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Research Article

Estimation of Marginal Productivity of Supply Chains for Capacity Planning and Resource Allocation: A Case Study of the Power Industry

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Abstract

Marginal productivity (MP) estimation is utilized to plan maximum output levels and allocate resources to address fluctuating demand for supply fuel in the power plant sector as well as adjust transferring and dispatching in the transmission and distribution networks. In this paper, a data envelopment analysis (DEA) model is introduced for estimating the directional marginal productivity of supply chain divisions. The proposed model for estimating the directional marginal productivity in the supply chain tries to find the optimal direction of efficient divisions on the frontier so that marginal profit is maximized. This model measures efficiency by maximizing marginal profit for multiple outputs in predetermined directions based on multiple inputs. The purpose of this study is to develop acceptable techniques for responding to demand fluctuations, especially in the energy and power plant sectors. This is when confronted with efficiency losses from climate change and critical conditions. The results suggested that the oil field division of one of the supply chains had fundamental capacities to respond to peak demand. Furthermore, the power plant division of this supply chain also had a considerable structure for the marginal profit maximization of outputs. Additionally, there were transmitters and distribution lines that obtained marginal profit maximization by adding one extra unit to the line's length in the determined direction

Keywords: Climate Change; Demand Fluctuation; Directional Marginal Productivity; Marginal Profit Maximization; Capacity Planning.

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1. Introduction

Climate change causes changes in temperature, the temporal pattern of precipitation, and the amount of rainfall. Moreover, temperature changes have a direct effect on energy production performance. Indeed, the gas turbine output of gas power plants and their productivity depend on the environment's temperature [1]. Additionally, the output power of gas power plants decreases by 5% to 10% for each 10 degrees increase up to 15 degrees [2]. The boiler is the main factor for efficiency reduction in thermal power plants, as the increase in environment temperature until 5 centigrade provides a thermal efficiency reduction of 2.35% and an energy efficiency decrease of 8.2%, which is a mean power plant productivity decline of about 0.45 for every one degree increase in weather temperature. Generally, climate change causes power plant productivity abatement as power plants' fuel consumption increases with increases in the environment's temperature.

The increase in temperature reduces the efficiency of fossil power plants, as they require more fossil fuel for power production.

Also, efficiency results of combined cycle power plants indicate that environmental temperature has significant effects on energy production abatement [3]. Similarly, environmental factors such as monsoon winds and storms cause transmission airline conductor vibrations, which create broken wire and a power cut in the transmitter network. In other words, fluctuations in distribution lines have bad and destructive effects on dispatching power.

The major factors that cause disturbances in the voltage of distribution networks are the installation of power sources such as electric motors, electric furnaces, and electric welding machines. Also, thunder and lightning, rainfall, and the wetting of the wiring paths create power fluctuations on distribution lines. Climate change and factors such as wind, storms, and frost, along with the galloping and swinging of power transmission wires, contribute to power loss and wasted energy. In response, it is fundamentally important to adjust the output level of supply chain energy and power plant sectors, transmitters, and distribution lines by using variable resources to adjust their output levels. Moreover, managing existing capacities and adjusting output levels by controlling available resources results in system efficiency enhancements in the electricity supply chain.

Differential characteristics of efficient frontiers are important for the analysis of production technologies in economic activities. The marginal rate plays an important role in economic theory and its applications. If the frontier is smooth, the partial derivatives of the production function that defines the efficient frontier can be used for the calculation of various elasticity measures to see how changing one variable affects the other production factors. Marginal productivity (MP) measures the response of an output to an extra unit of input. Indeed, MP characterizes how the dependent variable will be affected by changing one extra unit of the independent variable. In most cases, the estimation of the MP for a specific firm is not a fixed value, so the decision-making unit should select the direction to move in toward the direction of marginal profit maximization. Specifically, this study examines how electricity supply chain divisions are able to respond to demand fluctuations via variable resource management. Indeed, this paper aims to identify supply chain divisions that create MP maximization outputs in predetermined directions based on the increase of one unit of inputs, as the proposed model considers more than an input for supply chain divisions.

In this way, the MP estimate is based on the increment of each input of supply chain divisions and determining marginal profit maximization outputs in normalized directions. Directional marginal productivity (DMP) uses various directions in directional distance functions (DDF) to allocate resources as well as measure efficiency via marginally profitable orientation. An expectable trade-off between multiple products based on an extra unit of inputs is referred to as multi-output MP estimation.

In this case, the energy and power plant sectors and transmission and distribution networks should have the necessary patterns to adjust output levels when confronting increased demand based on climate change and other critical situations. In other words, the power plant sectors of supply chains face a special climate situation as they consume more fossil fuel to increase efficiency and productivity. Similarly, the transmitter and distributor lines of the electricity supply chain are meeting more power losses under climate changes and needing more electricity for economic return increment. Thus, the transmitter and distributer networks should have the necessary preparation for capacity adjustments and adjust output levels by controlling variable resources.

This study examines how electricity supply chain divisions, especially the energy and power plant sectors and transmission and distribution lines, will respond to demand fluctuations in critical situations such as climate change, defective equipment, power losses, unauthorized uses, and excessive domestic consumption in electricity production. In this case, DMP estimates marginal profit maximization based on predetermined directions in supply chain divisions. The direction vectors indicate weights between investigated areas that can be defined by decision-making in the production process.

For capacity planning and resource allocation, we have to move toward MP, which is essential for supply chain divisions. The current paper presents an MP model for multi-output based on a DDF, as the proposed model describes how the allocation of one unit of each input affects the multiple outputs. Indeed, benefit capacity adjustment under demand fluctuations is performed according to MP estimation. The proposed model estimates MP outputs simultaneously for more than one input in 11 normalized directions. The supply chain divisions that obtained marginal profit maximization have the necessary capacities for responsiveness to demand fluctuations in climate change and critical situations. For illustration, oil and gas fields should have acceptable techniques for responding to fuel demand fluctuations when the power plant sections confront efficiency losses in climate change conditions and other critical situations. Furthermore, the power plant sector needs improved engineering systems and specialized workers for more power production in order to respond appropriately to voltage losses in transmission lines. Similarly, transmission lines should be able to respond to demand for an increase in the distribution network in unforeseen events under various climate conditions.

The remainder of this paper is organized as follows: In Section 2, a literature review of how DEA has been used to respond to demand fluctuations in the energy and power plant sectors and transmission and distribution lines is presented. Section 3 is devoted to introducing the approach for calculating proportional reallocation for obtaining MP of supply chain divisions in the presence of inputs, desirable outputs, and sets of intermediate measures. The next section presents a case study to demonstrate the applicability of the proposed method to the Iranian power industry. Finally, the last section presents conclusions.

2. Literature Review

Below are brief overviews of various studies on the single-output MP DEA models, the DMP, and the sustainability of supply chains.

2.1 MP

Economists use the term ‘‘elasticity’’ to measure the percentage of how changing one variable affects other variables. In a DEA framework, the dual multiplier linear program to the primal envelopment model represents MP, and it also refers to shadow price. Indeed, an expectable trade-off between multiple products refers to multi-output MP estimation.

Banker and Thrall [4] and Førsund et al. [5] developed a range of scale elasticity (SE) to explicitly support the decision-maker since DEA may not have a unique shadow price scale. Fare et al. [6] applied a nonparametric approach to obtain the capacity measure from a cross-sectional dataset. Banker et al. [7] first defined the classical returns to scale (RTS) based on the production possibility set (PPS) in the DEA framework and proposed a piecewise linear production function estimated using DEA based on collected observations. However, a piecewise linear frontier forms a polyhedral set representing production technologies and is thus not differentiable.

Podinovski and Førsund [8] and Atici and Podinovski [9] pointed out that the derivative in the RTS may not always exist, and thus they replaced the classical derivative by directional derivatives, defined left-hand and right-hand SE, gave an explicit definition of differential characteristics on a non-differentiable efficient frontier, and proposed a directional-derivative approach to calculate elasticity measures without any simplifying assumptions. Examples of direct methods for the calculation of SE can be found in Podinovski and Førsund in Krivonozhko et al. [10] and Førsund et al. [11], and for various marginal rates in Rosen et al. [12] and Asmild et al. [13]. Podinovski et al. [14] and podinovski et al. [15] also noted the possible non-proportional changes in inputs or outputs in research organizations.

It should be noted that changes over time and RTS are different concepts. Lee [16] suggests that firms should select the direction via DMP to move in the direction of marginal profit maximization. Yang et al. [17] proposed research based on RTS. Yang et al. [18] verified the research on directional RTS. They analyzed the directional SE of production functions and the directional RTS of Chinese biological institutes based on the DEA method.

Moreover, Yang et al. [19] estimated directional RTS for two categories of inefficient and efficient decision-making (strongly and weakly efficient). The basic idea is to examine the ratio of the amount of change in outputs on the efficient frontier in the specified direction caused by an increase (or decrease) in a small enough amount of inputs in the specified direction. Lee [20] provided a theoretical foundation for DMP supporting the meta-DEA, which measures efficiency via a marginal-profit-maximizing orientation. Also, DMP investigated the differential characteristics of non-smooth piece-wise linear frontier estimates by DEA.

2.2 Sustainability of the Supply Chains

Tavana et al. [21] extended the Epsilon-Based Measures model proposed by Ton et al. [22] and proposed a new network EBM (NEMB). Nikfarjam et al. [23] proposed a new DEA method for evaluating supply chains with integrated approaches. They showed that the proposed model could be used for evaluating performance to identify benchmarking units for the inefficient supply chain.

Tajbakhsh et al. [24] proposed a multi-stage DEA model to evaluate the sustainability of a chain of business partners. They assessed supply chain sustainability in the banking and beverage sectors.

Khodakerami et al. [25] proposed a new two-stage DEA model of supply chain sustainability in a resin-producing company. Pouralizadeh [26] presented a radial model to study the investment regions of supply chain divisions. Also, she investigated whether the investment in the electricity supply chain division could effectively decrease the number of undesirable outputs or whether increasing the inputs under managerial disposability would have a limited effect on decreasing the number of undesirable outputs. Pouralizadeh [27] proposed two models for sustainability assessment of the electricity supply chain via reduction of wasted resources and pollution emissions management. She suggested that supply chains are generally evaluated under natural and management disposability based on unified operational and environmental efficiency. Also, the supply chain divisions with the necessary facilities and new technology to confront undesirable outputs can utilize more inputs (under managerial disposability) for more output production without increasing undesirable outputs. Those supply chain divisions that lack the adequate ability to reduce undesirable outputs should prevent the increase of undesirable outputs by using available capacities under natural disposability.

Pouralizadeh [28] presented a model to estimate the marginal profit maximization of desirable output. The proposed model is introduced for estimating the directional marginal profit maximization of supply chain divisions based on wasted energy and power losses. The proposed approach estimates the directional marginal productivity in the supply chains, which find the optimal direction of efficient divisions on the frontier.

Pouralizadeh et al. [29] proposed a new DEA-based model for the sustainability evaluation of an electricity supply chain in the presence of undesirable outputs. They planned a supply chain with five stages and fifteen divisions from different districts in Iran. Also, the weak disposability assumption was adopted for activity level control in the production activity. The proposed model could determine the type and size of inputs to control the undesirable outputs. They proposed a radial model for the performance assessment of the electricity supply chain. By scaling down the production levels, Pouralizadeh et al.'s model dramatically decreased harmful emissions in the energy and power plant sectors and harnessed power losses in transmission and distribution networks.

2.3 Single-output MP Model

Let us suppose $X_k = (x_{1k}, x_{2k}, ..., x_{mk})^T > 0$ and $Y_k = (y_{1k}, y_{2k}, ..., y_{sk})^T > 0$ show the column vectors of the inputs and desirable outputs. Also, let set I represent the inputs and index $i \in I$, and set J represent the outputs and index $j \in J$. The set K shows firm and index $k \in K$, and the index $r \in K$ is used for under consideration firm. Also, the column vectors of structural variables (λ) are used for connecting the input and output vectors by convex combination under variable returns to scale (VRS). Let y_j be the decision variable representing the maximum absolute level of output j.

Podinovski and Førsund [15] assessed the single-output MP of a non-differential efficient frontier constructed by the DEA estimator based on a directional-derivative technique. The maximum absolute level of one specific output j^*, given the level of one specific input i^* of one specific firm r, is calculated by model (1):

$$\begin{aligned} & Max\ y_{j^*r} \\ S.t \quad & \sum_{k=1}^{K} \lambda_k x_{i^*k} \le x_{i^*r} \\ & \sum_{k=1}^{K} \lambda_k x_{ik} \le x_{ir}, \quad i \ne i^*, i=1,\dots,m \\ & \sum_{k=1}^{K} \lambda_k y_{jk} \ge y_{j^*r} \\ & \sum_{k=1}^{K} \lambda_k y_{rk} \ge y_{jr}, \quad j \ne j^*, j=1,\dots,r \\ & \sum_{k=1}^{K} \lambda_k = 1, \quad k=1,\dots,K \\ & \lambda_k \ge 0, \quad y_{j^*}\ is\ free \end{aligned} \tag{1}$$

Let v_i, u_j, and u_o determine the dual variables inputs, desirable outputs, and convex combination constraints in model (1). The dual model of model (1) is presented as follows:

$$\begin{aligned} & Y_{j^*r}(X_{i^*r}) = min \sum_{i \in I} v_i x_{ir} - \sum_{\substack{j \in J \\ j \ne j^*}} u_r y_{jr} + u_o \\ s.t \quad & \sum_{i \in I} v_i x_{ik} - \sum_{j \in J} u_r y_{jk} + u_o \ge 0 \qquad k=1,\dots,K \\ & u_{j^*} = 1 \\ & v_i, u_j \ge 0 \quad u_0\ is\ free \end{aligned} \tag{2}$$

The evaluation firm r is on frontier because MP is one of the differential characteristics on the frontier; then $\sum_{i \in I} v_i x_{ir} - \sum_{j \in J} u_r y_{jr} + u_o = 0$

$$
\begin{aligned}
&\beta^{+DEA}_{i^*j^*r} = \min v_{i^*} \\
s.t \quad &\sum_{i\in I} v_i x_{ir} - \sum_{j\in J} u_r y_{jr} + u_o = 0 \\
&\sum_{i\in I} v_i x_{ik} - \sum_{j\in J} u_r y_{jk} + u_o \ge 0 \qquad (3) \\
&u_{j^*} = 1 \\
&v_i, u_j \ge 0 \quad u_0 \ is\ free
\end{aligned}
$$

Therefore, the MP approaching from the right sides with respect to one particular input i^* and one particular output j^* is defined as follows:

$$\beta^{+DEA}_{i^*j^*r} = \frac{\partial^+ Y_{j^*r}(X_r, Y_r)}{\partial X_{i^*r}} = \min_{v,u,u_0\in\Pi} v_{i^*}, \quad (4)$$

where Π is the optimal solutions set of the dual problem (3).

Proposition 1: The right-sight MP always exists and is finite: (see in Podinovski et al. [8]).

If input i^* unit (X_o, Y_o) can be reduced then the left-hand marginal productivity exists, is finite and

$$\beta^{-DEA}_{i^*j^*r} = \frac{\partial^- Y_{j^*r}(X_r, Y_r)}{\partial^- X_{i^*r}} = \max_{v,u,u_0\in\Pi} v_{i^*} \quad (5)$$

Also, the MP approaching from the left side is calculated by replacing the objective function as follows:

$$\beta^{-DEA}_{i^*j^*r} = \max_{v,u,u_0\in\Pi} v_{i^*}$$

2.4 DMP via DDF

DDF estimates efficiency by expanding outputs while reducing inputs simultaneously. Let $(g^{X_{i^*}}, g^{Y_{j^*}})$ define predetermined vectors for inputs and outputs. The DDF is defined as follows:

$$\begin{aligned} & Max\ \eta \\ & \sum_{k=1}^{K}\lambda_k x_{i^*k} \le x_{ir} + \eta g^{X_i}, && \forall i=1,\dots,m \\ & \sum_{k=1}^{K}\lambda_k y_{jk} \ge y_{jr} + \eta g^{Y_j}, && \forall j=1,\dots,r \\ & \sum_{k=1}^{K}\lambda_k = 1, && k=1,\dots,K \\ & \lambda_k \ge 0, \quad \eta \text{ is free} \end{aligned} \tag{6}$$

where η is the decision variable for the efficiency estimation. The firm r is efficient if $\eta=0$, and inefficient if $\eta>0$. Yen Lee [19] presented an MP model for multiple outputs based on DDF. The proposed model by Lee describes how a change in a single input i^* affects the multiple outputs $j^* \subset j$. Also, the DMP estimation is provided by the direction vectors $(g^{X_{i^*}}, g^{Y_{j^*}})$ where $g^{X_{i^*}}=0$ and $\sum_{j\in j^*} g^{Y_{j^*}}=1$

$$\begin{aligned} & Max\ \eta \\ & \sum_{k=1}^{K}\lambda_k x_{i^*k} \le x_{i^*r} \\ & \sum_{k=1}^{K}\lambda_k x_{ik} \le x_{ir}, && \forall i \ne i^* \\ & \sum_{k=1}^{K}\lambda_k y_{jk} \ge y_{jr} + \eta g^{Y_{j^*}}, && \forall j \in j^* \\ & \sum_{k=1}^{K}\lambda_k y_{rk} \ge y_{jr}, && \forall j \ne j^* \\ & \sum_{k=1}^{K}\lambda_k = 1, && k=1,\dots,K \\ & \lambda_k \ge 0, \quad y_{j^*} \text{ is free} \end{aligned} \tag{7}$$

The MP is one of the differential characteristics on the frontier, and the firm under evaluation is on frontier. Therefore, $\eta=0$ and $\sum_{i\in I} v_i x_{ir} - \sum_{j\in J} u_r y_{jr} + u_o = 0$. The dual model (7) is defined as follows:

$$\begin{aligned} &\min \; v_{i*} \\ &s.t \quad \sum_{i\in I} v_i x_{ir} - \sum_{j\in J} u_r y_{jr} + u_o = 0 \\ &\sum_{i\in I} v_i x_{ik} - \sum_{j\in J} u_r y_{jk} + u_o \ge 0 \qquad \forall k \ne r \qquad (8) \\ &\sum_{j\in j*} u_{j*} g^{y_j} = 1 \\ &v_i, u_j \ge 0 \quad u_0 \text{ is free} \end{aligned}$$

Lee [20] eliminated the unit of each factor for normalization as $X_i^{\max} = \max\left\{X_{ik}\right\}$ and $Y_j^{\max} = \max\left\{Y_{jk}\right\}$

$$\begin{aligned} &\min \; \frac{v_{i*}}{X_{i*}^{\max}} \qquad (9) \\ &s.t \quad \sum_{i\in I} \frac{v_i x_{ir}}{X_i^{\max}} - \sum_{j\in J} \frac{u_r\, y_{jr}}{Y_j^{\max}} + u_o = 0 \\ &\sum_{i\in I} \frac{v_i x_{ik}}{X_i^{\max}} - \sum_{j\in J} \frac{u_r\, y_{jk}}{Y_j^{\max}} + u_o \ge 0 \qquad \forall k \ne r \\ &\sum_{j\in j*} u_{j*} g^{y_j} = 1 \\ &v_i, u_j \ge 0 \quad u_0 \text{ is free} \end{aligned}$$

3. DDF for Efficiency Assessment of the Supply Chain

In this section, a DEA model for MP estimation of supply chain divisions is proposed. We suppose a supply chain contains an arbitrary number of suppliers, manufacturers, transmitters, distributors, and customers.

Assume a supply chain consists of five stages: supplier, manufacturer, transmitter, distributor, and customer. We treat each supply chain as a DMU. Let us consider h_s, h_m, h_t, h_d and h_c as the number of divisions in supplier, manufacturer, transmitter, distributor, and customer. The electricity supply chains are power suppliers in the power production process. They are comprised of fuel suppliers (oil and gas fields), power producers (power plants), electricity transmitters (transmission lines), power distributors (distribution lines), and final customers. These entities collaborate on power production and management in the economic sector.

The production factors of the kth supply chain (DMU) are summarized as follows:

$X_k^h = (x_{1k}^h, x_{2k}^h, ..., x_{mk}^h)^T > 0$: A column vector of m inputs from the hth division in the kth supply chain $h = 1, ..., H$, $k = 1, ..., K$.

$Y_k^h = (y_{1k}^h, y_{2k}^h, ..., y_{sk}^h)^T > 0$: A column vector of s desirable outputs from the hth division in the kth supply chain $h = 1,...,H,\ k = 1,...,K$.

$V_k^{(h,h')} = (v_{1k}^{(h,h')}, v_{2k}^{(h,h')}, ..., v_{Pk}^{(h,h')})^T > 0$: A column vector of P material flows or intermediate measures sent from the division h to the division h' in the kth supply chain $h = 1,...,H$, $k = 1,...,K$

$s_{pk}^{(h,h')}$: The slack variables of the pth intermediate measure from the division h to division h' in the kth supply chain (p = 1,…,P), (k = 1,…,K).

$\Lambda^h = (\lambda_1^h, \lambda_2^h, ..., \lambda_n^h)^T$: An unknown column vector.

η_r^h : Efficiency score of rth output from the hth division

The efficiency of the overall supply chain and its divisions can be estimated by a DDF based on expanding outputs and reducing inputs simultaneously.

Let $g = (g_{ir}^h,\ g_{jr}^h)$ be the predetermined directional vector for inputs and outputs of the hth division in the rth chain, where η^h is the decision variable for the efficiency estimate of the hth division. If $\eta^h = 0$; then the firm under consideration r is efficient, and if $\eta^h > 0$, it is inefficient. In this study, we considered the different weights for partners at a particular stage of the network supply chain as $\omega_h, (h = 1,...,H)$, which are weights for H divisions that were defined by decision makers in production activities.

Now let us suppose $i^* \subset I$ indicates the categories of inputs considered for more utilization and $j^* \subset J$ defines the outputs set whose marginal profit maximization is estimated via DMP. Let $g_{i^*r}^h$ and $g_{j^*r}^h$ be the given elements of the directional vectors of one specific input i^* and one specific output j^* from the hth division in the rth supply chain. Model (10) determines the maximum absolute level of the categories of outputs j^* for supply chain divisions.

$$
\begin{array}{lll}
\multicolumn{3}{l}{Max\ \omega_h(\sum_{h=1}^{H} y^h_{j^*r} + \eta^h g^h_r)} \\
\sum_{k=1}^{K} \lambda^h_k x^h_{i^*_k} & \le x^h_{i^*r} - \eta^h g^h_{i^*r} & i^* \subset I \quad h=1,\dots,H \\
\sum_{k=1}^{K} \lambda^h_k x^h_{ik} & \le x^h_{ir} & \forall i \ne i^*,\ h=1,\dots,H \\
\sum_{k=1}^{K} \lambda^h_k y^h_{j^*k} & \ge y^h_{j^*r} + \eta^h g^h_{j^*r} & j^* \subset I,\ h=1,\dots,H \\
\sum_{k=1}^{K} \lambda^h_k y^h_{jk} & \ge y^h_{jr} & \forall j \ne j^*,\ h=1,\dots,H \\
\sum_{k=1}^{K} \lambda^h_k v^{(h,h')}_{pk} & = \sum_{k=1}^{K} \lambda^{h'}_k v^{(h,h')}_{pk} & h=1,\dots,h_s,\ p=1,\dots,P_s,\ h'=h_s+1,\dots,h_m \\
\sum_{k=1}^{K} \lambda^h_k v^{(h,h')}_{pk} & = \sum_{k=1}^{K} \lambda^{h'}_k v^{(h,h')}_{pk} & h=h_s+1,\dots,h_m,\ p=1,\dots,P_m,\ h'=h_m+1,\dots,h_t \\
\sum_{k=1}^{K} \lambda^h_k v^{(h,h')}_{pk} & = \sum_{k=1}^{K} \lambda^{h'}_k v^{(h,h')}_{pk} & h=h_m+1,\dots,h_t,\ p=1,\dots,P_t,\ h'=h_t+1,\dots,h_d \\
\sum_{k=1}^{K} \lambda^h_k v^{(h,h')}_{pk} & = \sum_{k=1}^{K} \lambda^{h'}_k v^{(h,h')}_{pk} & h=h_t+1,\dots,h_d,\ p=1,\dots,P_d,\ h'=h_d+1,\dots,h_c \\
\sum_{k=1}^{K} \lambda^h_k & =1 & k=1,\dots,K,\ h=1,\dots,H \\
\lambda_k \ge 0,\ \eta\ free, & & k=1,\dots,K,\ h=1,\dots,H
\end{array} \tag{10}
$$

Because $y^h_{j^*r}$ is a constant as described by the maximum output level j^* for hth division in rth supply chain, this will not affect optimization results.

3.1 Directional MP Modeling of Supply Chain

We will estimate DMP by a directional vector $g=(g^h_{i^*r}, g^h_{j^*r})$ where $g^h_{i^*r}=0$ and $\sum_{j\in j^*} g^h_{jr}=1$.

Model (7) can be further developed as a network model by incorporating the set of intermediate measures for each supply chain division into an efficiency assessment of the overall supply chain via a marginal-profit-maximizing orientation.

$$
\begin{array}{lll}
Max\ \left(\sum_{h=1}^{H}\omega_h\eta^h\right) & & \\
\sum_{k=1}^{K} x^h_{i*_k}\lambda^h_k & \leq x^h_{i*r} & i^*\subset i,\ h=1,\dots,H \\
\sum_{k=1}^{K} x^h_{i_k}\lambda^h_k & \leq x^h_{ir} & \forall i\neq i^*,\ h=1,\dots,H \\
\sum_{k=1}^{K} y^h_{j*k}\lambda^h_k & \geq y^h_{j*r}+\eta^h\, g^h_{j*r} & j^*\subset j,\ \ h=1,\dots,H \\
\sum_{k=1}^{K} y^h_{jk}\lambda^h_k & \geq y^h_{jr} & \forall j\neq j^*,\ h=1,\dots,H \\
\sum_{k=1}^{K}\lambda^h_k v^{(h,h')}_{pk} & =\sum_{k=1}^{K}\lambda^{h'}_k v^{(h,h')}_{pk} & h=1,\dots,h_s,\, p=1,\dots,P_s,\, h'=h_s+1,\dots,h_m \\
\sum_{k=1}^{K}\lambda^h_k v^{(h,h')}_{pk} & =\sum_{k=1}^{K}\lambda^{h'}_k v^{(h,h')}_{pk} & h=h_s+1,\dots,h_m,\, p=1,\dots,P_m,\, h'=h_m+1,\dots,h_t \\
\sum_{k=1}^{K}\lambda^h_k v^{(h,h')}_{pk} & =\sum_{k=1}^{K}\lambda^{h'}_k v^{(h,h')}_{pk} & h=h_m+1,\dots,h_t,\, p=1,\dots,P_t,\, h'=h_t+1,\dots,h_d \\
\sum_{k=1}^{K}\lambda^h_k v^{(h,h')}_{pk} & =\sum_{k=1}^{K}\lambda^{h'}_k v^{(h,h')}_{pk} & h=h_t+1,\dots,h_d,\, p=1,\dots,P_d,\, h'=h_d+1,\dots,h_c \\
\sum_{k=1}^{K}\lambda^h_k & =1 & k=1,\dots,K,\, h=1,\dots,H \\
\lambda_k\geq 0\ ,\ \eta\ \ free, & & k=1,\dots,K,\ h=1,\dots,H
\end{array} \tag{11}
$$

The first categories constraints that correspond to inputs are controllable and are considered discretionary, while the second category constraints are related to inputs that are not controllable and are defined as non-discretionary inputs. Furthermore, the third category constraints that correspond to outputs estimate their MP in predetermined directions, and the fourth category constraints indicate other outputs. The fifth, sixth, seventh, and eighth category constraints correspond to intermediate measures sent from the supplier divisions to the manufacturer divisions, from the manufacturer divisions to the transmitter divisions, from the transmitter divisions to the distributor divisions, and from them to the customer divisions. The last category constraints are related to RTS in the production process.

In the proposed approach, the column vectors of structural variables (λ^h) are used for connecting the input, desirable output vectors, and the set of intermediate measures by convex combination under VRS in the hth division. Also, η^h presents the efficiency of the output variable from the hth division. The dual of model (9) is proposed as follows:

Let us consider $v^h_i(i=1,\dots,m),\ u^h_j(j=1,\dots,J),\ B'^h_p(p=1,\dots,P_s),\ \bar{B}^h_p(p=1,\dots,P_m),$ $\tilde{B}^h_p(p=1,\dots,P_t),\ \hat{B}^h_p(p=1,\dots,P_d)$ indicate the dual variables corresponding to the ith

the category constraints of the input, the r$^{\text{th}}$ category constraints of the desirable output, the dual variables of the categories constraints related to intermediate measures that are sent from the supplier divisions to the manufacture divisions, from the manufacture divisions to the transmitter divisions, and from them to the distributor divisions, and finally from the distributor divisions to the customer divisions.

$$Min\ \sum_{h=1}^{H}\omega_h\,(v_{i^*}^h) \tag{12}$$

$$\begin{aligned}
s.t\ &\sum_{i\in I}v_i^h x_{ir}^h-\sum_{j\in J}u_j^h y_{jr}^h+\sum_{p=1}^{P}B'_p v_{pr}^{(h(s),h(m))}+u_o^h=0\,, && h=1,\dots,h_s\,,\ p=1,\dots,p_s\\
&\sum_{i\in I}v_i^h x_{ir}^h-\sum_{j\in J}u_j^h y_{jr}^h+\sum_{p=1}^{P}\bar{B}_p v_{pr}^{(h(m),h(t))}-\sum_{p=1}^{P}B'_p v_{pr}^{(h(s),h(m))}+u_o^h=0, && h=1,\dots,h_m,\ p=1,\dots,p_m\\
&\sum_{i\in I}v_i^h x_{ir}^h-\sum_{j\in J}u_j^h y_{jr}^h+\sum_{p=1}^{P}\tilde{B}_p v_{pr}^{(h(t),h(d))}-\sum_{p=1}^{P}\bar{B}_p v_{pr}^{(h(m),h(t))}+u_o^h=0, && h=1,\dots,h_t\,,\ p=1,\dots,p_t\\
&\sum_{i\in I}v_i^h x_{ir}^h-\sum_{j\in J}u_j^h y_{jr}^h+\sum_{p=1}^{P}\hat{B}_p v_{pr}^{(h(d),h(c))}-\sum_{p=1}^{P}\tilde{B}_p v_{pr}^{(h(t),h(d))}+u_o^h=0, && h=1,\dots,h_d\,,\ p=1,\dots,p_d\\
&\sum_{i\in I}v_i^h x_{ir}^h-\sum_{j\in J}u_j^h y_{jr}^h-\sum_{p=1}^{P}\hat{B}_p v_{pr}^{(h(d),h(c))}+u_o^h=0, && h=1,\dots,h_c,\ p=1,\dots,p_c\\
&\sum_{i\in I}v_i^h x_{ik}^h-\sum_{j\in J}u_j^h y_{jk}^h+\sum_{p=1}^{P}B'_p v_{pk}^{(h(s),h(m))}+u_o^h\le 0, && k\ne r,h=1,\dots,h_s\,,\ p=1,\dots,p_s\\
&\sum_{i\in I}v_i^h x_{ik}^h-\sum_{j\in J}u_j^h y_{jk}^h+\sum_{p=1}^{P}\bar{B}_p v_{pk}^{(h(m),h(t))}-\sum_{p=1}^{P}B'_p v_{pk}^{(h(s),h(m))}+u_o^h\ge 0, && k\ne r,h=1,\dots,h_m,\ p=1,\dots,p_m\\
&\sum_{i\in I}v_i^h x_{ik}^h-\sum_{j=1}^{s}u_j^h y_{jk}^h+\sum_{p=1}^{P}\tilde{B}_p v_{pk}^{(h(t),h(d))}-\sum_{p=1}^{P}\bar{B}_p v_{pk}^{(h(m),h(t))}u_o^h\ge 0, && k\ne r,h=1,\dots,h_t,\ p=1,\dots,p_t\\
&\sum_{i\in I}v_i^h x_{ik}^h-\sum_{j\in J}u_j^h y_{jk}^h+\sum_{p=1}^{P}\hat{B}_p v_{pk}^{(h(d),h(c))}-\sum_{p=1}^{P}\tilde{B}_p v_{pk}^{(h(t),h(d))}+u_o^h\ge 0, && k\ne r,h=1,\dots,h_d,\ p=1,\dots,p_d\\
&\sum_{i\in I}v_i^h x_{ik}^h-\sum_{j\in J}u_j^h y_{jk}^h-\sum_{p=1}^{P}\hat{B}_p v_{pk}^{(h(d),h(c))}+u_o^h\ge 0\,, && k\ne r,h=1,\dots,h_c,\ p=1,\dots,p_c\\
&\sum_{j\in j^*}u_j^h g_i^h \qquad\qquad =1\,, && h=1,\dots,H\\
&\quad v_i^h,\ u_j^h\ge 0, B'_p,\bar{B}_p,\tilde{B}_p,\hat{B}_p\ free,\ \ h=1,\dots,H
\end{aligned}$$

For normalization, eliminate the measuring unit of production factors as follows:

$$(X_i^h)^{\max}=\max\left\{X_{ik}^h\right\},\ \ (Y_j^h)^{\max}=\max\left\{Y_{jk}^h\right\},\ \ (V_p^{(h(s),h(m))})^{\max}=\max\left\{V_p^{(h(s),h(m))}\right\}.$$

$$\alpha = Min \ \sum_{h=1}^{H} \omega_h \frac{v_{i*}^{h}}{(X_{ir}^{h})^{\max}} \qquad (13)$$

$$s.t \sum_{i\in I} \frac{v_i^h x_{ir}^h}{(X_{ir}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{(Y_{jr}^h)^{\max}} + \sum_{p=1}^{P} \frac{B'_p v_{pr}^{(h(s),h(m))}}{(V_p^{(h(s),h(m))})^{\max}} + u_o^h = 0 \ , \qquad h = 1,\dots,h_s \ , p = 1,\dots,p_s$$

$$\sum_{i\in I} \frac{v_i^h x_{ir}^h}{(X_{ir}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{(Y_{jr}^h)^{\max}} + \sum_{p=1}^{P} \frac{\overline{B}_p v_{pr}^{(h(m),h(t))}}{(V_p^{(h(m),h(t))})^{\max}} - \sum_{p=1}^{P} \frac{B'_p v_{pr}^{(h(s),h(m))}}{(V_p^{(h(s),h(m))})^{\max}} + u_o^h = 0, \qquad h = 1,\dots,h_m, p = 1,\dots,p_m$$

$$\sum_{i\in I} \frac{v_i^h x_{ir}^h}{(X_{ir}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{(Y_{jr}^h)^{\max}} + \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pr}^{(h(t),h(d))}}{(V_p^{(h(t),h(d))})^{\max}} - \sum_{p=1}^{P} \frac{\overline{B}_p v_{pr}^{(h(m),h(t))}}{(V_p^{(h(m),h(t))})^{\max}} + u_o^h = 0, \qquad h = 1,\dots,h_t \ , p = 1,\dots,p_t$$

$$\sum_{i\in I} \frac{v_i^h x_{ir}^h}{(X_{ir}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{(Y_{jr}^h)^{\max}} + \sum_{p=1}^{P} \frac{\hat{B}_p v_{pr}^{(h(d),h(c))}}{(V_p^{(h(d),h(c))})^{\max}} - \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pr}^{(h(t),h(d))}}{(V_p^{(h(t),h(d))})^{\max}} + u_o^h = 0, \qquad h = 1,\dots,h_d \ , p = 1,\dots,p_d$$

$$\sum_{i\in I} \frac{v_i^h x_{ir}^h}{(X_{ir}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jr}^h}{(Y_{ir}^h)^{\max}} + \sum_{p=1}^{P} \frac{\hat{B}_p v_{pr}^{(h(d),h(c))}}{(V_p^{(h(d),h(c))})^{\max}} + u_o^h = 0, \qquad h = 1,\dots,h_c, p = 1,\dots,p_c$$

$$\sum_{i\in I} \frac{v_i^h x_{ik}^h}{(X_{ik}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{(Y_{jk}^h)^{\max}} + \sum_{p=1}^{P} \frac{B'_p v_{pk}^{(h(s),h(m))}}{(V_p^{(h(s),h(m))})^{\max}} + u_o^h \le 0 \ , \qquad k \ne r, h = 1,\dots,h_s \ , p = 1,\dots,p_s$$

$$\sum_{i\in I} \frac{v_i^h x_{ik}^h}{(X_{ik}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{(Y_{jk}^h)^{\max}} + \sum_{p=1}^{P} \frac{\overline{B}_p v_{pk}^{(h(m),h(t))}}{(V_p^{(h(m),h(t))})^{\max}} - \sum_{p=1}^{P} \frac{B'_p v_{pk}^{(h(s),h(m))}}{(V_p^{(h(s),h(m))})^{\max}} + u_o^h \ge 0, \qquad k \ne r, h = 1,\dots,h_m, p = 1,\dots,p_m$$

$$\sum_{i\in I} \frac{v_i^h x_{ik}^h}{(X_{ik}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{(Y_{jk}^h)^{\max}} + \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pk}^{(h(t),h(d))}}{(V_p^{(h(t),h(d))})^{\max}} - \sum_{p=1}^{P} \frac{B'_p v_{pk}^{(h(s),h(m))}}{(V_p^{(h(s),h(m))})^{\max}} + u_o^h \ge 0, \qquad k \ne r, h = 1,\dots,h_t, p = 1,\dots,p_t$$

$$\sum_{i\in I} \frac{v_i^h x_{ik}^h}{(X_{ik}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{(Y_{jk}^h)^{\max}} + \sum_{p=1}^{P} \frac{\hat{B}_p v_{pk}^{(h(d),h(c))}}{(V_p^{(h(d),h(c))})^{\max}} - \sum_{p=1}^{P} \frac{\tilde{B}_p v_{pk}^{(h(t),h(d))}}{(V_p^{(h(t),h(d))})^{\max}} + u_o^h \ge 0, \qquad k \ne r, h = 1,\dots,h_d, p = 1,\dots,p_d$$

$$\sum_{i\in I} \frac{v_i^h x_{ik}^h}{(X_{ik}^h)^{\max}} - \sum_{j\in J} \frac{u_j^h y_{jk}^h}{(Y_{jk}^h)^{\max}} - \sum_{p=1}^{P} \frac{\hat{B}_p v_{pk}^{(h(d),h(c))}}{(V_p^{(h(d),h(c))})^{\max}} + u_o^h \ge 0 \ , \qquad k \ne r, h = 1,\dots,h_c, p = 1,\dots,p_c$$

$$\sum_{j\in j*} u_j^h g_j^h = 1 \ , \qquad h = 1,\dots,H$$

$$v_i^h, \ u_j^h \ge 0, B'_p, \overline{B}_p, \tilde{B}_p, \hat{B}_p \ is \ \ free, \ \ h = 1,\dots,H$$

Based on DEA framework, the dual multiplier to the primal envelopment model refers to shadow price and it also represents the MP. In estimation of DMP eliminating the measurement units of production factors, hence, the results are independent of the units of production indexes.

After measuring the ith input dual multiplier v_{i*}^{h} of supply chain divisions by model (13), the α_i^h defines as follows:

$$\alpha_i^h = \frac{v_{i*}^h}{(X_{ir}^h)^{\max}}, \qquad i = 1,\dots,m, \ h = 1,\dots,H, \quad r \in k = 1,\dots,K \quad (14)$$

The other word, MP of two outputs of the supply chain divisions is estimated based on the weight $(g_{Y_1}^h, \ g_{Y_2}^h)$. In this way, the estimation of DMP created based on the directional vector from outputs.

The vector of DMP with respect to output Y_j^h is estimated based on increasing one extra unit as follows:

$$\frac{\partial(Y_1^h,\ Y_2^h)}{\partial X_{ir}^{h}} = \alpha_i^h \times (\ Y_1^{\max} g_{Y_1}^h\ ,\ Y_2^{\max} g_{Y_2}^h\ ),\ \ \forall j \in j^*,\ i=1,\dots,m,\ h=1,\dots,H,\ \ r \in k=1,\dots,K \quad (15)$$

Also, The approximate value of DMP vector of the overall supply chain with respect to output Y_j^h of supply chain divisions is calculated by model (16).

$$\frac{\partial(Y_1^h,\ Y_2^h)}{\partial X_{ir}^{h}} = \sum_{h=1}^{H} \alpha_i^h \times (\ Y_1^{\max} g_{Y_1}^h\ ,\ Y_2^{\max} g_{Y_2}^h\ ), \qquad \forall j \in j^*,\ i=1,\dots,m,\ \ r \in k=1,\dots,K \quad (16)$$

3.2 A Real Case of the Power Industry

In this section, the proposed model is applied to the analysis of the power industry in Iran. The dataset, the inputs, and the outputs will be described in the following subsections, and in the next subsection, the results will be presented.

3.3 The Dataset

In our application, we consider 10 supply chains (or DMUs), including oil and gas fields (suppliers) supplying fuel to power stations, power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers. Two suppliers are assumed per supply chain: oil and gas companies that satisfy the fuel demand of power plants (the intermediate product) and sell fuel as the final output.

In the proposed model, suppliers use two inputs (capital and labor) to produce two desirable outputs: the amount of oil or gas sold and the percentage of share of oil and gas consumed in the whole country by the supplier. Each manufacturer includes at least three power plants with different technologies (thermal, combined cycle, gas, hydro, wind, and solar). They use fuel, capital, and labor to produce electricity, which they sell to regional power companies. To update and increase their capacity, manufacturers can replace the existing plants with more efficient ones or construct new ones. Also, the percentage of share of gross product in the whole country is considered the second desirable output. The transmitters transfer electricity from manufacturers to distributing companies, and the capacity and length of the lines are considered inputs, while the construction of new lines and the percentage of share of gross product in the whole country are desirable outputs.

Distribution companies receive electricity from transmitters and dispatch it to the final consumers. They use two additional capital inputs estimated as capacity and length of the distribution lines under natural disposability and two final desirable outputs as the meter of electricity and the percentage of the sale share of power in the whole country in the distribution lines. Finally, customers are classified as residential, agricultural, public, or industrial. They use two inputs and produce two desirable outputs and one undesirable output. More details concerning the parameters used to characterize this supply chain are as follows:

h_s : Numerator of divisions in the supplier level (h_s : 1, 2).

$x_{1k}^{h(s)}$: Capacity of oil (10^3 Barrels) and gas (10^6 m^3) fields of the h_s th supplier in the kth supply chain

$x_{2k}^{h(s)}$: Number of employees from the h_s th supplier in the kth supply chain.

$y_{1k}^{h(s)}$: Oil (10^3 Barrels) and gas (10^6 m^3) sold to other companies from the h_s th supplier in the kth supply chain.

$y_{2k}^{h(s)}$: Percentage of share of oil and gas consumption of the h_s th supplier in the whole country in the kth supply chain (%).

h_m : Numerator of division in the manufacturer level (h_m : 3, 4, and 5).

$x_{1k}^{h(m)}$: Power nominal of the h_m th manufacturer in the kth supply chain (10^6 kWh).

$x_{2k}^{h(m)}$: Number of employees of the h_m th manufacturer in the kth supply chain.

$y_{1k}^{h(m)}$: The total of produced electricity of the h_m th manufacturer in the kth supply chain (10^6 kWh).

$y_{2k}^{h(m)}$: Percentage of share of gross product of the h_m th manufacturer in the whole country in the kth supply chain (%).

h_t : Numerator of the divisions in the level of the transmitters (h_t : 6, 7).

$x_{1k}^{h(t)}$: Capacity of transmission lines of the h_t th transmitter in the kth supply chain (MWa).

$x_{2k}^{h(t)}$: Length of transmission line of the h_t th transmitter in the kth supply chain (km circuit).

$y_{1k}^{h(t)}$: The transferred electricity of the h_t th transmitter in the jth supply chain (10^6 kWh).

$y_{2k}^{h(t)}$: Percentage of share of gross product of the h_t th transmitter in the whole country in the kth supply chain (%).

h_d : Numerator of the division in the distributer level (h_d : 8, 9, 10, and 11).

$x_{1k}^{h(d)}$: Capacity of the distribution lines of the h_d th distributer in the kth supply chain (Mva).

$x_{2k}^{h(d)}$: Length of distribution line of the h_d th distributer in the kth supply chain (km).

$y_{1k}^{h(d)}$: The dispatched electricity of the h_t th distributer in the kth supply chain (10^6 kWh).

$y_{2k}^{h(d)}$: Percentage of sale share of the h_d th distributer in the whole country in the kth supply chain (%).

h_c : Numerator of the division in the customer level (h_c : 12, 13, 14, and 15).

$x_{1k}^{h(c)}$: Average cost with fuel subsidy of the h_c th customer in the kth supply chain (USD).

$y_{1k}^{h(c)}$: Number of customers of the h_c th customer in the kth supply chain.

$y_{2k}^{h(c)}$: Sales of electricity of the h_c th customer in the kth supply chain (10^6 kWh).

$v_{pk}^{(h,h')}$: Material flow from the division h to division h' in the kth supply chain (10^6 kVA).

We consider 10 supply chains (DMUs), including oil and gas fields (suppliers) that provide different fuels to power stations, power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers. All the data from the two oil and gas fields (suppliers), power plants (manufacturers), regional power companies (transmitters), distribution companies (distributors), and customers (residential, public, agricultural, industrial) are available on the TAVANIR website (2015). For each supply chain, we consider two suppliers: oil and gas companies that satisfy the fuel demand of power plants (intermediate products) and those that can also sell fuel as final output.

The dataset has been collected from the power industry companies in Iran, and the reference year is 2015 (see the TAVANIR website for the detailed data [30]). Desirable output is computed as the difference between the average annual production and the amount of oil and gas delivered to power plants. Information related to the demand for fuel for power plants is collected from TAVANIR Company (2015) in the power industry, and they are considered as intermediate measures from oil and gas fields to power plants. Desirable outputs of regional power companies were collected from the transmission section of TAVANIR Company in the power industry.

Distribution companies receive electricity from transmitters and dispatch it to the final consumers. All of the data for distribution companies was obtained from the dispatch section of TAVANIR Company in the power industry. Finally, customers were classified as residential, agricultural, public, or industrial. They use one input and produce two desirable outputs. The desirable outputs of customers are computed as the number of customers and total sale of electricity to residential, public, agricultural, and industrial divisions in 2015. The datasets corresponding to the 10 supply chains (DMUs) under analysis are presented in Tables 1–11.

Table 1. The first and second suppliers' input

DMU	Supplier 1 (division 1) capacity of oil (10^3 Barrels)	labor	Supplier 2 (division 2 capacity of gas field (10^6 m^3)	labor
	x_{1k}^{1}	x_{2k}^{1}	x_{1k}^{2}	x_{2k}^{2}
1	039352	1	0.33333	0.83333
2	0.94444	0.40625	1	0.83333
3	0.33333	1	0.5	0.8
4	0.5	0.971875	0.3	0.46667
5	0.19444	0.875	0.9	1
6	0.66667	0.6875	0.5	0.8
7	0.72222	0.75	0.5	0.46
8	0.61111	0.5	1	0.75
9	0.14444	0.671875	0.9	0.72667
10	1	0.78125	0.3	0.96667

Table 2. The first manufacturer's inputs

DMU	Manufacturer 1 (Division 3) Power nominal (10^6 kWh). x_{1k}^{3}	labor x_{2k}^{3}	Manufacturer 1 (Division 4) Power nominal (10^6 kWh) x_{1k}^{4}	labor x_{2k}^{4}	Manufacturer 1 (Division 5) Power nominal (10^6 kWh) x_{1k}^{5}	labor x_{2k}^{5}
1	1	1	1	0.445158	0.667029	0.300451
2	0.256232	0.669461	0.674974	0.195367	0.147211	0.644216
3	0.165254	0.447227	0.37001	0.9210	0.939209	0.495493
4	0.126889	0.297305	0.559605	0.921016	0.467587	0.502003
5	0.081994	0.266467	0.124642	0.249232	0.92003	0.70681
6	0.216261	0.688623	0.214953	0.687971	0.220568	0.2003

7	0.015284	0.434132	0.542056	0.744069	1	0.222834
8	0.023586	0.448802	0.669782	0.628244	0.941451	0.325488
9	0.061243	0.449102	0.687305	1	0.396306	1
10	0.181159	0.94012	0.440498	0.212113	0.115063	0.398097

Table 3. Transmitter level inputs

DMU	Transmitter 1 (division 6)		Transmitter 2 (division 7)	
	Capacity of transmission Lines (Mva)	Length of transmission line	Capacity of transmission Lines (Mva)	Length of transmission line
		(Km circuit)		(Km circuit)
	x_{1k}^{6}	x_{2k}^{6}	x_{1k}^{7}	x_{2k}^{6}
1	0.671576	0.592201501	0.611689547	1
2	1	0.621035944	0.12040672	0.152710968
3	0.333056985	0.588078407	1	0.621035944
4	0.403428348	0.70540969	1	0.621035944
5	0.167540416	0.193955517	0.333056985	0.588078407
6	0.343029919	0.759737918	0.12040672	0.152710968
7	0.34554144	0.393292828	0.213649996	0.304836811
8	0.263636585	0.562897596	0.375679696	0.414745164
9	0.611689547	1	0.179634732	0.256917749
10	0.263636585	0.562897596	0.188154398	0.098913435

Table 4. The distributor level inputs

DMU	Distributor 1 (Division 8)		Distributor 2 (Division 9)	
	Capacity of distribution lines (Mva)	Length of distribution line (km)	Capacity of distribution lines (Mva)	Length of distribution line (km)
	x_{1k}^{8}	x_{2k}^{9}	x_{1k}^{9}	x_{2k}^{9}
1	0.686580315	0.624972953	0.270315091	0.160869061
2	1	1	0.552001895	0.381887479
3	1	1	0.726841981	0.542543724
4	0.758833377	0.191740595	0.423359394	0.172999536
5	0.079302141	0.206840592	0.587538498	0.517915183
6	1	1	0.752191424	0.304345303
7	0.320644991	0.57421718	0.342099029	0.266696332
8	0.183628514	0.798862477	1	0.477654388
9	0.686580315	0.624972953	0.448708837	0.351377496
10	0.237025289	0.550307564	0.493721867	1

Table 5. The distributor level inputs

DMU	Distributor 3 (Division 10)		Distributor 4 (Division 11)	
	Capacity of distribution lines (Mva)	Length of distribution line (km)	Capacity of distribution lines (Mva)	Length of distribution line (km)

	x_{1k}^{10}	x_{2k}^{10}	x_{1k}^{11}	x_{2k}^{11}
1	0.817555938	0.228087914	0.395805798	0.155358412
2	0.439390214	0.300371279	0.116662261	0.171571203
3	0.897713302	0.539232911	0.079302141	0.206840592
4	0.460781903	0.200142545	0.279760331	0.868350283
5	0.974920089	0.539232911	0.270332188	0.433417823
6	0.325547086	0.183998541	0.16688695	0.280702297
7	0.221293337	0.221822582	1	1
8	1	1	0.475372279	0.808939445
9	0.817555938	0.228087914	0.358357565	0.932459584
10	1	1	0.475372279	0.808939445

Table6. The customer level inputs

DMU	Customer 1	Customer 2	Customer 3	Customer 4
	(Division 12)	(Division 13)	(Division 14)	(Division 15)
	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)	Average cost with fuel subsidy ($)
	x_{1k}^{12}	x_{1k}^{13}	x_{1k}^{14}	x_{1k}^{15}
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	1	1	1	0.5
5	1	1	1	1
6	1	1	1	1
7	1	1	1	1
8	1	1	1	1
9	1	1	1	0.5
10	1	1	1	1

Table 7. The level of desirable outputs of suppliers 1 and 2 and manufacturers 1 and 2

DMU	Supplier 1		Supplier 2		Manufacturer 1	
	(Division 1)		(Division 2)		(Division 3)	
	Sold oil (10^3 Barrels)	The share of oil consumption (%)	Sold gas (10^6 mm^3)	The share of oil consumption (%)	Produced electricity (10^6 kWh)	Percentage of share of gross product
	y_{1k}^{1}	y_{2k}^{1}	y_{1k}^{2}	y_{2k}^{2}	y_{2k}^{3}	y_{2k}^{3}
1	0.042878107	0.012315271	0.113641925	0.564356436	1	1
2	1	0.295566502	0.6900842	0.623762376	0.961117016	0.067226891
3	0.221720938	0.445812808	0.356977886	0.712871287	0.406302835	0.067226891
4	0.653813956	0.561576355	0.184900351	1	0.279998751	0.151260504
5	0.112213989	0.179802956	1	0.198019802	0.095396722	0.050420168
6	0.574874748	1	0.321001534	0.297029703	0.479940493	0.084033613
7	0.420981259	0.556650246	0.225434678	0.366336634	0.014743364	0.411764706
8	0.391218668	0.600985222	0.905818346	0.920792079	0.031328434	0.445378151
9	0.14942874	0.039408867	0.943611201	0.217821782	0.159054402	0.327731092
10	0.63104532	0.426108374	0.211570684	0.485148515	0.414665747	0.588235294

Table 8. The level of desirable outputs of manufacturers 2 and 3 and distributor 1

DMU	Manufacturer 2		Manufacturer 3		Transmitter 1	
	(Division 4)		(Division 5)		(Division 6)	
	Produced electricity (10^6 kWh)	Percentage of share of gross product	Produced electricity (10^6 kWh)	Percentage of share of gross product	The transferred electricity (10^{6}kWh)	Percentage of share of gross product
	y^4_{1k}	y^4_{2k}	y^5_{1k}	y^5_{2k}	y^6_{1k}	y^6_{2k}
1	0.03364106	0.804597701	0.064122104	0.784810127	1	0.509433962
2	0.909102556	1	0.090205742	0.189873418	0.702111903	0.075471698
3	0.566828191	0.344827586	0.568130132	0.189873418	0.344664193	1
4	0.830666795	1	0.361670863	0.189873418	0.644984209	1
5	0.159877478	0.563218391	0.696620498	0.316455696	0.133162769	0.283018868
6	0.181616247	0.091954023	0.199053321	0.493670886	0.500863544	0.075471698
7	0.892480848	1	0.900088586	0.53164557	0.879767022	0.113207547
8	1	0.609195402	1	1	0.734554814	0.943396226
9	0.811220613	0.804597701	0.306534562	0.784810127	0.537211931	0.301886792
10	0.366038283	0.609195402	0.108720399	0.240506329	0.578066119	0.264150943

Table 9. The level of desirable outputs of transmitter 2 and distributers 1 and 2

DMU	Transmitter 2		Distributor 1		Distributor 2	
	(Division 7)		(Division 8)		(Division 9)	
	The dispatched electricity (10^{6}kWh)	Percentage of sale share	The dispatched electricity (10^{6}kWh	Percentage of sale share	The dispatched electricity (10^{6}kWh	Percentage of sale share
	y^7_{1k}	y^7_{2k}	y^8_{1k}	y^8_{2k}	y^9_{1k}	y^9_{2k}
1	0.140600711	1	1	1	0.061068074	0.878787879
2	1	0.654320988	0.702111903	0.777777778	0.42054083	0.575757576
3	0.969653421	0.185185185	0.70313936	0.777777778	0.421156242	0.606060606
4	0.318352158	0.50617284	0.230851485	0.777777778	0.901421058	0.393939394
5	0.715439166	0.074074074	0.057069758	0.111111111	0.310741615	0.575757576
6	0.301192342	0.24691358	0.031201161	0.777777778	1	0.181818182
7	0.167807535	0.49382716	0.879767022	0.523809524	0.170065003	0.393939394
8	0.530449123	0.481481481	0.734554814	0.349206349	0.537585107	1
9	0.228903718	0.333333333	0.071137883	1	0.231983096	0.575757576
10	0.105229386	0.481481481	0.076306566	0.333333333	0.807897257	.666666667

Table 10. The level of desirable outputs for distributor 3, distributor 4, and customer 1

DMU	Distributor 3		Distributor 4		Customer 1	
	(Division 10)		(Division 11)		(Division 12)	
	The dispatched electricity (10^{6}kWh)	Percentage of sale share	The dispatched electricity (10^{6}kWh)	Percentage of sale share	Sold electricity (106 Kwh)	Number of customers
	y^{10}_{1k}		y^{11}_{1k}	y^{11}_{2k}	y^{12}_{1k}	y^{12}_{2k}
1	.196523643	0.793103448	0.583443768	0.734693878	0.221352722	0.984586012
2	0.599032663	0.448275862	0.987189815	0.204081633	0.778772766	0.882165357
3	0.66435251	0.448275862	0.201092175	0.142857143	0.950989497	0.93620145

4	0.659948435	0.586206897	0.044896287	0.163265306	0.793115567	0.782549977
5	1	0.448275862	0.181283638	0.408163265	0.459903894	0.582519333
6	0.420989452	0.344827586	0.084952575	0.387755102	0.968278496	0.642661673
7	0.238776041	0.24137931	0.513294586	1	1	0.894786666
8	0.317756351	1	1	0.775510204	0.435502188	1
9	0.443783629	0.793103448	0.731343557	0.591836735	0.388538913	0.628302091
10	0.477532731	1	0.04452059	0.775510204	0.445317007	0.584675109

Table11. The level of desirable outputs for customers 2, 3, and 4

DMU	Customer 2		Customer 3		Customer 4	
	(Division 13)		(Division 14)		(Division 15)	
	Sold electricity (106 Kwh)	Number of customers	Sold electricity (106 Kwh)	Number of customers	Sold electricity (106 Kwh)	Number of customers
	y_{1k}^{13}	y_{2k}^{13}	y_{1k}^{14}	y_{2k}^{14}	y_{1k}^{15}	y_{2k}^{15}
1	0.157976366	0.984586008	0.02620202	0.984586008	0.103555453	0.984586008
2	0.809577548	0.882165357	0.060437255	0.882165357	0.779537561	0.882165357
3	0.987088938	0.93620145	0.08237208	0.93620145	0.878795659	0.93620145
4	0.815401683	0.782549977	0.060012938	0.782549977	0.723104366	0.782549977
5	0.389603413	0.582519333	0.067986909	0.782549977	0.391640428	0.582519333
6	0.946065522	0.642661673	0.072787303	0.642661673	1	0.642661673
7	1	0.894786666	1	0.894786666	0.977445641	0.894786666
8	0.437993719	1	0.170761466	1	0.327450371	1
9	0.314668545	0.628302091	0.103341382	0.628302091	0.407762267	0.628302091
10	0.433864838	0.584675109	0.176219954	0.584675109	0.318409708	0.584675109

4. Results

In this section, we describe the results obtained by applying the proposed method. Model (10) is applied to estimate the MP of two outputs under two inputs for 15 divisions of 10 supply chains (DMU$_S$). Also, the 11 directions were investigated as intervals between $(g_{Y_1}^h,\ g_{Y_2}^h)=(1,0)$ and $(g_{Y_1}^h,\ g_{Y_2}^h)=(0,1)$ to identify marginal profit maximization of supply chain divisions. Tables 12–20 show the directional marginal profit of 15 divisions of those supply chains, with marginal profit maximization between 10 supply chains. The α_i^h is computed by model (13) and defined as follows:

$$\alpha_i^h = \frac{v_{i*}^h}{(X_{ir}^h)^{\max}} \qquad i=1,2,\ r=1,\dots,K,\ h=1,\dots,H$$

Also, the DMP of outputs in the $(g_h^{Y_1},\ g_h^{Y_2})$ direction that is estimated based on a more one-unit allocation of ith input in the hth division.

$$\frac{\partial(Y_1^h,\ Y_2^h)}{\partial X_{ir}^h} = \alpha_i^h \times (\ Y_1^{\max} g_{Y_1}^h,\ Y_2^{\max} g_{Y_2}^h\ ) \qquad r=1,\dots,K,\ h=1,\dots,H$$

Table 12 shows the DMP of oil field outputs for supply chain 3 created by the utilization of one extra unit of oil resources and the DMP for supply chain 9 based on using one more unit of specialist workforce. According to Table 12, supply chain 3 obtained marginal profit maximization for the utilization of one extra unit of resources from 10 supply chains when using the $(g_{Y_1}^3, g_{Y_2}^3) = (0.9, 0.1)$ direction. Also, the increase of one unit from the specialist workforce in the oil field in supply chain number 9 creates significantly different amounts of oil sold to other companies. The selection of $(g_{Y_1}^9, g_{Y_2}^9) = (0.1, 0.9)$ direction is useful for more output generation while using one extra unit from the specialist workforce.

Table 12. DMP of suppliers 1 of supply chains 3 and 9 under 11 normalized directions

Direction (normalized)	Objective function of oil field of DMU$_3$	DMP	Objective function of oil field of DMU$_9$	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	$\alpha_1^{S_1}$	$\frac{\partial(Y_1^{S_1}, Y_2^{S_1})}{\partial X_1^{S_1}}$	$\alpha_2^{S_1}$	$\frac{\partial(Y_1^{S_1}, Y_2^{S_1})}{\partial X_2^{S_1}}$
(1, 0)	0.000049	(1.98, 0.000000)	0.000000	(0.000,0.000000)
(0.9, 0.1)	0.000061	(2.24, 0.000249)	0.000000	(0.000,0.000000)
(0.8, 0.2)	0.000055	(1.777032, 0.000445)	0.000048	(1.58, 0.000394)
(0.7, 0.3)	0.000072	(2.045134, 0.000877)	0.000035	(1.007131, 0.000432)
(0.6, 0.4)	0.000054	(1.314565, 0.000877)	0.000028	(0.420661, 0.000281)
(0.5, 0.5)	0.000043	(0.876153, 0.000877)	0.000022	(0.449562, 0.000450)
(0.4, 0.6)	0.000036	(0.583894, 0.000876)	0.000130	(2.110932, 0.003168)
(0.3, 0.7)	0.000031	(0.375320, 0.000866)	0.000231	(2.816212, 0.006575)
(0.2, 0.8)	0.000027	(0.216301, 0.000866)	0.000371	(3.010000, 0.012065)
(0.1, 0.9)	0.000024	(0.0977, 0.000880)	0.000965	(3.916430, 0.035271)
(0, 1)	0.003	(0.0000, 0.000743)	0.002359	(0.000, 0.1218)

According to Table 12, using one more unit from oil field capacities in supply chain 3, an increase of 2.24 units of oil sold to other companies and an increment of 0.0002 units from the share of oil consumption of the first supplier in the whole country in the $(g_{Y_1}^3, g_{Y_2}^3) = (0.9, 0.1)$ direction was provided. Also, utilization of more units of workforce in supply chain number 9 creates a notable increment in sold oil to other companies, as the increase in one unit of oil resource capacity provides an increment of 3.9 units of sold oil and 0.035 units of share of oil consumption in the $(g_{Y_1}^9, g_{Y_2}^9) = (0.1, 0.9)$ direction. Similarly, the MP of Y_1 is null using the direction $(g_{10}^{Y_1}, g_{10}^{Y_2}) = (1, 0)$, and the MP of Y_2 is 0.12 using the direction of $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (0,1)$. Indeed, the DMP of Y_1 and Y_2 is estimated by equation (14) when one extra unit of input X_i is used in the oil field of supply chains as follows:

$$\frac{\partial(Y_1^{S_1}, Y_2^{S_1})}{\partial X_i^{S_1}} = \alpha_i^{S_1} \times (40572.1 g_{Y_1}^{S_1}, 40.6 g_{Y_1}^{S_1}) ,$$

$$[\,(Y_1^{S_1})^{\max}, (Y_2^{S_1})^{\max}] = (40572.1, 40.6) \quad (17)$$

Table 13 indicates the DMP of gas field outputs in supply chain 3 with the utilization of one extra unit of gas resources and the DMP of supply chain 10 based on the use of one more unit of specialist workforce.

Table13. DMP of suppliers 2 of supply chains 3 and 10 under 11 normalized directions

	Objective function of gas field, DMU$_3$	DMP	Objective function of gas field of DMU$_{10}$	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	$\alpha_1^{s_2}$	$\frac{\partial(Y_1^{S_2}, Y_2^{S_2})}{\partial X_1^{S_2}}$	$\alpha_2^{s_2}$	$\frac{\partial(Y_1^{S_2}, Y_2^{S_2})}{\partial X_2^{S_2}}$
(1, 0)	0.000000	(0.000000, 0.00000)	0.000044	(0.410, 0.000000)
(0.9, 0.1)	0.000070	(0.654806, 0.000330)	0.000045	(0.412845,0.000208)
(0.8, 0.2)	0.000037	(0.310256, 0.000352)	0.000048	(0.398146 ,0.000452)
(0.7, 0.3)	0.000033	(0.241414, 0.000470)	0.000068	(0.498487, 0.000970)
(0.6, 0.4)	0.000029	(0.179902, 0.000545)	0.000075	(0.472038,0.002086)
(0.5, 0.5)	0.000018	(0.095457, 0.000433)	0.000110	(0.688920,0.0020856)
(0.4, 0.6)	0.000019	(0.0785160, 0.000535)	0.000068	(0.354053,0.001608)
(0.3, 0.7)	0.000028	(0.088573, 0.000938)	0.00000	(0.000000,0.000000)
S (0.2, 0.8)	0.000030	(0.062631, 0.001138)	0.00000	(0.000000,0.000000)
(0.1, 0.9)	0.000027	(0.027957, 0.001142)	0.00000	(0.000000,0.000000)
(0, 1)	0.000025	(0.000000, 0.001194)	0.00000	(0.000000,0.000000)

Similarly, the increase of one extra unit of gas resources in supply chain 3 creates marginal profit maximization of two outputs between 10 supply chains. Hence, the selection of the $(g_{Y_1}^3, g_{Y_2}^3) = (0.9, 0.1)$ direction among 11 directions is appropriate because the most increase in gas sold to other companies happened in supply chain 3, while the highest share of gas consumption in supply chain 3 in the whole country is provided in the $(g_{Y_1}^3, g_{Y_2}^3) = (0, 1)$ direction. Also, the increase of one extra unit of specialist workforce in the gas field of supply chain number 10 causes marginal profit maximization of two outputs in the $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (0.5, 0.5)$ direction. According to Table 13, the MP of Y_1 is null using the direction $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (1, 0)$ and the MP of Y_2 is 0.13 using the direction $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (0,1)$, respectively. Let's consider that the DEA frontier includes a free disposable portion with respect to outputs, so we can increase the MP of Y_1 to obtain 0.65 by shifting the direction from $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (0,1)$ to (0.9,0.1). Therefore, we prefer selecting $(g_{Y_1}^{10}, g_{Y_2}^{10}) = (0.9, 0.1)$ rather than (0, 1) for more generation of outputs.

Table 14 shows the DMP of manufacturer 1 for supply chains 9 and 7 has the DMP between 10 supply chains.

Table 14. DMP of Manufacturer 1 of supply chains 9 and 10 under 11 normalized directions

	Objective function of power plant1 of, DMU$_9$	DMP	Objective function of power plant1 of DMU$_7$	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	$\alpha_1^{M_1}$	$\frac{\partial(Y_1^{M_1}, Y_2^{M_1})}{\partial X_1^{M_1}}$	$\alpha_2^{M_1}$	$\frac{\partial(Y_1^{M_1}, Y_2^{M_1})}{\partial X_1^{M_1}}$
(1, 0)	0.000000	(0.000000,0.000000)	0.000072	(1.270000,0.000000)
(0.9, 0.1)	0.000000	(0.000000,0.000000)	0.000088	(0.699000,0.000105)

(0.8, 0.2)	0.000008	(0.125264,0.000021)	0.000100	(1.404460 ,0.000238)
(0.7, 0.3)	0.000010	(1.226693,0.000356)	0.000085	(1.049898,0.000304)
(0.6, 0.4)	0.000147	(1.553064,0.000701)	0.000099	(1.045211,0.000040)
(0.5, 0.5)	0.000230	(2.027199,0.001372)	0.000085	(0.749927,0.000507)
(0.4, 0.6)	0.000291	(2.043942,0.002075)	0.000075	(0.526822,0.000535)
(0.3, 0.7)	0.000390	(2.055929,0.003246)	0.000109	(0.527308,0.000912)
(0.2, 0.8)	0.000427	(1.502725,0.002904)	0.000024	(0.085270,0.000231)
(0.1, 0.9)	0.000305	(0.536350,0.003267)	0.000000	(0.000000,0.000000)
(0, 1)	0.000079	(0.000000,0.000945)	0.000000	(0.000000,0.000000)

The third column of Table 14 indicates the DMP of the first power plant in supply chain 9 that obtained marginal profit maximization of outputs in the direction of $(g_{Y_1}^{9},\ g_{Y_2}^{9}) = (0.3, 0.7)$ between 10 supply chains based on the allocation of one extra unit from the nominal capacity of the power plant (the first input). Indeed, the utilization of more than one unit from the nominal capacity of the first power plant increases the produced electricity by 2.05 units, while the share of the gross product of the power plant in the whole country increases by 0.003 units. Also, the first power plant of supply chain 7 obtained marginal profit maximization of outputs in the $(g_{Y_1}^{7},\ g_{Y_2}^{7}) = (0.8, 0.2)$ direction between 10 supply chains using more than one unit of specialist workforce (the second input). According to column 5 of Table 14, the increase of one extra unit of the second input creates 1.40 units of electricity flow generation and 0.0002 units from the share of gross product of manufacturer 1 in the whole country in the $(g_{Y_1}^{7},\ g_{Y_2}^{7}) = (0.8, 0.2)$ direction.

Table 15 indicates that the DMPs of the second and third power plants in supply chains 3 and 10 have marginal profit maximization of outputs in 10 supply chains. The increase of one extra unit from the nominal capacity of the second power plant in supply chain 3 creates 0.36 units of produced power and increases 0.0001 of the share of gross product of the power plant 3 in the whole country in the $(g_{Y_1}^{3},\ g_{Y_2}^{3}) = (0.8, 0.2)$ direction, while the utilization of one more unit of the nominal capacity of the third power plant in supply chain 10 produces marginal profit maximization of outputs in the $(g_{Y_1}^{10},\ g_{Y_2}^{10}) = (0.6, 0.4)$ direction and creates the increase of 0.34 units of power production.

Table 15. DMP of manufacturers 2 and 3 of supply chains 3 and 10 under 11 normalized directions

	Objective function of manufacturer 2 DMU$_3$	DMP	Objective function of manufacturer 3 DMU$_{10}$	DMP
$(g_{Y_1}^{h},\ g_{Y_2}^{h})$	$\alpha_1^{M_2}$	$\frac{\partial(Y_1^{M_2}, Y_2^{M_2})}{\partial X_1^{M_2}}$	$\alpha_1^{M_3}$	$\frac{\partial(Y_1^{M_3}, Y_2^{M_3})}{\partial X_1^{M_3}}$
(1, 0)	0.000000	(0.0000000,0.00000)	0.000057	(0.108000,0.000000)
(0.9, 0.1)	0.000001	(0.058848,0.000008)	0.000050	(0.085928,0.000040)
(0.8, 0.2)	0.000067	(0.359070,0.000117)	0.000218	(0.146775 ,0.000153)
(0.7, 0.3)	0.000065	(0.304357,0.000170)	0.000297	(0.288283 ,0.000516)
(0.6, 0.4)	0.000063	(0.254452,0.000221)	0.000297	(0.337566,0.000694)
(0.5, 0.5)	0.000074	(0.246500,0.000320)	0.000338	(0.319638,0.001335)

(0.4, 0.6)	0.000086	(0.231209,0.000451)	0.000334	(0.260568,0.001632)
(0.3, 0.7)	0.000081	(0.162961,0.000494)	0.000532	(0.301674,0.002940)
(0.2, 0.8)	0.000093	(0.124293,0.000647)	0.000483	(0.182578,0.003050)
(0.1, 0.9)	0.000106	(0.071128,0.000832)	0.000439	(0.083083,0.003123)
(0, 1)	0.000256	(0.000000,0.002228)	0.000417	(0.000000,0.003297)

Table 16. DMP of transmitters 1 and 2 of the supply chain 10 under 11 normalized directions

$(g_{Y_1}^h,\ g_{Y_2}^h)$	Objective function of transmitter 1 DMU₁₀	DMP	Objective function of transmitter 2 DMU₁₀	DMP
	$\alpha_1^{T_1}$	$\frac{\partial(Y_1^{T_1},Y_2^{T_1})}{\partial X_1^{T_1}}$	$\alpha_1^{T_2}$	$\frac{\partial(Y_1^{T_2},Y_2^{T_2})}{\partial X_1^{T_2}}$
(1, 0)	0.0000000	(0.000000,0.000000)	0.000031	(0.366000,0.000000)
(0.9, 0.1)	0.0000000	(0.000000,0.000000)	0.000020	(0.219000,0.000017)
(0.8, 0.2)	0.0000000	(0.000000,0.000000)	0.000086	(0.819389,0.000140)
(0.7, 0.3)	0.0000000	(0.000000,0.000000)	0.000089	(0.735885 ,0.000215)
(0.6, 0.4)	0.0000000	(0.000000,0.000000)	0.000066	(0.470000,0.000214)
(0.5, 0.5)	0.0000000	(0.000000,0.000000)	0.000061	(0.364152,0.000249)
(0.4, 0.6)	0.0000000	(0.000000,0.000000)	0.000055	(0.263197,0.000270)
(0.3, 0.7)	0.0000000	(0.000000,0.000000)	0.000049	(0.175383,0.000280)
(0.2, 0.8)	0.0000008	(0.026764,0.000035)	0.000020	(0.0493,890.000135)
(0.1, 0.9)	0.0000001	(0.001977,0.000006)	0.000036	(0.042733,0.000263)
(0, 1)	0.000000	(0.000000,0.000016)	0.000022	(0.000000,0.000179)

Table 16 shows the DMP of transmitters 1 and 2 that obtained the most marginal profit of outputs in the supply chain. According to Table 16, the utilization of more than one unit from the capacity of the first transmitter line provides an increase of 0.027 unit of transferred electricity flow and an increment of 0.0003 unit from the share of gross product of the first transmitter in the whole country in the $(g_{Y_1}^{10},\ g_{Y_2}^{10}) = (0.2, 0.8)$ direction, while the increase of one extra unit of line capacity of transmitter 2 creates about an 0.82 percent increase in transferred electricity to distribution networks and a 0.0001 percent increment in the share of transferred power of the second transmitter in the whole country in the $(g_{Y_1}^{10},\ g_{Y_2}^{10}) = (0.8, 0.2)$ direction.

Tables 17–20 show the DMP of four distribution networks for 11 directions under supply chains that have the most marginal profit among 10 supply chains. Table 17 indicates the DMP of the first distribution outputs of supply chains 5 and 10 under the allocation of more than one unit of length of distribution lines. According to the third column of Table 17, the one extra unit from line length of distribution 1 of DMU 5 creates the marginal profit maximizing of outputs between 10 supply chains in the $(g_{Y_1}^{5},\ g_{Y_2}^{5}) = (1, 0)$ direction. Indeed, the dispatched electricity increment is 2.99 units, while the increase in the sale share of the first distributor network with regard to the whole country is null in the $(g_{Y_1}^{5},\ g_{Y_2}^{5}) = (1, 0)$ direction. Also, the allocation of more than one-unit capacity to the first distributor lines did not affect the dispatch of energy to power consumers and the sale share of the distribution network in the whole country.

Table 17. DMP of distributor 1 of supply chains 5 and 10 under 11 normalized directions

	Objective function of distributer 1 DMU$_5$	DMP	Objective function of distributer 1 DMU$_{10}$	DMP
$(g^h_{Y_1}, g^h_{Y_2})$	$\alpha_2^{D_1}$	$\frac{\partial(Y_1^{D_1}, Y_2^{D_1})}{\partial X_2^{D_1}}$	$\alpha_2^{D_1}$	$\frac{\partial(Y_1^{D_1}, Y_2^{D_1})}{\partial X_1^{D_1}}$
(1, 0)	0.000261	(2.990000,0.000000)	0.0000001	(0.001440,0.000000)
(0.9, 0.1)	0.000008	(0.084600,0.000005)	0.0000000	(0.000000,0.000000)
(0.8, 0.2)	0.000178	(1.633104,0.000225)	0.0000000	(0.000000,0.000000)
(0.7, 0.3)	0.000011	(0.084575,0.000020)	0.0000230	(0.181000 ,0.000004)
(0.6, 0.4)	0.000012	(0.084578,0.000031)	0.0000009	(0.006390,0.000002)
(0.5, 0.5)	0.000015	(0.000000,0.000000)	0.0000010	(0.005770,0.000003)
(0.4, 0.6)	0.000013	(0.058533,0.000048)	0.0000030	(0.018000,0.000012)
(0.3, 0.7)	0.000018	(0.062610,0.000080)	0.0000000	(0.000000,0.000000)
(0.2, 0.8)	0.000376	(0.859056,0.001892)	0.0000000	(0.000000,0.000000)
(0.1, 0.9)	0.000000	(0.000000,0.000000)	0.0000000	(0.000000,0.000000)
(0, 1)	0.000008	(0.000000,0.000053)	0.0000000	(0.000000,0.000000)

According to Table 18, supply chain number 4 obtained marginal profit maximization among 10 supply chains when using an extra unit of distribution 2 line capacity. Indeed, the utilization of more than one unit of capacity of distributor lines 2 of supply chain 4 creates an increase of 0.68 units of the dispatched electricity flow to power consumers and the increment of 0.0001 units from the sale share of distribution network power regard to the whole country in the $(g^9_{Y_1}, g^9_{Y_2}) = (0.7, 0.3)$ direction.

Table 18. DMP of distributor 2 of supply chains 9 and 4 under 11 normalized directions

	Objective function of distributer2 DMU$_4$	DMP	Objective function of distributer 2 DMU$_4$	DMP
$(g^h_{Y_1}, g^h_{Y_2})$	$\alpha_1^{D_2}$	$\frac{\partial(Y_1^{D_2}, Y_2^{D_2})}{\partial X_1^{D_2}}$	$\alpha_2^{D_2}$	$\frac{\partial(Y_1^{D_2}, Y_2^{D_2})}{\partial X_2^{D_2}}$
(1, 0)	0.000092	(0.751, 0.000000)	0.000000	(0.573940,0.000000)
(0.9, 0.1)	0.000877	(0.579353,0.000026)	0.000000	(0.579353,0.000026)
(0.8, 0.2)	0.0000339	(0.260967, 0.000026)	0.000048	(0.266907,0.000026)
(0.7, 0.3)	0.000119	(0.681864,0.000117)	0.000018	(0.681864 ,0.000118)
(0.6, 0.4)	0.000028	(0.135659, 0.000036)	0.000053	(0.135659, 0.000036)
(0.5, 0.5)	0.000052	(0.212291, 0.000085)	0.000115	(0.212291,0.000085)
(0.4, 0.6)	0.000083	(0.271455, 0.000164)	0.000004	(0.271456,0.000164)
(0.3, 0.7)	0.000074	(0.180993, 0.000170)	0.000052	(0.180993,0.000170)
(0.2, 0.8)	0.000149	(0.243547, 0.000393)	0.000311	(0.243745,0.000393)
(0.1, 0.9)	0.000270	(0.221278, 0.000803)	0.000144	(0.221278,0.000803)
(0, 1)	0.000327	(0.000000,0.001080)	0.000053	(0.000000,0.001080)

Also, the increase of an extra one of unit distribution line length in the second distribution of supply chain 4 provides 0.68-unit dispatched electricity flow and an increase of 0.0001 unit from the sale share of the second distribution line in the whole country in the $(g_4^{Y_1}, g_4^{Y_2}) = (0.7, 0.3)$ direction. The third distribution line of supply chain 9 obtained the marginal profit maximizing of outputs between 10 supply chains when it utilized the extra unit of the distribution line's length in the $(g_Y^9, g_{Y_2}^9) = (0.8, 0.2)$ direction. Also, we can increase the MP of Y_2 to obtain 0.0.0003 by shifting the direction from $(g_Y^9, g_{Y_2}^9) = (0.8, 0.2)$ to (0.2, 0.8).

Therefore, we prefer selecting $(g_Y^9, g_{Y_2}^9) = (0.8, 0.2)$ rather than (0.2, 0.8) for more generation of dispatched power as 0.50 units.

Table 19. DMP of distributor 3 of supply chains 6 and 9 under 11 normalized directions

	Objective function of distributer 3 DMU6	DMP	Objective function of distributer 3 DMU9	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	$\alpha_2^{D_3}$	$\frac{\partial(Y_1^{D_3}, Y_2^{D_3})}{\partial X_2^{D_3}}$	$\alpha_2^{D_3}$	$\frac{\partial(Y_1^{D_3}, Y_2^{D_3})}{\partial X_2^{D_3}}$
(1, 0)	0.000018	(0.110000,0.000000)	0.000069	(0.410000,0.000000)
(0.9, 0.1)	0.000020	(0.110000,0.000059)	0.000047	(0.250000,0.000013)
(0.8, 0.2)	0.000023	(0.109639,0.000013)	0.000106	(0.503000,0.000061)
(0.7, 0.3)	0.000023	(0.095934,0.000020)	0.000076	(0.313871 ,0.000066)
(0.6, 0.4)	0.000031	(0.109641,0.000036)	0.000074	(0.262884,0.000086)
(0.5, 0.5)	0.000037	(0.109639,0.000053)	0.000077	(0.229840,0.000112)
(0.4, 0.6)	0.000045	(0.106185,0.000078)	0.000074	(0.176146,0.000129)
(0.3, 0.7)	0.000045	(0.081100,0.000092)	0.000099	(0.171467,0.000195)
(0.2, 0.8)	0.000056	(0.066100,0.000129)	0.000120	(0.142241,0.000275)
(0.1, 0.9)	0.000045	(0.026453,0.000116)	0.000105	(0.062456,0.000275)
(0, 1)	0.000044	(0.000000,0.000129)	0.000062	(0.000000,0.000181)

Table 20. DMP of distributer 4 of supply chain 3 under 11 normalized directions

	Objective function of distributer4 DMU3	DMP	Objective function of distributer 4 DMU3	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	$\alpha_1^{D_4}$	$\frac{\partial(Y_1^{D_4}, Y_2^{D_4})}{\partial X_1^{D_4}}$	$\alpha_2^{D_4}$	$\frac{\partial(Y_1^{D_4}, Y_2^{D_4})}{\partial X_1^{D_4}}$
(1, 0)	0.000000	(0.000, 0.000000)	0.000012	(0.102, 0.000000)
(0.9, 0.1)	0.000000	(0.000, 0.000000)	0.000015	(0.117, 0.000008)
(0.8, 0.2)	0.000000	(0.000, 0.000000)	0.000017	(0.117, 0.000017)
(0.7, 0.3)	0.000108	(0.637, 0.000159)	0.000014	(0.0806, 0.000020)
(0.6, 0.4)	0.000263	(1.32, 0.000515)	0.000009	(0.0466, 0.000018)
(0.5, 0.5)	0.000554	(2.33, 0.001360)	0.000019	(0.0818, 0.000048)
(0.4, 0.6)	0.000757	(2.54, 0.002220)	0.000015	(0.0491, 0.000043)
(0.3, 0.7)	0.000561	(1.41, 0.001920)	0.000094	(0.0238, 0.000032)
(0.2, 0.8)	0.000306	(0.514, 0.001200)	0.000018	(0.00295, 0.000007)
(0.1, 0.9)	0.000140	(0.117, 0.000616)	0.000000	(0.00000, 0.000000)
(0, 1)	0.000000	(0.000, 0.000000)	0.000020	(0.00000,0.000039)

Table 20 indicates the DMPs of the fourth distribution output of supply chain 3. We obtained the marginal profit maximizing lines in 10 supply chains based on the allocation of more than one unit of capacity and length of distribution.

According to the third column of Table 20, utilization of the extra one unit of the distribution lines capacity in supply chain 3 creates production of 2.54 units of dispatched electricity flow to power consumers, while the sale share of the fourth distribution line in the whole country raises 0.002 units. Therefore, the output generation in the $(g_3^{Y_1}, g_3^{Y_2}) = (0.4, 0.6)$ direction provides a significant increase in economic return.

Let us now try to detect the determinants of the success of supply chains in terms of the marginal profit maximization of 15 divisions. Indeed, the DMP of supply chains is based on 11 directions for direction determination and provides the most increase in output in order to maximize the utilization of one extra unit resource.

Tables 21–22 indicate the value obtained from the objective function of model 10 and the DMP of supply chains 3, 7, 9, and 10 under 11 normalized directions. According to Table 21, the allocation of one extra unit of the capacities of oil and gas fields and specialist workforce, nominal capacity of power plants and labor, capacity and length of transmission and distribution lines creates a 35.4 unit increase in the amount of oil and gas sold to other companies and produced and transferred electricity to distribution networks and dispatching power flow to power consumers in the $(g_3^{Y_1}, g_3^{Y_2}) = (0.4, 0.6)$ direction, while the utilization of more one unit of input provides an increase of 0.11 units of the share of oil and gas consumption, share of gross product of power plants, share of gross product of transmitter lines, and sale share of distribution lines in the whole country in the $(g_3^{Y_1}, g_3^{Y_2}) = (0,1)$ direction. Therefore, we prefer selecting $(g_3^{Y_1}, g_3^{Y_2}) = (0.4, 0.6)$ rather than (0, 1) for more generation of outputs.

Table 21. DMP of supply chains 3 and 7 under 11 normalized directions

	Objective function of supply chain 3	DMP	Objective function of supply chain 7	DMP
$(g^h_{Y_1}, g^h_{Y_2})$	α_3	$\frac{\partial(Y_1^{\,3}, Y_2^{\,3})}{\partial X^{\,3}}$	α_7	$\frac{\partial(Y_1^{\,7}, Y_2^{\,7})}{\partial X^{\,7}}$
(1, 0)	0.000035	(1.96, 0.000)	0.000008	(0.498, 0.000)
(0.9, 0.1)	0.000136	(8.09, 0.00148)	0.000145	(6.35, 0.000918)
(0.8, 0.2)	0.000169	(8.89, 0.00354)	0.000196	(8.04, 0.00157)
(0.7, 0.3)	0.000263	(18.5, 0.01)	0.000223	(9.78, 0.00535)
(0.6, 0.4)	0.000382	(23, 0.175)	0.0000237	(10.1, 0.00789)
(0.5, 0.5)	0.000642	(31.9, 0.0416)	0.000251	(6.43, 0.00924)
(0.4, 0.6)	0.001	(35.4, 0.0654)	0.000270	(7.43, 0.0135)
(0.3, 0.7)	0.000514	(13.7, 0.0423)	0.000250	(4.90, 0.0160)
(0.2, 0.8)	0.000829	(14.7, 0.0871)	0.000275	(3.60, 0.0176)
(0.1, 0.9)	0.000641	(5.69, 0.0680)	0.000365	(2.16, 0.0244)
(0, 1)	0.000946	(0.000, 0.111)	0.002	(0.00, 0.1302)

According to Table 21, the MP of Y_1 is 0.498 using the direction $(g_7^{Y_1}, g_7^{Y_2}) = (1, 0)$, and the MP of Y_2 is 0.13 using the direction $(g_7^{Y_1}, g_7^{Y_2}) = (0,1)$ in supply chain number 7, respectively. Let's assume that the DEA frontier includes a free disposable portion with respect to outputs, so we can increase the MP Y_1 to obtain 10.1 by shifting the direction from $(g_7^{Y_1}, g_7^{Y_2}) = (0,1)$ to (0.6,0.4).

Similarly, Table 22 shows the value α calculated from the objective function of Model 10 and the marginal profit maximization of supply chains 9 and 10 under normalized direction 11. The supply chain number 9 obtained the marginal profit maximization in the $(g_{Y_1}^9, g_{Y_2}^9) = (0.3, 0.7)$ direction as the allocation of one more unit input of energy and power plant sectors, transmission and distribution lines, and power customers creates 57.3374 units of oil and gas sold to other companies, produced electricity and transferred to distribution networks, and dispatched power flow to power consumers in the $(g_{Y_1}^9, g_{Y_2}^9) = (0.3, 0.7)$ direction. Also, utilization of one extra unit of inputs in this direction creates approximately 0.11 unit shares of oil and gas consumption, share of gross product of power plants, share of gross product of transmitter lines, and sale share of distribution lines in the whole country, while the increase of one unit of inputs in 15 divisions of supply chain 9 provides 0.517 unit shares of oil and gas consumption, share of gross product of power plants and transmitter lines, and sale share of distribution lines in the $(g_{Y_1}^9, g_{Y_2}^9) = (0,1)$ direction.

Table 22. DMP of supply chains 9 and 10 under 11 normalized directions

	Objective function of supply chain 9	DMP	Objective function of supply chain 10	DMP
$(g_{Y_1}^h, g_{Y_2}^h)$	α^9	$\frac{\partial(Y_1^9, Y_2^9)}{\partial X^9}$	α^{10}	$\frac{\partial(Y_1^{10}, Y_2^{10})}{\partial X^{10}}$
(1, 0)	0.0000122	(0.000881, 0.00)	0.0000328	(1.74, 0.00)
(0.9, 0.1)	0.0000540	(3.91, 0.000344)	0.000110	(4.12, 0.000926)
(0.8, 0.2)	0.000188	(0.150, 0.00453)	0.000275	(11.3, 0.00432)
(0.7, 0.3)	0.000358	(32.0, 0.0142)	0.000422	(22.3, 0.0113)
(0.6, 0.4)	0.000477	(28.6, 0.0166)	0.000550	(19.5, 0.0191)
(0.5, 0.5)	0.000666	(32.6, 0.0223)	0.000634	(17.8, 0.0274)
(0.4, 0.6)	0.000950	(29.5, 0.365)	0.000700	(17.9, 0.0201)
(0.3, 0.7)	0.002	(57.3374, 0.10934)	0.000835	(0.18, 0.0211)
(0.2, 0.8)	0.002	(35.0, 0.0699)	0.000737	(11.6, 0.0250)
(0.1, 0.9)	0.003	(33.9438, 0.23436)	0.000713	(5.77, 0.0396)
(0, 1)	0.006	(0.000, 0.517)	0.000678	(0.000,0.0424)

Therefore, we prefer selecting $(g_{Y_1}^3, g_{Y_2}^3) = (0.4, 0.6)$ rather than (0, 1) for more generation of outputs. Also, the increase of one unit of inputs in the energy and power plant sectors, transmitter and distribution lines, and power customers of supply chain 10 creates an increase of 22.3 units of oil and gas sold to other companies, produced electricity, and transferred and dispatched to power customers in $(g^{10}, g_{Y_2}^{10}) = (0.7, 0.3)$ the direction, while the sum of the shares of oil and gas consumption, share of gross product of power plants and transmission lines, and sale share of distribution lines in supply chain 10 is slight

Overall, supply chain 9 has appropriate capacities for production at the maximum marginal profit among the 10 supply chains. According to the obtained results, an increase of one unit of oil field workforce in the supply chain 9 creates approximately four units of oil sold to other companies in a predetermined direction, while the share of oil consumption in the whole country increases to 0.12. Moreover, supply chain 9 is the only DMU in which increments of output happen when the specialist workforce is increased.

Also, the utilization of one more unit of oil resources in supply chain 9 provides 0.82 units of sold oil to other companies, while applying one unit of oil resources in supply chain 3 creates an increase of 2.24 units of sold oil. In this case, supply chain management enables the choice of directions in which the increment of inputs provides the marginal profit maximization in a determined direction. Similarly, allocation of one extra unit of gas field labor to supply chain 10 creates gas sales maximization between supply chains, while increases in the labor of other supply chains have no effective result on capacity increases or economic returns. On the other hand, there is only a supply chain as increases in inputs to a power plant's division create increments in outputs in a predetermined direction. There are also supply chains in which distribution lines have appropriate opportunities to increase outputs as line capacity and length increase significantly.

5. Conclusion

Climate change and critical weather conditions have a direct effect on energy production performance, as a one-degree Celsius increase in ambient temperature reduces the productivity and performance of the energy and power plant sectors and causes efficiency losses in the transmission and distribution networks. The MP plays a fundamental role in economic theory and its applications. This study provides a DEA model for estimation of DMP of supply chains, which measures the divisions' efficiency via a marginal-profit maximization orientation. The current paper estimates the MP of 10 supply chains and their 15 divisions under two categories of inputs by 11 normalized directions. Indeed, the proposed model, in addition to identifying outputs in different directions, enables distinguishing inputs with significant effects on the production of more outputs. Therefore, the divisions' recognition in the electricity supply chain and choice of the right direction for production increment not only cause resource control and capacity management and have a marketable effect on economic return but also protect the environment from the negative effects of greenhouse gases. This study has two empirical results in the energy and power plant sectors and transmission and distribution lines. First, the results show that only one supply chain in the oil field has a high level of effectiveness opportunity where an increase of one unit from the specialist workforce creates a significant increment in outputs, so this supply chain needs investment in labor for more capacity generation. It is worth mentioning that the utilization of one more unit of distribution line length in the first distribution provided a significant increase in the dispatched flow to power customers. Second, inputs determine and play a fundamental role in MP. The inputs' accurate selection and the resources' appropriate allocation create a desirable output increment and performance productivity.

Indeed, the proposed model, in addition to identifying outputs in different directions, enables distinguishing inputs with significant effects on the production of more outputs. Therefore, the divisions' recognition in the electricity supply chain and choice of the right direction for production increment not only cause resource control and capacity management and have a marketable effect on economic growth but also protect the environment from the negative effects of greenhouse gases. The supply chain divisions that obtained marginal profit maximization have the necessary capacities for responsiveness to demand fluctuations in climate change and critical situations.

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