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Research Article

Data Envelopment Analysis and Malmquist Index for Measuring Productivity of Inefficient DMUs

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Abstract

Data envelopment analysis (DEA), is a non-parametric mathematical programming technique to evaluate the efficiency of a set of homogeneous decision-making units (DMUs), so that DMUs are evaluated into two groups, efficient and inefficient. According to the staggering costs in order to managing DMUs or organizations, maintaining some loss-making organizations are not cost-effective. Therefore, one of the concerns of managers in the discussion related to the financial problems of organizations is the maintenance or merger or elimination of inefficient organizations (inefficient DMUs). However, this article focuses on the performance of inefficient units. Therefore, we measure the productivity of inefficient DMUs using the revised Malmquist productivity index (MPI) to make a decision based on the maintenance or merger or elimination of these DMUs by decision makers

Keywords : Data Envelopment Analysis; Efficiency; Productivity; Malmquist productivity index; Decision-making.

1 Introduction

DEA is currently a popular technique for analyzing technical efficiency and it has been used in a number of applications. The applications of DEA present a range of issues relating to the homogeneity of the units under assessment. In order to achieve the correct insight into productivity (a function of technical efficiency and effectiveness) DMUs, in order to improve their situation, recognizing their performance is a priority.

DEA models divide DMUs into two groups, efficient and inefficient. Inefficient DMUs can be improved their performance by decreasing their current input levels or either increasing their current output levels in order to reach the efficient frontier. One of the application problems that concerns management of DMs is solving the problem of financial and credit institutions regarding the maintenance or merger or elimination of inefficient organizations (inefficient DMUs). Due to the staggering costs of organizations, it is not

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cost-effective to maintain some loss-making organizations. Therefore, DMs seek to merge or eliminate such organizations. In some application problems, the data appear as a ratio from input to output, or vice versa, where DEA models are unable to calculate efficiency, and we use DEA-R models instead of DEA models [8, 16 - 20].

The Malmquist productivity index (MPI) is one of the powerful tools, so that it can help to managers. This index was first suggested by Malmquist [15] and later extended for productivity analysis by Caves et al. [4]. The advantage of this method is measuring the position of an inefficient unit in relation to a combination of all efficient units and also in relation to a combination of all other inefficient units. MPI is the most important indices for evaluating the productivity the set of DMUs at the two time periods or two groups of DMUs at the one time period [2].

This paper uses the revised MPI to compute the productivity between two groups efficient and inefficient at the one time period in order to determine the productivity performance of inefficient DMUs. In other words, the purpose of this article is to identify the effect of inefficient units on the efficient units using the productivity index. However, this information can be useful for managers in making decisions and improving in the managerial and operational tasks in their organizational units. For this purpose, consider n DMUs with multiple inputs and multiple outputs, where they have been divided into two groups of efficient and inefficient units. Suppose that A be the set of efficient units and B be the set of inefficient units. The value of efficiency is unity for efficient units and in order to the inefficient units is less than 1. Suppose that p is an inefficient unit, then in order to evaluate its productivity we compute the amount of the product of the performance of relative to the two efficiency frontier of efficient units A and the frontier of the inefficient units B.

This paper unfolds as follows. Section 2 discusses about the scope and purpose. Section 3 gives a background of DEA and MPI. Sections 4 introduce a revised method for measuring the productivity of inefficient DMUs. Section 5 uses an application real example to calculate the productivity of inefficient units from 40 industrial banks in Iran. Some concluding remarks follow in Section 6.

2 Scope and purpose

Consider several decision-making units engaged in educational and economic activities under the supervision of a central organization. Each decision-making unit (DMU) includes multiple inputs and multiple outputs. Suppose that after several decades of operation, it is determined that some units are efficient and some are inefficient. One group of DMs wants to eliminate the collection of inefficient units and allocate their related budgets to other units, but another group has a different view and wants to maintain inefficient units and encourage these units to become efficient. For this purpose, in order to reach a logical solution, we measure the productivity of inefficient DMUs, and then make a final decision about maintaining or merger or eliminating these units.

3 Background

3.1 Data Envelopment Analysis (DEA)

DEA is a mathematical programming approach for evaluating the relative efficiency of homogenous DMUs. DEA was first proposed by Charnes et al. [6], which provide a measure of efficiency of each DMU. The relative efficiency for each DMU with multiple inputs and multiple outputs is defined as the ratio of the weighted outputs to the weighted inputs. The proposed original model by Charnes et al. [5, 6], is based on constant returns to scale in the production process. Banker et al. [1] also, introduced an alternative model based on variable returns to scale.

Consider n DMUs , so that each DMU consumes m inputs for produce s outputs. Assume that $x_j = (x_{1j}, ..., x_{mj}) \in R^m_+$ and $y_j = (y_{1j}, ..., y_{sj}) \in R^s_+$ be the input vector and output vector of DMU_j respectively, with $y_j \geq 0, y_j \neq 0, x_j \geq$

$0, x_j \neq 0$. The CCR DEA model evaluate each DMU with the composite unit that consumes the lowest possible fraction of that DMU's current input levels to produce at least that DMU's current output levels, separately. The measure of efficiency $DMU_o, o \in \{1, ..., n\}$ in the envelopment form CCR input-oriented model obtains as follows:

$$\begin{aligned} & min \;\; \theta \\ \text{s.t.} \;\; & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \quad j = 1, \cdots, n. \end{aligned} \tag{3.1}$$

Assume that θ^* be the optimal value in evaluating DMU_o , If $\theta^* = 1$, then DMU_o is efficient, otherwise it is inefficient.

3.2 The Malmquist productivity index (MPI)

Productivity and technical change between periods can be measured by Malmquist productivity index (MPI). The productivity growth in target achievement for an individual unit can be measured by the MPI as improved efficiency relative to the benchmark frontier. The MPI in DEA, compute the distance function, for measurement the productivity change among two time periods or two groups at the one time period. This index is based on two factors of efficiency change index and the technological change index. Fare extended the MPI and then presented it for each unit with combine the Farrell views for measurement of efficiency [9, 13, 14]. MPI is divided on two factors of efficiency change index which indicates increase or decrease of efficiency and technological change index, which measure the value of frontier movement.

*3.2.*1 The Malmquist productivity index at two time periods

Consider a set of homogeneous DMUs as $DMU_j = (x_j^t, y_j^t), j \in \{1, ..., n\}$, at the time period t . The DEA efficiency is obtained using the CCR model in evaluating $DMU_o, o \in \{1, ..., n\}$ as follows:

$$\begin{aligned} & D^t(x_o^t, y_o^t) = min \; \theta \\ \text{s.t.} & \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^t, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^t, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \; j = 1, \cdots, n. \end{aligned} \tag{3.2}$$

If $D^t(x_o^t, y_o^t) = 1$, then DMU_o is efficient at the time period t , otherwise it is the inefficient unit. By replacing $t+1$ to t in model (3.2), we can evaluate technical efficiency $D^{t+1}(x_o^{t+1}, y_o^{t+1})$ for DMU_o at the time period $t+1$. MPI requires calculating distance function at two single period and two mixed period measures. Therefore, in order to compute distance function at two mixed period, the following models are presented:

$$\begin{aligned} & D^t(x_o^{t+1}, y_o^{t+1}) = min \; \theta \\ \text{s.t.} & \sum_{j=1}^{n} \lambda_j x_{ij}^t \leq \theta x_{io}^{t+1}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^t \geq y_{ro}^{t+1}, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \; j = 1, \cdots, n, \end{aligned} \tag{3.3}$$

and

$$\begin{aligned} & D^{t+1}(x_o^t, y_o^t) = min \; \theta \\ \text{s.t.} & \sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \leq \theta x_{io}^t, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \geq y_{ro}^t, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \; j = 1, \cdots, n, \end{aligned} \tag{3.4}$$

where, model (3.3) provides the measure of productivity efficiency by data of time period $t+1$ relative to technology frontier at the time period

t. Alternatively, model (3.4) gives the measure of productivity efficiency by data of time period t relative to technology frontier at the time period $t+1$. Hence, the input oriented MPI introduced as follows [4]:

$$MPI = \left[\frac{D^t(x^{t+1}, y^{t+1})D^{t+1}(x^{t+1}, y^{t+1})}{D^t(x^t, y^t)D^{t+1}(x^t, y^t)}\right]^{\frac{1}{2}} \tag{3.5}$$

3.2.2 The Malmquist productivity index for two groups at one time period

Consider two set A and B, such that $(x_j^A, y_j^A), j = 1, \dots, n_A$ and $(x_j^B, y_j^B), j = 1, \dots, n_B$.$D(x_j^B, y_j^B)$, denotes the input distance function for evaluating DMU_j in group B with respect to the efficiency frontier of group A. Camanho et al. [3] defined an index for the comparison of performance between two groups of DMUs (group A and B), according to the Malmquist productivity index that had been developed by Fare et al. [10], as follows:

$$I^{AB} = \left[\frac{\left(\prod_{j=1}^{n_A} D^A(x_j^A, y_j^A)\right)^{\frac{1}{n_A}}}{\left(\prod_{j=1}^{n_B} D^A(x_j^B, y_j^B)\right)^{\frac{1}{n_B}}} \times \frac{\left(\prod_{j=1}^{n_A} D^B(x_j^A, y_j^A)\right)^{\frac{1}{n_A}}}{\left(\prod_{j=1}^{n_B} D^B(x_j^B, y_j^B)\right)^{\frac{1}{n_B}}}\right]^{\frac{1}{2}} \tag{3.6}$$

The first ratio inside square brackets evaluates the average distance DMUs from group A divided by the average distance DMUs from group B. The second ratio is a similar quotient. In terms of the interpretation of the overall index, a value less than unity indicates better performance in group A than in group B.

4 Measuring the productivity of inefficient DMUs

The relation (3.6) has two weaknesses. First, it is not any taken preference from group A to group B, for this reason, the geometric mean had been used. Second, this formula compares two groups and is unable to calculate the productivity of a member of one group with another group frontier. For this purpose, we assume that group A performs better than group B. Thus, to evaluate a DMU such as $(x_p, y_p) \in B$ relative to group A and group B, we introduce a distance index (DI) the following relation:

$$DI_p^{AB} = \left[\frac{\left(\prod_{j=1}^{n_A} D^A(x_j^A, y_j^A)\right)^{\frac{1}{n_A}}}{D^A(x_p^B, y_p^B)} \times \frac{\left(\prod_{j=1}^{n_A} D^B(x_j^A, y_j^A)\right)^{\frac{1}{n_A}}}{D^B(x_p^B, y_p^B)}\right]^{\frac{1}{2}} \tag{4.7}$$

Thus, the value of $DI_p^{AB} < 1$, indicates (x_p^B, y_p^B) has good productivity, $DI_p^{AB} > 1$, bad productivity and $DI_p^{AB} = 1$, indicates that its performance stayed constant in a system. In order to describe the method proposed in this paper, consider n decision making units (DMUs), as (x_j, y_j) , $j \in \{1, \dots, n\}$ each consumes m inputs $x_j = (x_{1j}, \dots, x_{mj}) \in R_+^m$ to produce s outputs $y_j = (y_{1j}, \dots, y_{sj}) \in R_+^s$. First, using CCR DEA model (3.1), we compute the efficiency of the DMUs and categorize them into two efficient and inefficient groups. Assume that the set A includes efficient DMUs and the set B including inefficient DMUs. The efficiency value of members A is the same and equal to 1, and the efficiency value of members B is less than 1. Let us, (x_j^A, y_j^A) , $j = 1, \dots, n_A$ be DMUs in group A and (x_j^B, y_j^B),$j = 1, \dots, n_B$ be DMUs in group B, such that $n_A + n_B = n$. Also, we have $D^A(x_j^A, y_j^A) = 1$, $j = 1, \dots, n_A$, and $D^A(x_j^B, y_j^B) < 1$, $j = 1, \dots, n_B$. However, in order to measurement the productivity of an inefficient DMU (x_p^B, y_p^B) by relation (4.7), where we have $D^A(x_j^A, y_j^A) = 1$ and the value $\left(\prod_{j=1}^{n_A} D^B(x_j^A, y_j^A)\right)^{\frac{1}{n_A}}$ is constant value in evaluating all inefficient DMUs. Therefore, it is enough that, we obtain the distance function $D^A(x_p^B, y_p^B)$, which represents the input distance for DMU_p in group B with respect to the efficiency frontier of group A and $D^B(x_p^B, y_p^B)$, which relates the input distance for DMU_p in group B relative to the efficiency frontier B as it has shown in (4.8).

$$DI_p^{AB} = \frac{1}{D^A(x_p^B, y_p^B).D^B(x_p^B, y_p^B)} \tag{4.8}$$

Therefore we define productivity for inefficient (PI) DMU_p as follows:

$$PI_p^{AB} = \frac{1}{DI_p^{AB}} \tag{4.9}$$

Table 1: Average values of inputs and outputs from 2017 to 2018.

DMUs	y_1	y_2	y_3	x_1	x_2
1	609.85	811.50	550.50	2952.49	1031.00
2	998.95	982.45	832.05	4459.63	1140.00
3	340.80	980.90	639.50	2135.65	696.20
4	1441.00	2914.50	894.40	6565.92	1090.00
5	441.90	1284.50	739.40	2722.38	707.40
6	1129.00	1420.50	800.25	4551.81	1097.00
7	3149.00	781.70	3315.50	8026.07	1150.00
8	1035.00	823.80	771.90	4357.19	1032.00
9	1154.00	1313.00	659.80	4021.79	1070.00
10	828.70	1244.00	720.15	4562.83	1022.00
11	1414.00	2358.00	1000.00	6907.83	1061.00
12	2454.00	10896.50	2178.00	11601.31	674.80
13	1124.05	615.70	1021.60	3977.59	1008.00
14	1001.90	1623.50	762.65	3754.29	1068.00
15	1015.00	1007.10	542.50	3866.61	1048.00
16	909.20	600.15	1396.50	3786.60	1010.00
17	1205.00	1933.00	818.95	5621.49	1092.00
18	1508.50	2364.50	1099.00	6647.73	1089.00
19	1810.50	797.40	2235.50	7265.68	1062.00
20	880.60	1207.00	720.20	4729.05	988.00
21	895.95	1018.90	637.90	2948.40	1006.00
22	1380.50	1416.50	965.45	5228.99	1029.00
23	1015.60	1197.50	625.40	2961.06	1023.00
24	820.25	841.20	847.00	3215.65	980.00
25	716.40	718.45	712.20	2742.78	938.70
26	1483.50	659.35	611.75	2537.32	968.60
27	1099.15	338.90	1305.50	4131.15	951.20
28	1062.20	1015.90	601.30	2968.56	927.90
29	1103.00	383.25	861.35	3440.38	919.80
30	711.35	1003.45	814.05	3079.24	924.40
31	987.25	987.10	906.05	4698.42	894.00
32	518.15	825.85	544.90	2791.18	918.40
33	984.55	438.80	442.85	2716.58	941.80
34	766.90	554.10	346.40	2610.33	925.50
35	340.40	448.90	287.85	2092.02	937.70
36	530.85	427.50	295.95	2195.39	852.00
37	890.95	670.90	533.80	3110.52	819.10
38	241.80	543.40	362.40	1958.45	813.20
39	626.50	236.85	173.95	2159.22	813.20
40	198.20	204.55	372.15	1628.37	813.20
G-Mean	889.2	904.8	722.1	3673.5	955.7

5 An application in the Tehran banking industry

In this section, we describe a real application example from the article on achieving the products of bank branches (see Tavallaaee, Alirezaee) [18]. We suppose all bank branches have two inputs and three outputs, which given in Table 1.

Inputs: human resource and location index of branches.
Outputs: deposits, facilities, and services. The input of human resources has related to the employees of the branch and the input of the place is related to the physical location of the branch. The output index of branch deposits includes all types of methods of collecting cash by that

Table 2: The efficiency score 40 of the Iran industrial banks.

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	0.61	11	0.60	21	0.77	31	0.58
2	0.59	12	1.00	22	0.66	32	0.62
3	0.96	13	0.71	23	0.83	33	0.65
4	0.65	14	0.77	24	0.76	34	0.57
5	0.90	15	0.58	25	0.75	35	0.46
6	0.65	16	0.93	26	1.00	36	0.52
7	1.00	17	0.59	27	0.77	37	0.62
8	0.57	18	0.64	28	0.80	38	0.59
9	0.67	19	0.77	29	0.70	39	0.50
10	0.54	20	0.53	30	0.80	40	0.59

Table 3: The efficiency score for DMUs in group B.

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	0.73	11	0.96	21	0.93	31	0.81
2	0.76	12	-	22	0.98	32	0.73
3	1.00	13	0.94	23	1.00	33	1.00
4	1.00	14	1.00	24	0.90	34	0.82
5	1.00	15	0.81	25	0.90	35	0.55
6	0.87	16	1.00	26	-	36	0.67
7	-	17	0.87	27	1.00	37	0.88
8	0.77	18	0.99	28	1.00	38	0.64
9	0.91	19	1.00	29	1.00	39	0.80
10	0.74	20	0.73	30	0.93	40	0.63

Table 4: The PI index of all inefficient DMUs in group B.

DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency	DMUs	Efficiency
1	0.4453	11	0.5760	21	0.7161	31	0.4698
2	0.4484	12	-	22	0.6468	32	0.4526
3	0.9600	13	0.6674	23	0.3800	33	0.6500
4	0.6500	14	0.7700	24	0.6840	34	0.4674
5	0.9000	15	0.4698	25	0.6750	35	0.2530
6	0.5655	16	0.9300	26	-	36	0.3484
7	-	17	0.5133	27	0.7700	37	0.5456
8	0.4389	18	0.6336	28	0.8000	38	0.3776
9	0.6097	19	0.7700	29	0.7000	39	0.4000
10	0.3996	20	0.3869	30	0.7440	40	0.3717

branch. It is an output because workforce, advertising, and money have been spent to obtain it. The output of the facility includes all the funds that have been paid by the branch in the form of various types of facilities. Also, the output of services is an indicator that includes all kinds of fee services in the form of card issuance, types of guarantees and opening of visual and long-term documentary credit, foreign exchange transaction fee and funds transfer fee by providing weighted coefficients by the branch to its customers. By using model (3.1), we estimate of technical efficiency for 40 of the Iran industrial banks is reported in Table 2. Table 2 denote the efficient DMUs have efficiency score 1 and inefficient DMUs have efficiency less than one. Thus, we obtain two set A and B, where the set A contain DMU7, DMU12 and DMU26 as the efficient DMUs and the other units lie in the set B as the inefficient DMUs. This means that, according Table 2, $D^A(x_j^A, y_j^A) = 1$, $j \in A$ and $D^A(x_j^B, y_j^B) < 1$, $j \in B$. Now, we obtain the value of distance function (efficiency) $D^B(x_j^B, y_j^B)$, for DMU_j, $j \in B$ by CCR DEA model (3.1), which represented in Table 3. The productivity for inefficient (PI) units (x_j^B, y_j^B) ,$j = \{1, \ldots, n_B\}$ have been denoted in Table (4).

6 Conclusions

Financial cost for some loss-making organizations is one of the concerns of managers. Therefore, identifying loss-making organizations that are less productive is of particular importance. In this article, we first categorized a set of organizations into two groups: efficient and inefficient, and then we determined the productivity of inefficient organizations, so that central organization management can suggest decisions for retention, merger, or removal for less productivity. In this article, we studied 40 industrial banks in Iran, of which 3 became efficient banks and 37 became inefficient. Using these two sets efficient and inefficient, the productivity index determined for inefficient units. Thus, according to the obtained PI index (say average PI), a decision maker can make decisions to maintain, merge, or eliminate low-productivity banks. One can also determine the productivity index for efficient units so that more funds can be allocated to efficient units that have more productivity.

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