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Research Article

Non-Convex Cost and Revenue Efficiency Analysis in Two-Stage Networks and Its Application to Iranian Airlines

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Abstract

In the real word, the decision maker may want to use revenue and cost efficiency analysis for existing units compared to virtual units, traditional DEA models can no longer be used and FDH models must be used to evaluate the efficiency of decision-making units. In this paper, we develop the revenue and cost efficiency models based on the FDH model and obtain the efficiency scores based on pair wise comparisons. In the following, we will develop the proposed models for the two-stage network structure and obtain the desired scores of inputs and outputs by considering the input and output prices. An algorithm for measuring the revenue and cost efficiency is presented based on the ratio of inputs and outputs. Finally, we use the proposed algorithm to evaluate the efficiency of 13 airports in Iran with a two-stage network structure without considering the constraint of convexity and present the results of the research.

Keywords : DEA; Free Disposal Hull; Cost and Revenue efficiency; Airlines; Two-stage network.

1 Preliminaries and problem formulation

THe airline industry is considered as one of the main backbones of economy in all countries, as well as being an integral part of the universal economy. Policy-makers, the mass media and different industries almost always have an eye on the airlines; furthermore, the economic situation of airlines can, under specific conditions, have positive or negative effects on other associated industries, such as the aircraft industry and the tourism industry. Nowadays, in most societies, almost everyone has at least one flying experience during their lifetime. Due to the importance of the airline industry, a large number of researchers have recently employed numerous methods to measure the efficiency of airlines. Oum et al. [24] made an assessment of productivity in the worlds major airports in 2003. Yu et al. [35] did a study on the productivity growth of Taiwans domestic airports. Merkert and Mangia [23] used different DEA models to determine the cost efficiency of airports. However, all these studies consider each airline as a black box and analyze the companys performance through a compari-

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Table 1: DEA-based studies on airport efficiency.

Authors	Methodology
Suzuki and Nijkamp [29]	DEA DFM with fixed factors
Fernandes and Pacheco [11]	DEA CRS/VRS envelopment, output-oriented models
Yoshida and Fujimoto [34]	Two-stage DEA model:CRS/VRS envelopment, input oriented models/ Tobit regression to assess the impact of contextual variables
Pacheco et al. [26]	DEA VRS envelopment, output-oriented model due to different airport sizes
Barros and Dieke [1]	Two-stage DEA model:CRS/VRS envelopment, output oriented models/balanced panel data with Tobit regression for bootstrapped estimates
Barros and Dieke [2]	Two-stage DEA models CRS/VRS envelopment output oriented models/bootstrapped DEA scores with a truncated regression
Yuen and Zhang [36]	Two-stage DEA model: VRS envelopment, output-oriented model/balanced panel data with Tobit regression for a random effects model
Gillen and Lall [12]	Two-stage DEA model. First stage:CRS/VRS envelopment, output oriented models. Second stage: balanced panel data with To bit regression for a random effects model
Lin and Hong [15]	DEA VRS multiplier input-oriented model
Liu [17, 18]	Network DEA
Peter Wanke and Barros [33]	DEA Cost structure
Wanke [32]	Network-DEA with two stages

son of inputs and outputs in the whole system. For instance, in the studies conducted by Fan et al. [8] and Shao and Sun [28] relating performance evaluation of airports, the common feature is that they both consider each airport as a black box and assess the operational performance of the airports without considering the influence of sub-processes on the networks overall operational efficiency, while disregarding the efficiency of internal sub-processes in the airlines.

Table 1 presents a number of DEA-based studies on efficiency evaluation of airports, along with their general specifications in terms of inputs, outputs and employed methodologies.

After comparison of all existing approaches to performance measurement, DEA (Charnes et al. [5]) was identified as the best method for data organization and analysis (Lim and Zhu [16]). Technically speaking, DEA is a non-parametric method that measures the relative efficiency of homogeneous decision-making units (DMUs) based on similar objectives and similar variables that can be categorized as inputs or outputs (Lim and Zhu [16]). In particular, we can find the applications of DEA in various service industries, such as airlines (duygun et al. [7]; Liu [17, 18]), banking (Paradi et al. [25]), financial services (Kaffash et al. [13]), hotels (Manasakis et al. [21]), rail transport (Bhanot et al. [3]) and telecommunication (Masson et al. [22]). DEA defines efficiency as a weighted sum of outputs divided by a weighted sum of inputs. To evaluate a DMU, we select the best possible set of weights for that DMU and calculate the efficiency score for all DMUs in the set using those weights. DMUs with the highest efficiency score are categorized as efficient and the others are deemed inefficient. We must note that conventional DEA methods do not consider the intermediate activities in their evaluation; in this regard, DEA treats such activities as a black box and only includes the initial inputs and final outputs in the analysis. Meanwhile, the activity of intermediate measures can be of great significance in the evaluation of certain decision-making units, such as hospitals, companies, airports and production lines; for this exact reason, researchers such as Fare and Grosskopf [10] and Tone and Tsutsui [30] decided to study the network DEA model aiming to involve the relationships between intermediate variables. Furthermore, Lozano et al. [19] and Maghbouli et al. [20] studied the performance of airports by categorizing the airport operations under two sub-activities and measuring the efficiency of each sub-activity in the course of one year. Fare and Grosskopf [9] extended the network DEA model to intermediate products. Seiford and Zhu [27] presented a two-stage network structure for evaluation of commercial banks in the United States. Castelli et al. [4] engaged in a discussion of units with a two-stage, or two-level, structure. Tone and Tsutsui [30] proposed a network model that evaluated the intermediate products.

Free Disposal Hull (FDH) is an experimental approach to the production possibility set (PPS) based on the unique assumption of Free Disposability (FD), in which the PPS applies. These models, initially formulated by Deprins et al. [6], are a special variation of DEA that contrary to DEA, do not require the assumptions of convexity and feasibility in formation of the reference technology. When using the FDH formula, each DMU is evaluated in comparison to other DMUs. The DMU under evaluation is considered efficient only when it is not dominated by any other DMUs. Since a convex combination of airlines is impossible and each company works independently with affiliation to the Civil Aviation Organization (CAO), we will make use of DEA-based FDH models in our evaluation of Iranian airlines. It is quite important to consider all input, output and intermediate data in the two-stage network DEA; in this regard, the network considers the airlines before take-off in stage 1, and stage 2 evaluates the airlines after the flight. Obviously, an accurate assessment of the personnel (pilots, co-pilots, etc.) would show different results before and after flight, and the outputs of both stages would be very important in the performance evaluation of airlines. Passenger load factor, which is defined as a ratio of passenger-kilometers traveled to seat-kilometers provided, is a significant parameter in the flight process; thus, we will determine the standard for inputs in the pre- and post-flight stages using cost efficiency; similarly, revenue efficiency will be used to determine the standard for outputs based on their prices.

Cost and revenue efficiency measurements are both among the possible evaluations of the network structure. Managers everywhere are always contemplating new ways to increase efficiency and productivity in their organizations. First introduced and discussed by Farrell, cost efficiency is a type of efficiency measure mostly significant to industrial and manufacturing units. According to Farrells proposed approach, cost efficiency of a decision-making unit is composed of two parts, technical efficiency and allocative efficiency, the combination of which would produce the overall efficiency, or the cost efficiency in this context. In cases where the input costs are available, the remaining question is: to which degree should we use the inputs in order to achieve the minimum overall costs. After answering that question, we proceed to measure the cost efficiency of our DMUs as a ratio of minimum costs to actual costs for the considered output vector.

The remainder of the study is organized as follows. The second section presents a brief review of basic concepts related to cost efficiency and DEA-FDH. In the third section, we propose our cost and revenue efficiency models in two-stage network FDH. In section four, we engage in a performance evaluation of 13 Iranian airlines in 2014 using the proposed models, and the conclusions are drawn in the final section.

2 A Review of Basic Concepts

In this section, we will provide a brief review of basic efficiency concepts in FDH-based models and models of cost and revenue efficiency.

2.1 Efficiency in FDH-based Models

Deprins et al. [6] were the first ones to formulate the FDH model; later on, Tulkens [31] developed and extended the model. Our main purpose here is to ensure that the evaluations of efficiency are exclusively affected by the observed performances.

Consider a set of decision-making units including $DMU_1, DMU_2, \cdots, DMU_n$ where DMU_j, $j = 1, \cdots, n$ uses m positive inputs x_{ij}, $i = 1, \cdots, m$ to produce s positive outputs y_{rj}, $r = 1, \cdots, s$. Let $x_j = (x_{1j}, \cdots, x_{mj}) \in R^m_+$ and $y_j = (y_{1j}, \cdots, y_{sj}) \in R^s_+$ be the input and output vectors of DMU_j, $j = 1, \cdots, n$ respectively. Suppose, $x_j = (x_{1j}, \cdots, x_{mj})$, $x_{ij} \geq 0$, $i = 1, \cdots, m$, and $y_j = (y_{1j}, \cdots, y_{sj})$, $y_{rj} \geq 0$, $r = 1, \cdots, s$, $j = 1, \cdots, n$.

In the following, we will present an input-oriented FDH model for evaluation of radial efficiency in DMU_o using mixed-integer programming (Kerstens and Eeckaut [14]):

$$\begin{aligned} \max \quad & \theta_o^{FDH} \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta_o^{FDH} x_{io}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \qquad r = 1, \cdots, s, \\ & \sum_{j=1}^{n} \mu_j = 1, \qquad j = 1, \cdots, n, \\ & \lambda_j = \delta \mu_j,\ \mu_j \in \{0,1\}, \delta \geq 0. \end{aligned} \tag{2.1}$$

Definition 2.1. *DMU_o would be an FDH efficient point if θ_o^{FDH}=1, and equally, DMU_o would be an FDH efficient point if (x_o, y_o) was a point on the production possibility frontier.*

We can produce the (Productivity possibility Set) PPS using our observations as following.

$$T_{FHD}=\{(x,y) \mid \sum_{j=1}^{n} \lambda_j x_j \leq x, \sum_{j=1}^{n} \lambda_j y_j \geq y,\ \lambda_j = \delta\mu_j,\ \mu_j \in \{0,1\}, \sum_{j=1}^{n} \mu_j = 1,\ j = 1, \cdots, n\}.$$

2.2 Cost Efficiency Model in DEA

In data envelopment analysis, if we consider the input prices and have access to the input and output vectors for all DMUs, we can determine the standard for inputs using the cost efficiency model. Model (2.2) is a linear programming problem that calculates the cost efficiency of DMU_o through the equation $CE = w^*/(\sum_{i=1}^{m} c_i x_{io})$.

$$\begin{aligned} w^* = \min \quad & \sum_{i=1}^{m} c_i x_i \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_i, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{ro}, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \quad j = 1, \cdots, n. \end{aligned} \tag{2.2}$$

2.3 Revenue Efficiency Model in DEA

In DEA, if we consider the output prices and have access to the input and output vectors of all our decision-making units, we will be able to calculate the standard for outputs via model (2.3), commonly known as the revenue efficiency model.

$$\begin{aligned} v^* = \max \quad & \sum_{i=1}^{m} p_r y_r \\ s.t. \quad & \sum_{j=1}^{n} \lambda_j x_{ij} \leq x_{io}, \quad i = 1, \cdots, m, \\ & \sum_{j=1}^{n} \lambda_j y_{rj} \geq y_r, \quad r = 1, \cdots, s, \\ & \lambda_j \geq 0, \quad j = 1, \cdots, n. \end{aligned} \tag{2.3}$$

model (2.3) is a linear programming problem that calculates the revenue efficiency of DMU_o through the equation

$$RE = (\sum_{r=1}^{s} p_r y_{ro})/v^*.$$

3 Cost and Revenue Efficiency in Two-stage Network FDH

In this section, we will propose cost and revenue efficiency models in two-stage network FDH, as well as a model for overall efficiency. It is important to be able to calculate the cost and revenue efficiency scores in a two-stage network through comparison of ratios, without having to solve a zero-one programming model.

3.1 Cost Efficiency in Two-stage Network FDH

Consider a two-stage network where the first stage uses the input vector to produce the output vector z_j. Vector z_j is known as the intermediate vector for DMU_j. Thus, the corresponding model for cost efficiency in the first stage of the FDH network is proposed as follows:

$$\begin{aligned}
\min \ & \sum_{i=1}^{m} c_i x_i \\
s.t. \ & \sum_{j=1}^{n} \lambda_j x_{ij} \le x_i, \quad i=1,\cdots,m, \\
& \sum_{j=1}^{n} \lambda_j z_{fj} \ge z_{fo}, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n} \mu_j = 1, \quad j=1,\cdots,n, \\
& \lambda_j = \delta\mu_j, \ \mu_j \in \{0,1\}, \delta \ge 0.
\end{aligned} \tag{3.4}$$

We use pair wise comparison to solve model (3.4). Now, instead of solving the nonlinear model (3.4), consider the following algorithm.

Step one: Using the second set of constraints in model (3.4), we calculate the value of λ_j^* through the following equation $\lambda_j^* = \max\{z_{fo}/z_{fj} \mid f=1,\cdots,l\}$.

Step Two: Using the first set of constraints in model (3.4), we calculate the value of x_i^* by considering λ_j^*.
Similarly, we can arrive at the cost efficiency of stage 2 in our input-oriented two-stage network structure by considering the cost vectors, weight vectors and intermediate measures of stage 1 and the final outputs of stage 2 in model (3.5), as follows:

$$\begin{aligned}
\min \ & \sum_{f=1}^{l} p_f z_f \\
s.t. \ & \sum_{j=1}^{n} \lambda_j z_{fj} \le z_f, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n} \mu_j = 1, \quad j=1,\cdots,n, \\
& \lambda_j = \delta\mu_j, \ \mu_j \in \{0,1\}, \delta \ge 0.
\end{aligned} \tag{3.5}$$

In a similar way, the following steps are suggested to calculate the cost efficiency in stage 2.

Step One: we calculate the value of λ_j^* through the following equation $\lambda_j^* = \max\{y_{ro}/y_{rj} \mid r=1,\cdots,s\}$.

Step Two: since the constraints related to $\sum_{j=1}^{n} \lambda_j^* z_{fj} \le z_f, \quad f=1,\cdots,l$, are all binding in optimality, then by considering λ_j^*, we will arrive at $z_f^* = \lambda_j^* z_{fj}, f=1,\cdots,l$.

Step Three: to calculate the value of z_f^*, we obtain the minimum calculated ratio by considering the vector p_f.
Similarly, we can arrive at the overall cost efficiency by considering a convex combination of inputs in both stage 1 (vector x) and stage 2 (vector z) in the equation $q^*/(\sum_{i=1}^{m} c_i x_{io} + \sum_{f=1}^{l} p_f z_{fo})$, where q^* is the optimal solution of model (3.6). Suppose $\alpha \in [0,1]$ is a real number.

$$\begin{aligned}
\min \ & (1-\alpha)\sum_{i=1}^{m} c_i x_i + \alpha \sum_{f=1}^{l} p_f z_f \\
s.t. \ & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \le x_i, \quad i=1,\cdots,m, \\
& \sum_{j=1}^{n} \lambda_j^1 z_{fj} \ge z_{fo}, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n} \lambda_j^2 z_{fj} \le z_f, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{ro}, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n} \mu_j^1 = 1, \quad j=1,\cdots,n, \\
& \lambda_j^1 = \delta\mu_j^1, \ \mu_j^1 \in \{0,1\}, \delta \ge 0, \\
& \sum_{j=1}^{n} \mu_j^2 = 1, \quad j=1,\cdots,n, \\
& \lambda_j^2 = \delta\mu_j^2, \ \mu_j^2 \in \{0,1\}, \delta \ge 0.
\end{aligned} \tag{3.6}$$

We propose the following algorithm for calculation of overall efficiency.
Step One: If we consider the constraint $\sum_{j=1}^{n} \lambda_j^2 y_{rj} \ge y_{ro}, \quad r=1,\cdots,s$, we will have

$\lambda_j^{2*} = \max \{y_{ro}/y_{rj} \mid r = 1,\cdots,s\}$ and by considering the constraint
$\sum_{j=1}^{n} \lambda_j^1 z_{fj} \geq z_{fo}, \quad f = 1,\cdots,l,$
we will arrive at
$\lambda_j^{1*} = \max \{z_{fo}/z_{fj} \mid f = 1,\cdots,l\}.$

Step Two: using λ_j^{1*}, λ_j^{2*} from step one and the binding constraints
$\sum_{j=1}^{n} \lambda_j^1 z_{fj} = z_{fo}, \quad f = 1,\cdots,l,$ and
$\sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_i, \quad i = 1,\cdots,m,$
we will have
$z_f^* = \max \{\lambda_j^{2*} z_{fj} \mid j = 1,\cdots,n\}$, and
$x_i^* = \max \{\lambda_j^{1*} x_{ij} \mid j = 1,\cdots,n\}.$

Step Three: we calculate the minimum value by considering x_i^* and z_f^* in the equation

$$(1-\alpha)\sum_{i=1}^{m} c_i x_i^* + \alpha \sum_{f=1}^{l} p_f z_f^*.$$

3.2 Revenue Efficiency in a Two-stage Network

In this section, by considering the output revenues in the first stage. Then, we calculate the objective function of model (3.7) using a and b, without having to solve the model.

$$\begin{aligned}
R_l^* = \max \quad & \sum_{f=1}^{l} \beta_f z_f \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_{io}, \quad i = 1,\cdots,m, \\
& \sum_{j=1}^{n} \lambda_j^1 z_{fj} \geq z_f, \quad f = 1,\cdots,l, \\
& \sum_{j=1}^{n} \mu_j^1 = 1, \quad j = 1,\cdots,n, \\
& \qquad \lambda_j^1 = \delta\mu_j^1, \ \mu_j^1 \in \{0,1\}, \delta \geq 0.
\end{aligned} \tag{3.7}$$

We can obtain the revenue efficiency of DMU_o in the FDH model from the equation $\sum_{f=1}^{l} \beta_f z_{fo}/R_l^*$ in this regard, R_l^* is calculated via model (3.7). It is difficult to solve model (3.7). Therefore, we solve it using the following equation, which represents the relationship between prices.

$a) \sum_{j=1}^{n} \lambda_j^1 x_{ij} \leq x_{io}$
$\rightarrow \lambda_j^{1*} = \max \{x_{io}/x_{ij} \mid i = 1,\cdots,m\}.$
$b) \sum_{j=1}^{n} \lambda_j^1 z_{fj} = z_f.$

Pair wise comparison is used to solve model (3.7). Now, instead of solving the nonlinear model (3.7), we consider the following algorithm:

Step One: using the input constraint set from model (3.7), we calculate the value of λ_j^{1*} through the following equation
$\lambda_j^{1*} = \max \{x_{io}/x_{ij} \mid i = 1,\cdots,m\}.$

Step Two: the value of R_l^* is calculated using the output constraint set and considering λ_j^{1*}.

In a similar manner, we calculate the revenue efficiency score of DMU_o in the second stage of the FDH network through the equation $\sum_{f=1}^{l} \varphi_r y_{ro}/R_2^*$ where R_2^* is the optimal solution of model (3.8).

$$\begin{aligned}
R_2^* = \max \quad & \sum_{r=1}^{s} \varphi_r y_r \\
s.t. \quad & \sum_{j=1}^{n} \lambda_j^2 z_{fj} \leq z_{fo}, \quad f = 1,\cdots,l, \\
& \sum_{j=1}^{n} \lambda_j^2 y_{rj} \geq y_r, \quad r = 1,\cdots,s, \\
& \sum_{j=1}^{n} \mu_j^2 = 1, \quad j = 1,\cdots,n, \\
& \qquad \lambda_j^1 = \delta\mu_j^2, \ \mu_j^2 \in \{0,1\}, \delta \geq 0.
\end{aligned} \tag{3.8}$$

Obviously, in order to obtain R_2^* from model (3.8), we first calculate $\lambda_j^{*2} = \max \{z_{fo}/z_{fj} \mid f = 1,\cdots,l\}$,
and then arrive at the value of R_2^* through the equation $\sum_{j=1}^{n} \lambda_j^{*2} y_{rj} = y_r, \quad r = 1,\cdots,s.$

Step Three: to calculate the value of R_2^*, we obtain the maximum calculated ratio by considering the vector φ_r.

Model (3.9) is proposed for calculation of overall revenue efficiency, as follows:

$$
\begin{aligned}
\max \ \ & (1-\alpha)\sum_{i=1}^{m}\beta_f z_f+\alpha\sum_{r=1}^{s}\varphi_r y_r \\
s.t. \ \ & \sum_{j=1}^{n}\lambda_j^1 x_{ij}\le x_{io}, \quad i=1,\cdots,m, \\
& \sum_{j=1}^{n}\lambda_j^1 z_{fj}\ge z_f, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n}\lambda_j^2 z_{fj}\le z_{fo}, \quad f=1,\cdots,l, \\
& \sum_{j=1}^{n}\lambda_j^2 y_{rj}\ge y_r, \quad r=1,\cdots,s, \\
& \sum_{j=1}^{n}\mu_j^1=1, \quad j=1,\cdots,n, \\
& \lambda_j^1=\delta\mu_j^1,\ \mu_j^1\in\{0,1\},\delta\ge 0, \\
& \sum_{j=1}^{n}\mu_j^2=1, \quad j=1,\cdots,n, \\
& \lambda_j^2=\delta\mu_j^2,\ \mu_j^2\in\{0,1\},\delta\ge 0.
\end{aligned}
\tag{3.9}
$$

In model (3.9), if we consider the first stage, i.e. $\lambda_j^{1*}=\max\{x_{io}/x_{ij}\mid i=1,\cdots,m\}$.
then $\sum_{j=1}^{n}\lambda_j^{1*}z_{fj}=z_f, \quad f=1,\cdots,l,$
furthermore, in the second stage, meaning $\lambda_j^{2*}=\max\{z_{fo}/z_{fj}\mid f=1,\cdots,l\}$,
we have $\lambda_j^{2*}y_{rj}=y_r$. Therefore, the overall efficiency is calculated through a convex combination of optimal solutions in the first and second stages. We propose the following algorithm for calculation of overall efficiency.

Step One: if we consider the constraint $\sum_{j=1}^{n}\lambda_j^1 x_{ij}\le x_{io}, \quad i=1,\cdots,m,$
we will have $\lambda_j^{1*}=\max\{x_{io}/x_{ij}\mid i=1,\cdots,m\}$,
and by considering the constraint $\sum_{j=1}^{n}\lambda_j^2 z_{fj}\le z_{fo}, \quad f=1,\cdots,l,$
we will arrive at $\lambda_j^{2*}=\max\{z_{fo}/z_{fj}\mid f=1,\cdots,l\}$.

Step Two: using $\lambda_j^{1*},\lambda_j^{2*}$ obtained in step one and the binding constraints $\sum_{j=1}^{n}\lambda_j^2 z_{fj}=z_{fo}, \quad f=1,\cdots,l,$
$\sum_{j=1}^{n}\lambda_j^2 y_{rj}=y_r, \quad r=1,\cdots,s,$
we will have
$z_f^*=\max\{\lambda_j^{1*}z_{fj}\mid j=1,\cdots,n\}$.
and $y_r^*=\max\{\lambda_j^{2*}y_{rj}\mid j=1,\cdots,n\}$.

Step Three: we calculate the maximum value by considering z_f^* and y_r^* in the equation $(1-\alpha)\sum_{i=1}^{m}\beta_f z_f^*+\alpha\sum_{r=1}^{s}\varphi_r y_r^*$.

4 Applied Study

To assess the study objectives, in this section, we engage in a study of 13 Iranian Airlines during the year 2014. To this end, we will apply the presented models in Section 3 to data from the 13 airlines, including inputs, intermediate measures and final outputs as presented in Tables 2, 3 and 4, respectively. The airlines each have a two-stage network production process, where the first stage has four inputs (seat-kilometers provided, airport service personnel, administrative personnel and other employees) and four intermediate products (Number of flights, times of departure, flight attendants and flight engineers), considered as outputs for the first stage and inputs for the second; stage two also has three outputs, including ton-kilometers provided, passengers transported and total collected duties collected. Table 5 presents the more significant results obtained in this study. The airlines were each considered as a two-stage network as follows. Table 2 shows the input data related to the two-stage network structures of the Iranian airlines under study in the year 2014, which enter the network in the first stage. The first input was the seat-kilometers provided

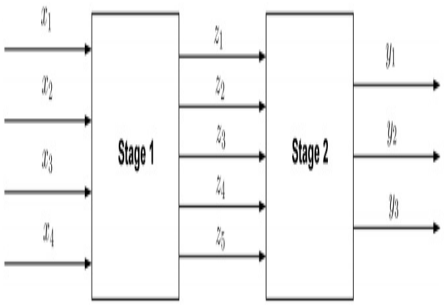

Figure 1: Two-stage FDH network related to Iranian airlines.

Table 2: Input data related to the two-stage networks of the 13 Iranian airlines in 2014.

DMU	x_1	x_2	x_3	x_4
	Seat-Kilometers Provided (in thousands)	Airport Service Personnel	Administrative Personnel	Other Personnel
IranAir(Homa)	1838923	1802	657	2575
Iran Airtour	1216026	101	54	164
Ata	1371597	72	34	97
Aseman	1925524	282	180	645
Taban	1248996	105	56	282
Zagros	2210951	51	30	174
PuyaAir	11509	3	8	10
GhesmAir	662626	86	243	1
Kaspian	1070398	31	24	232
KishAir	1517847	108	128	174
mahanair	3295267	227	146	1182
meraj	26319	44	12	75
naftiran	493163	88	41	174

Table 3: Intermediate data related to the two-stage networks of the Iranian airlines in 2014.

DMU	z_1	z_2	z_3	z_4	z_5
	Number of Flights	Departure Times	Pilots Co-pilots	Flight Attendants	Flight Engineers
IranAir(Homa)	20212	29267	326	696	26
Iran Airtour	9439	13656	61	160	1
Ata	13294	18600	55	108	1
Aseman	25697	37242	209	328	14
Taban	10225	12314	100	109	1
Zagros	16239	20082	95	122	17
PuyaAir	567	683	30	12	15
GhesmAir	9913	13205	90	157	1
Kaspian	8849	10165	48	97	28
KishAir	14005	19895	65	119	1
mahanair	20730	26194	293	983	11
meraj	2162	12265	31	74	1
naftiran	1025	11553	45	53	32

(numbers presented in thousands), the second input involved the airport service personnel and the third and fourth inputs were related to the administrative personnel and other employees, respectively. Table 3 includes the intermediate data produced in stage one of the two-stage network to be used as inputs in the second stage. The four intermediate measures included number of flights, times of departure, pilots and co-pilots, flight attendants and flight engineers.

Table 4 presents the output data produced by the two-stage network FDH structures under study, which included the ton-kilometers provided, passengers transported and total duties collected. As the first input, the ton-kilometers provided is obtained from a multiplication of the total weight transported at each departure and arrival (in tons) by the distance between the flight origin and destination. The second column shows the output data corresponding to passengers transported, which is the number of profitable passengers transported multiplied by the distance traveled in each flight stage. Finally, the third column of the table provides data related to total customs duties collected per each airline under study. Table 5 includes the cost and revenue efficiency scores calculated for each stage, as well as the whole system. The second and third columns

Table 4: Output data related to the two-stage networks of the Iranian airlines in 2014.

DMU	O_1	O_2	O_3
	Ton-Kilometers Provided	Passengers Transported	Total Collected Duties
IranAir(Homa)	155781	2173667	283411100000
Iran Airtour	96164	1292895	1874250000
Ata	130267	1855066	21726250000
Aseman	152382	2226634	51821350000
Taban	113569	1423173	98233800000
Zagros	141894	1839755	37750300000
PuyaAir	17043	15062	255500000
GhesmAir	47212	802684	1386521500000
Kaspian	89139	1077191	30725100000
KishAir	126949	1368319	555810500000
mahanair	234907	2934851	359038750000
meraj	23304	300939	25826150000
naftiran	34808	645130	3155050000

Table 5: Cost, revenue and overall efficiency in two-stage network FDH.

DMU	Cost efficiency Model (3.4)	Cost efficiency Model (3.5)	Overall cost Model (3.6)	Revenue efficiency Model (3.7)	Revenue efficiency Model (3.8)	Overall revenue Model (3.9)
IranAir (Homa)	0.36	0.68	0.52	0.58	0.03	0.3
Iran Airtour	0.09	0.88	0.48	0.21	0.57	0.39
Ata	0.11	0.92	0.51	0.27	0.41	0.34
Aseman	0.19	0.56	0.37	1	0.99	0.99
Taban	0.1	1	0.55	0.21	0.23	0.22
Zagros	0.15	0.8	0.47	0.5	0.69	3.7
PuyaAir	1	0.2	0.6	0.003	0.03	0.01
GhesmAir	0.18	0.54	0.36	0.17	0.36	0.26
Kaspian	0.16	0.89	0.52	0.12	0.56	0.34
KishAir	0.11	0.64	0.37	0.42	0.39	0.4
mahanair	0.1	0.97	0.53	1	1	1
meraj	1	0.3	0.66	0.01	0.04	0.02
naftiran	0.39	0.81	0.6	0.06	0.31	0.18

show the cost efficiency scores in stages one and two corresponding to models (3.4) and (3.5), respectively. The efficiency scores varied between 0.09 - 1 in stage one, where 0.09 belonged to the most inefficient unit, meaning AirTour airlines, and 1 pertained to PuyaAir and Meraj airlines. In the second stage, Taban airlines was the only unit deemed cost efficient. As can be observed by the results of our cost efficiency evaluations in stage 2, MahanAir airlines is quite close to being cost efficient; in order to achieve cost efficiency, the airline needs to reduce the first input of stage 2 from 20730 to 10225, the second input from 26194 to 12314, the third input from 293 to 100 and the fourth input from 983 to 109. Moreover, the fifth input needs to be reduced from 11 to 1 in this airline. MahanAir airlines had the best performance among units in stages one and two, as well as the overall network, and was deemed a revenue efficient unit. Column 4 presents the overall cost efficiency scores corresponding to model (3.6). None of the units were found efficient in our overall cost efficiency evaluations; in this relation, GheshmAir was deemed the most inefficient airline with an efficiency sore of 0.36. In Table 4, columns five and six provide the revenue efficiency scores in the first and second stages of the network, corresponding to models (3.7) and

(3.8), respectively. Aseman and MahanAir airlines were evaluated as revenue efficient units in the first stage of the two-stage FDH network and PuyaAir, with an efficiency score of 0.003, was the most inefficient unit among airlines. In stage two, MahanAir was found as the most efficient airline and IranAir and PuyaAir were the most inefficient units, both with the efficiency score of 0.03. Comparing the revenue efficiency scores related to Aseman airlines in Table 4, we find that the airline had its best performance in the first stage, where it was deemed revenue efficient; this unit is also very close to efficiency in stage 2, as well as the overall network, and needs to make changes in its outputs in order to achieve this goal. For instance, the unit should increase its first and second outputs from 0.648691 and 0.758687 to 1, respectively, and increase the third output from 0.003738 to 0.025898, if it wants to become efficient. The Tables last column presents the overall revenue efficiency scores corresponding to model (3.9); in this regard, Zagros airlines was the only unit considered revenue efficient in the evaluations, and PuyaAir was determined the most inefficient unit with a score of 0.01.

5 Conclusions

Nowadays, many countries consider the aviation industry a most significant factor in tourist attraction and income generation, and pay special attention to the sector as a result. In this study, we evaluated a number of Iranian airlines using two-stage network DEA based on FDH models. Elimination of the convexity constraint in the production possibility set (PPS) and use of cost and revenue efficiency models with consideration to input and output prices have led to a more accurate evaluation of the airlines. Pre-flight services (Stage 1) referred to the airport personnel and the seats provided, which determine the number of flights and departure times. In the post-flight process (Stage 2), number of passengers traveled, duties collected from the passengers and the transported cargo are of great significance. In this study, the cost and revenue efficiency scores determined the standard for inputs and outputs, respectively. Furthermore, based on the results of our cost and revenue efficiency evaluations, its obvious that the efficient units had inputs or outputs corresponding to the determined standards (although, in comparison with the other airlines). Considering the Iranian airlines in 2014, we can observe that 16 persent of the units are cost efficient in the first stage; in this regard, PuyaAir and Meraj airlines had appropriate inputs in stage one. However, only 7 persent of the airlines were cost efficient in stage two, where Taban airlines was the only one with standard inputs. In terms of revenue efficiency, only 16 persent of the units were found efficient; in this relation, Aseman and Meraj airlines had appropriate inputs in the first stage, but Meraj was the only revenue efficient unit in stage two. With a comparison of overall revenue and cost efficiency scores, we can see that Meraj airlines was revenue efficient in both pre- and post-flight processes, respectively; i.e. the airline had appropriate outputs in both stages. However, none of the airlines had a desirable overall revenue efficiency. For future research, we suggest calculating the Malmquist productivity index and determining the returns to scale assumption in two-stage airline processes (pre- and post-flight).

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