Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 13, No. 3, 2021 Article ID IJIM-1324, 9 pages
Research Article

Numerical Solution of a SIR Fractional Model of the Distribution of Computer Viruses Using Dickson Polynomials

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Received Date: 2019-05-15 Revised Date: 2019-11-25 Accepted Date: 2020-09-25

Abstract

In this paper, a numerical method is presented using a Dickson-based collocations method to solve a fractional model of computer virus propagation. The model presented in this paper is a system of differential equations of fraction. By using the Dickson-based collocation method and using Chebyshev's spatial points, we transform the system of deficit differential equations into a system of algebraic equations. In this way, an approximate solution can be found for the proposed model. By introducing the error functions for the expressed fractional model, the accuracy and convergence of the obtained solutions are investigated. Some of the approximate results obtained using this method is displayed in the numerical results section.

Keywords : Dickson Polynomials; Fractional model of computer virus; Collocation method.

1 Introduction

BEcause of the interconnection of different computer networks and the abundance of users of these networks, if one of the computers connected to the network is infected, the virus can easily be transmitted to other related computers and damage them. Damage incurs can be the destruction of the general or partial information of the host computer, or unauthorized access, without the user's knowledge, to sensitive information of the user system, such as bank information, user accounts, etc., or interfere with Host computer function is to occupy part of the mainframe or main memory or disable this part. To learn more about the performance of viruses in computer networks, we can refer to [1, 2, 3]. Antivirus is essential for users to prevent virus disruptions. Because of the importance of this, many scientists and researchers have investigated the function of viruses in computer networks and provided models of computer virus performance in computer networks, for example, Ren et al.[4], presented a mathematical model of computer virus dispersal. Yang et al.[5] Presented a model of computer virus performance and then examined the sustainability of the proposed model. Some other models presented can be found in [6] and [2]. The performance of computer viruses, in terms of dispersal and transmission, is similar to the performance of biological viruses. In the same way that

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biological viruses are transmitted from one animal to another, computer viruses are also transmitted from one computer to another and spread throughout the network [7]. Because of the similarity of computer viruses and biological viruses, we can use the SIR model to check the performance of computer viruses, as defined below [5]

$$\begin{cases} \frac{dS}{dt} = b - \gamma S(t)I(t) - dS(t), \\ \frac{dI}{dt} = \gamma S(t)I(t) - \epsilon I(t) - dI(t), \\ \frac{dR}{dt} = \epsilon I(t) - dI(t) \end{cases} \tag{1.1}$$

The initial conditions of the model are as follows:

$$S(0) = S_0, I(0) = I_0, R(0) = R_0.$$

The parameters used in this model are presented in the Table 1. Refer to [5] for details on this model. The similarity between computer and biological viruses is a known fact. The biological viruses are similar to the laws of fractal geometry, and because of the similarity between biological viruses and fractal geometry, in [8], the relation between fractal geometry and computations a deficit is indicated. For this reason, we use a fractional calculation to check computer viruses and we describe the model (1.2) as a fractional model:

$$\begin{cases} D_*^\alpha S(t) = b - \gamma S(t)I(t) - dS(t), \\ D_*^\alpha I(t) = \gamma S(t)I(t) - \epsilon I(t) - dI(t), \\ D_*^\alpha R(t) = \epsilon I(t) - dI(t) \end{cases} \tag{1.2}$$

In the above model, $D^\alpha S$ is the Caputo derivative of order $0 < \alpha \leq 1$, and its initial conditions are $S(0) = S_0, I(0) = I_0, R(0) = R_0$. We also use the Caputo formula for calculating the fractional derivative of the formula in this paper, which is defined as follows:

$$D_*^\alpha f(x) = \begin{cases} \frac{1}{\Gamma(n-\alpha)} \int_0^x \frac{f^{(n)}(x)}{(x-\tau)^{\alpha-n+1}} d\tau, \\ \qquad n-1 \leq \alpha < 1 \\ \frac{d^n f(x)}{d^n x}, \quad \alpha = n \end{cases} \tag{1.3}$$

where $n \in N, \ x > 0, \ \alpha > 0.$

Also we have

$$D_*^\alpha c = 0, \qquad c \ \ is \ \ constant$$

$$D_*^\alpha x^j = \begin{cases} 0, \quad j \in N \cup \{0\} \ and \ j < \lceil \alpha \rceil \\ \frac{\Gamma(j+1)}{\Gamma(j+1-\alpha)} x^{j-\alpha}, \\ \qquad j \in N \cup \{0\} \ \ and \ \ j \geqslant \lceil \alpha \rceil \\ \qquad or \ \ j \notin N \ \ and \ \ j > \lfloor \alpha \rfloor \end{cases} \tag{1.4}$$

In recent years, numerous studies have been done in the field of calculating the deficit and used various numerical methods for solving differential deficit methods. For example, Li and Wang [9] presented a numerical algorithm based on Adomian decomposition for deficit differential equations. Also, Jafari and Daftardar in [10] solved the nonlinear fractional differential equations using adomian decomposition method. Ismaili et al.[11] have proposed a collocation method based on the Müntz-Legendre polynomial to find approximate solutions of fractional differential equations. Tavassoli and Rasouli, [12], have proposed a numerical method for solving differential fractional differential equations based on Müntz-Legendre polynomials. In addition, they developed a model of HIV infection as a system of fractional differential equations and provided a spatial method based on the Müntz-Legendre polynomials [13]. Some of the methods used to solve differential equations of fraction can be studied in [14, 15, 16, 17]. Kurkcu et al. [18] used Dickson polynomials to solve differential-integral equations.

In this paper, we use the method of collocation on the basis of Dickson polynomials to obtain the approximate solutions of the system (1.2). This article is comprised of the following sections: In the section *2.1*, we describe the definition of Dickson polynomials and express some of their properties. Then, in the section *2.2* the approximation of a function is expressed by using the Dixon polynomial. In section 3, a collocation method is proposed based on the Dickson polynomials for finding approximate solutions of the model (1.2). In the section 4, an error analysis has been performed with the introduction of error functions. In the section 5, the approximate results obtained using the method presented in this paper are investigated in the section on numerical results. Finally, in section 6 we will conclude and discuss the results obtained using the method presented in this paper.

2 Preliminaries

2.1 Dickson polynomials

Suppose R is a substitutable loop that has the same member. In this case, the first-order Dick-

Table 1: Parameters used in the proposed model

Parameters	descriptions
$R(t)$	The number of computers retrieved at time t
$I(t)$	Number of infected computers at time t
$S(t)$	The number of computers exposed at time t
b	Connecting computers from outside the network to the network
ϵ	Recovery rate of infected computers is affected by antivirus network
d	Exit speed of computers
γ	The rate of infecting computers

son polynomials of degree n and with the parameter $\lambda \in R$ are defined as follows [18, 19]

$$d_n(x,\lambda) = \sum_{m=0}^{[\frac{n}{2}]} \frac{n}{n-m} \binom{n-m}{m} (-\lambda)^m x^{n-2m} \qquad (2.5)$$

where $d_0(x,\lambda) = 2$ and $d_1(x,\lambda) = x$. The special modes of Dickson polynomials for different values of λ are shown in Table 2 [18]. We can also have the following recurrence relation for Dickson polynomials.

$$d_n(x,\lambda) = xd_{n-1}(x,\lambda) - \lambda d_{n-2}(x,\lambda) \quad n \geq 2$$

here $d_0(x,\lambda) = 2$ and $d_1(x,\lambda) = x$. By solving the second-order differential equation,

$$(x^2 - 4\lambda)\ddot{d} + x\dot{d} - n^2 d = 0 \quad n = 0,1,2,3,\ldots$$

we can also obtain Dickson polynomials.

2.2 Function approximation using Dickson polynomials

Suppose the function $f(x)$ is available, which is approximate as follows.

$$\forall \lambda \in R, \quad f(x) \approx g(x) = \sum_{i=0}^{n} c_i d_i(x,\lambda)$$

In which c_i are unknown values, we obtain the equations for the following equations.

$$f(\theta_i) = g(\theta_i), \quad i = 0,1,2,...,n$$

where θ_i are the Chebyshev points transmitted to the interval $[0,T]$ defined as follows.

$$\theta_i = \frac{T}{2} - \frac{T}{2}cos(\frac{\pi i}{n}) \quad i = 0,1,2,...,n.$$

Thus, the system $f(\theta_i) = g(\theta_i)$ is a system of algebraic equations with $n+1$ equations and $n+1$ unknowns, then by solving this system,we can obtain unknown coefficients c_i, $\quad i = 0,1,2,...,n$.

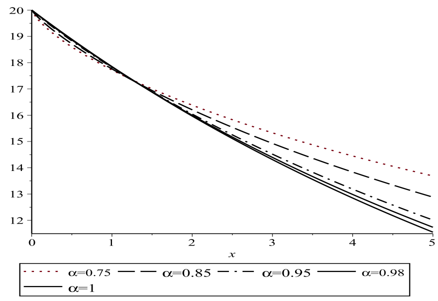

Figure 1: The graph of the approximate results obtained for the function $S(t)$ for $n = 15$ and for different values of α

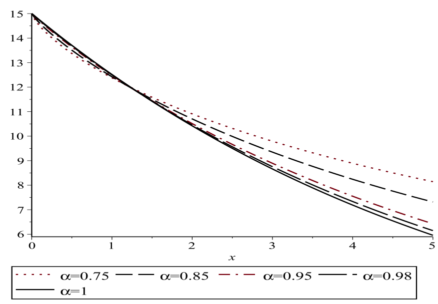

Figure 2: The graph of the approximate results obtained for the function $I(t)$ for $n = 15$ and for different values of α

Table 3: Values assigned to model parameters.

Parameters	values
b	0
γ	0.001
d	0.1
ϵ	0.1
S_0	20
I_0	15
R_0	10

Table 2: Special modes of Dickson polynomials.

Parameter λ	polynomials
-1	Pell-Lucas polynomials $d_n(x,-1) = Q_n(\frac{x}{2})$
0	Power polynomials $d_n(x,0) = x^n$
1	First kind Chebyshev polynomials $d_n(x,1) = 2T_n(\frac{x}{2})$
2	Ferma-Lucas polynomials $d_n(x,1) = Fl_n(\frac{x}{3})$

Table 4: The values obtained for $S(t)$ for $n = 15$ and various values of α.

t	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1$
0	20	20	20	20
1	17.7438038503926	17.7904127886923	17.8187416285726	17.8501096330141
2	16.2954315301640	16.1244189602330	16.0441937687261	15.9677579935880
3	15.1319188635607	14.7251411304448	14.5190139540304	14.3113893818910
4	14.1486561937741	13.5165530458748	13.1866965475440	12.8475369563923
5	13.2952956422631	12.4562322110827	12.0114043181920	11.5489823289406

Table 5: The values obtained for $I(t)$ for $n = 15$ and various values of α

t	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1$
0	15	15	15	15
1	12.4043167595789	12.4520132198786	12.4818225286824	12.5152884547679
2	10.8075369451851	10.6072935325303	10.5123369424508	10.4211717691036
3	9.57500214604886	9.1241086194401	8.8944150126676	8.66213933060665
4	8.5740918590018	7.9016758829060	7.5504218970789	7.18881514330027
5	7.7392384673274	6.8808100339458	6.4277113904543	5.95786292716376

Table 6: The values obtained for $R(t)$ for $n = 15$ and various values of α

t	$\alpha = 0.8$	$\alpha = 0.9$	$\alpha = 0.95$	$\alpha = 1$
0	10	10	10	10
1	10.3364805155120	10.3460354667580	10.3495977610140	10.3522857238360
2	10.3771859192211	10.4136461781185	10.4333746759717	10.4539541258174
3	10.3011655538718	10.3275795370758	10.3441966123531	10.3632912181796
4	10.1568323480806	10.1396721764775	10.1329832893884	10.1280499719111
5	9.9695901242754	9.8827037798238	9.8358200118934	9.78703443096404

Table 7: The values obtained for $S(2.5)$ and $\alpha = 1$ for different values of n and λ

λ	$n = 10$	$n = 20$	$n = 25$
-1	15.113582935427220	15.113582935424576	15.113582935424576
0	15.11358293542722023	15.1135829354245769	15.1135829354245769
1	15.11358293542722023	15.11358293542457698	15.1135829354245769
2	15.11358293542722023	15.11358293542457698	15.1135829354245769

2.3 Generalized Dickson operational matrix to fractional order

In this subsection, we introduce the operational matrix of fractional derivative of Dickson polynomials. Using (1.4), (2.5) and the linear property

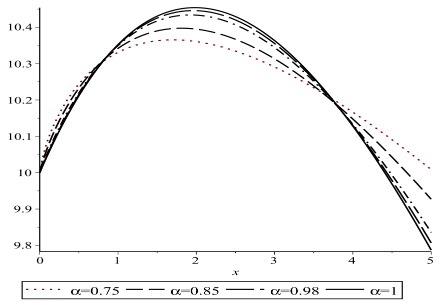

Figure 3: The graph of the approximate results obtained for the function $R(t)$ for $n = 15$ and for different values of α

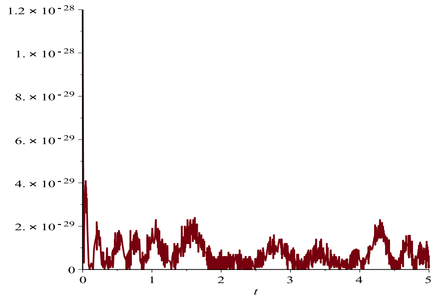

Figure 4: Error diagram for the $I(t)$ function for $n = 25$ and for, $\alpha = 1, \lambda = 0$

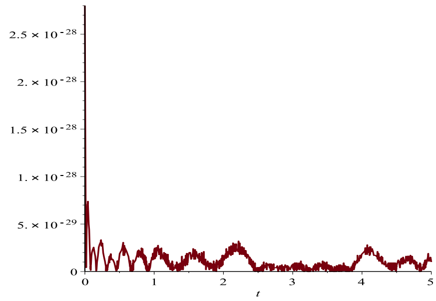

Figure 5: Error diagram for the $I(t)$ function for $n = 25$ and for, $\alpha = 1, \lambda = 0$

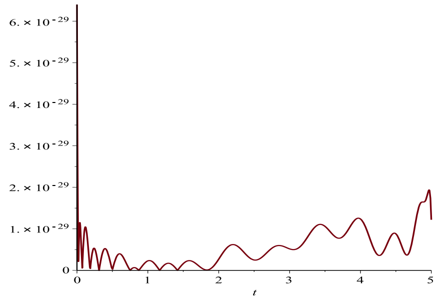

Figure 6: Error diagram for the $R(t)$ function for $n = 25$ and for, $\alpha = 1, \lambda = 0$

of Caputo operator, we have:

$$D^{(\alpha)}d_i(x,\lambda) = \sum_{j=0}^{[\frac{i}{2}]} \frac{i}{i-j}\binom{i-j}{j}(-\lambda)^j D^{(\alpha)}(x^{i-2j})$$
$$= \sum_{j=0}^{[\frac{i}{2}]} \frac{i}{i-j}\binom{i-j}{j}(-\lambda)^j \frac{\Gamma(i-2j+1)}{\Gamma(i-2j-\alpha+1)}x^{i-2j-\alpha} \quad (2.6)$$

Further, we can approximate the terms $x^{i-2j-\alpha}$ using Dickson polynomials as

$$x^{i-2j-\alpha} \cong \sum_{l=0}^{n} a_{l,i}d_l(x,\lambda). \quad (2.7)$$

Noteworthy, the orthogonality of Dickson polynomials is only obtained on the interval $[-2\sqrt{\lambda}, 2\sqrt{\lambda}]$ that is not suited to obtain the values of $a_{l,i}$. However, $a_{l,i}$'s for each value of i, λ can be obtained using symbolic software "Mathematica". Therefore, (2.6) and (2.7) imply,

$$D^{(\alpha)}d_i(x,\lambda) \cong \sum_{j=0}^{[\frac{i}{2}]} \frac{i}{i-j}\binom{i-j}{j}(-\lambda)^j \frac{\Gamma(i-2j+1)}{\Gamma(i-2j-\alpha+1)}\sum_{l=0}^{n} a_{l,i}d_l(x,\lambda). \quad (2.8)$$

Then,

$$D^{(\alpha)}d_i(x,\lambda) \cong \sum_{l=0}^{n}\sum_{j=0}^{[\frac{i}{2}]} \frac{i}{i-j}\binom{i-j}{j}(-\lambda)^j \frac{\Gamma(i-2j+1)}{\Gamma(i-2j-\alpha+1)}a_{l,i}d_l(x,\lambda).$$
$$= \sum_{l=0}^{n}(\sum_{j=0}^{[\frac{i}{2}]} w_{i,j,l}d_l(x,\lambda). \quad (2.9)$$

where

$$w_{i,j,l} = \frac{i}{i-j}\binom{i-j}{j}(-\lambda)^j \frac{\Gamma(i-2j+1)}{\Gamma(i-2j-\alpha+1)}a_{l,i}$$

Rewrite (2.9) as a vector form we have,

$$D^{(\alpha)}d_i(x,\lambda) \cong [\sum_{j=0}^{[\frac{i}{2}]} w_{i,j,0}, ..., \sum_{j=0}^{[\frac{i}{2}]} w_{i,j,n}]B(x,\lambda) \quad (2.10)$$

Hence,

$$D^{(\alpha)}B(x,\lambda) \cong [D^{(\alpha)}d_0(x,\lambda), ..., D^{(\alpha)}d_n(x,\lambda)]^T \quad (2.11)$$

Substituting (2.10) into (2.11), yieids,

$$D^{(\alpha)}B(x,\lambda) \cong D^{\alpha}B(x,\lambda) \quad (2.12)$$

where,

$$D^{\alpha}_{(n+1)(n+1)} = \begin{bmatrix} \sum_{j=0}^{[\frac{i}{2}]} w_{0,j,0} & \dots & \sum_{j=0}^{[\frac{i}{2}]} w_{0,j,n} \\ \dots & \dots & \dots \\ \sum_{j=0}^{[\frac{i}{2}]} w_{n,j,0} & \dots & \sum_{j=0}^{[\frac{i}{2}]} w_{n,j,n} \end{bmatrix}$$

3 Dickson collocation method

The approximation of the functions S, I, and R in terms of the Dickson polynomials is as follows.

$$\begin{cases} S(t,\lambda) \approx S_n(t,\lambda) = P^T B(t,\lambda) \\ I(t,\lambda) \approx I_n(t,\lambda) = Q^T B(t,\lambda) \\ R(t,\lambda) \approx R_n(t,\lambda) = R^T B(t,\lambda), \end{cases} \tag{3.13}$$

where,

$$P^T = [p_0, ..., p_n]$$

$$Q^T = [q_0, ..., q_n]$$

$$R^T = [r_0, ..., r_n]$$

$$B(t,\lambda) = [d_0(t,\lambda), ..., d_n(t,\lambda)]^T.$$

Now we apply the above approximations in Equation (1.2) and we obtain the equations of the following.

$$\begin{cases} P^T D^{(\alpha)} B(t,\lambda) = b - \gamma P^T B(t,\lambda) Q^T \\ \quad B(t,\lambda) - dP^T B(t,\lambda), \\ \\ Q^T D^{(\alpha)} B(t,\lambda) = \gamma P^T B(t,\lambda) Q^T \\ \quad B(t,\lambda) - \epsilon Q^T B(t,\lambda) - dQ^T B(t,\lambda), \\ \\ R^T D^{(\alpha)} B(t,\lambda) = \epsilon Q^T B(t,\lambda) \\ \quad -dQ^T B(t,\lambda) \end{cases} \tag{3.14}$$

$$P^T B(0,\lambda) = S_0, \quad Q^T B(0,\lambda) = I_0, \quad R^T B(t,\lambda) = R_0.$$

Now, by inserting the collocation points θ_j in the above equation,we obtain:

$$\begin{cases} P^T D^{(\alpha)} B(\theta_j,\lambda) = b - \gamma P^T B(\theta_j,\lambda) Q^T \\ \quad B(\theta_j,\lambda) - dP^T B(\theta_j,\lambda), \\ \\ Q^T D^{(\alpha)} B(\theta_j,\lambda) = \gamma P^T B(\theta_j,\lambda) Q^T \\ \quad B(\theta_j,\lambda) \epsilon Q^T - B(\theta_j,\lambda) \\ \qquad -dQ^T B(\theta_j,\lambda), \\ \\ R^T D^{(\alpha)} B(\theta_j,\lambda) = \epsilon Q^T B(\theta_j,\lambda) \\ \quad -dQ^T B(\theta_j,\lambda) \end{cases} \tag{3.15}$$

It is noticeable that the systems (3.15) and initial conditions in (3.14) have unknown $3(n+1)$ and $3(n+1)$ equations, which is solved with one of the common methods to obtain unknown values p_i and q_i and r_i, in which $i = 0, 1, ..., n$. By inserting them into relations (3.13) we can obtain the approximate solutions of S_n, I_n and R_n.

To determine the error assesment, we define the functions $E_1(t)$ and $E_2(t)$ and $E_3(t)$ for each of the equations of the model presented in an enormous way so that the approximate solutions obtained in the model are valid.

$$\begin{cases} E_n^1(t) = |D_*^\alpha S_n(t) - b \\ \quad +\gamma S_n(t) I_n(t) + dS_n(t)|, \\ \\ E_n^2(t) = |D_*^\alpha I_n(t) - \gamma S_n(t) I_n(t) \\ \quad +\epsilon I_n(t) + dI_n(t)|, \\ \\ E_n^3(t) = |D_*^\alpha R_n(t) \\ \quad -\epsilon I_n(t) + dI_n(t)|. \end{cases} \tag{3.16}$$

With respect to the above, we must have $E_n^1(t) \approx 0$ and $E_n^2(t) \approx 0$ and $E_n^3(t) \approx 0$ which are discussed below and are clearly visible in Figs. 4, 5 and 6.

4 Numerical results

To determine the error assesment, we define the functions $E_1(t)$ and $E_2(t)$ and $E_3(t)$ for each of the equations of the model presented in an enormous way so that the approximate solutions obtained in the model are valid.

$$\begin{cases} E_n^1(t) = |D_*^\alpha S_n(t) - b + \gamma S_n(t) I_n(t) \\ \quad +dS_n(t)|, \\ \\ E_n^2(t) = |D_*^\alpha I_n(t) - \gamma S_n(t) I_n(t) \\ \quad +\epsilon I_n(t) + dI_n(t)|, \\ \\ E_n^3(t) = |D_*^\alpha R_n(t) - \epsilon I_n(t) + dI_n(t)|. \end{cases} \tag{4.17}$$

With respect to the above, we must have $E_n^1(t) \approx 0$ and $E_n^2(t) \approx 0$ and $E_n^3(t) \approx 0$ which are discussed below and are clearly visible in Figs. 4, 5 and 6.

In this section we apply the proposed method for numerical solution of model (1.2) and present the obtained results. The values for the parameters used in the model are expressed in Table 3. In this section, the numerical results obtained for different values of the fractional derivative are presented. In Tables 2, 3 and 4 for $n = 15$ and different values of α on interval $[0, 5]$, the numerical results obtained for functions S, I and R are presented, respectively. In Tables 5, 6 and 7, the approximate values obtained for $S(2.5)$ and $I(2.5), R(2.5)$ for $\alpha = 1$ and different values of λ, n are shown, respectively. The results show that when α goes to 1, the approximate solutions go to the approximate solution obtained for $\alpha = 1$. Also, Tables 5, 6 and 7 show that with increasing n, we obtain more accurate approximate solutions.Tables 8, 9, 10, 11, 12 and 13 show that with increasing n, and for different values of

Table 8: The values obtained for $I(2.5)$ and $\alpha = 1$ for different values of n and λ

λ	$n = 10$	$n = 20$	$n = 25$
-1	9.50300025751504153	9.50300025752024391	9.503000257520243916
0	9.50300025751504153	9.50300025752024391	9.50300025752024391
1	9.50300025751504153	9.50300025752024391	9.50300025752024391
2	9.50300025751504153	9.50300025752024391	9.50300025752024391

Table 9: The values obtained for $R(2.5)$ and $\alpha = 1$ for different values of n and λ

λ	$n = 10$	$n = 20$	$n = 25$
-1	10.42945204527095843	10.429452045268398	10.42945204526839817
0	10.429443435608457996	10.42945204526839817	10.4294520452683981717
1	10.42945204527095843	10.42945204526839817	10.429452045268391717
2	10.42945204527095843	10.429452045268398171	10.42945204526839817

Table 10: The Max Error values obtained for $\alpha = 0.5$ and $\lambda = -1$

	$n = 10$	$n = 20$	$n = 30$
Max E_n^1	1.5×10^{-5}	4.1×10^{-7}	2.0×10^{-9}
Max E_n^2	6.7×10^{-6}	3.1×10^{-8}	4.4×10^{-9}
Max E_n^3	1.2×10^{-5}	5.3×10^{-7}	7.1×10^{-9}

Table 11: The Max Error values obtained for $\alpha = 0.9$ and $\lambda = -1$

	$n = 10$	$n = 20$	$n = 30$
Max E_n^1	2.1×10^{-6}	3.6×10^{-9}	4.7×10^{-12}
Max E_n^2	5.5×10^{-5}	4.0×10^{-8}	1.2×10^{-10}
Max E_n^3	3.2×10^{-6}	3.3×10^{-9}	2.2×10^{-11}

Table 12: The Max Error values obtained for $\alpha = 0.5$ and $\lambda = 1$

	$n = 10$	$n = 20$	$n = 30$
Max E_n^1	3.4×10^{-5}	3.9×10^{-7}	4.1×10^{-9}
Max E_n^2	7.2×10^{-6}	5.5×10^{-8}	7.0×10^{-9}
Max E_n^3	5.1×10^{-5}	6.2×10^{-7}	1.7×10^{-9}

Table 13: The Max Error values obtained for $\alpha = 0.9$ and $\lambda = 1$

	$n = 10$	$n = 20$	$n = 30$
Max E_n^1	5.1×10^{-5}	1.4×10^{-7}	1.0×10^{-9}
Max E_n^2	7.0×10^{-6}	1.3×10^{-8}	3.2×10^{-9}
Max E_n^3	2.2×10^{-5}	3.5×10^{-7}	1.7×10^{-9}

α, λ we obtain more accurate approximate solutions. In Figs. 1, 2 and 3, the graphs for the approximate solutions $S_n(t)$ and $I_n(t)$ and $R_n(t)$ for $n = 15$ and various values of are shown, respectively. As it is expected, with the approach of α to the 1, the graphs of the obtained approximations are also converge to the solution graph for $\alpha = 1$. Figs 4, 5 and 6 show the error function graphs for $n = 25$ and, $\alpha = 1, \lambda = 0$, for $S(t), I(t), R(t)$ go to zero.

5 Conclusion

In this paper, a numerical method has been presented using a Dickson-based collocation method. Using this method, the differential deficit differential equation system was converted to algebraic equations and this resulted in ease of resolution of the device, thus obtaining approximate solutions for the deficit model. In this paper, based on numerical results, it can be concluded that the Dickson-based collocation method is a suitable method for solving fractional differential equations. According to Tables 5, 6 and 7, in the results of the numerical results, it can be concluded that by increasing n, more accurate answers are obtained for this model. As shown in Figs. 1 and 2, with the onset of antivirus in a computer network, over time, the number of infected computers and computers exposed to pollution decreases, and according to Fig. 3, it can be seen that the number of computers Recovery initially increases and then, over time, and the number of infected computers decreases, the number of recovered computers is also reduced, and these results are achieved for all values of $0 \leq \alpha \leq 1$. Approaching to a solution is also expected to be $\alpha = 1$, as shown in Figs. 1, 2 and 3.

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