Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 11, No. 4, 2019 Article ID IJIM-1063, 13 pages
Research Article

Constrained Non-Renewable Resource Allocation in Metagraphs with Discrete Random Times

N. Akhoundi Shiviyari *, S. S. Hashemin †‡

Received Date: 2017-04-28 Revised Date: 2019-03-19 Accepted Date: 2019-08-31

Abstract

In this paper, it is supposed that a project can be shown as a metagraph. The duration of activities are discrete random variables with known probability function, also one kind of consumable and constrained resource is required to execute each activity of the project. Besides, the probability functions of activity durations depend on the amount of the resource allocated to it. Clearly, the amount of resource which can be allocated to each activity is limited to specific values. In this paper, it is assumed that the due date of project is known and definite. The objective of this paper is to maximize the probability of completion of stochastic metagraph before the due date of the project. As respects, solving the problem by using the analytical method is very challenging; therefore, we developed a new heuristic method including two sub-algorithms for solving the problem. Twenty examples were designed for examining the proposed algorithm.

Keywords : Constrained Resource Allocation; Stochastic Metagraph; Project Completion Time.

---

1 Introduction

The main objective of project managers is to complete the project on time. Achieving this goal is closely related to the optimal allocation of constrained resource. Indeed, if resources are allocated properly and principally, the project can be completed with minimum delay. So, allocation of the limited resource is one of the most important issues in the project planning and control.

For modeling, analysis and evaluation of the problems of resource allocation and scheduling the project activities, we require graphical tools. Networks are common tools for displaying and analysis of projects. According to the nature of the projects, different networks can be utilized for their analysis. It is obvious that accuracy in selection of networks for analysis of the problems related to resources planning and scheduling leads to obtaining an efficient algorithm for solving such problems. One of the graphical tools which has gained the attention of the researchers of recent decades being a powerful graphic tool for modeling and analysis of most problems -such as project management problems- is metagraph. Indeed, a metagraph is a graphical structure that represents directed relationships between sets of elements [3, 7].

Since the available resources are limited and in most project management problems the nature of

*Department of Industrial Engineering, Ardabil Branch, Islamic Azad University, Ardabil, Iran.

†Corresponding author. s.s.hashemin@gmail.com, Tel: +98(914)3521356.

‡Department of Industrial Engineering, Ardabil Branch, Islamic Azad University, Ardabil, Iran.

non-renewable resources is in a way that by ending one activity, the predicted value of resource for completion of the activity is ended, proper allocation of resources is very important. In real world projects, determining the exact completion time of activities is not possible all the times, particularly in case of activities that are accomplished for the first time in which there is no specific former experience. In this case, due to shortage of information on the activities completion time, their related parameters are estimated according to the former experience and by experts. Hence in modeling, it is proper to assume that the completion time of activities are random variables. Also, the structure of most project management problems is such that they can be modeled in the form of metagraphs, thus, the researchers have tried to present solutions for these problems. In this paper for modeling of allocation of non-renewable limited resources, the relations between activities were shown by metagraphs and a new heuristic method was proposed for solving these problems. In other words, for solving this problem by the aim of maximizing the probability of completion of stochastic metagraph before the due date of project, a new heuristic method including two sub-algorithms was proposed. For evaluation of the efficiency of this algorithm, twenty examples were designed and solved. Also, all feasible resource allocations are generated by computer and evaluated by simulation programs. The results of obtained answers were compared to each other by means of simulation programs and proposed algorithm which depicted good performance of the proposed algorithm.

2 Reveiw of Literature

The concept of metagraph was introduced for the first time by [1]. They used metagraphs for modeling Enterprise for the first time. Matagraphs as a graphic tool are capable of modeling and analyzing most systems [8] and they have been used in different areas such as decision support systems [2, 4] and workflow management [5, 6, 7].

Application of metagraphs in the project planning and control scope has no long history and a few studies have been done in this regard. According to capabilities of metagraphs, different studies have been conducted in project management area which used metagraphs as a tool for project planning and control. These studies have considered the completion time of activities in certain and uncertain states. Also, a project with uncertain activity time has been studied as a fuzzy project and stochastic . Thus, these investigations can be categorized into studies by certainty, fuzzy and random approaches according to the nature of the time of activities and solving methods.

2.1 Certainty Approach

Application of the metagraphs as modeling tool in project planning and control scope was proposed for the first time by [7]. They showed that forward and backward calculations can be carried out in certain metagraphs and determination of critical path and critical and floating time can be determined accordingly.

2.2 Fuzzy Approach

Examining the issue of allocation of limited resources in the projects that can be shown in the metagraph format was done for the first time by [10]. In this research, it was assumed that the completion time of each edge is a positive trapezoidal fuzzy number. In this study, we tried to propose a new algorithm for allocation of nonrenewable resource among the activities of the metagraph by the aim of reducing the project completion time by ranking the paths and edges.

In other study, metagraphs were used as a tool for planning and controling uncertain and fuzzy projects. In this research, it was assumed that the resource is limited and non-renewable [11]. Also, it was assumed that the completion time of each edge is a positive trapezoidal fuzzy number. Furthermore forward and backward calculations were done for fuzzy metagraphs and as a result, completion time of project, earliest and latest start and finish time and also floating time of the activities of the metagraph were obtained as trapezoidal fuzzy numbers. In this research, we have tried to develop a new method for allocation of non-renewable resources among metagraph activities by using the min-slack method.

Time cost trade-off problem was studied in projects that can be shown as fuzzy metagraphs [12]. In this research, it was assumed that completion time of each activity is a positive trapezoidal fuzzy number and for executing the edges, they need one kind of consumable resource. In the current study, a new method was developed for exchange of time and cost in order to access early completion time of project and reduction of the total cost of metagraph.

A new approach for allocation of constrained non-renewable resource in fuzzy metagraphs was suggested [16]. In this research, it was assumed that the time of activities is uncertain and fuzzy which was depicted by positive trapezoidal fuzzy numbers. In this study, we tried to develop a new method for allocation of non-renewable resource among the activities of the metagraph by the aim of reducing computational efforts.

2.3 Random Approach

The stochastic metagraphs as a tool for planning and control of projects with random time of activities was used for the first time [13]. In this research, it was assumed that time of each activity is a known random variable. We attempted to estimate the cumulative function of project completion time by proposed algorithm and conditional Mont Carol simulation.

3 Metagraph

Definition 3.1 *[8] The generating set of a metagraph is the set of elements $X = \{x_1, x_2, ..., x_n\}$, which represent variables of interest, and which occur in the edges of the metagraph .*

Definition 3.2 *[8] An edge e in a metagraph is a pair $e = \langle V_e, W_e \rangle \in E$ (where E is the set of edges) consisting of an invertex $V_e \subset X$ and an outvertex $W_e \subset X$, each of which may contain any number of elements. The different elements in the invertex (outvertex) are coinputs (cooutputs) of each other .*

Definition 3.3 *[8] A metagraph $S = \langle X, E \rangle$ is then a graphical construct specified by its generating set X and a set of edges E defined on the generating set).*

3.1 Simple Path

Definition 3.4 *[10] An element $x \in X$ is connected to element $x^{'} \in X$ if the sequence of edges $\left(e^{'}_k, k = 1, 2, ..., K^{'}\right)$ exists such that, $x \in V^{'}_1$, $x^{'} \in W^{'}_{K^{'}}$ and $\forall k = 1, 2, ..., K^{'}-1$, $W^{'}_k \cap V^{'}_{k+1} \neq \varnothing$. This sequence of edges is called a simple path from x to $x^{'}$. x is called source and $x^{'}$ is called target. $K^{'}$ is called the length of simple path .*

3.2 Stochastic Metagraph

Definition 3.5 *[13] A stochastic metagraph is identified with $(F(X, E, D))$. $(X = \{x_i, 1, 2, \ldots, I\})$ is called the generating set. (x_i) is called the element of (X) . $(E = \{e_j, j = 1, 2, \ldots, n\})$ is the set of edges. Each edge is an ordered pair as $((V_j, W_j))$. $(V_j \subset X)$ is called the invertex of (e_j) and $(W_j \subset X)$ is called the outvertex of (e_j) such that $(\forall j, V_j \cap W_j = \varnothing)$.*

It is supposed that the duration of edge (e_j) is a known discrete random variable and is represented by (D_j) such that $(D_j \in D)$.

4 Allocation of Non-Renewable Resource in Random Metagraphs

In previous section it was mentioned that a projectin which the completion time of activities is a discrete random variable, can be shown as a stochastic metagraph. In other words, a project with random completion time of activities can be modeled as stochastic metagraph. When the nonrenewable resource is required for completion of the activities, the problem will be constrained non-renewable resource allocation in stochastic metagraph;so that, the probability of completion of stochastic metagraph before the due date of project should be maximized. In this section, the problem of allocation of non-renewable resources in stochastic metagraphs is explained and the algorithm to solve it, is introduced.

4.1 Problem Description

Consider a project containing n activities, which have precedence relationships between them.

These relationships are represented by a stochastic metagraph. Required time for accomplishment of each activity of project is a known discrete random variable. Their probability function depend on level of allocated resource to that activity. Each of metagraph activity (e_j) needs non-renewable resources for accomplishment, and the amount of resource which can be allocated to each activity is limited. The aim is to determine how to allocate non-renewable resources among metagraph activities, in a way that, the probability of completion of stochastic metagraph before the due date of project is maximized.

The algorithms designed for allocation of non-renewable resources are based on rational inferences. Although, solving the problem by mathematical approach offers optimal solution [14, 15] but it is impossible in large scale projects. Thus, in this paper, a heuristic method was proposed which most of the time requires efficiency to achieve the optimal solution. Next, the necessary notations and assumptions for the proposed algorithm are explained.

4.1.1 Assumptions

- The metagraph of project has a source invertex and a target outvertex.
- Stop is not allowed after beginning the activity.
- Interruption is not permitted in the paths of the metagraph which begins from the source invertex and ends to the end outvertex.
- Activities accomplishments need one kind of constrained resource.
- The resource is non-renewable.
- Available resource is limited and defined.
- Required duration for accomplishment of each activity is a discrete random variable with a given probability function, probability function of which depends on the amount of resources allocated to it.
- The due date of project is known and definite.
- When the invertex of an activity is available, the activity can be accomplished and it does not need preparation time.
- Difference between levels of allocable resource for each activity is considered a unit, thus in each loop of proposed algorithm implementation, a unit from available resource is added to the amount of resource allocated to selected activity or is reduced from its amount.

4.1.2 Notations

e_j j-th activity (edge), j=1,2,...,n.

E Set of activitiesedges of the metagraph, $E = \{e_j, j = 1, 2, ..., n\}$

D_j Random variable of activity completion time,

S_{L_j} Amount of resource allocated to activity ,$e_j, L_j = 1, 2, ..., k_j$

$P_j\left(S_{L_j}, D_j\right)$ Probability function of duration of activity e_j , when the allocated resource to this activity is S_{L_j}

t Desired metagraph completion time due date

T Random variable of project completion time

Rs Available value of constrained resource

R_{max} Maximum value of required resource

R_{min} Minimum value of required resource

P Set of paths of the metagraph, $P = \{p_j, j = 1, 2, ..., m\}$

P_r rth path of the metagraph from source invertex to target of metagraph and m=| P| is the number of above paths

T_r Random variable of Completion time of the rth path

ΔR^+ Amount of increased resource from R_{min} to Rs

ΔR^- Amount of decreased resource from R_{max} to Rs

$R^{(i)}$ n tuple ordered of allocated resource to activities 1 to n in ith loop of algorithm,

$\mu_j\left(S_{L_j}\right)$ Average of completion time of activity e_j , when the allocated resource to that is

N'_j Number of paths that activity e_j lie on them

$N"_j$ Number of paths that activities e_j lie on them and equation $P(T_r \leq t) = 1$ is held for them

$\Delta\mu_j^-\left(S_{L_j}, S_{L_{j+1}}\right)$ Amount of reduction of completion time mean of activity e_j when the allocated resource is increased from S_{L_j} to $S_{L_{j+1}}$

$\Delta\mu_j^+\left(S_{L_j}, S_{L_{j-1}}\right)$ Amount of increase in completion time mean of activity e_j when the allocated resource is reduced from S_{L_j} to $S_{L_{j-1}}$

ET_j Effective time reduction coefficient of activity e_j

ET_j' Effective time increase coefficient of activity e_j

Relations among parameters and the parameters calculation are as follows:

$$\Delta R^+ = Rs - R_{min} = K \tag{4.1}$$

$$\Delta R^- = R_{max} - Rs = K' \tag{4.2}$$

$$N_j = N_j' - N"_j \tag{4.3}$$

$$\Delta\mu_j^-\left(S_{L_j}, S_{L_{j+1}}\right) = \mu_j\left(S_{L_j}\right) - \mu_j\left(S_{L_{j+1}}\right) \tag{4.4}$$

$$\Delta\mu_j^+\left(S_{L_j}, S_{L_{j-1}}\right) = \mu_j\left(S_{L_{j-1}}\right) - \mu_j\left(S_{L_j}\right) \tag{4.5}$$

$$ET_j = \Delta\mu_j^-(S_{L_j}, S_{L_{j+1}}) * N_j \tag{4.6}$$

$$ET_j' = \Delta\mu_j^+\left(S_{L_j}, S_{L_{j-1}}\right) * N_j' \tag{4.7}$$

Relations 4.1 and 4.2 show amount of necessary increase of resource from (R_{min}) and amount of necessary decrease of resource from (R_{max}) to Rs, respectively. 4.3 depicts number of paths that activity (e_j) was done on them except the paths that activity (e_j) was done on them and equation $(P(T_r \le t) = 1)$ is held related them. 4.4 depicts amount of reduction of completion time mean of activity (e_j) when the allocated resource is increased from (S_{L_j}) to $(S_{L_{j+1}})$ and 4.5 shows amount of increase of completion time mean of activity (e_j) when the allocated resource is reduced from (S_{L_j}) to (S_{L_j-1}) . 4.6 and 4.7 show effective time reduction and increase coefficient of activity (e_j) , respectively.

5 Proposed Algorithm for Allocation of Constrained Non-Renewable Resource in Stochastic Metagraphs

Steps of proposed algorithm are as follows:

Step 1. Calculate the (ΔR^+) and (ΔR^-) .

Step 2. If $(min(\Delta R^+,\ \Delta R^-) = \Delta R^+)$ then go to step 3, if $(min(\Delta R^+, \Delta R^-) = \Delta R^-)$ then go to step 4.
Note: if ΔR^+ and ΔR^- are equal, go to step 3 or step 4 arbitrary, but which offers a better answer cannot be predicted.

Step 3. Implement the sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ and go to step 5.

Step 4. Implement the sub-algorithm $Algorithm2\left(K', R^{(K')}\right)$ and go to step 5.

Step 5. Considering the results gained from implementation of proposed algorithm ($R^{(i)}$), obtain the $P = (T \le t|Rs)$ by using the simulation method.

Figure 1 shows the flow chart of the proposed algorithm.

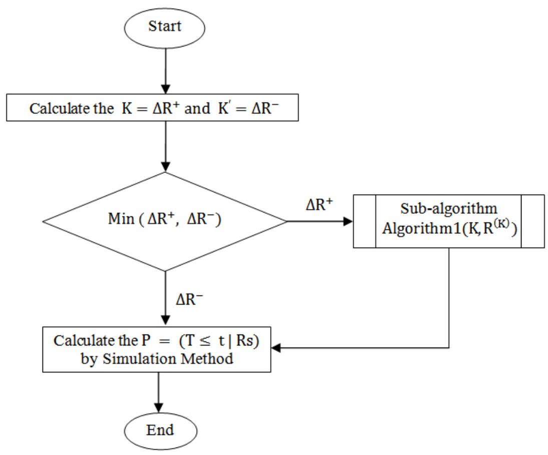

Figure 1: Flow chart of the proposed algorithm

Sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$

Step 1. $\Delta R^+ = K$, $L^{(i)} = (L_1 = 1, L_2 = 1, \ldots, L_n = 1), i = 0$, $R^{(i)} = \left(S_{L_1}^{(i)}, S_{L_2}^{(i)} \ldots, S_{L_n}^{(i)}\right)$ and consider $R^{(0)}$ as initial allocation.

Step 2. According to $R^{(i)}$, calculate the average of completion time of project paths. Choose a path that has maximum completion time

mean. If there are two or more paths with equal completion time mean, choose the path with maximum S.D. (standard deviation).
Note: If we have $p(T_r \leq t) = 1$ for a path, it will be removed from the calculations.

Step 3. Compute the ET_j for all activities of chosen path in the step 2

Step 4. based on the activities of chosen path in the step 2, select the activity that has the maximum ET_j . If there are two or more activities with equal ET_j , determine the activity according to the following prioritization:

(a) Choose an activity that lies on many effective paths.

(b) Choose an activity that has maximum $\Delta\mu_j^-\left(S_{L_j}, S_{L_{j+1}}\right)$.

Step 5. Choose L_j of selected activity in step 4 and set $L_j = L_j+1$ and $i = i+1$. According to L_j , determine $L^{(i)}$ and $R^{(i)}$. If $i < K$ then return to step 2 otherwise stop. It is obvious that $R^{(i)}$ specifies final allocation. Then return to main algorithm.

Sub-algorithm $Algorithm2\left(K', R^{(K')}\right)$

Step 1. $\Delta R^- = K'$, i = 0, $L^{(i)} = (L_1 = k_1, L_2 = k_2, \ldots, L_n = k_n)$, $R^{(i)} = \left(S_{L_1}^{(i)}, S_{L_2}^{(i)} \ldots, S_{L_n}^{(i)}\right)$ and consider $R^{(0)}$ as initial allocation, so that ($\sum_{j=1}^{n} S_{L_j}^{(0)} = R_{max}$).

Step 2. According to $R^{(i)}$, calculate the average of completion time of project paths. Choose the path with minimum completion time mean. If there are two or more paths with equalcompletion time mean, choose the path with minimum S.D..

Step 3. Compute the $ET_j^{'}$ for all activities of chosen path in the step 2.

Step 4. Based on the activities of chosen path in the step 2, select the activity that has the minimum $ET_j^{'}$. If there are two or more activities with equal $ET_j^{'}$, determine the activity according to the following prioritization:

1. Choose the activity that lies on few effective paths.
2. Choose the activity that has minimum $\Delta\mu_j^+\left(S_{L_j}, S_{L_{j-1}}\right)$.

Step 5. choose L_j of selected activity in step 4 and set $L_j = L_j-1$ and $i = i+1$. According to L_j , determine $L^{(i)}$ and $R^{(i)}$. If $i < K^{'}$ then return to step 2 otherwise stop. It is obvious that $R^{(i)}$ specifies final allocation. Then return to main algorithm.

6 Examples

In this section, for evaluating the efficiency of the proposed algorithm related to solving problems of allocation of non-renewable limited resource in stochastic metagraph, twenty exampleswere designed and solved by the proposed algorithm and simulation method. An example is explained in the following:

Suppose that a metagraph of a project has 8 activities, as shown in Figure 2, with RS = 30 and t = 17. The probability function of activities depends on the resource allocated to them and has been presented in Table 1.

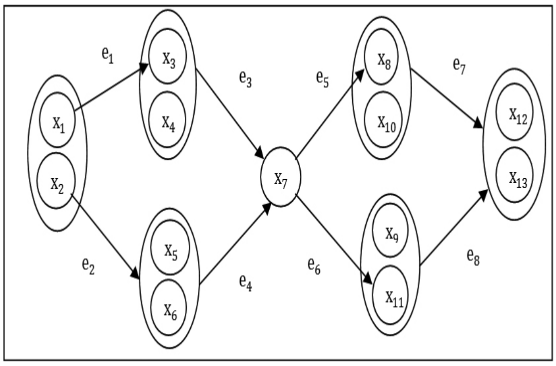

Figure 2: Flow chart of the proposed algorithm

Steps of proposed method for solving the aforementioned example are as follows:

Step 1. Compute (ΔR^+) and (ΔR^-)

$$\Delta R^- = R_{max} - Rs = 33 - 30 = 3 \quad (6.8)$$

$$\Delta R^+ = Rs - R_{min} = 30\ 25 = 5 \quad (6.9)$$

Step 2. Since min ($\Delta\ R^+$, $\Delta\ R^-$) = min (3,5) = 3 = $\Delta\ R^- = K^{'}$ then go to step 4.

Table 1: Probability function of metagraph activities of Figure 2

L_1	S_{L_1}	$P_1(S_{L_1}, D_1))$	L_2	S_{L_2}	$P_2(S_{L_2}, D_2)$
1	5	$=\frac{1}{6}\quad D_1 = 3$ $=\frac{5}{6}\quad D_1 = 4$	1	2	$=\frac{1}{3}\quad D_2 = 4$ $=\frac{2}{3}\quad D_2 = 5$
2	6	$=\frac{5}{6}\quad D_1 = 2$ $=\frac{1}{6}\quad D_1 = 3$	2	3	$=\frac{2}{3}\quad D_2 = 3$ $=\frac{1}{3}\quad D_2 = 4$
L_3	S_{L_3}	$P_3(S_{L_3}, D_3)$	L_4	S_{L_4}	$P_4(S_{L_4}, D_4)$
1	1	$=\frac{1}{2}\quad D_3 = 6$ $=\frac{1}{2}\quad D_3 = 7$	1	2	$=\frac{1}{4}\quad D_4 = 4$ $=\frac{3}{4}\quad D_4 = 5$
2	2	$=\frac{1}{3}\quad D_3 = 5$ $=\frac{2}{3}\quad D_3 = 6$	2	3	$=\frac{3}{4}\quad D_4 = 3$ $=\frac{1}{4}\quad D_4 = 4$
L_5	S_{L_5}	$P_5(S_{L_5}, D_5))$	L_6	S_{L_6}	$P_6(S_{L_6}, D_6)$
1	4	$=\frac{2}{7}\quad D_5 = 5$ $=\frac{5}{7}\quad D_5 = 6$	1	6	$=\frac{1}{2}\quad D_6 = 4$ $=\frac{1}{2}\quad D_6 = 5$
2	5	$=\frac{2}{5}\quad D_5 = 4$ $=\frac{2}{5}\quad D_5 = 5$	2	7	$=\frac{3}{5}\quad D_6 = 6$ $=\frac{2}{5}\quad D_6 = 4$
L_7	S_{L_7}	$P_7(S_{L_7}, D_7))$	L_8	S_{L_8}	$P_8(S_{L_8}, D_8)$
1	2	$=\frac{1}{3}\quad D_7 = 3$ $=\frac{2}{3}\quad D_7 = 4$	1	3	$=\frac{1}{6}\quad D_8 = 3$ $=\frac{5}{6}\quad D_8 = 4$
2	3	$=\frac{1}{4}\quad D_7 = 2$ $=\frac{3}{4}\quad D_7 = 3$	2	4	$=\frac{5}{6}\quad D_8 = 2$ $=\frac{1}{6}\quad D_8 = 3$

Step 4. Implement the sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$.

Steps of sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$ for solving the aforementioned example are as follows:

Step 1. $K' = 3, i = 0, L^{(i)} = (L_1 = 2, L_2 = 2, L_3 = 2, L_4 = 2, L_5 = 2, L_6 = 2, L_7 = 2, L_8 = 2), (R^{(i)} = (S^{(i)}_{L_1}, S^{(i)}_{L_2}, \dots, S^{(i)}_{L_n})$ and consider $R^{(0)} = (6, 3, 2, 3, 5, 7, 3, 4)$ as initial allocation.

Step 2. According to $R^{(i)}$, calculate the average of completion time of project paths and choose the shortest path. The results of calculations of step 2 have been shown in Table 2.

Step 3. The ET'_j of all activities of selected path in step 2 is calculated and the results have been displayed in Table 2.

Step 4. According to Table 2, path $e_2-e_4-e_6-e_8$ has the minimum completion time mean and activity e_6 on the this path has the minimum ET'_j . Thus one unit is reduced from the amount of resources allocated to activity e_6 .

Step 5. $L_6 = L_6 - 1 = 1$ and $i = i + 1 = 1.L^{(1)} = (L_1 = 2, L_2 = 2, L_3 = 2, L_4 = 2, L_5 = 2, L_6 = 1, L_7 = 2, L_8 = 2)$ and $R^{(1)} = (6, 3, 2, 3, 5, 6, 3, 4)$. since $i < 3$ then go to step 2.

Step 2. According to $R^{(1)}$, calculate the average of completion time of project paths and choose the shortest path. The results of calculations of step 2 have been shown in Table 3.

Step 3. The ET'_j of all activities of selected path in step 2 is calculated and the results can be seen in Table 3.

Step 4. According to Table 3, path $e_2-e_4-e_6-e_8$ has the minimum completion time mean and activity e_2 on the this path has the minimum $ET_j^{'}$. Thus one unit is reduced from the amount of resources allocated to activity e_2.

Step 5. $L_2 = L_2 - 1 = 1$ and $i = i+1 = 2.L^{(2)} = (L_1 = 2, L_2 = 1, L_3 = 2, L_4 = 2, L_5 = 2, L_6 = 1, L_7 = 2, L_8 = 2)$ and $R^{(2)} = (6,2,2,3,5,6,3,4)$. since $i < 3$ then go to step 2.

Step 2. According to $R^{(2)}$, calculate the average of completion time of project paths and choose the shortest path. The results of calculations of step 2 have been shown in Table 4.

Step 3. The $ET_j^{'}$ of all activities of selected path in step 2 is calculated and the results have been displayed in Table 4.

Step 4. According to Table 4, path $e_1-e_3-e_6-e_8$ has the minimum completion time mean and activity e_3 on the this path has the minimum $ET_j^{'}$. Thus one unit is reduced from the amount of resources allocated to activity e_3 .

Step 5. $L_3 = L_3 - 1 = 1, i = i+1 = 3, L^{(3)} = (L_1 = 2, L_2 = 1, L_3 = 1, L_4 = 2, L_5 = 2, L_6 = 1, L_7 = 2, L_8 = 2)$ and $R^{(3)} = (6,2,1,3,5,6,3,4)$. since $i = 3$ then stop and return to step 5 of main algorithm.

Step 5. The $R^{(3)} = (6,2,1,3,5,6,3,4)$ is allocation of available resource to project activities that was obtained by $Algorithm2\left(K', R^{\left(K'\right)}\right)$.

Considering allocation $R^{(3)}$, equation 6.10 has been calculated by simulation method:

$$P = (T \leq 17 | Rs = 30) = 0.9587 \qquad (6.10)$$

The proposed algorithm has been used for solving 20 problems in which the number of activities of their metagraphs varied from 5 to 15. For evaluation of the efficiency of the proposed algorithm, the resource allocations obtained by means of the proposed algorithm were compared to the optimal resource allocation which was gained by using examining all feasible resource allocations.

In 11 examples out of those 20, new heuristic method obtained the optimal resource allocation and the examples for which the optimal resource allocation wasnt achieved are: 1, 6,7,10,12,13,14,19 and 20. Results have been shown in Table 5.

In some of 9 problems for which the optimal allocations have not achieved, the probability of completion time of their metagraph, is close to optimal probability of completion time of their metagraph according to obtained resource allocation. This error was less than 0.01 in some case and in one case it was 0.2.

This issue has caused this curiosity that in these 9 examples, what sub-algorithm has been used by the proposed algorithm. Examinations indicate that in 8 examples out of these 9, the sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$ has been used. It seems that the efficiency of these sub-algorithms is not the same. Thus, all examples solved by sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$ were resolved by sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ and the results have been shown in table 6. The results indicate that although sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ in some of 8 examples could not obtain the optimal resource allocation, but it has not offered answers worse than answers of sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$. Sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ in 7 examples could not obtain the optimal resource allocation; however, the probability of completion time of their metagraphs, according to obtained resource allocation compared to sub-algorithm 2 is close to optimal probability of completion time of their metagraphs. So that maximum error has reached from 0.2 to less than 0.05. In other words, out of 20 examples solved by the sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ only in one example the error is less than 0.05 and in other 19 examples the error is insignificant.

It is obvious that both sub-algorithms reduce the calculations significantly. Table 7 depicts this

Table 2: Calculations of steps 2 and 3 in first iteration

ET'_j	N'_j	Increasable paths				Activity
		$e_2-e_4-e_6-e_8$	$e_2-e_4-e_5-e_7$	$e_1-e_3-e_6-e_8$	$e_1-e_3-e_5-e_7$	
-	-					e_1
2.6667	2					e_2
-	-					e_3
3	2					e_4
-	-					e_5
2.2	2					e_6
-	-					e_7
3.3333	2					e_8
Selected path:						Completion
$e_2-e_4-e_6-e_8$						time mean
Selected activity: e_6		12.15	13.9333	13.4	15.1833	of path

Table 3: Calculations of steps 2 and 3 in second iteration

ET'_j	N'_j	Increasable paths				Activity
		$e_2-e_4-e_6-e_8$	$e_2-e_4-e_5-e_7$	$e_1-e_3-e_6-e_8$	$e_1-e_3-e_5-e_7$	
-	-					e_1
2.6667	2					e_2
-	-					e_3
3	2					e_4
-	-					e_5
2.2	2					e_6
-	-					e_7
3.3333	2					e_8
Selected path:						Completion
$e_2-e_4-e_6-e_8$						time mean
Selected activity: e_2		13.25	13.9333	14.5	15.1833	of path

Table 4: Calculations of steps 2 and 3 in third iteration

ET'_j	N'_j	Increasable paths				Activity
		$e_2-e_4-e_6-e_8$	$e_2-e_4-e_5-e_7$	$e_1-e_3-e_6-e_8$	$e_1-e_3-e_5-e_7$	
3.3333	2					e_1
-	-					e_2
1.6667	2					e_3
-	-					e_4
-	-					e_5
-	-					e_6
-	-					e_7
3.3333	2					e_8
Selected path:						Completion
$e_1-e_3-e_6-e_8$						time mean

fact. For comparison of simulation method and proposed algorithm, consider the example 18 according to information shown in Tables 5, 6 and 7. In this example, the number of all resource allocation states is 4196 that among these states, 495 states are feasible. In other words, in these feasible states we have ($S_2 + \ldots + S_{12} = 42$).

For solving this example by using simula-

Table 5: Comparison of the proposed algorithm and simulation methods response

Example Number	Number of Activity	R_s	T	Obtained allocation by proposed algorithm	$P(T \leq t \mid Rs)$ related to obtained allocation by proposed algorithm	Obtained optimal allocation by simulation method	$P(T \leq t \mid Rs)$ related to by allocation by simulation method
1	5	18	6	(3,3,3,4,5)	0.7985	(4,3,3,3,5)	0.8071
2	5	17	7	(3,4,3,3,4)	0.9429	(3,4,3,3,4)	0.9431
3	5	16	8	(4,3,2,4,3)	0.9001	(4,3,2,4,3)	0.8999
4	5	16	7	(3,3,2,5,3)	0.6376	(3,3,2,5,3)	0.6375
5	6	18	11	(2,3,5,2,2,4)	0.9571	(2,3,5,2,2,4)	0.9571
6	6	20	7	(3,3,4,3,5,2)	0.75	(3,3,3,3,5,3)	0.8572
7	6	23	8	(4,2,5,3,5,4)	0.5446	(4,2,5,2,6,4)	0.5902
8	6	23	10	(3,5,3,4,4,4)	0.6809	(3,5,3,4,4,4)	0.6807
9	8	30	17	(6,2,1,3,5,6,3,4)	0.9587	(6,2,1,3,5,6,3,4)	0.9588
10	8	29	15	(2,4,4,5,3,2,4,5)	0.9315	(3,3,4,4,3,3,4,5)	0.9796
11	8	28	13	(2,4,3,4,2,6,3,4)	0.9484	(2,4,3,4,2,6,3,4)	0.9484
12	8	38	15	(2,5,3,2,6,8,5,7)	0.5312	(2,6,3,2,5,8,5,7)	0.5314
13	10	38	11	(5,3,1,4,6,1,3,4,7,4)	0.8278	(5,3,2,4,5,2,2,4,7,4)	0.9248
14	10	51	14	(5,4,3,9,9,6,1,6,3,5)	0.9267	(5,4,3,9,9,6,2,6,2,5)	0.9636
15	10	38	15	(8,6,2,4,4,2,1,4,4,3)	0.7021	(8,6,2,4,4,2,1,4,4,3)	0.7018
16	10	40	14	(6,6,6,2,2,3,3,7,3,2)	0.8812	(6,6,6,2,2,3,3,7,3,2)	0.8813
17	12	40	15	(3,2,4,4,3,4,1,5,3,3,2,6)	0.9861	(3,2,4,4,3,4,1,5,3,3,2,6)	0.9862
18	12	42	14	(2,3,2,4,5,5,3,5,2,3,6,2)	0.9644	(2,3,2,4,5,5,3,5,2,3,6,2)	0.9643
19	12	43	13	(2,2,3,5,5,4,1,7,3,4,4,3)	0.5677	(2,2,3,5,5,4,1,7,4,4,3,3)	0.7770
20	12	40	14	(3,3,4,5,5,2,4,3,3,3,2,3)	0.8709	(4,3,4,5,5,2,4,3,2,3,2,3)	0.9175

tion method 495 states should be investigated and finally, a state that maximizes the $P = (T \leq 14|Rs = 42)$ is chosen as optimal resource allocation. However, if we want to solve this example by using the proposed algorithm by four repetitions it can obtain the optimal resource allocation $(R_{max} - Rs = 46 = 42 = 4)$. It is obvious that solving the mentioned example by using the simulation method requires relatively long calculation time, but by using the proposed algorithm, calculation time is reduced significantly.

7 Discussion and Conclusion

In this paper, we proposed a new heuristic algorithm for allocation of constrained nonrenewable resource to edges (activities) of a stochastic metagraph to maximize the probability of completion time of stochastic metagraph before the due date of project. The results of calculations in 20 sample examples by using sub-algorithms $Algorithm1\left(K, R^{(K)}\right)$ and $Algorithm2\left(K', R^{\left(K'\right)}\right)$ show that in most problems th e proposed algorithm has been successful and the optimal probability of completion time of metagraph has been achieved approximately.

Also if we condone the increase of computational efforts and consider the quality of sub-algorithms solutions, the sub-algorithm $Algorithm1\left(K, R^{(K)}\right)$ is prior to sub-algorithm $Algorithm2\left(K', R^{\left(K'\right)}\right)$. It is obvious that the number of implementation loops of the proposed algorithm depends on the amount of available resource and if the number of specific levels of resource allocation for activities increase, as a result, the total number of n tuples ordered and the total number of feasible n tuples ordered of resource allocation will be increased.

The proposed algorithm in this paper requires less computational efforts compared to optimiza-

Table 6: Solving problem by using sub-algorithm Algorithm1 $(K, R^{(k)})$

Example Number	Number of Activity	R_s	t	Obtained allocation by sub-algorithm Algorithm1 $(K, R^{(k)})$	$P(T \leq t \mid Rs)$ related to obtained allocation by sub-algorithm Algorithm1 $(K, R^{(k)})$
1	5	18	6	(3,3,3,4,5)	0.7985
2	5	17	7	(3,4,3,3,4)	0.9429
3	5	16	8	(4,3,2,4,3)	0.9001
4	5	16	7	(3,3,2,5,3)	0.6376
5	6	18	11	(2,3,5,2,2,4)	0.9571
6	6	20	7	(3,3,3,3,5,3)	0.8573
7	6	23	8	(4,2,5,3,5,4)	0.5446
8	6	23	10	(3,5,3,4,4,4)	0.6809
9	8	30	17	(6,2,1,3,5,6,3,4)	0.9587
10	8	29	15	(3,3,4,5,3,2,4,5)	0.9795
11	8	28	13	(2,4,3,4,2,6,3,4)	0.9484
12	8	38	15	(2,5,3,2,6,8,5,7)	0.5312
13	10	38	11	(5,3,2,4,6,1,2,4,7,4)	0.9212
14	10	51	14	(5,4,3,9,9,6,2,6,2,5)	0.9636
15	10	38	15	(8,6,2,4,4,2,1,4,4,3)	0.7021
16	10	40	14	(6,6,6,2,2,3,3,7,3,2)	0.8812
17	12	40	15	(3,2,4,4,3,4,1,5,3,3,2,6)	0.9861
18	12	42	14	(3,3,2,4,5,5,3,4,2,3,6,2)	0.9643
19	12	43	13	(2,2,3,4,5,5,1,7,4,4,3,3)	0.7713
20	12	40	14	(4,3,4,5,5,2,4,3,2,3,2,3)	0.9176

Table 7: Volume of calculations in simulation method, proposed algorithm and sub-algorithm Algorithm1 $(K, R^{(k)})$

Example Number	Total number of allocation states	Number of feasible allocation states	R_{max}	R_{min}	Number of iterations of the proposed algorithm	Number of iterations of the sub-algorithm Algorithm1 $(K, R^{(k)})$
1	32	18	21	14	3	4
2	32	10	19	14	2	3
3	32	10	18	13	2	3
4	32	10	19	14	2	2
5	64	1	22	16	2	2
6	64	15	22	16	2	4
7	64	15	25	19	2	4
8	64	20	26	20	3	3
9	256	56	33	25	3	5
10	256	56	32	24	3	5
11	256	8	29	21	1	7
12	256	56	43	35	3	3
13	1024	120	41	31	3	7
14	1024	120	54	44	3	7
15	1024	210	44	34	4	4
16	1024	210	46	36	4	4
17	4096	495	44	32	4	8
18	4096	495	46	34	4	8
19	4096	792	48	36	5	7
20	4096	495	44	32	4	8

tion methods and examining all feasible resource allocations by using simulation method. so, the time of solving large-scaled problems is reduced significantly.

In this paper, according to the assumptions mentioned, by removing or reshaping each of the assumptions, we can define a new form of problem that is close to modeling reality, thus for the future studies, the following options are recommended:

1. In this paper, the difference between levels of allocable resource for each activity is considered a unit, for future studies it can be changed in any desired way.
2. This research can be conducted with renewable resource.
3. Considering the activities completion time as continuous random variables.
4. Considering preparation time for activities.
5. Allocation of limited resource to the activities can be done when we have more than one kind of non-renewable resource.
6. Allocation of constrained resource to the activities can be done when we have multiple resources (renewable and non-renewable).

References

[1] A. Basu, R. W. Blanning, Enterprise modeling using metagraphs, *In Proceedings of the IFIP TC8/WG8. 3 Working Conference on Decision Support Systems: Experiences and Expectations*,(pp. 183-199). North-Holland Publishing Co.(1992, June).

[2] A. Basu,R. W. Blanning, Metagraphs: A tool for modeling decision support systems, *Management Science* 40 (1994) 1579-1600.

[3] A. Basu, R. W. Blanning, Metagraphs, *Omega* 23 (1995) 13-25.

[4] A. Basu, R. W. Blanning, A metagraph-based DSS analysis workbench, In System Sciences, 1996, *Proceedings of the Twenty-Ninth Hawaii International Conference on* ,(1996, January), (Vol. 2, pp. 386-395). IEEE.

[5] A. Basu, R. W. Blanning, Metagraph Transformations and Workflow Management, *In Proceedings of the 30th Hawaii International Conference on System Sciences: Information Systems TrackInternet and the Digital Economy*-(1997, January) Volume 4 (p. 359). IEEE Computer Society.

[6] A. Basu , R. W. Blanning, Metagraphs in workflow support systems, *Decision Support Systems* 25 (1999) 199-208.

[7] A. Basu, R. W. Blanning, Workflow Analysis using Attributed Metagraphs, In *Proceedings of the 34th Annual Hawaii International Conference on System Sciences (HICSS-34)-Volume 9-Volume 9*(2001, January) (p. 9040). IEEE Computer Society.

[8] A. Basu, R. W. Blanning, Metagraphs and their applications (2007) (Vol. 15), *Springer Science & Business Media.*

[9] D. Gaur, A. Shastri, R. Biswas, Metagraph: a new model of data structure, *In Computer Science and Information Technology, 2008. ICCSIT'08. International Conference on*(2008, August) (pp. 729-733). IEEE.

[10] S. S. Hashemin, Constrained resource allocation in fuzzy metagraph, *10 th Iranian Conference on Fuzzy Systems, Shahid Beheshti University, Tehran, Iran.*(2010, July 13-15).

[11] S. S. Hashemin, Constrained Renewable Resource Allocation in Fuzzy Metagraphs via Min-Slack, *International Journal of Applied Operational Research* 1 (2011) 7-17.

[12] S. S. Hashemin, Time cost trade-off in uncertain metagraphs with trapezoidal fuzzy edge time, *Annals of Fuzzy Sets, Fuzzy Logic and Fuzzy System* 1 (2011) 117-127.

[13] S. S. Hashemin, Completion Time of Stochastic Metagraphs by Sampling from Edge Time and using the Conditional Monte-Carlo Simulation, *12th International*

Symposiumon Econometrics, Operations Research and statistis, Pamukkale University, Denizli, Turkey. (2011, May 26-29).

[14] S. S. Hashemin, S. M. T. Fatemi Ghomi, M. Modarres, Optimal constrained nonrenewable resource allocation in PERT networks with discrete activity times, *Scientia Iranica* 19 (2012) 841-848.

[15] M. Modarres, S. S. Hashemin, S. M. T. Fatemi Ghomi, Optimal constrained resource allocation in PERT: a hybrid approach, *International Journal of Engineering Science* 17 (2006) 61-68.

[16] H. Vahedi, S. S. Hashemin, A New Method for Allocation of Constrained Non-Renewable Resource in Fuzzy Metagraphs, *International Journal of Industrial Engineering* 2 (2015) 14-19.

Nastaran Akhoundi shiviyari is a graduate of M.Sc.Eng from industrial engineering department, Ardabil Branch, Islamic Azad University, Ardabil, Iran. Her field of interest is constrained resource allocation in schocastic metagraphs.

Seyed Saeid Hashemin is a graduate of Eng. D. from Industrial Engineering Department of Science and Research Brach, Islamic Azad University, Tehran, Iran. He is assisstant professor of Industrial Enginering Department, Ardabil Brach, Islamic Azad University, Ardabil, Iran. His field of interest is constrained cosumable and renewable resource allocation in uncertain projects which means projects with fuzzy activity times or stochastic activity times. In addition, he has conducted multiple researches regarding multi period production planning with stochastic demands. He has so far worked as advisor of more than 30 graduates of M.Sc. Eng. in industrial engineering.