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Research Article

Performance Assessment of Two-stage Network Structures in the Presence of Contextual Variables

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Abstract

This research aims to evaluate the efficiency of a two-stage network structure by examining the impact of contextual variables on its performance. To accomplish this, a two-stage approach is implemented, which involves network data envelopment analysis (NDEA) in the first step and ordinary least squares (OLS) analysis in the second step. In the first step, a non-cooperative (Leader-Follower) model is employed to assess the efficiency of each sub-section within the two-stage process based on their inputs and outputs. In the second step, the logarithm of the estimated efficiency scores is regressed on the contextual variables to refine the network-specific efficiency. The performance of various Spanish airports is analyzed to demonstrate the effectiveness of this approach.

Keywords : Network data envelopment analysis; Two-stage network; Non-cooperative; Contextual variable; Undesirable output.

1 Introduction

In data envelopment analysis (DEA), a firm's performance is evaluated based on its inputs and outputs. However, other factors, such as contextual or explanatory variables, can also have a significant impact on firm performance. Therefore, it is important to examine the effects of these variables on efficiency, even if they are not directly observable, as they can provide valuable insights to managers. Researchers have proposed various methods over the years to estimate the effects of contextual variables. In an early attempt, Ray [17] employed a two-stage DEA method. In the first step, efficiency scores were calculated, and in the second step, the estimated efficiency scores were subjected to regression analysis with contextual variables. Subsequently, Wang and Schmidt [20] and Simar and Wilson [18] introduced one-step and two-step approaches, respectively, to estimate the effects of contextual variables on efficiency levels. Banker and Natarajan [1, 2] demonstrated in their work that a two-step process can consistently estimate the parameters of contextual variables. The initial step involves evaluating efficiency scores using the DEA approach. In the subsequent stage, the computed scores are subjected to regression analysis along

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with contextual factors, utilizing ordinary least squares (OLS). However, it is crucial to acknowledge that this methodology can only provide reliable estimates when the input and output data conform to a production function that exhibits monotonic increasing and concave properties.

The main objective of this paper is to enhance the accuracy of firm-specific efficiency estimates in two-stage network structures. To achieve this, we propose a non-cooperative approach. In this approach, we calculate the efficiency of the first stage (referred to as the leader) for each firm based on specific inputs and outputs. In the second step, we regress the logarithm of the efficiency scores obtained from the leader stage on contextual variables. This regression analysis helps refine the efficiency estimates. Once we have refined the leader stage efficiency, we proceed to compute the efficiency of the second stage (referred to as the follower) while maintaining the efficiency of the previous stage. Similarly, we regress the logarithm of the estimated efficiency on contextual variables to further enhance the accuracy of the estimates. By employing this approach, we aim to provide a more precise and comprehensive understanding of the efficiency of two-stage structures while considering the influence of contextual factors on this system.

The rest of this paper is organized as follows: Section 2 provides a brief review of current research on NDEA. In Section 3, we introduce our proposed technique for evaluating the performance of two-stage network processes. Section 4 applies the proposed method to assess the performance of various Spanish airports, demonstrating the practical usefulness of the methodology. Finally, Section 5 summarizes our findings and conclusions.

2 A review of studies on two-stage structures

DEA is a commonly employed method for evaluating the relative efficiency of decision-making units (DMUs). However, traditional DEA models exhibit limitations as they solely focus on primary inputs and final outputs when assessing DMU performance. These models treat each process as a black box, disregarding the internal structures within the DMUs. Consequently, the impact of internal performance on overall efficiency is overlooked in certain cases. To tackle this issue, Fre and Grosskopf [7] introduced the concept of network DEA (NDEA), which takes into account the internal structure of processes. By incorporating these internal factors, NDEA provides a more comprehensive framework for assessing DMU efficiency.

Various studies have proposed models with multiplicative and additive objective functions to analyze two-stage processes [3, 4, 9]. However, these models have limitations in that improving the performance of one sub-process in a two-stage process may result in inefficiency in another sub-process. To address this issue, Liang et al. [12] developed non-cooperative and centralized models that overcome the shortcomings of previous approaches. These newly proposed models determine the optimal efficiency of each sub-process within a two-stage process and provide a comprehensive evaluation of the overall system efficiency. They offer a more accurate assessment of units with complex structures, taking into account their internal processes. Building upon the method initially introduced by Liang et al. [12], Li et al. [11] expanded the approach by incorporating additional inputs for the second stage. To measure the efficiency of two-stage processes with freely shared flows, Zha and Liang [28] utilized a non-cooperative model. In contrast, Du et al. [6] assessed the efficiency of two-stage networks using the Nash bargaining game theory. Yu and Shi [25] proposed a non-cooperative model for a two-stage process with additional inputs in the second stage, where intermediate products were treated as final inputs. Evaluating a two-stage network configuration with additional inputs for the second stage, Chen and Zhu [5] applied an additive slack-based measure.

Previous models for evaluating the efficiency of two-stage processes did not account for the influence of undesirable factors. However, the significance of these factors in the manufacturing industry has motivated researchers to propose new models. These novel models utilize different methodologies to address undesirable inputs, intermediate variables, or final outputs, allowing for a more comprehensive evaluation of two-stage process performance. By incorporating the impact of undesirable factors, this improved modelling framework offers a more holistic assessment of efficiency. In light of this, we will conduct a literature review of selected studies in the field to explore the advancements made in addressing the impact of undesirable factors on the efficiency of two-stage processes.

Fukuyama and Weber [8] introduced a slack-based measure approach model, while Lozano et al. [14] presented a directional model specifically designed for evaluating two-stage structures with undesirable outputs. Maghbouli et al. [15] incorporated the weak disposability assumption to handle these outputs effectively. Wu et al. [22] made a valuable contribution to the literature by developing an additive DEA model suitable for network structures, which they applied to assess Chinese industrial production. Furthermore, they extended their model to propose a centralized approach for evaluating the efficiency of two-stage structures with shared inputs, successfully employed to assess industrial performance in China [23]. Finally, Wu et al. [24] introduced a model based on the Nash bargaining game to evaluate sustainable manufacturing in Chinese industries.

In addition to the previously mentioned studies, Nematizadeh et al. [16] developed a two-stage network model that incorporated return flows using the directional distance function. Similarly, Zeng et al. [27] introduced a novel two-stage model to assess the environmental efficiency of firms. Wang et al. [21] investigated the Chinese high-tech industry, building upon previous models proposed by [12]. Li et al. [13] applied a two-stage DEA methodology to evaluate the efficiency of China's Internet banking industry, with a specific focus on the growing significance of Internet banking in enhancing competitiveness. Finally, Yu and See [26] adopted the SBM-NDEA approach to assess the technical efficiency of 29 global airlines in 2018, considering the presence of undesirable factors.

3 Analyzing the impact of contextual variables on the two-stage network efficiency

This paper aims to examine the efficiency of subsection units (sub-DMUs) within two-stage structures through an analysis of contextual variables that may influence their performance. To achieve this objective, we adopt a two-step approach. First, we assess the sub-DMUs' efficiency under non-cooperative conditions by examining their inputs and outputs. Second, we employ the OLS method to determine the influence of contextual variables on efficiency, based on the estimated efficiencies from the first step. The algorithm for our proposed approach is presented in Figure 1.

In this section, we will provide a detailed description of the proposed method. Our scenario involves DMUs, with each $DMU_j : j = 1, , J$ having a two-stage structure as illustrated in Figure 2.

Figure 2 illustrates the first stage of the system, where an input vector $x_j = (x_{1j}, ..., x_{Ij}) \geq 0$ is used to generate two sets of outputs: the final undesirable output vector $w_j = (w_{1j}, ..., w_{Mj}) \geq 0$ and the desirable output vector $v_j = (v_{1j}, ..., v_{Rj}) \geq 0$. In the second step, the desirable output vectorv_j is employed as input, along with an additional input vector$z_j = (z_{1j}, ..., z_{Sj}) \geq 0$, to produce the desirable output vector $y_j = (y_{1j}, ..., y_{Tj}) \geq 0$. It is important to recognize that the desirable intermediate measure, v_j, plays a dual role in the two-stage process. It serves as both an output in the initial stage and an input in the subsequent stage simultaneously. Furthermore, each stage of the system incorporates a distinct set of contextual variables, denoted as $c_{nj} \forall n, j$, in addition to the specific inputs and outputs.

3.1 Efficiency evaluation of sub-DMUs

Based on the assumption that stage 1 (i.e. the leader) holds greater significance than stage 2 (i.e. the follower), this section introduces a method for evaluating the efficiency of sub-sections within decision-making units.

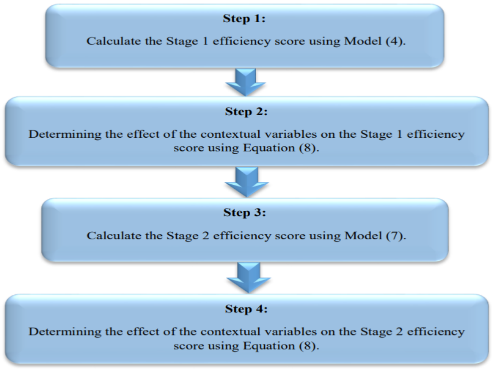

Figure 1: The proposed approach algorithm.

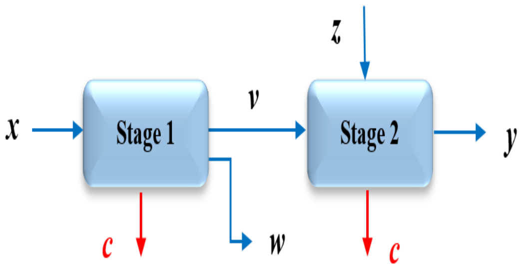

Figure 2: The configuration of a two-stage process.

Stage 1

In accordance with Shephard's weak disposability assumption [19], the production possibility set of Stage 1 can be mathematically expressed under the assumption of variable returns to scale as follows

$$\begin{aligned}
&T_{Stage1} = \{(x,v,w): \\
&\sum_{j=1}^{J} \xi_j x_ij \leq x_{io}, i=1,,I, \\
&\sum_{j=1}^{J} \delta_j \xi_j v_r j \geq v_{ro}, r=1,,R, \\
&\sum_{j=1}^{J} \delta_j \xi_j w_m j = w_{mo}, m=1,,M, \\
&\sum_{j=1}^{J} \xi_j = 1, \\
&0 \leq \delta_j \leq 1, \xi_j \geq 0, j=1,,J\}
\end{aligned} \tag{3.1}$$

The δ_j in the production technology set represent a contraction coefficient assigned to each DMU, as postulated by Kuosmanen [10], resulting in the nonlinearity of the technology set. When considering $\xi_j = \tau_j + \kappa_j$ as a composite of $\tau_j = \delta_j \xi_j$ and $\kappa_j = (1-\delta_j)\xi_j$, it can be mathematically expressed in the following linear form

$$\begin{aligned}
&T_{Stage1}^{New} = \{(x,v,w): \\
&\sum_{j=1}^{J} (\tau_j + _j) x_{ij} \leq x_{io}, i=1,,I, \\
&\sum_{j=1}^{J} \tau_j v_r j \geq v_{ro}, r=1,,R, \\
&\sum_{j=1}^{J} \tau_j w_m j = w_{mo}, m=1,,M, \\
&\sum_{j=1}^{J} (\tau_j + \kappa_j) = 1, \\
&\tau_j, \mu_j \geq 0, j=1,,J\}.
\end{aligned} \tag{3.2}$$

To evaluate the efficiency score of Stage 1 for DMU_o, the following model is utilized to minimize the level of undesirable outputs

$$\begin{aligned}
&\min \ \varphi_0 \\
&s.t. \\
&\sum_{j=1}^{J} (\tau_j + _j) x_{ij} \leq x_{io}, i=1,,I, \\
&\sum_{j=1}^{J} \tau_j v_r j \geq v_{ro}, r=1,,R, \\
&\sum_{j=1}^{J} \tau_j w_m j = \varphi_o w_{mo}, m=1,,M, \\
&\sum_{j=1}^{J} (\tau_j + \kappa_j) = 1, \\
&\tau_j, \mu_j \geq 0, j=1,,J, \\
&\varphi_0 \text{ is free.}
\end{aligned} \tag{3.3}$$

Therefore, the linear programming model or the dual form of Model (3.3) computes the efficiency of Stage 1 for DMU_o, as depicted below:

$$\begin{aligned}
&E_o^{\star Stage1} = \max \sum_{r=1}^{R} f_r v_r o + u_o^{(1)} \\
&s.t. \\
&\sum_{m=1}^{M} g_m w_{mo} + \sum_{i=1}^{I} h_i x_{io} = 1, \\
&\sum_{i=1}^{I} h_i x_{ij} + u_0^{(1)} \leq 0, j = 1,, J, \\
&\sum_{r=1}^{R} f_r v_{rj} - \sum_{m=1}^{M} g_m w_{mj} - \sum_{i=1}^{I} h_i x_i j \\
&+ u_0^{(1)} \leq 0, j = 1,, J, \\
&f_r^{(1)}, h_i \geq 0, g_m \text{ are free in sign } \forall r, i, m.
\end{aligned} \tag{3.4}$$

In the aforementioned model, the optimal objective value $0 < E_o^{\star Stage1} \leq 1$ indicates the optimal efficiency score of Stage 1, whereas $h_i^\star, f_r^\star, g_m^\star \ \forall i, r, m$ represent the corresponding optimal weights.

Definition 3.1. *The first sub-section of DMU_o is said to be efficient if and only if $E_o^{\star Stage} = 1$; Otherwise, it is considered inefficient.*

Stage 2

The production technology set for Stage 2 can be expressed based on the structure depicted in Figure 1 as follows:

$$\begin{aligned}
&T_{Stage2} = \{(v, z, y) : \\
&\sum_{j=1}^{J} \mu_j v_r j \geq v_{ro}, r = 1,, R, \\
&\sum_{j=1}^{J} \mu_j z_s j \leq z_{so}, s = 1,, S, \\
&\sum_{j=1}^{J} \mu_j y_t j \geq y_{to}, t = 1,, T, \\
&\sum_{j=1}^{J} \mu_j = 1, \\
&\mu_j \geq 0, j = 1,, J\}
\end{aligned} \tag{3.5}$$

In the above technology, μ_j is an unknown variable. It is noteworthy that the two stages of this technology are interconnected through an intermediate measure, denoted as v_j which is deemed desirable. As v_j is a useful indicator that can be employed by the system, it is reasonable to increase its value system-wide.

To assess the efficiency of Stage 2, we retain the efficiency score achieved in Stage 1 ($E_o^{\star Stage1}$) while striving to enhance the desirable final outputs. Furthermore, we assume that the weights assigned to the desirable intermediate measure v_j in Stage 1 correspond to those utilized in Stage 2. Consequently, we ascertain the efficiency score of Stage 2 for DMU_o by employing the following linear programming model:

$$\begin{aligned}
&\max \beta_0 \\
&s.t. \\
&\sum_{j=1}^{J} \mu_j v_r j \geq v_{ro}, r = 1,, R, \\
&\sum_{j=1}^{J} \mu_j z_s j \leq z_{so}, s = 1,, S, \\
&\sum_{j=1}^{J} \mu_j y_t j \geq beta_0 y_{to}, t = 1,, T, \\
&\sum_{j=1}^{J} \mu_j = 1, \\
&beta_0 \geq 1, \mu_j \geq 0, j = 1,, J
\end{aligned} \tag{3.6}$$

Likewise, the dual form of Model (6) calculates the efficiency of Stage 2 for

$$\begin{aligned}
&E_o^{\star Stage2} = \min \sum_{s=1}^{S} k_s z_s o + u_o^{(2)} \\
&s.t. \\
&\sum_{t=1}^{T} p_t y_{to} + \sum_{r=1}^{R} f_r v_{ro} = 1, \\
&\sum_{s=1}^{S} k_s z_{sj} - \sum_{t=1}^{T} p_t y_{ti} - \sum_{r=1}^{R} f_r v_r j \\
&+ u_0^{(2)} \geq 0, j = 1, ..., J \\
&\sum_{r=1}^{R} f_r v_{rj} - \sum_{m=1}^{M} g_m w_{mj} - \sum_{i=1}^{I} h_i x_{ij} \\
&+ u_0^{(1)} \leq 0, j = 1,, J,
\end{aligned} \tag{3.7}$$

$$\sum_{i=1}^{I} h_i x_{ij} + u_0^{(1)} \leq 0 j = 1, , J,$$
$$\sum_{r=1}^{R} f_r v_{ro} - \sum_{i=1}^{I} h_i x_{io} + u_0^{(1)} = E_o^{\star Stage1}$$
$$f_r, k_s, p_t, h_i \geq 0, \forall r, s, t, i,$$
$$g_m \text{is free in sign } \forall m.$$

In Model (7), the optimal objective value $E_o^{\star Stage2} \geq 1$ indicates the optimal efficiency score of Stage 1, whereas $h_i^\star, f_r^\star, g_m^\star, p_t^\star, k_s^\star \forall i, r, m, t, s$ represent the corresponding optimal weights. It is noteworthy that the fifth constraint guarantees the maintenance of efficiency during the first stage.

Definition 3.2. *The second sub-section of DMU_o is said to be efficient if and only if $E_o^{\star Stage2} = 1$; Otherwise, it is considered inefficient.*

3.2 Estimation of the effects of contextual variables

After calculating the efficiency of each sub-section of DMU_o, we employ the following regression model to examine the influence of contextual variables on the aforementioned efficiency:

$$\log_{10}(E_j^{\star Sub-DMU}) = \beta_0 + \sum_{n=1}^{N} \beta_n c_{nj} + \epsilon_o; j = 1, , J \tag{3.8}$$

In the aforementioned regression model, the intercept and error terms are represented by $\beta_n : n = 1, , N$ and ϵ_o, respectively. The coefficients β_n are associated with the contextual variables $(c_{nj} : n = 1, , N)$, which can have positive or negative values. The positive or negative coefficients of β_n in Equation 3.8 indicate the direct or inverse effect of the contextual variable on the performance of the unit under evaluation. This model performs regression analysis by regressing the base-10 logarithm of the efficiency of each component of $DMU_j : j = 1, , J$ on the contextual variables.

In practical application, the efficiency score of each sub-section is measured by considering its inputs and outputs, and any contextual factors that may affect performance are removed from the estimated efficiency. The resulting modified efficiency score is then used to determine the efficiency of subsequent stages. The modified efficiencies of both stages are denoted as $\tilde{E}_o^{\star Stage1}$, and $\tilde{E}_o^{\star Stage2}$. It should be noted that in Model (7), the efficiency of Stage 2 of DMU_o is optimized while keeping the modified efficiency of Stage 1 ($\tilde{E}_o^{\star Stage1}$) unchanged.

4 An application for airports assessment

The proposed methodology is implemented using a dataset consisting of 39 Spanish airports, as documented by Lozano et al. [14]. Figure 3 presents a graphical representation illustrating the configuration of each airport. As depicted in

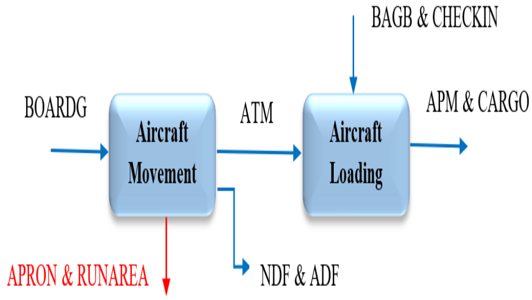

Figure 3: The airport structure.

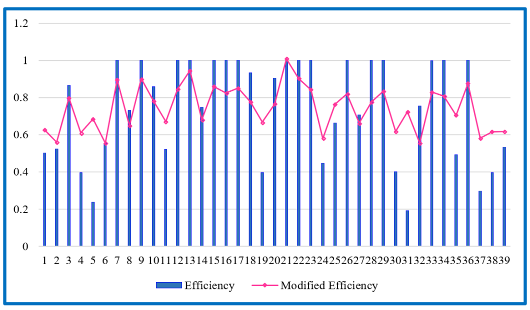

Figure 4: Stage 1 efficiency changes and its modified efficiency.

Figure 3, the structure of each of the 39 airports is divided into two distinct stages: a primary stage comprising the aircraft movement process (Stage

1), and a secondary stage encompassing the aircraft loading process (Stage 2). The corresponding inputs, outputs, and contextual variables of both stages are listed below.

- **Stage 1: Aircraft movement process**
  Input:
  Number of boarding gates (BOARDG).
  Desirable output:
  Aircraft Traffic Movement (ATM).
  Undesirable output:
  Number of delayed flights (NDF), Accumulated flight delays (ADF).
  Contextual variables:
  Apron capacity (APROAN), Total runway aria (RUNAREA).

- **Stage 2: Aircraft loading process**
  Input:
  Number of baggage belts (BAGB), Number of check-in counters (CHECKIN).
  Desirable output:
  Annual passenger movements (APM), Cargo handled (CARGO).

It is noteworthy that Air Traffic Management (ATM) serves as an intermediate measure, playing both input and output roles in the introduced structure.
In the following section, the dataset, comprising inputs, outputs, and contextual variables, is collected and presented in Table 1.

4.1 Efficiency results

In the initial phase, the efficiency of the first component of 39 airports was determined using a single input factor (BORDAGE), one desirable output factor (ATM), and two undesirable output factors (NDF, ADF) through the utilization of Model (4). It is important to note that the aircraft movement process was designated as the leader in Stage 1, while the aircraft loading process was regarded as the follower in Stage 2. Subsequently, in the second step, the logarithm in base 10 of the efficiency scores of the first component for all 39 airports was regressed on contextual variables (APRON, RUNAREA), as specified in the following model:

$$Log_10(E^{*Stage1}) = \beta_0 + \beta_1(''APRON'') \\ + \beta_2(''RUNAREA'') + \epsilon_o$$

According to the findings of the aforementioned regression model, the influence of the contextual variables mentioned was found to be eliminated from the efficiency value of the first component. As a result, the modified efficiencies obtained were retained and utilized to assess the performance of the second component in 39 airports, using Model (7). The outcomes of these procedures have been gathered in Tables 2 and 3, which will be explained in Table 2.
Based on the findings presented in Table 2, it becomes evident that 15 airports demonstrate efficiency in the aircraft movement process (stage 1). Nevertheless, the revised performance values in the third column signify that there have been alterations in the performance of these units. Specifically, certain units have exhibited an increase in efficiency, whereas others have experienced a decrease following the refinement. The modified efficiency value reflects the actual performance of the first component in airports. In essence, a higher value of modified efficiency signifies a superior performance of the first component in comparison to other airports, whereas a lower value indicates poorer performance.
To illustrate, among the efficient airports, only the 21st airport has demonstrated enhanced performance in its first component, indicating that it outperforms the first component of other airports. On the other hand, some efficient airports have experienced a decline in efficiency after the correction, thereby exhibiting the weakest performance among the efficient units. It is worth noting that changes in efficiency, whether an increase or decrease, do not solely affect efficient units; the efficiency of inefficient units is also impacted.
After the refinement process, the performance of the first components of Airports 1, 2, 4, 5, 6, 11, 19, 24, 25, 30, 31, 35, 37, 38, and 39 has increased, while that of Airports 3, 8, 10, 14, 18, 20, 27, 32, and 33 has decreased. These fluctuations in the efficiency values indicate improvements or deteriorations in the actual performance of these units,

Table 1: The inputs, outputs and contextual variables.

Airport	Boardg	Ndf	Adf	Atm	Bagb	Chechin	APM	Cargo	Apron	Runarea
Airport	x	w_1	w_2	v	z_1	z_2	y_1	y_2	c_1	c_2
1	4	1218	23783.4	17.719	3	10	1174.97	283.571	5	87300
2	2	58	1376.5	2.113	1	4	19.254	8.924	2	162000
3	16	7642	142446	81.097	9	42	9578.3	5982.313	31	135000
4	5	1114	20149.1	18.28	4	17	1024.3	21.322	15	144000
5	9	1310	23893.5	18.371	3	11	1530.25	139.465	7	99000
6	2	137	2365.4	4.033	1	4	81.01	0	1	171000
7	65	33036	645925	321.693	19	143	30272.1	103996.49	121	475000
8	12	4592	80848.2	61.682	7	36	4172.9	3178.758	21	207000
9	1	14	254.4	9.604	0	1	22.23	0	23	62100
10	2	27	641.6	4.775	1	5	195.425	171.717	3	37500
11	10	3920	72179.7	44.552	8	34	4492	2722.661	34	153000
12	7	4992	100306	49.927	3	18	5510.97	184.127	17	108000
13	38	7463	136381	116.252	19	86	10212.1	33695.248	55	139500
14	3	951	17868.8	19.279	3	12	1422.01	66.889	11	134550
15	12	6193	152840	57.233	8	48	4647.36	3928.387	25	126000
16	5	1174	19292.2	50.551	3	13	1303.82	90.428	9	103500
17	2	17	420.7	3.393	1	5	41.89	7.863	3	45000
18	5	423	8286	20.109	2	13	1151.36	1277.264	5	99000
19	16	5104	101686	53.375	8	49	5438.18	5429.589	24	108000
20	2	442	7191.5	5.705	1	3	123.183	15.979	5	94500
21	230	52526	908361	469.746	53	484	50846.5	329186.63	263	917000
22	30	15548	277664	119.821	16	85	12813.5	4800.271	43	144000
23	2	218	2979.6	10.959	1	4	314.643	386.34	5	64260
24	5	1344	24103.1	19.339	4	18	1876.26	2.73	5	138000
25	68	26038	501486	193.379	16	204	22832.9	21395.791	86	295650
26	2	666	11691.8	12.971	1	4	434.477	52.942	7	99315
27	5	943	18240.8	26.676	3	8	1278.07	119.848	5	110475
28	2	427	6626.1	12.45	2	4	60.103	0	6	150000
29	3	713	11184	12.282	2	6	403.191	63.791	6	78930
30	5	1004	17842	19.198	2	8	856.606	37.482	8	104400
31	12	2007	34322.3	21.945	5	19	1917.47	2418.798	16	144000
32	3	1095	19547.6	14.584	2	6	594.952	21438.894	12	302310
33	10	2567	51084.9	65.067	6	42	4392.15	6102.264	23	151200
34	16	1783	32637	67.8	5	37	4236.62	20781.674	16	153000
35	22	5254	110819	60.779	14	87	8251.99	8567.093	44	144000
36	18	4998	102719	96.795	8	42	5779.34	13325.799	35	144000
37	5	843	14760.6	13.002	2	8	479.689	34.65	7	180000
38	6	1535	25593.6	17.934	3	12	1278.76	1481.939	8	108000
39	3	669	11585.8	12.225	2	7	67.818	34989.727	18	157000

respectively.

Therefore, based on the aforementioned findings, it can be inferred that Airport 21 exhibits the best performance in the first component, while Airport 32 displays the weakest performance.

The efficiency values of the second component for the 39 airports are presented in the fourth column of Table 2. The results reveal that certain airports demonstrate efficient or inefficient performance in both components, while others exhibit efficiency in only one component. Specifically, airports 7, 9, 12, 16, and 21 demonstrate efficiency in both components, whereas airports 32 and 39 are only efficient in their second component. Notably, the second airport obtains the lowest efficiency value among all the airports' sec-

Table 2: The results of efficiencies.

Airport	$E^{\star Stage1}$	$\tilde{E}^{\star Stage1}$	$E^{\star Stage2}$
1	0.5016	0.6242	1.9163
2	0.523	0.5591	9.3751
3	0.8658	0.7983	1.0717
4	0.3946	0.6093	3.0418
5	0.2358	0.6835	1.8993
6	0.5537	0.5547	4.8587
7	1	0.8949	1
8	0.7301	0.6476	1.5862
9	1	0.9012	1
10	0.8577	0.78	4.3771
11	0.5216	0.6698	1.9096
12	1	0.8457	1
13	1	0.9442	1.7375
14	0.7469	0.679	2.0243
15	1	0.8575	2.1138
16	1	0.8232	1
17	1	0.8497	6.849
18	0.9318	0.7763	1.9877
19	0.3948	0.6668	2.1426
20	0.9043	0.7653	2.7863
21	1	1.0086	1
22	1	0.9018	1.4659
23	1	0.8411	1.6948
24	0.4466	0.5798	2.6732
25	0.6625	0.7635	1.1225
26	1	0.82	1.4249
27	0.706	0.662	1.1318
28	1	0.777	1.5936
29	1	0.8328	2.0596
30	0.4015	0.6171	1.5949
31	0.1918	0.7231	2.4383
32	0.7552	0.554	1
33	0.9968	0.8294	1.6151
34	1	0.8071	1.3176
35	0.491	0.7049	2.3242
36	1	0.8758	1.1471
37	0.2979	0.5805	2.4284
38	0.3954	0.6148	2.1257
39	0.534	0.6169	1

ond components.
Figure 2 provides a detailed illustration of the modifications that have occurred in the efficiency values of the initial component. These changes vary in magnitude, with airports 6 and 31 experiencing the least and most significant changes, respectively. It is worth noting that the efficiency of the first component has increased for both airports after the modification. Additionally, the 28th airport exhibits the highest reduction in efficiency among all the airports.

4.2 Regression analysis

Table 3 displays the coefficients related to the efficiency value and their corresponding contextual variables. Positive coefficients indicate a direct relationship between efficiency and the contextual variable, while negative coefficients sug-

Table 3: Regression results of logarithm of the stage 1 efficiency on the contextual variables.

Variables	Coefficients	Standard Error	t- Stat	P-value
Intercept	-0.12468394	0.05988725	-2.08197802	0.04451321
APRON	0.00324548	0.00185813	1.74663382	0.08922918
RUNAREA	-0.00000079	0.00000058	-1.35258101	0.18462467
R-Square	0.0904	0.0904	0.0904	0.0904
Standard Error	0.1938	0.1938	0.1938	0.1938

gest an inverse relationship. The study's findings are noteworthy in that they have revealed a statistically significant positive association between the airport's APRON and efficiency. Additionally, the study has identified a negative correlation between the airport's RUNAREA and efficiency, which suggests that increasing the runway area may result in decreased efficiency. The coefficient of 0.00324548 for APRON denotes that a one-unit increase in this variable results in a $100 \times (e^{0.00324548} - 1) = 0.325$ change in efficiency. This coefficient indicates that the variable APRON is nearly statistically significant. Furthermore, the coefficient of -0.000000079 for RUNAREA suggests that a one-unit increase in this variable leads to a -7.910^{-6} change in efficiency. Furthermore, the R-Square value of approximately 0.09 suggests that the regression model accounts for more than 9% of the observations.

5 Conclusion

This study presents a novel two-stage methodology for evaluating two-stage network structures that consider contextual variables along with firm-specific inputs and outputs, which can have a substantial impact on firm performance. The first stage involves estimating the efficiency values of sub-sections within the aforementioned two-stage network structure using the non-cooperative method. Subsequently, to enhance the network-specific relative efficiency, the estimated efficiency scores were regressed against the contextual variables using the OLS method. The proposed theoretical framework was then applied to analyze the performance of 39 Spanish airports. The results of our study indicate that the variables APRON and RUNAREA have a significant impact on airport efficiency. Specifically, we found a positive association between APRON and airport efficiency, while RUNAREA exhibited a negative association.

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