Available online at http://sanad.iau.ir/journal/ijim/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol.16, No. 1, 2024, Article ID IJIM-1629, 17 pages
Research Article

Proposing a new method to introduce the closest target based of inputs to the evaluated unit

M. Khanmohammadi[*] [1], V. Rezaie[2]

[1] Department of Mathematics, Islamshahr Branch, Islamic Azad University, Islamshahr, Tehran, Iran,

[2] Department of Mathematics, Yasooj Branch, Islamic Azad University, Yasooj, Iran.

Submission Date: 2022/09/30, Revised Date: 2023/09/18, Date of Acceptance: 2024/08/05

Abstract

DEA is a nonparametric method for calculating the relative efficiency of a DMU that yields to a reference target for an inefficient DMU. However, it is very hard for inefficient DMUs to be efficient by benchmarking a target DMU which has different inputs. Finding appropriate benchmarks based on the similarity of inputs makes it easier for an inefficient DMU to try to be like its target DMUs. But it is rare to discover a target DMU, which is both the most efficient and similar in inputs, in real situation. Therefore, it is necessary to find the most similar and closest real DMU in terms of inputs on the strong efficiency frontier, which has the highest possible output. In this paper, a combination of the Enhanced Russell model and the additive model is proposed as a new model to improve the efficiency of the inefficient DMUs. The proposed model is applied on a dataset of a large Canadian Bank branches. The target introduced by the proposed method is more practical target for the evaluated unit. The inefficient unit can improve its efficiency more easily by this benchmark.

Keywords: DEA, DMU, Benchmarking, Closest target, Enhanced Russell measure.

[*] Corresponding author: Email: kh_khanmohamady@yahoo.com

1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric technique based on mathematical programming for evaluating the technical efficiency of a set of decision-making units (DMUs) that consumes inputs to produce outputs. The efficiency score is got from the distance between the evaluated DMU and a point on the frontier of the technology that assists as efficient target. Information on targets can express an important role since they show keys for inefficient units to improve their performance.

Traditional DEA models calculate the efficient targets which have the furthest projection on the efficiency frontier. On the other hand, improving the inputs and outputs of the assessed unit relative to the closed target requires less effort than reaching the furthest target. Therefore, a number of authors [1,2] discuss that the distance to the efficient projection point should be minimized, instead of maximized, until the resulting targets be as similar as possible to the inputs and outputs of the evaluated unit. Determining closest targets has been one of the essential subjects in the DEA literature, which is problematic enough and needs new ways in order to overcome it.

In the meantime, finding the closest targets for DMUs to be efficient has involved growing interests of many researchers in DEA area. Different researchers gave different definitions about the closest targets. Therefore, several methods for finding the closets targets have been suggested in the literature. A number of papers minimized the selected distance, while the others minimized the chosen efficiency measure. Frei and Harker [3] gave the closest targets by minimizing the Euclidean distance to the efficient frontier. In [4,5] the weighted versions of the Euclidean distance is applied to obtain the closest targets. In [6] a technique is proposed for obtaining the minimum distance of DMUs from the frontier of the PPS by $\|\bullet\|_1$. In [7], Ando et al. pointed out that least distance measures based on holder norms. Aparicio and Pastor [8] obtained a solution for output-oriented models based on an extended PPS that is strongly monotonic. Fukuyama et al, [9] calculated smallest distance p-norm inefficiency measures which satisfy strong monotonicity over the strongly efficient frontier to obtain the benchmarks. In [10] enhanced Russell measure and closet targets are combined to provide the closest targets, which is briefly explained in the next section.

Gonzalez and Alvarez [11] state that an inefficient DMU “could be more interested in visiting firm that uses more or less the same quantities of inputs (or outputs) than in visiting a firm that is using the same proportions of inputs but a different scale”. When an inefficient company compares itself with the most similar efficient company, in this case, it easily realizes its work mistakes and can improve efficiency by correcting them. But the question that arises here is what the best measure of similarity is? Many scientists defined the best measure of similarity to be proximity, which can be measured relative to inputs.

Gonzalez and Alvarez [11], in the context of an input-oriented efficiency assessment, minimize the sum of input contractions required to reach the efficient subset of the production frontier, which is equivalent to maximizing the input-oriented Russell efficiency measure [12].

Aparicio et al. [1] proposed a mixed integer mathematical programming program whose obtain all the efficient points dominating the assessed DMU then build efficient frontier by them. After that they calculated closer targets of inefficient DMU by finding the minimum distance of it to the efficient frontier. We will explain their method in the section 2.

In the current research, a new definition is introduced for the target of the evaluated DMU, and then, a combination of the Enhanced Russell model and the additive model is proposed as a new model.

The paper is organized as follows: in Section 2, some necessary preliminaries such as introduction of the additive model, Enhanced Russell measure and closet targets model based on Enhanced Russell measure is reviewed and in the last part of it, the introduction of Aparicio et al. [1]'s model and the difference between that model and the model proposed in this article have been discussed. The motivation of the research is introduced in section 3. In section 4, the proposed method for finding the most similar and closest real DMU in terms of inputs on the strong efficiency frontier is explained. A small example for verification of the proposed method is shown in the section 5 and an Empirical example is provided in section 6 And in this section, the difference between the results of Estrada et al. [13]'s model and our proposed model has been discussed. Finally, the paper is concluded in section 7.

2. Background

2.1 The additive model

Assume that there are a set of n DMUs, and each $DMU_j, (j = 1,2,\dots,n)$ produces s different outputs using m different inputs, which are denoted as $y_{rj}\ (r = 1,2,\dots,s)$ and $x_{ij}\ (i = 1,2,\dots,m)$, respectively. Additive model which has been provided by Charnes et al. [7] to evaluate decision making units is defined as follows:

$$
\begin{aligned}
E_{VRS} = \min \quad & \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} + s_i^- = x_{io}, \quad i = 1,\cdots,m, \qquad (1) \\
& \sum_{j=1}^{n} \lambda_j y_{rj} - s_r^+ = y_{ro}, \quad r = 1,\cdots,s, \\
& \sum_{j=1}^{n} \lambda_j = 1, \\
& s_i^- \geq 0, \quad i = 1,\cdots,m, \\
& s_r^+ \geq 0, \quad r = 1,\cdots,s, \\
& \lambda_j \geq 0, \quad j = 1,\cdots,n.
\end{aligned}
$$

2.2 Enhanced Russell measure

Assume that there are a set of n DMUs, and each $DMU_j, (j = 1,2,\dots,n)$ produces s different outputs using m different inputs which are denoted as $y_{rj}\ (r = 1,2,\dots,s)$ and $x_{ij}\ (i = 1,2,\dots,m)$, respectively.

The Russell measure of technical efficiency is non-orientation efficiency measure that proposed by Fare and Lovell [12]. This model has computational and explanatory problems;

therefore Pastor et al. [14] constructed an enhanced measure model for measuring the efficiency, as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & \frac{1/m \sum_{i=1}^{m} \theta_i}{1/s \sum_{r=1}^{s} \varphi_r} \\
s.t \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_i x_{io}, \quad i = 1, \cdots, m, \qquad (2) \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_r y_{ro}, \quad r = 1, \cdots, s, \\
& 0 \le \theta_i \le 1, \quad i = 1, \cdots, m, \\
& \varphi_r \ge 1, \quad r = 1, \cdots, s, \\
& \lambda_j \ge 0, \quad j = 1, \cdots, n.
\end{aligned}
$$

E_{CRS} is the efficiency of DMU_O. If $E_{CRS} = 1$, then DMU_O is Pareto efficient DMU also if $E_{CRS} < 1$ then DMU_O is inefficient DMU. This model is under the assumption of constant return to scale (CRS), this model is simply extended to non-decreasing (NDRS), non-increasing (NIRS) and variable return to scale (VRS) by addition $\sum_{j=1}^{n} \lambda_j \le 1, \sum_{j=1}^{n} \lambda_j \ge 1, \sum_{j=1}^{n} \lambda_j = 1$ in the constraints of model (1), respectively.

Model (2) can be easily changed into a same linear programming, shown as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & 1/m \sum_{i=1}^{m} \Theta_i \\
s.t \quad & \sum_{r=1}^{s} \Phi_r = s, \\
& \sum_{j=1}^{n} \Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i = 1, \cdots, m, \qquad (3) \\
& \sum_{j=1}^{n} \Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r = 1, \cdots, s, \\
& 0 \le \Theta_i \le w, \quad i = 1, \cdots, m, \\
& \Phi_r \ge w, \quad r = 1, \cdots, s, \\
& \Lambda_j \ge 0, \quad j = 1, \cdots, n, \\
& 0 \le w \le 1.
\end{aligned}
$$

Note: If model (2) is in states non-decreasing (NDRS), non-increasing (NIRS) or variable return to scale (VRS), then constrain related to the type of return to the desired scale, i.e.

$\sum_{j=1}^{n}\lambda_j \le 1, \sum_{j=1}^{n}\lambda_j \ge 1, \sum_{j=1}^{n}\lambda_j = 1$ will be converted to constrain $\sum_{j=1}^{n}\Lambda_j \le w, \sum_{j=1}^{n}\Lambda_j \ge w, \sum_{j=1}^{n}\Lambda_j = w$ in model (3), respectively.

2.3 Closet targets model based on enhanced Russell measure

An et al. [2] firstly constructed an enhanced Russell measure model in the existence of undesirable output, and then built the closet targets for the evaluated DMU_O under the enhanced Russell model. Therefore, they showed the input, desirable output and undesirable output of DMU_j by x_j, y_j, z_j, respectively. Then their model for measuring the efficiency in the presence of the undesirable output obtained as follows:

$$
\begin{aligned}
E_{CRS} = \min \quad & \tfrac{1}{m}\sum_{i=1}^{m}\Theta_i + \tfrac{1}{p}\sum_{t=1}^{p}\phi_t \\
s.t \quad & \sum_{r=1}^{s}\Phi_r = s, \\
& \sum_{j=1}^{n}\Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i=1,\cdots,m, \qquad (4) \\
& \sum_{j=1}^{n}\Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n}\Lambda_j z_{tj} \le \phi_r z_{to}, \quad t=1,\cdots,p, \\
& 0 \le \Theta_i \le w, \quad i=1,\cdots,m, \\
& \Phi_r \ge w, \quad r=1,\cdots,s, \\
& 0 \le \phi_t \le w, \quad t=1,\cdots,p, \\
& \Lambda_j \ge 0, \quad j=1,\cdots,n, \\
& 0 \le w \le 1.
\end{aligned}
$$

E_{CRS} is the efficiency of DMU_O. If $E_{CRS} = 1$, then DMU_O is Pareto efficient DMU also if $E_{CRS} < 1$ then DMU_O is inefficient DMU. Unfortunately, the above model finds the maximum distance between the evaluated DMU and the efficient production frontier. So, they proposed the closest target method, to find the closest target for the evaluated DMUs.

Assuming that E is the set of efficient units of model (4), they proved under hypothesis constant returns to scale, any virtual and real DMU formed by the Russell efficient DMUs set

E concluded $\sum_{j\in E}\Lambda_j x_j = X$, $\sum_{j\in E}\Lambda_j y_j = Y$, $\sum_{j\in E}\Lambda_j z_j = Z$ is efficient. Because of this theorem, they found the closet targets to the evaluated DMU_O by model (5):

$$
\begin{aligned}
E'_{CRS} = \max \quad & \tfrac{1}{m}\sum_{i=1}^{m}\Theta_i + \tfrac{1}{p}\sum_{t=1}^{p}\phi_t \\
s.t \quad & \sum_{r=1}^{s}\Phi_r = s, \\
& \sum_{j\in E}\Lambda_j x_{ij} \le \Theta_i x_{io}, \quad i=1,\cdots,m, \qquad (5)\\
& \sum_{j\in E}\Lambda_j y_{rj} \ge \Phi_r y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j\in E}\Lambda_j z_{tj} \le \phi_r z_{to}, \quad t=1,\cdots,p, \\
& 0 \le \Theta_i \le w, \quad i=1,\cdots,m, \\
& \Phi_r \ge w, \quad r=1,\cdots,s, \\
& 0 \le \phi_t \le w, \quad t=1,\cdots,p, \\
& \Lambda_j \ge 0, \quad j\in E, \\
& 0 \le w \le 1.
\end{aligned}
$$

They measured the optimal solution of model (4) as $(\theta^*, \Phi^*, \phi^*, \Lambda^*, w^*)$, then obtained the proportion of the inputs (α^*), the desirable outputs (Φ^*) and the undesirable outputs (ϕ^*) of DMU that be obtained by $\left(\theta^* = \Theta^*/w^*, \varphi^* = \Phi^*/w^*, \gamma^* = \phi^*/w^*\right)$. According to $(\theta^*, \varphi^*, \gamma^*)$, they could catch the closest targets for the inefficient DMUs to be efficient.

2.4 Closet targets model based on method of Aparicio et. al. [1]

Aparicio et. al. [1] proposed a mixed integer linear programming (MILP) program whose determines all the efficient points dominating the DMU being assessed, then, they calculated all the efficient facets of the frontier by them. At the end, they to find the closest targets for DMU_o, minimized the distance between its inputs and outputs to the efficient facet.

Let E be the set of extreme efficient units, They obtained a linear combination of extreme efficient units and dominate DMU_o by conditions $X = \sum_{j\in E}\lambda_j X_j$, $Y = \sum_{j\in E}\lambda_j Y_j$ in their model. $\sum_{j\in E}\lambda_j y_{rj} = y_{ro} + s^+_{ro}$, $r=1,\ldots,s$, $\sum_{j\in E}\lambda_j x_{ij} = x_{io} + s^-_{io}$, $i=1,\ldots,m$,

Also, With constraints $-\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} + d_j = 0$, $j\in E$, that $v_i \ge 1$, $i=1,\ldots,m$,

$u_r \geq 1, \quad r = 1,\ldots,s,$ they allow for all the hyperplanes such that all the points of PPS lie on or below these hyperplanes. Finally, the constraints $d_j \leq M b_j, \quad j \in E$, $\lambda_j \leq M(1-b_j), \quad j \in E$, (M is a big positive quantity) are the key conditions that connect the two previously entioned groups.

Therefore, the points (X, Y) satisfying in the above conditions are only those of PPS dominating $\left(X_o, Y_o\right)$ that can be expressed as a combination of extreme efficient units lying on the efficient facet of the frontier.

Then, they to find the closest targets for DMU_o, minimized the distance between its inputs and outputs to the efficient facet by different metric for example L_1-distance, mADD problem and mERG problem that are MILP problem. In fact, they solved MILP problems for getting to the closest targets. In the following, we show model mADD, which is very similar to our proposed model.

$$
\begin{aligned}
\text{mADD: Max} \quad & \sum_{i=1}^{m} s_{io}^{-} + \sum_{r=1}^{s} s_{ro}^{+} \\
\text{s.t} \quad & \sum_{j \in E} \lambda_j x_{ij} = x_{io} + s_{io}^{-}, \quad i = 1,\ldots,m, \\
& \sum_{j \in E} \lambda_j y_{rj} = y_{ro} + s_{ro}^{+}, \quad r = 1,\ldots,s, \\
& v_i \geq 1, \quad i = 1,\ldots,m, \\
& u_r \geq 1, \quad r = 1,\ldots,s, \\
& d_j \leq M b_j, \quad j \in E \\
& \lambda_j \leq M(1-b_j), \quad j \in E \\
& \lambda_j \geq 0, \quad j \in E \\
& s_{io}^{-} \geq 0, \quad i = 1,\ldots,m, \\
& s_{ro}^{+} \geq 0, \quad r = 1,\ldots,s.
\end{aligned}
$$

As you can see the model mADD, you can see that this model is a MILP model, which has used a large M number in its conditions, while our proposed model is a simple LP model that does not have the Computational complexity of the above model. Model mADD calculates the closest targets in terms of both inputs and outputs to the DMU under evaluation, but our proposed model looks for one real closest target in terms of input that has the highest output. In fact, it introduces the most ideal target that is most similar to the assessed DMU in terms of inputs.

3. Research Motivation

The majority of papers about targeting consider the image of an inefficient DMU on Farrell frontier as reference or target DMU. The efficient DMUs are considered as their own target. Sometimes the introduced target for an inefficient DMU is virtual and non-real which is obtained by calculating the linear combination of several efficient DMUs. The most of the managers and decision makers look for a real target. Actually, they want to compare their DMU's inputs and outputs with a real and efficient DMU.

Another drawback can be considered about the definition of closest targets in targeting articles. The defined target for a DMU may be real, but their inputs and outputs do not be similar to inputs and outputs of the DMU under evaluation. Here, the DMU under evaluation will not be able to improve its conditions based on its target because it is very far from of DMU under evaluation such that its inputs do not look like with inputs of DUM under evaluation, therefore clearly its outputs will not be similar to outputs of DUM under evaluation too. Hence, a target must be having the nearest inputs to inputs of an inefficient DMU. In these similar conditions, it can be stated that the DMU under evaluation does not have acceptable performance and must improve its performance with regard to its benchmark. Actually, this benchmark will be a rational and valid benchmark for an inefficient DMU.

On the other hand, a number of inputs in the evaluation of the DMUs are non-discretionary. For example, suppose a branch of a bank as a DMU and consider the area and the number of personnel as two of its inputs. Sometimes it is impossible that a branch of bank extends its area. Also, it is impossible that a branch of bank decreases its number of personnel with oust them.

Therefore, it cannot be introduced a benchmark for an inefficient DMU which its inputs are very different with the inputs of DMU under evaluation. It also cannot be observed great distance between their inputs.

For instance, for a bank with small and determined area which there is no conditions for extending it, cannot introduce a benchmark with very large area or good space.

Generally, we are looking for a real benchmark which has nearest inputs to the inputs of DMU under evaluation and at least there is no need to change inputs in large amount.

In this paper, we are going to get an idea from an enhanced Russell measure model and the additive model for obtaining a real benchmark which has the most similar inputs to the DMU under evaluation also has the maximum outputs. This target is the best because of:

i) This target is a real DMU, not a virtual DMU.
ii) Comparing the evaluated DMU with this target is easier and more realistic.

The proposed method will be discussed in the next Section.

4. Proposed Method (Methodology)

In this work, to determine the efficient DMUs, the Enhanced Russell Measure model is considered. The enhanced Russell measure yields to determination of the strong efficient DMUs which are located on the strong frontier. This model is unable to assign the weak efficient DMUs. So, this model is used due to that weak efficient DMUs may be dominated by some DMUs and the benchmark can be introduced for them.

Therefore, the aim of this work is to find a benchmark for DMUs that are evaluated as inefficient DMUs by enhanced Russell measure model with $E^*_{VRS} < 1$.

We consider smallest neighborhood around DMU under evaluation based on its inputs. So we find a real efficient DMU in this neighborhood which its inputs have smallest distance to DMU under evaluation's inputs, i.e. its inputs is could be as close to the inputs of the DMU under evaluation as possible, while it has the largest outputs. For example, may be there are several efficient DMUs in this neighborhood but the DMU with the largest and best outputs is considered as a benchmark. Since we are looking for a benchmark's DMU among real DMUs not virtual DMUs, so the proposed model is a binary model as below:

$$\begin{aligned} \min \quad & \frac{1}{m}\sum\nolimits_{i=1}^{m} s_i - \frac{1}{s}\sum\nolimits_{r=1}^{s} \varphi_r \\ s.t. \quad & \sum_{j\in E} \lambda_j x_{ij} + s_i = x_{io}, \quad \mathrm{i} = 1,\dots,\mathrm{m}, \qquad (6) \\ & \sum_{j\in E} \lambda_j y_{rj} \geq \varphi_r y_{ro}, \quad r = 1,\dots,s, \\ & \sum_{j\in E} \lambda_j = 1, \\ & \lambda_j \in \{0,1\}. \end{aligned}$$

In which E is a set of efficient DMUs by obtaining from model (3) and s_i is a slack of input $i\,(i = 1,\dots,m)$. It means that, the minimum s_i is searched that can be deducted from inputs and maximum outputs that can be given to the DMU in a combination of real and efficient DMUs ($j \in E$).

Since, only one real benchmark is searched, the constraint $\lambda_j \in \{0,1\}$ must be considered. If we look for a virtual benchmark DMU, the constraint $\lambda_j \geq 0$ is used instead of $\lambda_j \in \{0,1\}$.

Important note: In solving model (6), we can substitute constraint $\lambda_j \geq 0$ instead of constraint $\lambda_j \in \{0,1\}$, then, in the optimal answer, if one DMU has λ^*_j with a value of one, $(\lambda^*_j = 1)$, then the same DMU is selected as the benchmark, if there are more than one DMU that have a positive λ^*_j $(\lambda^*_j > 0)$, then we choose the DMU that has the largest λ^*_j as a benchmark of the evaluated DMU.

5. Verification Example

We consider 10 DMUs with one input and one output. The data of these DMUs are shown in Table 1. The Production Possibility Set (PPS) that these 10 DMUs are created, is shown in Figure1.

Table 1. Inputs and Outputs of 10 DMUs

DMUs	Input	Output
DMU_1	1	2
DMU_2	2	5
DMU_3	4	7
DMU_4	8	8
DMU_5	4	4
DMU_6	2	3
DMU_7	8	7.5
DMU_8	5	7
DMU_9	7	7
DMU_{10}	6	7

As seen in Figure 1. DMU_1, DMU_2, DMU_3, DMU_4 are evaluated as efficient and others DMUs are inefficient. We want to calculate the benchmarks of these inefficient DMUs based on proposed model in this paper. The results of running model (5) are shown in Table 2.

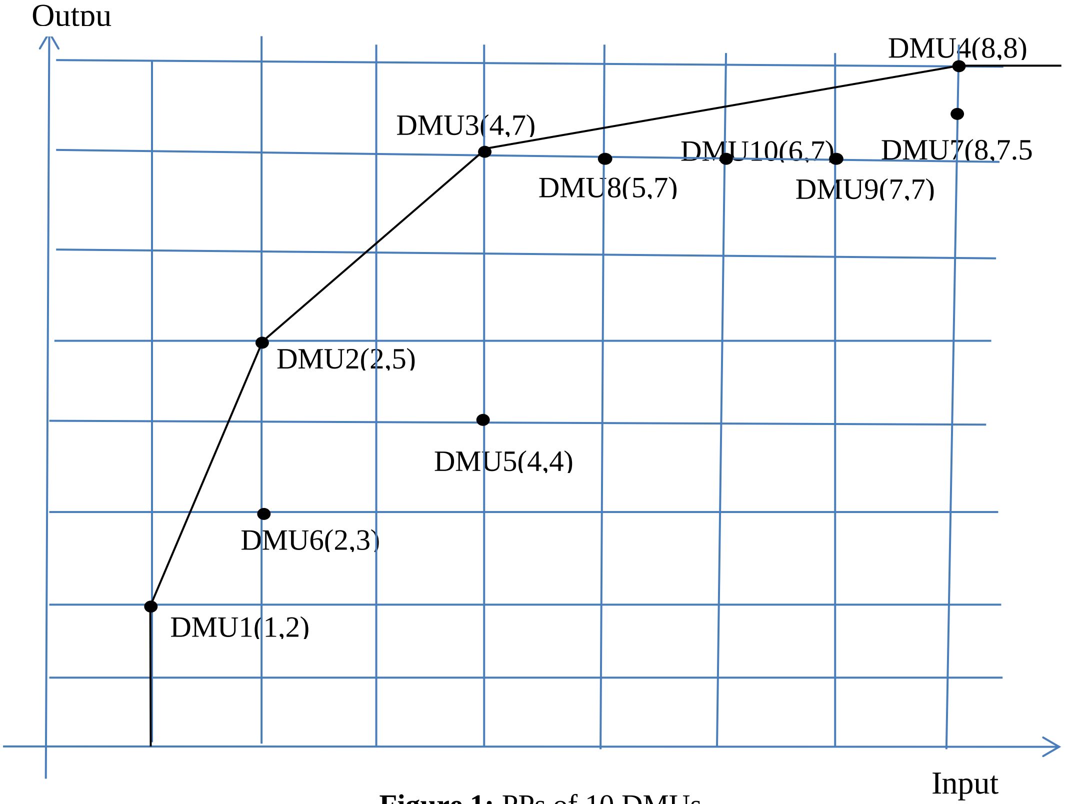

Figure 1: PPs of 10 DMUs

As seen in Table 2, the benchmark of DMU_5, is DMU_3. It means that the input of DMU_3 is similar to DMU_5 input, but it has better output rather than the output of DMU_5. So, in order to DMU_5 reach to an efficient status, there is no need to increase or decrease its input and with this input can reach to an efficient DMU. The DMU_5 must be compared with DMU_3 and accept it as a benchmark. Also DMU_5 should increase its output same as DMU_3.

DMU_2 and DMU_4 are as the benchmark for DMU_6 and DMU_7 with same inputs respectively. DMU_3, DMU_4 are as two benchmarks for DMU_8, DMU_9, and DMU_{10}.

As seen in Table 2, DMU_8 has $\lambda_3 = 0.75,\ \lambda_4 = 0.25$, it means that this DMU is %75 similar to DMU_3 and %25 similar to DMU_4. So, for DMU_8 is easy to choose DMU_3 as its real target, because input of DMU_8 is closer to input of DMU_3 rather than the input of DMU_4.

For DMU_9, $\lambda_3 = 0.25,\ \lambda_4 = 0.75$ which implies that the data of DMU_9 is closer to DMU_4 and reach to this DMU is easier in compare to DMU_3.

Finally, DMU_{10} has same λ ($\lambda_3 = \lambda_4 = 0.5$). This DMU has more freedom of action and can move itself to the place of DMU_3 or DMU_4 to be efficient. In the other words, can choose whichever DMU_3 or DMU_4 is easier and more capable for itself as a benchmark.

Table 2. The results of determination Benchmarking from inefficient DMUs.

DMUs	$\lambda^*_j \geq 0 \ \ (j \in E)$	**Benchmark**
DMU_5	$\lambda_3 = 1$	DMU_3
DMU_6	$\lambda_2 = 1$	DMU_2
DMU_7	$\lambda_4 = 1$	DMU_4
DMU_8	$\lambda_3 = 0.75,\ \lambda_4 = 0.25$	DMU_3
DMU_9	$\lambda_3 = 0.25,\ \lambda_4 = 0.75$	DMU_4
DMU_{10}	$\lambda_3 = \lambda_4 = 0.5$	$DMU_3,\ DMU_4$

6. Empirical Example

We apply our proposed method on a large Canadian Bank branches dataset [13]. There are 79 branches (DMUs) located in Canada in the dataset. The inputs are three types of full-time equivalent number of employees and the outputs are, respectively, loans, mortgages, registered retirement saving plans and letter of credit. Estrada et al. [13] used this data for their dynamic proposed method to obtain the appropriate benchmark based on the similarity of inputs for inefficient DMUs to improve their efficiency gradually. At the end, we compare the targets of the first 6 banks of these 79 banks, which were obtained by these two methods.

Table 3 shows the raw data of the three inputs and four outputs and the 12 efficient DMUs and 67 inefficient DMUs with their corresponding targets DMUs as results of our proposed model.

It is noteworthy that DMU_{62}, DMU_{70} are not the targets of any DMUs.

Table 3. Inputs and Outputs of 79 branches of bank and their benchmark based on the proposed method

DMU	Input 1	Input 2	Input 3	Output 1	Output 2	Output 3	Output 4	$\lambda_j^* > 0$	Benchmark
1	45.34	40.93	5.09	263	137	935	425	λ_4^*	DMU_4
2	9.02	1.34	0.1	42	6	176	32	$\lambda_{36}^* > \lambda_4^*$	DMU_{36}
3	26.12	8.24	1.01	130	20	679	101	$\lambda_4^* > \lambda_{36}^*$	DMU_4
4	10.94	4.87	1.03	134	37	437	80	efficient	DMU_4
5	49.52	32.28	7.21	308	46	726	227	λ_4^*	DMU_4
6	10.82	1.09	0	27	2	181	36	λ_{36}^*	DMU_{36}
7	11.52	1.98	0	44	5	337	47	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
8	8.11	3.91	0	34	1	245	33	λ_{36}^*	DMU_{36}
9	5.08	0	0	20	2	142	40	$\lambda_{34}^* > \lambda_{54}^*$	DMU_{34}
10	9.96	5.26	0	29	2	202	49	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
11	9.86	1.01	0	67	10	161	52	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
12	7.49	1	0	34	0	249	36	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
13	4	1.58	0	42	2	159	17	$\lambda_{65}^* > \lambda_{36}^*$	DMU_{65}
14	5.78	1.52	0.26	85	1	196	78	efficient	DMU_{14}
15	4.87	1.05	0	52	4	237	52	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
16	2.93	1.97	0	6	2	127	18	$\lambda_{49}^* > \lambda_{36}^*$	DMU_{49}
17	2.93	1.97	0	9	5	60	31	$\lambda_{65}^* > \lambda_{36}^* > \lambda_{49}^*$	DMU_{65}
18	5.99	0.97	0	61	0	133	24	$\lambda_{69}^* > \lambda_{36}^* > \lambda_{54}^*$	DMU_{69}
19	6.61	0.87	0.79	28	0	375	37	$\lambda_{36}^* > \lambda_{69}^* > \lambda_{34}^* > \lambda_4$	DMU_{36}
20	2.96	1.58	0	21	2	103	23	$\lambda_{65}^* > \lambda_{36}^*$	DMU_{65}

21	5.3	0	0	25	4	168	38	$\lambda_{54}^* > \lambda_{42}^* > \lambda_{34}^*$	DMU_{54}
22	9.84	5.02	0	25	1	301	50	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
23	16.06	1.99	0.67	143	7	551	187	$\lambda_{49}^* > \lambda_{4}^*$	DMU_{49}
24	25.6	7.76	005	151	13	808	211	$\lambda_{36}^* > \lambda_{4}^*$	DMU_{36}
25	5.31	1.06	0.06	35	3	250	40	$\lambda_{36}^* > \lambda_{65}^* > \lambda_{49}^* > \lambda_{4}$	DMU_{36}
26	6.46	1.59	0	37	3	323	35	$\lambda_{36}^* > \lambda_{65}^* > \lambda_{69}^*$	DMU_{36}
27	4.4	0.91	0.33	28	2	178	42	$\lambda_{54}^* > \lambda_{36}^* > \lambda_{4}^* > \lambda_{71}^*$	DMU_{54}
28	3.63	0	1.23	21	1	161	24	$\lambda_{42}^* > \lambda_{34}^* > \lambda_{54}^*$	DMU_{42}
29	6.16	0.27	0	34	6	227	142	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
30	29.22	6.66	1.29	135	13	760	161	$\lambda_{14}^* > \lambda_{4}^*$	DMU_{14}
31	8.46	0.67	0.87	48	1	293	50	$\lambda_{36}^* > \lambda_{71}^* > \lambda_{14}^* > \lambda_{4}^*$	DMU_{36}
32	4.87	2.65	0.35	41	6	313	30	$\lambda_{69}^* > \lambda_{65}^* > \lambda_{36}^* > \lambda_{4}^*$	DMU_{69}
33	10.69	3.17	0	93	3	393	77	λ_{49}^*	DMU_{49}
34	3.87	0	0	34	1	227	47	efficient	DMU_{34}
35	2.69	0.45	0	22	0	112	30	$\lambda_{49}^* > \lambda_{34}^* > \lambda_{69}^* > \lambda_{3}^*$	DMU_{49}
36	7.65	052	0	119	8	366	41	efficient	DMU_{36}
37	4.81	1.05	0	16	2	142	18	$\lambda_{65}^* > \lambda_{36}^*$	DMU_{65}
38	7.54	1.17	0	29	1	164	36	$\lambda_{36}^* > \lambda_{65}^*$	DMU_{36}
39	17.11	5.85	0	93	24	684	162	λ_{36}^*	DMU_{36}
40	5.91	0.66	0	40	3	177	42	$\lambda_{36}^* > \lambda_{65}^* > \lambda_{49}^*$	DMU_{36}
41	4024	1.08	0	21	0	107	42	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{49}
42	3.67	0	0	55	2	162	22	efficient	DMU_{42}
43	8.33	2.39	0	54	4	347	53	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}
44	2021	0.06	0	5	0	74	13	$\lambda_{42}^* > \lambda_{54}^* > \lambda_{49}^*$	DMU_{42}
45	3	0	0	18	1	77	21	$\lambda_{42}^* > \lambda_{54}^*$	DMU_{42}

46	3.71	1.17	0.12	12	2	148	52	$\lambda_{49}^{*} > \lambda_{65}^{*} > \lambda_{36}^{*} > \lambda_{4}^{*}$	DMU_{49}
47	10.1	3.53	0.64	76	7	329	54	$\lambda_{4}^{*} > \lambda_{36}^{*}$	DMU_{4}
48	7.79	2.33	0.09	39	1	207	55	$\lambda_{36}^{*} > \lambda_{14}^{*} > \lambda_{49}^{*}$	DMU_{36}
49	1	0.42	0	6	1	62	65	efficient	DMU_{49}
50	3.2	0.97	0	13	1	140	39	$\lambda_{65}^{*} > \lambda_{36}^{*} > \lambda_{49}^{*}$	DMU_{65}
51	12.05	0.9	0.08	69	2	410	186	$\lambda_{36}^{*} > \lambda_{4}^{*}$	DMU_{36}
52	4.55	0.17	0.73	36	5	171	42	$\lambda_{36}^{*} > \lambda_{54}^{*} > \lambda_{71}^{*} > \lambda_{34}^{*}$	DMU_{36}
53	9.42	1.88	1	59	3	420	97	$\lambda_{36}^{*} > \lambda_{4}^{*}$	DMU_{36}
54	0.76	0	0	1	4	31	23	efficient	DMU_{54}
55	7.95	1.45	0	52	2	432	77	λ_{36}^{*}	DMU_{36}
56	3.52	0.4	0	12	2	57	39	$\lambda_{36}^{*} > \lambda_{54}^{*} > \lambda_{49}^{*} > \lambda_{65}^{*}$	DMU_{36}
57	3	0	0	8	1	134	20	$\lambda_{42}^{*} > \lambda_{54}^{*} > \lambda_{34}^{*}$	DMU_{42}
58	622	0.95	0	37	0	135	59	$\lambda_{49}^{*} > \lambda_{36}^{*} > \lambda_{14}^{*}$	DMU_{49}
59	35.35	11.8	2.07	214	27	1090	225	λ_{4}^{*}	DMU_{4}
60	14.77	2.66	0.01	36	9	425	73	$\lambda_{36}^{*} > \lambda_{4}^{*}$	DMU_{36}
61	6.12	0	0.14	28	1	176	38	$\lambda_{34}^{*} > \lambda_{42}^{*}$	DMU_{34}
62	3.81	0.02	0	49	1	180	42	efficient	DMU_{62}
63	10.46	0.58	0	73	0	461	83	λ_{36}^{*}	DMU_{36}
64	3.72	1.22	0	33	1	136	23	$\lambda_{65}^{*} > \lambda_{36}^{*}$	DMU_{65}
65	2	1	0	18	5	157	26	efficient	DMU_{65}
66	5.42	0.63	0	42	2	199	31	$\lambda_{36}^{*} > \lambda_{65}^{*} > \lambda_{54}^{*}$	DMU_{36}
67	3.03	0.95	0	14	1	79	16	$\lambda_{65}^{*} > \lambda_{36}^{*}$	DMU_{65}
68	7.75	1.81	0	39	2	369	56	$\lambda_{36}^{*} > \lambda_{49}^{*}$	DMU_{36}
69	4.53	1.66	0	19	1	337	25	efficient	DMU_{69}
70	1	0	0	2	1	31	36	efficient	DMU_{70}

71	1.25	0	0.33	0	1	38	64	efficient	DMU_{71}
72	15.79	2.44	1	1.29	10	464	127	$\lambda_{14}^* > \lambda_4^* > \lambda_{71}^*$	DMU_{14}
73	73	1.95	0.09	118	1	359	109	$\lambda_{36}^* > \lambda_4^* > \lambda_{14}^*$	DMU_{36}
74	7.97	0.12	0.03	60	1	301	142	$\lambda_{36}^* > \lambda_{34}^*$	DMU_{36}
75	2	0.1	0	1	1	6	11	$\lambda_{42}^* > \lambda_{54}^* > \lambda_{49}^*$	DMU_{42}
76	20.42	10.19	0.83	107	16	408	238	λ_{36}^*	DMU_{36}
77	9.75	1.76	0	47	3	511	63	$\lambda_{65}^* > \lambda_{36}^*$	DMU_{65}
78	5.04	0	0.03	31	3	189	30	$\lambda_{42}^* > \lambda_{54}^* > \lambda_{34}^*$	DMU_{42}
79	7.17	0.95	0	40	1	207	43	$\lambda_{36}^* > \lambda_{49}^*$	DMU_{36}

In this part, we are going to compare the results obtained from our proposed method with the Estrada et al. method. Due to the size of the sample, we only present the results of the first 6 DMUs of Model Estrada et al. in Table 4. You can refer to their article to see the complete results of this model [13].

As you can see, in Estrada et al. method, two or more targets are introduced for each DMU, but our method introduces only one target for each DMU, and this target is among the targets of Estrada et al. method. As an example, we compare the targets of DMU_1 with two methods. In our proposed model, it firmly introduces DMU_4 as the target, but in the method of Estrada et al. it introduces DMU_4 and 54 as the targets. With a simple comparison of inputs and outputs, it is easy to see that inputs of DMU_4 are closer to inputs of DMU_1 than inputs of DMU_{54}, and DMU_4 has more outputs than DMU_{54} [13].

Also, by comparing the inputs and outputs of DMU_6 with DMU_{36}, DMU_{34}, DMU_{49} and DMU_{65}, we realize that the inputs of DMU_{36} are closer to DMU_6 and among these 5 DMUs, it has the most outputs. Our proposed method firmly introduces DMU_{36} as the target, but Estrada et al. method introduces four DMUs 34, 36, 49 and 65 as targets [13].

Table 4: Comparison of the objectives of the first 6 DMUs of Estrada et al. applied example and our proposed model [13].

DMU	Targets by Estrada et al. [8]'s model	Target by the proposed model
DMU_1	DMU_4, DMU_{54}	DMU_4
DMU_2	$DMU_4, DMU_{36}, DMU_{49}, DMU_{54}, DMU_{65}$	DMU_{36}
DMU_3	$DMU_4, DMU_{36}, DMU_{49}, DMU_{65}$	DMU_4
DMU_4	Efficient	DMU_4

DMU_5	$DMU_4, DMU_{36}, DMU_{49}$	DMU_4
DMU_6	$DMU_{34}, DMU_{36}, DMU_{49}, DMU_{65}$	DMU_{36}

7. Conclusion

The main purpose of this study was to propose a method to obtain the best benchmark for inefficient DMUs based on similarity in inputs. The benchmark is selected from the real DMUs and it is not a virtual DMU that does not exist externally. It is rare to find out a target DMU with input endowments similar to that of an inefficient DMU. An enhanced measure model was used for measuring the efficiency of DMUs. The model determines the strong efficient DMUs and gives all DMUs located on the strong frontier. The smallest neighborhood around DMU under evaluation was considered based on its inputs. So, a real efficient DMU was determined in this neighborhood which its inputs have smallest distance to DMU under evaluation's inputs, i.e., its inputs is could be as close to the inputs of the DMU under evaluation as possible, while it has the largest outputs. Therefore, the proposed model is a combination of the Enhanced Russell model and the additive model.

To evaluate the proposed model, an empirical study with a Canadian Bank branches dataset was used. The target introduced by the proposed method is more practical target for the evaluated unit. The inefficient unit can improve its efficiency more easily by this benchmark.

References

[1] J. Aparicio, J. L. Ruise, and I. Sirvent. (2007)." Closest targets and minimum distance to the Pareto-efficient frontier in DEA." Journal of Productivity Analysis, 28: 209-218.

[2] M. C. A. S. Portela, P. C. Borges, and E. Thanassoulis. (2003) "Finding closest targets in non-oriented DEA models: The case of convex and non-convex technologies. "Journal of Productivity Analysis, 19:251-269.

[3] F. X. Frei and P. T. Harker. (1999)." Projections onto efficient frontiers: theoretical and computational extensions to DEA." Journal of Productivity Analysis, 11:275–300.

[4] A. Amirteimoori and S. Kordrostami. (2010). "A Euclidean distance-based measure of efficiency in data envelopment analysis." Optimization, 59:985–996.

[5] J. Aparicio and J. T. Pastor. (2014a)." On how to properly calculate the Euclidean distance-based measure in DEA." Optimization, 63:421–432.

[6] G. R. Jahanshahloo, J. Vakili, and S. M. Mirdehghan. (2012)."Using the minimun distance of DMUs from the frontier of the PPS for evaluating group performance of DMUs in DEA." Asia-Pacific Journal of Operational Research, 29(2):1250010-1-1250010-25.

[7] K. Ando, A. Kai,Y. Maeda,K. Sekitani, (2012)." Least distance-based inefficiency measures on the Pareto-efficient frontier in DEA." J. Oper. Res. Soc. Jpn. 55 (1), 73–91.

[8] J. Aparicio and J. T. Pastor. (2014b)." Closest targets and strong monotonicity on the strongly efficient frontier in DEA." Omega, 44:51–57.

[9] H. Fukuyama, Y. Maeda, K. Sekitani, J. M. Shi, (2014)." Input–output substitutability and strongly monotonic p-norm least distance DEA measures." Eur. J. Oper. Res. http://dx.doi.org/10.1016/j.ejor.2014.02.033.

[10] Q. An, Z. Pang, H. Chen,L. Liang. (2015). "Closest targets in environmental efficiency evaluation based on enhanced Russell measure." Ecological Indicators, 51, 59-66.

[11] E. González, and A. Álvarez. (2001) "From efficiency measurement to efficiency improvement: the choice of a relevant benchmark", European Journal of Operational Research, 133, 512-520.

[12] R. Fare, C. A. K. Lovell, (1978). " Measuring the technical efficiency of production." J. Econ, Theory, 19(1), 150-162.

[13] S.A. Estrada, H.S. Song, Y. A. Kim, S. H. Namn, Sh. Ch. Kang. (2009). " A method of stepwise benchmarking for inefficient DMUs based on the proximity-based target selection " Expert Systems with Applications, 36, 11595–11604.

[14] M. C. A. S. Pastor, J. I. Ruiz, I. Sirvent. (1999). " An Enhanced Russell graph efficiency measure." Eur, J, Oper, Res. 115(3), 596-607.

[15] A. Charnes, W.W. Cooper, B. Golany, L. Seiford, J. (1985)" Stutz Foundations of data envelopment analysis for pareto-koopmans efficient empirical production functions." J. Econom, 30, pp. 91-107.