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Research Article

A New Practical Common Weights Approach to Rank Decision-Making Units in Data Envelopment Analysis

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Abstract

There exist several approaches for deriving a common set of weights in data envelopment analysis (DEA) literature. However, most of these approaches are based on complicated models. In this paper, a new practical approach is proposed to provide a common set of weights. The results of the new approach are compared with some of the existing models through several numerical examples.

Keywords : Data envelopment analysis; Common set of weights; Ranking; Efficiency; Multiple inputs and outputs

1 Introduction

DAta envelopment analysis (DEA) is a non-parametric technique to evaluate the efficiency and rank a set of DMUs that are comparable in terms of multiple inputs and outputs. DEA introduced by Charnes et al.[3] and developed rapidly so that this approach is one of the powerful tools in operations research and management science. Some of the application areas that DEA has been used more frequently are banking, health care, agriculture and farm, transportation, and education [20].
The efficiency measure of a DMU is defined as the ratio of a weighted sum of its outputs to a weighted sum of its inputs. The traditional DEA models allow each DMU to choose a set of favorable weights for inputs and outputs so that its maximum efficiency is attained. As a strength, this approach provides an opportunity for DMUs to show their performance at the highest possible level. However, this is a weakness as well because evaluation based on different weights may be unacceptable for all decision-makers. Further, it can decrease the discrimination between DMUs because several DMUs may be identified as efficient. The developments in the DEA context for dealing with the problems caused by dissimilar weights can be divided into three parts: cross efficiency [8, 24], weights restrictions [4, 29], and common weights [6, 22]. In cross efficiency, in addition to evaluation by its favorable weights, each DMU also is evaluated by the favorable weights of other DMUs, and then an average score is assigned to it. Weights restriction is an auxiliary approach that can be used to prevent an unreasonable weights

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assignment, since some DMUs may be efficient with an unrealistic set of weights. The common weights approach was developed to evaluate the efficiency of DMUs in a common framework. The topic that we want to pursue here is the latter, i.e., the common weights approach.

Common weights approach first proposed by Cook et al. [6] and Roll et al. [22] in which they imposed some bounds on weights and tried to reduce the variations of weights assigned to inputs and outputs among various DMUs. Sinuany-Stern et al. [26] and Sinuany-Stern and Friedman [27] proposed some discriminant analysis approaches to find a common set of weights to rank DMUs. Friedman and Sinuany-Stern [12] proposed a canonical correlation analysis approach in which a common set of weights is found for all DMUs in such a way that the coefficient of correlation between the vector of the weighted sum of the inputs and the vector of the weighted sum of the outputs of all DMUs is maximized. Kao and Hung [15] proposed a compromise solution approach to provide a common set of weights for all DMUs. They used the DEA efficiency results as an ideal solution and tried to find a common set of weights such that the new efficiency vector is as close as possible to this ideal solution. Cook and Zhu [7] proposed a nonlinear programming common weights approach for analysis of power plant efficiency for clustered power units. Jahanshahloo et al. [13] proposed a nonlinear model to provide a common set of weights to rank DMUs. Note that nonlinear models need some computational considerations, such as searching for a global optimal solution and dealing with alternative optimal solutions. Liu and Peng [19] provided a comprehensive analysis to rank efficient DMUs with common weights in which the efficiency score 1 is set as a benchmark level to find a set of weights that shows the efficiency of the group of efficient DMUs as close as possible to this benchmark level. Jahanshahloo et al. [14] proposed an approach to obtain a common set of weights by comparing the efficient DMUs with an ideal line and a special line. Zohrehbandian et al. [36] proposed an improvement to models of Kao and Hung [15] in such a way that, in some cases, the models are linear. Wang et al. [32] proposed a new approach based on regression analysis to find a common set of weights in which the efficiencies determined by common weights are fitted to the traditional CCR efficiencies. Chiang et al. [5] proposed a new approach to obtain a common set of weights using a multiple objective fractional linear programming model, including the efficiency ratios as objectives. They converted the proposed multiple objective fractional linear programming model into a linear one using a separation vector. Ramón et al. [23] proposed an approach for deriving a set of common weights that is the most similar to the profiles of weights of the efficient DMUs. Sun et al. [28], considering ideal and anti-ideal DMU, proposed two models for generating common weights in order to compare the efficiency scores of DMUs from two different perspectives. Khalili-Damghani and Fadaei [18] proposed an alternative approach related to the use of ideal and anti-ideal DMU to producing a common set of weights. Ramezani-Tarkhorani et al. [21] demonstrated the existence of alternative optimal solutions in Liu and Peng's approach [19] and proposed a lexicographic model to find a unique optimal solution. Wu et al. [34] introduced the concept of satisfaction degree of a DMU related to a common set of weights and proposed a nonlinear model to find a common set of weights that maximizes the minimum level of the satisfaction degree of all DMUs. They also proposed two algorithms to find a unique optimal solution for the nonlinear model. Yekta et al. [35] proposed a model that, besides providing a common set of positive weights to evaluate DMUs, tried to reduce weights dissimilarity.

Some other researchers used common weights to identify the most efficient DMU. Among them, we can refer to Karsak and Ahiska [16, 17], Foroughi [9, 10, 11], Wang and Jiang [33], and Toloo [30, 31].

One of the other approaches that have been developed to rank DMUs is the super-efficiency method introduced by Andersen and Petersen [2]. This approach evaluates each DMU based on a reduced production possibility set obtained by removing the DMU under assessment from the set of DMUs. Note that this approach does not consider the problems related to dissimilar weights.

In this paper, we propose a new approach in which the common set of weights is obtained using a linear programming model. The purpose of the model is to find a common weights hyperplane that is as close as possible to the parts of the efficient frontier that include the projection points of DMUs. In spite of the simplicity, the model contains significant information that helps to produce a reasonable common framework for evaluation of the DMUs.

The rest of the paper is organized as follows. In the next section, we present some preliminaries. Section 3 presents the new common weights approach. In section 4, we provide some numerical examples and discussions related to the applicability of the new approach. Section 5 concludes the paper.

2 Preliminaries

Consider a set of n DMUs to be evaluated. Suppose that each DMU uses m inputs to produce s outputs. Let the input and output vectors of DMU j $(j = 1, \ldots, n)$ be $(x_{1j}, \ldots, x_{mj})^t$ and $(y_{1j}, \ldots, y_{sj})^t$, respectively. The CCR model for measuring the efficiency of DMU o $(o = 1, \ldots, n)$, as proposed by Chernes et al. [3], is as follows:

$$
\begin{array}{lll}
\max & \theta_o = \dfrac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{r=1}^{s} v_i x_{io}} & \\
s.t. & \dfrac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1 & j = 1, \ldots, n \\
& v_i \geq 0 & i = 1, \ldots, m \\
& u_r \geq 0 & r = 1, \ldots, s
\end{array}
\tag{2.1}
$$

In this model, v_i $(i = 1, \ldots, m)$ and u_r $(r = 1, \ldots, s)$ are the weights of inputs and outputs, respectively. The optimal value of the objective function is called the CCR efficiency of DMU o, which is obviously less than or equal to 1. DMU o is known as efficient if its efficiency value is 1. Otherwise, it is inefficient. The fractional programming problem (2.1) can be converted to the following linear program, namely, the input-oriented CCR model:

$$
\begin{array}{lll}
\max & \theta_o = \sum_{r=1}^{s} u_r y_{ro} & \\
s.t. & \sum_{i=1}^{m} v_i x_{io} = 1 & \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 & j = 1, \ldots, n \\
& v_i \geq 0 & i = 1, \ldots, m \\
& u_r \geq 0 & r = 1, \ldots, s
\end{array}
\tag{2.2}
$$

This model is solved for each DMU separately, and the optimal weights may be different across DMUs. Indeed, this model finds a set of favorable weights that are specific to DMU under evaluation and often these weights exhibit worse efficiency scores for other DMUs than by their own optimal weights when they are under evaluation. This may cause some discordance among decision-makers since the evaluation is not based on a common framework. Another issue that arises from choosing weights individually by DMUs is that several DMUs may become efficient. In this situation, we cannot provide a ranking of DMUs. These issues create motivation for searching for a common set of weights.

Wang et al. [32] proposed two common weights models based on regression analysis as follow:

$$
\begin{array}{ll}
\min & \sum_{j=1}^{n} \left(\theta_j^* - \sum_{r=1}^{s} u_r y_{rj} / \sum_{i=1}^{m} v_i x_{ij} \right)^2 \\
s.t. & v_i \geq 0 \quad i = 1, \ldots, m \\
& u_r \geq 0 \quad r = 1, \ldots, s
\end{array}
\tag{2.3}
$$

$$
\begin{array}{ll}
\min & \sum_{j=1}^{n} \left(\sum_{r=1}^{s} u_r y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i x_{ij} \right)^2 \\
s.t. & \sum_{r=1}^{s} u_r \left(\sum_{j=1}^{n} y_{rj} \right) + \sum_{i=1}^{m} v_i \left(\sum_{j=1}^{n} x_{ij} \right) = n \\
& v_i \geq 0 \quad i = 1, \ldots, m \\
& u_r \geq 0 \quad r = 1, \ldots, s
\end{array}
\tag{2.4}
$$

Here, θ_j^* is the CCR efficiency of DMU j $(j = 1, \ldots, n)$. The common weights efficiency is obtained for each DMU as $\bar{\theta}_j = \sum_{r=1}^{s} u_r^* y_{rj} / \sum_{i=1}^{m} v_i^* x_{ij}$, where $(v_1^*, \ldots, v_m^*; u_1^*, \ldots, u_s^*)$ is an optimal solution of the corresponding model.

Model (2.3) is a direct fitting to the CCR efficiencies, while model (2.4) is an alternative regression model to indirect fitting. The optimal solutions of these models are not necessarily identical. Note that, these models are nonlinear and need special optimization software packages.

In the next section, we present our common weights approach. The proposed model is a linear programming model that tries to minimize the difference in common weights efficiencies and CCR efficiencies of all DMUs. The new model

Table 1: Data and results of the new common weights approach (Example 4.1)

DMU	x_1	x_2	y	CCR efficiency	CW efficiency	Rank
A	2	5	1	1.0000	0.6617	7
B	3	2.5	1	1.0000	1.0409	1
C	4.2	3.2	1	0.7702	0.7895	4
D	5	2.2	1	1.0000	0.9197	2
E	4	4	1	0.7143	0.6873	6
F	5.5	3	1	0.7712	0.7421	5
G	4	2.6	1	0.9219	0.9180	3

Table 2: Data for 12 DMUs, two inputs (x_1 and x_2), and four outputs (y_1, y_2, y_3, and y_4) along with CCR efficiencies (Example 4.2)

DMU	x_1	x_2	y_1	y_2	y_3	y_4	CCR Efficiency.
1	17.02	5.0	42	45.3	14.2	30.1	1
2	16.46	4.5	39	40.1	13	29.8	1
3	11.76	6.0	26	39.6	13.8	24.5	0.9824
4	10.52	4.0	22	36.0	11.3	25.0	1
5	9.50	3.8	21	34.2	12	20.4	1
6	4.79	5.4	10	20.1	5	16.5	1
7	6.21	6.2	14	26.5	7	19.7	1
8	11.12	6.0	25	35.9	9	24.7	0.9614
9	3.67	8.0	4	17.4	0.1	18.1	1
10	8.93	7.0	16	34.3	6.5	20.6	0.9536
11	17.74	7.1	43	45.6	14	31.1	0.9831
12	14.85	6.2	27	38.7	13.8	25.4	0.8012

Table 3: The results of different models and the new approach

DMU	Agg.[a]	rank	Benev.[b]	rank	Model (2.3)	rank	Model (2.4)	rank	CWkn[c]	rank
1	0.8483	2	0.9550	5	0.9866	5	1.0071	4	0.9922	6
2	0.8391	4	0.9355	6	0.9708	7	0.9784	8	0.9547	8
3	0.7767	5	0.9245	8	0.9633	8	0.9798	7	1.0080	5
4	0.8441	3	0.9812	2	1.0264	3	1.0009	5	1.0248	4
5	0.8668	1	0.9770	3	1.0085	4	1.0100	3	1.0588	2
6	0.7273	8	0.9556	4	1.0655	2	1.0684	2	1.0355	3
7	0.7581	6	0.9879	1	1.0818	1	1.0973	1	1.0814	1
8	0.7243	9	0.9308	7	0.9862	6	0.9974	6	0.9820	7
9	0.5638	12	0.7487	12	0.9390	10	0.9048	10	0.7845	12
10	0.6178	11	0.8147	10	0.8752	11	0.8858	11	0.8949	10
11	0.7472	7	0.9077	9	0.9460	9	0.9769	9	0.9516	9
12	0.6675	10	0.7734	11	0.8042	12	0.8076	12	0.8260	11

[a] Aggressive model
[b] Benevolent model
[c] New common weights model

aims to produce a common weights hyperplane that nearly passes through a part of the production possibility set that contains the more number of the projection points of DMUs.

Table 4: The correlation coefficient between the results of the new common weights approach and other models

	Aggressive	Benevolent	Model (2.3)	Model (2.4)	CW
Aggressive	1	0.7273	0.5874	0.6014	0.6713
Benevolent		1	0.9510	0.9510	0.9301
Model (2.3)			1	0.9720	0.9161
Model (2.4)				1	0.9371
CW					1

3 The new common weights approach

Let the set of efficient and inefficient DMUs identified by model (2.2) be E and E^c, respectively, with $E^c \neq \varnothing$. We propose the following minimax model to determine a common set of weights:

$$\begin{aligned} \min \quad & \Delta \\ s.t. \quad & \Delta \geq \sum_{r=1}^{s} u_r y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i x_{ij} \quad j \in E \\ & \Delta \geq \theta_j^* \sum_{i=1}^{m} v_i x_{ij} - \sum_{r=1}^{s} u_r y_{rj} \quad j \in E^c \\ & v_i \geq \varepsilon \quad i = 1, \dots, m \\ & u_r \geq \varepsilon \quad r = 1, \dots, s \end{aligned} \tag{3.5}$$

Here, ε is a small positive non-Archimedean element imposed to provide positive weights. Based on an optimal solution $(v_1^*, \dots, v_m^*; u_1^*, \dots, u_s^*)$ of the model, the common weights efficiency of DMU o $(o = 1, \dots, n)$ is obtained as $\hat{\theta}_o = \sum_{r=1}^{s} u_r^* y_{ro} / \sum_{i=1}^{m} v_i^* x_{io}$.
Note that $(\theta_j^* x_{1j}, \dots, \theta_j^* x_{mj}; y_{1j}, \dots, y_{sj})^t$ is the radial projection of DMU j onto the efficient frontier of the CCR model. However, for an efficient DMU j, $\theta_j^* = 1$, and this projection point is just itself. Defining $\hat{x}_{ij} = \theta_j^* x_{ij}$ $(i = 1, \dots, m)$ for all j, we can rewrite model (3.5) as the following model:

$$\begin{aligned} \min \quad & \Delta \\ s.t. \quad & \Delta \geq \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i \hat{x}_{ij} \quad j \in E \\ & \Delta \geq \sum_{i=1}^{m} v_i \hat{x}_{ij} - \sum_{r=1}^{s} u_r y_{rj} \quad j \in E^c \\ & v_i \geq \varepsilon \quad i = 1, \dots, m \\ & u_r \geq \varepsilon \quad r = 1, \dots, s \end{aligned} \tag{3.6}$$

The first set of constraints related to the efficient DMUs allows some efficient DMUs to have an efficiency score greater than or equal to one. This is a result of the following theorems.

Theorem 3.1. *The optimal objective function value of model (3.6) is nonnegative.*

Proof. Suppose, on the contrary, that the optimal value of Δ, i.e., Δ^*, is negative. Hence, we must have:

$$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} < 0 \, (j \in E)$$
$$\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* \hat{x}_{ij} > 0 \, (j \in E^c).$$

We show that this is a contradiction. Suppose that $j \in E^c$. The dual problem of model (2.2) for evaluating DMU j is as follows:

$$\begin{aligned} \min \quad & \theta_j \\ s.t. \quad & \sum_{k=1}^{n} \lambda_k x_{ik} + s_i^- = \theta_j x_{ij} \quad i = 1, \dots, m \\ & \sum_{k=1}^{n} \lambda_k y_{rk} - s_r^+ = y_{rj} \quad r = 1, \dots, s \\ & \lambda_k \geq 0 \quad k = 1, \dots, n \\ & s_i^- \geq 0 \quad i = 1, \dots, m \\ & s_r^+ \geq 0 \quad r = 1, \dots, s \end{aligned} \tag{3.7}$$

As we know, at optimality of this model for $k \in E^c$ we have $\lambda_k^* = 0$. Hence we have:

$$\sum_{k \in E} \lambda_k^* x_{ik} + s_i^{-*} = \theta_j^* x_{ij} \; (i = 1, \dots, m)$$
$$\sum_{k \in E} \lambda_k^* y_{rk} - s_r^{+*} = y_{rj} \; (r = 1, \dots, s).$$

Considering the inequality $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* \hat{x}_{ij} >$

0 and $\hat{x}_{ij} = \theta_j^* x_{ij}$ for $j \in E^c$, we have:

$$\sum_{r=1}^{s} u_r^* (\sum_{k \in E} \lambda_k^* y_{rk} - s_r^{+*}) - \sum_{i=1}^{m} v_i^* (\sum_{k \in E} \lambda_k^* x_{ik} + s_r^{-*}) > 0.$$

Applying some settings, we have:

$$\sum_{k \in E} \lambda_k^* (\sum_{r=1}^{s} u_r^* y_{rk} - \sum_{i=1}^{m} v_i^* x_{ik}) > \sum_{r=1}^{s} u_r^* s_r^{+*} + \sum_{i=1}^{m} v_i^* s_i^{-*}.$$

The right-hand side of this inequality is nonnegative. This contradicts the inequalities at the beginning of the proof. Therefore, the optimal value of the objective function is nonnegative.

Theorem 3.2. *The common weights efficiency for at least one efficient DMU is greater than or equal to one.*

Proof. We prove that at least one of the constraints related to efficient DMUs is active at optimality in model (3.6). The dual problem of model (3.6) is as follows:

$$\begin{aligned} \max \quad & \varepsilon(\sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+) \\ s.t. \quad & \sum_{j \in E} \lambda_j \hat{x}_{ij} + s_i^- = \sum_{j \in E^c} \lambda_j \hat{x}_{ij} \quad & i = 1, \ldots, m \\ & \sum_{j \in E} \lambda_j y_{rj} - s_r^+ = \sum_{j \in E^c} \lambda_j y_{rj} \quad & r = 1, \ldots, s \\ & \sum_{j \in E} \lambda_j + \sum_{j \in E^c} \lambda_j = 1 & \\ & \lambda_j \geq 0 & j = 1, \ldots, n \\ & s_i^- \geq 0 & i = 1, \ldots, m \\ & s_r^+ \geq 0 & r = 1, \ldots, s \end{aligned} \tag{3.8}$$

Clearly, the constraint $\sum_{j \in E} \lambda_j + \sum_{j \in E^c} \lambda_j = 1$ implies that at least one λ_j is positive. Considering the first set of constraints in model (3.8), we can conclude that at least for one $j \in E^c$, λ_j is positive. Now, the second set of constraints implies that for at least one $j \in E$ also λ_j is positive. According to the complementary slackness condition, at optimality in model (3.6), at least one of the constraints related to efficient DMUs is active. This means that for some $j \in E$ we have $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* \hat{x}_{ij} \geq 0$ or equivalently $\sum_{r=1}^{s} u_r^* y_{rj} - \sum_{i=1}^{m} v_i^* x_{ij} \geq 0$. Hence, we have $\sum_{r=1}^{s} u_r^* y_{rj} / \sum_{i=1}^{m} v_i^* x_{ij} \geq 1$.

Note that Wang et al. [32] proved that in the case of a single input and a single output, their models could generate a common set of weights that perfectly fit the CCR efficiencies. The same result exists for our approach, as it is proved in the following theorem.

Theorem 3.3. *In the case of a single input and a single output, the new common weights efficiencies are just the CCR efficiencies.*

Proof. In the case of a single input and a single output, the efficient frontier is a line passing through the efficient DMUs, and the projection points all lie on this line. Hence, there are positive multipliers $\bar{v}_1$ and $\bar{u}_1$ such that $\bar{u}_1 y_{1j} - \bar{v}_1 \hat{x}_{1j} = 0$ $(j = 1, \ldots, n)$. It follows that $\Delta^* = 0$ is the optimal value of Δ. Now, we have:

$$\hat{\theta}_j = \frac{\bar{u}_1 y_{1j}}{\bar{v}_1 x_{1j}} = \frac{\bar{u}_1 y_{1j}}{\bar{v}_1 (\hat{x}_{1j} / \theta_j^*)} = \frac{\theta_j^* \bar{u}_1 y_{1j}}{\bar{v}_1 \hat{x}_{1j}} = \theta_j^*.$$

Hence, the proof is complete.

Note that we can replace ε with any positive real number without any changes in efficiency values. In fact, the optimal solutions of models (3.5) and (3.6) can be obtained easily by setting $\varepsilon = 1$ and then multiplying the solutions obtained by ε. It is to be noted that model (3.5) is not an L_∞-norm version of model (2.4), because the constraints here are as simple inequalities, while in an L_∞-norm version of model (2.4), the constraints are as absolute value inequalities or double constraints, e.g., $-\Delta \leq \sum_{r=1}^{s} u_r y_{rj} - \theta_j^* \sum_{i=1}^{m} v_i x_{ij} \leq \Delta$ $(j = 1, \ldots, n)$. Further, there is no sufficient reason to impose a normalization constraint such as $\sum_{r=1}^{s} u_r (\sum_{j=1}^{n} y_{rj}) + \sum_{i=1}^{m} v_i (\sum_{j=1}^{n} x_{ij}) = n.$

An advantage of the new approach over the other approaches to determining a common set of weights is that the new approach uses a linear programming model while other approaches

mostly use nonlinear programming.
The next section presents some numerical examples and analytical discussions on the results of the new approach and comparisons with other approaches.

4 Numerical examples

In this section, two numerical examples are provided to assess the capability of the new approach. The first example considers the treatment of the new approach with situations in which some DMUs use the proportion of the resources in contrast to the majority of the DMUs. The second example is a real data application.

Example 4.1. *Consider the data for seven DMUs with two inputs and one output as presented in table 1. Figure 1 shows DMUs in the input space. It is easily seen that DMUs A, B and D are CCR efficient. The optimal common weights by setting* $\varepsilon = 1$ *are* $v_1^* = 1.0000$, $v_2^* = 2.5481$ *and* $u^* = 9.7539$. *The common weights hyperplane in the input space is a line by equation* $x_1 + 2.5481x_2 = 9.7539$ *as depicted in the figure. The CCR efficiency scores and the new common weights efficiency scores (indicated here by CW) are respectively presented in the fifth and sixth columns of table 1. The point that is notable here, is that DMU A has been received a low efficiency score by the common weights approach. This is a usual issue, because most of the projection points of DMUs lie on the line segment BD of the efficient frontier and hence the common weights line is closer to this line segment. In fact, using simple (not double) constraints associated to the efficient DMUs in models (3.5) and (3.6) prevents DMUs like A from influencing the results. The other point that we may state is that DMU A has been used the input resources with a different proportion than the other DMUs. The favorable weights that may cause DMU A to be efficient are highly unfavorable to other DMUs. The common weights approaches usually penalize such DMUs. It is noted by Ahn et al. [1] that DMUs like A are unbalanced and should not be considered as the reference for other DMUs.*

Example 4.2. *Table 2 presents the data related to 12 flexible manufacturing systems (FMSs) with two inputs and four outputs. These data are taken from Shang and Sueyoshi [25] and have been used by some other researchers, e.g., Wang et al. [32] and Sun et al. [28]. The inputs and outputs, respectively, are:*
x_1: *Annual operating and depreciation costs (one hundred thousand dollars)*
x_2: *The floor space requirements of each specific system (thousands of square feet)*
y_1: *The improvements in qualitative benefits*
y_2: *WIP*
y_3: *Average number of tardy jobs*
y_4: *Average yield*
Seven of the DMUs (FMSs) are CCR efficient, as it is seen from the last column of table 2. The results of aggressive and benevolent cross-efficiency of Doyle and Green [8], regression models of Wang et al. [32] (models (2.3) and (2.4)) and the new common weights approach (CW) are presented in table 3 for comparison. Except for the aggressive cross efficiency model, the other models rank DMU 7 in the first place. Table 4 shows the Spearman's rank correlation coefficient between different models given by the formula $\rho_{A,B} = 1 - \frac{6\sum_{j=1}^{n}(r_j^A - r_j^B)^2}{n(n^2-1)}$, *where* r_j^A *and* r_j^B *are the ranks of DMU j with the ranking approaches A and B respectively. As it is seen, the correlation coefficient between the new common weights approach and other approaches is considerable.*

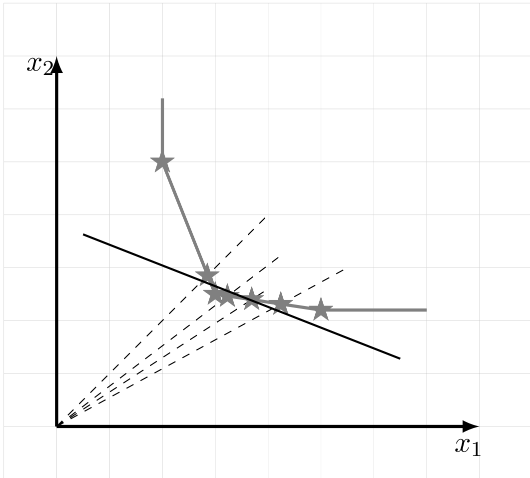

Figure 1: DMUs and the common weights hyperplane in the inputs space (star shapes represent the projection points of DMUs onto the efficient frontier)

5 Conclusion

The traditional data envelopment analysis models evaluate each decision-making unit by its own favorable weights. Sometimes the favorable weights of a DMU are highly unfavorable to other DMUs. This can make some discordance among decision-makers. Evaluating decision-making units based on a common set of weights is an important concern in data envelopment analysis. In this paper, we have proposed a new approach based on linear programming to find a common set of weights that define a hyperplane that is as close as possible to the parts of the efficient frontier where a larger number of projection points of DMUs accumulate. This is a meaningful idea because the projection points are lied on faces of the production possibility set that are used to evaluate DMUs. The validity of the new approach has been tested with numerical examples and an illustration. The examples have showed a high correlation between the results of the new approach and other existing approaches. However, the new approach needs less computational effort.

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