q_j}{2})|z_j|, (\frac{p_j + s_j}{2})w_j + (\frac{s_j - p_j}{2})|w_j|).$$

$$(f_{ij}, p_{ij}, q_{ij}, r_{ij}) = ((\frac{a_{ij} + d_{ij}}{2})x_j - (\frac{d_{ij} - a_{ij}}{2})|x_j|, (\frac{b_{ij} + c_{ij}}{2})y_j - (\frac{c_{ij} - b_{ij}}{2})|y_j|, (\frac{b_{ij} + c_{ij}}{2})z_j + (\frac{c_{ij} - b_{ij}}{2})|z_j|, (\frac{a_{ij} + d_{ij}}{2})w_j + (\frac{d_j - a_j}{2})|w_j|).$$

Case 3:

If $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \approx 0$, $(p_j, q_j, r_j, s_j) \approx 0$ that is $a_{ij} < b_{ij} < 0 < c_{ij} < d_{ij}, p_j < q_j < 0 < r_j < s_j$.

$$(l_{ij}, m_{ij}, n_{ij}, o_{ij}) = ((\frac{p_j w_j + q_j x_j}{2}) - |\frac{p_j w_j - q_j x_j}{2}|, (\frac{q_j z_j + r_j y_j}{2}) - |\frac{q_j z_j - r_j y_j}{2}|, (\frac{q_j y_j + r_j z_j}{2}) + |\frac{q_j y_j - r_j z_j}{2}|, (\frac{s_j w_j + pj x_j}{2}) + |\frac{s_j wj - p_j xj}{2}|).$$

$(f_{ij}, p_{ij}, q_{ij}, r_{ij}) =$

$$((\frac{a_{ij} w_j + d_{ij} x_j}{2}) - |\frac{a_{ij} w_j - d_{ij} x_j}{2}|, (\frac{b_{ij} z_j + c_{ij} y_j}{2}) - |\frac{b_{ij} z_j - c_{ij} y_j}{2})|, (\frac{b_{ij} y_j + c_{ij} z_j}{2}) + |\frac{b_{ij} y - j - c_{ij} z - j}{2}|, (\frac{d_{ij} w_j + aij x_j}{2}) + |\frac{d_{ij} wj - a_{ij} x_j}{2}|).$$

Case 4:

If $(a_{ij}, b_{ij}, c_{ij}, d_{ij}) \approx 0$, $(p_j, q_j, r_j, s_j) \approx 0$ that is $a_{ij} < b_{ij} < c_{ij} < 0 < d_{ij}, p_j < q_j < r_j < 0 < s_j$.

$$(l_{ij}, m_{ij}, n_{ij}, o_{ij}) = ((\frac{p_j w_j + q_j x_j}{2}) - |\frac{p_j w_j - q_j x_j}{2}|, r_j z_j, q_j z_j, (\frac{s_j w_j + p_j x_j}{2}) + |\frac{s_j w_j - p_j x_j}{2}|),$$

$$(f_{ij}, p_{ij}, q_{ij}, r_{ij}) = ((\frac{a_{ij} w_j + d_{ij} x_j}{2}) - |\frac{a_{ij} w_j - d_{ij} x_j}{2}|, c_{ij} z_j, b_{ij} z_j, (\frac{d_{ij} w_j + a_{ij} x_j}{2}) + |\frac{d_{ij} w_j - a_{ij} x_j}{2}|).$$

Step 7:

Put the values of $\tilde{x}_j$, obtained in step 6, in $\sum_{j=1}^{n} \tilde{c}_j \otimes \tilde{x}_j$ to find the fuzzy optimal value.

6 Conclusion

In this paper a new method is proposed for solving fully fuzzy linear programming problems, having arbitrary fuzzy coefficients and unrestricted fuzzy variables. The proposed methods are very easy to understand and to apply for fully fuzzy linear systems occurring in real life situation.

References

[1] T. Allahviranloo, S. Abbasbandy and R. Saneifard, A method for ranking of fuzzy numbers using new weighted distance, *Mathematical and Computational Applications* 2 (2011) 359-369.

[2] T. Allahviranloo, S. Abbasbandy and R. Saneifard, An approximation approach for ranking fuzzy numbers based on weighted interval-value, *Mathematical and Computational Applications* 3 (2011) 588-597.

[3] S. J. Chen, S. M. Chen, Fuzzy risk analysis based on similarity of generalized fuzzy numbers,*IEEE Transactions on Fuzzy Systems* 11 (2003) 45-56.

[4] C. H. Cheng, A new approach for ranking fuzzy numbers by distance method, *Fuzzy Sets and Systems* 95 (1998) 307-317.

[5] L. H. Chen, H. W. Lu, An approximate approach for ranking fuzzy numbers based on left and right dominance, *Computers and Mathematics with Applications* 41 (2001) 1589-1602.

[6] L. H. Chen, H. W. Lu, The preference order of fuzzy numbers, *Computers and Mathematics with Applications* 44 (2002) 1455-1465.

[7] T. Chu, C. Tsao, Ranking fuzzy numbers with an area between the centroid point and original point, *Computers and Mathematics with Applications* 43 (2002) 11-117.

[8] D. Dubois, H. Prade, Ranking of fuzzy numbers in the setting of possibility theory, *Information Science* 30 (1983) 183-224.

[9] H. W. Lu, C. B. Wang, An index for ranking fuzzy numbers by belief feature, *Information and Management Sciences* 16 (2005) 57-70.

[10] S. Murakami, S. Maeda, S. Imamura, Fuzzy decision analysis on the development of a centralized regional energy control system, *In Proceedings of the IFAC symposium on fuzzy information, knowledge representation and decision analysis* (1983) 363-368, Tokyo, Japan.

[11] Rahim Saneifard, Rasoul Saneifard, The Median Value of Fuzzy Numbers and its Applications in Decision Making, *Journal of Fuzzy Set Valued Analysis* http://dx.doi.org/10.5899/2012/jfsva-00051/.

[12] R. Saneifard, T. Allahviranloo, F. Hosseinzadeh and N. Mikaeilvand, Euclidean ranking DMU's with fuzzy data in dea, *Applied Mathematical Sciences* 60 (2007) 2989-2998.

[13] R. Saneifard, A method for defuzzification by weighted distance, *International Journal of Industrial Mathematics* 3 (2009) 209-217.

[14] R. Saneifard, Ranking L-R fuzzy numbers with weighted averaging based on levels, *International Journal of Industrial Mathematics* 2 (2009) 163 - 173.

[15] R. Saneifard, Defuzzification method for solving fuzzy linear systems, *International Journal of Industrial Mathematics* 4 (2009) 321-331.

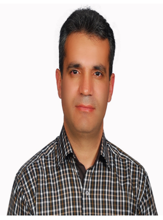
[16] R. Saneiafrd, Some properties of neural networks in designing fuzzy systems, *Neural Computing and Appllications* http://dx.doi.org/10.1007/s00521-011-0777-1/.

[17] R. Saneiafrd, Designing an algorithm for evaluating decision-making units based on neural weighted function, *Neural Computing and Appllications* http://dx.doi.org/10.1007/s00521-012-0878-5/.

[18] Y. M. Wang, J. B. Yang, D. L. Xu, K. S. Chin, On the centroids of fuzzy numbers, *Fuzzy Sets and Systems* 157 (2006) 919-926.

[19] R. R. Yager, On a general class of fuzzy connectives,*Fuzzy Sets and Systems* 4 (1980) 235-242.

[20] D. Yong, L. Qi, A TOPSIS-based centroid-index ranking method of fuzzy numbers and its application in decision-making, *Cybernetics and Systems* 36 (2005) 581-595.



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