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Research Article

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A Novel Transformation Method for Solving Complex Interval Matrix

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Abstract

Since complex interval matrix have many applications in different fields of science, in this paper interval complex matrix system as $[W][Z] = [K]$ in which $[W], [K]$ are $n \times n$ known interval complex matrices and $[Z]$ is $n \times n$ unknown interval complex matrix is studied.

Using operations on interval complex numbers and matrices and defining a theorem, two auxiliary addition and subtraction complex systems are introduced and proved. Then, using the equality property of two complex numbers, the auxiliary interval complex systems are transformed to real crisp systems. Then the new system is solved and $[Z]$ is achieved. Finally, some numerical examples are given to illustrate the applicability and ability of the proposed approach.

Keywords : Interval linear system; Interval complex number; Matrix system; Complex number; Crisp systems.

1 Introduction

Modeling of many problems in various sciences leads to matrix systems. In particular, when the variables are complex interval, we deal with interval complex matrix system. The study of complex systems has recently been considered by the authors. [?]-[?].
Introduction of interval computations is studied in [?]. In 1998, Petkovic considered complex interval arithmetic and its applications [?]. Candau and et al expressed Complex interval arithmetic in [?] using polar form. Gay and Hladik solved interval and complex systems ([?, ?]). Recently, Behera and Chakraverty solved the fuzzy complex system of linear equations in [?] using the addition and the subtraction systems. In [?], the definition of the multiplication of the imaginary interval number proposed in [?] was modified. Ghanbari in [?] expressed and corrected the defects in [?]. Chelabi in [?] solved fuzzy dual complex linear systems using the addition and the subtraction systems.
In this paper, we solve the interval complex matrix system using auxiliary systems (addition and subtraction).
This paper is organized as follows: In Section 2 we present some preliminaries of interval number and system. Interval complex matrix system of linear equations with the proposed method are explained in Sections 3. In Section ??, numerical examples are presented which verify the efficiency and applicability of the proposed method. Conclusion are drawn in Section ??.

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2 Basic Definitions

Definition 2.1 *[?] An interval number* $[x]$ *is defined as the set of real numbers such that* $[x] = [\underline{x}, \overline{x}] = \{x' \in R : \underline{x} \leq x' \leq \overline{x}\}$ *where* $\underline{x} \leq \overline{x}$.

Definition 2.2 *(Interval arithmetic) For arbitrary interval numbers* $[x] = [\underline{x}, \overline{x}]$ *and* $[y] = [\underline{y}, \overline{y}]$, *we define addition and multiplication by a scalar as:*

$$[x] + [y] = [\underline{x}+\underline{y}, \overline{x}+\overline{y}], \quad (2.1)$$

$$\lambda\,[x] = \begin{cases} [\lambda\underline{x}, \lambda\overline{x}], & \text{if } \lambda > 0 \\ [\lambda\overline{x}, \lambda\underline{x}], & \text{if } \lambda < 0 \end{cases} \quad (2.2)$$

$$[x] \times [y] = [\min\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\overline{y}, \overline{x}\underline{y}\}, \max\{\underline{x}\underline{y}, \underline{x}\overline{y}, \overline{x}\overline{y}, \overline{x}\underline{y}\}] \quad (2.3)$$

3 Solution of interval complex matrix system

Definition 3.1 *An arbitrary interval complex number may be represented as* $X = [p] + i[q]$, *where* $[p] = [\underline{p}, \overline{p}]$ *and* $[q] = [\underline{q}, \overline{q}]$ *are interval numbers.*

Definition 3.2 *(Interval complex arithmetic) Let* $X = [\underline{p}, \overline{p}] + i\,[\underline{q}, \overline{q}]$, $Y = [\underline{w}, \overline{w}] + i\,[\underline{v}, \overline{v}]$, *so we have:*

$$X + Y = [\underline{p}+\underline{w}, \overline{p}+\overline{w}] + i\,[\underline{q}+\underline{v}, \overline{q}+\overline{v}] \quad (3.4)$$

$$X * Y = (\,[\underline{p}, \overline{p}] \times [\underline{w}, \overline{w}]) + i\,([\underline{q}, \overline{q}] \times [\underline{v}, \overline{v}]\,) \quad (3.5)$$

Definition 3.3 *Let* $X = [p] + i\,[q]$:

1. If $[p] \geq 0$, $[q] \geq 0$, we say $X \geq 0$.
2. If $[p] \leq 0$, $[q] \leq 0$, we say $X \leq 0$.
3. If $[p] = [\underline{p}, \overline{p}]$, $[q] = [\underline{q}, \overline{q}]$, $(\underline{p}, \underline{q} \leq 0), (\overline{p}, \overline{q} \geq 0)$, we say $0 \in X$.

Definition 3.4 (W) *is called interval complex matrix if at least one of its elements is an interval complex number.*

Definition 3.5 *The* $n \times n$ *matrix system*

$$\begin{pmatrix} [w_{11}] & [w_{12}] & \cdots & [w_{1n}] \\ [w_{21}] & [w_{22}] & \cdots & [w_{2n}] \\ \vdots & & \vdots & \\ [w_{n1}] & [w_{n2}] & \cdots & [w_{nn}] \end{pmatrix} \begin{pmatrix} [z_{11}] & [z_{12}] & \cdots & [z_{1n}] \\ [z_{21}] & [z_{22}] & \cdots & [z_{2n}] \\ \vdots & & \vdots & \\ [z_{n1}] & [z_{n2}] & \cdots & [z_{nn}] \end{pmatrix} = \begin{pmatrix} [k_{11}] & [k_{12}] & \cdots & [k_{1n}] \\ [k_{21}] & [k_{22}] & \cdots & [k_{2n}] \\ \vdots & & \vdots & \\ [k_{n1}] & [k_{n2}] & \cdots & [k_{nn}] \end{pmatrix} \quad (3.6)$$

Where $(W) = ([w_{ij}])_{n\times n}$ & $(Z) = ([z_{ij}])_{n\times n}$, $(K) = ([k_{ij}])_{n\times n}$ are $n \times n$ interval complex matrix, is called interval complex matrix system (ICMS).

Proposed method

System (3.6) can be represented as follow:

$$\sum_{k=1}^{n} [w_{ik}][z_{kj}] = [k_{ij}] \qquad 1 \leq i,j \leq n \quad (3.7)$$

By substuting

$[w_{ik}] = [a_{ik}] + i\,[b_{ik}]$, $[z_{kj}] = [p_{kj}] + i\,[q_{kj}]$, $[k_{ij}] = [u_{ij}] + i\,[v_{ij}]$ in (3.7), we have:

$$\sum_{k=1}^{n} (\,[a_{ik}] + i\,[b_{ik}]\,)\,(\,[p_{kj}] + i\,[q_{kj}]\,) =$$

$$1 \leq i,j \leq n (3.8)$$

Therefore

$$\sum_{k=1}^{n} \Big([a_{ik}]\,[p_{kj}] + i\,[a_{ik}]\,[q_{kj}] + i\,[b_{ik}]\,[p_{kj}] - [b_{ik}]\,[q_{kj}]\Big)$$

$$= [u_{ij}] + i\,[v_{ij}] \qquad 1 \leq i,j \leq n \quad (3.9)$$

Now we consider three cases and using interval arithmetic we obtain: Case 1: $[z_{kj}] \geq 0$

$$
\begin{aligned}
&\sum_{[a_{ik}] \geq 0} [\underline{a}_{ik}\underline{p}_{kj}, \overline{a}_{ik}\ \overline{p}_{kj}] \\
&+\sum_{[a_{ik}] \leq 0} [\underline{a}_{ik}\overline{p}_{kj}, \overline{a}_{ik}\underline{p}_{kj}]+ \\
&\sum_{0\in[a_{ik}]} [\underline{a}_{ik}\overline{p}_{kj}, \overline{a}_{ik}\overline{p}_{kj}] \quad + \\
&i\sum_{[a_{ik}] \geq 0} [\underline{a}_{ik}\underline{q}_{kj}, \overline{a}_{ik}\ \overline{q}_{kj}] \\
&+i\sum_{[a_{ik}] \leq 0} [\underline{a}_{ik}\overline{q}_{kj}, \overline{a}_{ik}\underline{q}_{kj}] \\
&+i\sum_{0\in[a_{ik}]} [\underline{a}_{ik}\overline{q}_{kj}, \overline{a}_{ik}\overline{q}_{kj}] \\
&+\mathrm{i}\sum_{[b_{ik}] \geq 0} [\underline{b}_{ik}\underline{p}_{kj}, \overline{b}_{ik}\ \overline{p}_{kj}] \\
&+i\sum_{[b_{ik}] \leq 0} [\underline{b}_{ik}\overline{p}_{kj}, \overline{b}_{ik}\underline{p}_{kj}]+ \\
&i\sum_{0\in[b_{ik}]} [\underline{b}_{ik}\overline{p}_{kj}, \overline{b}_{ik}\overline{p}_{kj}] \\
&-\sum_{[b_{ik}] \geq 0} [\underline{b}_{ik}\underline{q}_{kj}, \overline{b}_{ik}\ \overline{q}_{kj}] \\
&-\sum_{[b_{ik}] \leq 0} [\underline{b}_{ik}\overline{q}_{kj}, \overline{b}_{ik}\underline{q}_{kj}] - \\
&\sum_{0\in[b_{ik}]} [\underline{b}_{ik}\overline{q}_{kj}, \overline{b}_{ik}\overline{q}_{kj}] \\
&= [\underline{u}_{ij}, \overline{u}_{ij}] + \mathrm{i}\ [\underline{v}_{ij}, \overline{v}_{ij}]
\end{aligned}
\tag{3.10}
$$

Case 2: $[z_{kj}] \leq 0$

$$
\begin{aligned}
&\sum_{[a_{ik}] \geq 0} [\overline{a}_{ik}\underline{p}_{kj}, \underline{a}_{ik}\overline{p}_{kj}]+ \\
&\sum_{[a_{ik}] \leq 0} [\overline{a}_{ik}\overline{p}_{kj}, \underline{a}_{ik}\underline{p}_{kj}] \\
&+\sum_{0\in[a_{ik}]} [\overline{a}_{ik}\underline{p}_{kj}, \underline{a}_{ik}\underline{p}_{kj}] \\
&+\mathrm{i}\sum_{[a_{ik}\ ]\geq 0} [\overline{a}_{ik}\underline{q}_{kj}, \underline{a}_{ik}\overline{q}_{kj}]+ \\
&i\sum_{[a_{ik}] \leq 0} [\overline{a}_{ik}\overline{q}_{kj}, \underline{a}_{ik}\underline{q}_{kj}]+ \\
&i\sum_{0\in[a_{ik}]} [\overline{a}_{ik}\underline{q}_{kj}, \underline{a}_{ik}\underline{q}_{kj}] \\
&+\mathrm{i}\sum_{[b_{ik}] \geq 0} [\overline{b}_{ik}\underline{p}_{kj}, \underline{b}_{ik}\overline{p}_{kj}]+ \\
&i\sum_{[b_{ik}] \leq 0} [\overline{b}_{ik}\overline{p}_{kj}, \underline{b}_{ik}\underline{p}_{kj}]+ \\
&i\sum_{0\in[b_{ik}]} [\overline{b}_{ik}\underline{p}_{kj}, \underline{b}_{ik}\underline{p}_{kj}] \\
&-\sum_{[b_{ik}] \geq 0} [\overline{b}_{ik}\underline{q}_{kj}, \underline{b}_{ik}\overline{q}_{kj}] - \\
&\sum_{[b_{ik}] \leq 0} [\overline{b}_{ik}\overline{q}_{kj}, \underline{b}_{ik}\underline{q}_{kj}] - \\
&\sum_{0\in[b_{ik}]} [\overline{b}_{ik}\underline{q}_{kj}, \underline{b}_{ik}\underline{q}_{kj}] \\
&= [\underline{u}_{ij}, \overline{u}_{ij}] + \mathrm{i}\ [\underline{v}_{ij}, \overline{v}_{ij}]
\end{aligned}
\tag{3.11}
$$

Case 3: $0\in[z_{kj}]$

$$
\begin{aligned}
&\sum_{[a_{ik}] \geq 0} [\overline{a}_{ik}\underline{p}_{kj}, \overline{a}_{ik}\overline{p}_{kj}] \\
&+\sum_{[a_{ik}] \leq 0} [\underline{a}_{ik}\overline{p}_{kj}, \overline{a}_{ik}\overline{p}_{kj}] \\
&+\sum_{0\in[a_{ik}]} [\underline{a}_{ik}\overline{p}_{kj}, \underline{a}_{ik}\underline{p}_{kj}] \\
&+\mathrm{i}\sum_{[a_{ik}] \geq 0} [\overline{a}_{ik}\underline{q}_{kj}, \overline{a}_{ik}\overline{q}_{kj}]+ \\
&i\sum_{[a_{ik}] \leq 0} [\underline{a}_{ik}\overline{q}_{kj}, \overline{a}_{ik}\overline{q}_{kj}]+ \\
&i\sum_{0\in[a_{ik}]} [\underline{a}_{ik}\overline{q}_{kj}, \underline{a}_{ik}\underline{q}_{kj}] \\
&+\mathrm{i}\sum_{[b_{ik}] \geq 0} [\overline{b}_{ik}\underline{p}_{kj}, \overline{b}_{ik}\overline{p}_{kj}]+ \\
&i\sum_{[b_{ik}] \leq 0} [\underline{b}_{ik}\overline{p}_{kj}, \overline{b}_{ik}\overline{p}_{kj}]+ \\
&i\sum_{0\in[b_{ik}]} [\underline{b}_{ik}\overline{p}_{kj}, \underline{b}_{ik}\underline{p}_{kj}] \\
&-\sum_{[b_{ik}] \geq 0} [\overline{b}_{ik}\underline{q}_{kj}, \overline{b}_{ik}\overline{q}_{kj}] - \\
&\sum_{[b_{ik}] \leq 0} [\underline{b}_{ik}\overline{q}_{kj}, \underline{b}_{ik}\underline{q}_{kj}] \\
&-\sum_{0\in[b_{ik}]} [\underline{b}_{ik}\overline{q}_{kj}, \overline{b}_{ik}\overline{q}_{kj}] \\
&= [\underline{u}_{ij}, \overline{u}_{ij}] + \mathrm{i}\ [\underline{v}_{ij}, \overline{v}_{ij}]
\end{aligned}
\tag{3.12}
$$

In the following by defining a theorem, two auxiliary addition and subtraction systems are introduced and proved for solving matrix system (3.6).

Theorem 3.1 *Let* $(A) = [a_{ij}],\ (B) = [b_{ij}], (P) = [p_{ij}],$ $(Q) = [q_{ij}],\ (U) = [u_{ij}],\ (V) = [v_{ij}]$, *so we have:*

1. $$((\underline{(A)\ \ (P)}+\overline{(A)\ \ (P)}) - ((\underline{(B)\ \ (Q)}+\overline{(B)\ \ (Q)}))+ i\ ((\underline{(A)\ \ (Q)}+\underline{(B)\ \ (P)})+ \overline{(A)\ \ (Q)}+\overline{(B)\ \ (P)}))= ((\underline{(U)}+\overline{(U)}) + \mathrm{i}\ ((\underline{(V)}+\overline{(V)}) \tag{3.13}$$

2. $$((\underline{(A)\,(P)} - \overline{(A)\,(P)}) + ((\underline{(B)\,(Q)} - \overline{(B)\,(Q)}))+ i\ ((\underline{(A)\,(Q)}+\underline{(B)\,(P)}) - \overline{(A)\,(Q)}+\overline{(B)\,(P)}) = ((\underline{(U)}-\overline{(U)}) + \mathrm{i}\ ((\underline{(V)} - \overline{(V)}) \tag{3.14}$$

Proof. Using matrixes and interval arithmetic and ((3.10)-(3.12)), the proof is obvious.

Proposed method:

for solving interval complex matrix system (3.6), auxiliary systems (??) and (??) are defined using operations on interval complex numbers and matrix. As two complex numbers are equal, if their integer sections are equal with each other, and their imaginary sections are equal with each other, we obtain:

$$(\underline{(A)\ (P)}+\overline{(A)\,(P)})-(\underline{(B)\,(Q)}+\overline{(B)\,(Q)})$$
$$=\underline{(U)}+\overline{(U)}$$

$$(\underline{(A)\,(Q)}+\underline{(B)\,(P)})+(\overline{(A)\,(Q)}+\overline{(B)\,(P)})$$
$$=\underline{(V)}+\overline{(V)}$$

$$(\underline{(A)\,(P)}-\overline{(A)\,(P)})+((\underline{(B)\,(Q)}-\overline{(B)\,(Q)})$$
$$=\underline{(U)}-\overline{(U)}$$
$$(\underline{(A)\,(Q)}+\underline{(B)\,(P)})-(\overline{(A)\,(Q)}+\overline{(B)\,(P)})=\underline{(V)}-\overline{(V)} \tag{3.15}$$

By solving the system (**??**), which is a crisp system, the solution of the system (3.6) is obtained.

4 Numerical example

In this section the performance of the proposed method is shown by presenting some examples. The proposed method provides the solution with less complexity and computations in comparison with similar methods in this regard.

Example 4.1 *Consider the following system*

$$\begin{pmatrix} [-1,3]+\mathrm{i}\,[0,2] & [1,4] \\ \mathrm{i}[-2,-1] & [0,3]+\mathrm{i}\,[-2,5] \end{pmatrix}\begin{pmatrix} [x_{11}] & [x_{12}] \\ [x_{21}] & [x_{22}] \end{pmatrix}$$
$$=\begin{pmatrix} [-4,42]+\mathrm{i}\,[3,41] & [-14,74]+\mathrm{i}\,[-2,65] \\ [-35,25]+\mathrm{i}\,[-32,65] & [-29,59]+\mathrm{i}\,[-42,65] \end{pmatrix}$$

We put:

$$A=\begin{pmatrix} [-1,3] & [1,4] \\ [0,0] & [0,3] \end{pmatrix},$$

$$B=\begin{pmatrix} [0,2] & [0,0] \\ [-2,-1] & [-2,5] \end{pmatrix},$$

$$P=\begin{pmatrix} [\underline{p}_{11},\overline{p}_{11}] & [\underline{p}_{12},\overline{p}_{12}] \\ [\underline{p}_{21},\overline{p}_{21}] & [\underline{p}_{22},\overline{p}_{22} \end{pmatrix}$$

$$,\ Q=\begin{pmatrix} [\underline{q}_{11},\overline{q}_{11}] & [\underline{q}_{12},\overline{q}_{12}] \\ [\underline{q}_{21},\overline{q}_{21}] & [\underline{q}_{22},\overline{q}_{22} \end{pmatrix}$$

$$,\ U=\begin{pmatrix} [-4,42] & [-14,74] \\ [-35,25] & [-29,59] \end{pmatrix}$$

$$,\ V=\begin{pmatrix} [3,41] & [-2,65] \\ [-32,65] & [-42,65] \end{pmatrix}$$

Using theorem 3.1, we obtain:

$$(2\overline{p}_{11}+\underline{p}_{21}+4\overline{p}_{21}-2\overline{q}_{11})$$
$$+i\ (2\overline{q}_{11}+\underline{q}_{21}+4\overline{q}_{21}+2\overline{p}_{11})$$
$$=38+44i$$

$$(2\overline{p}_{12}+\underline{p}_{22}+4\overline{p}_{22}-2\overline{q}_{12})$$
$$+i\ (2\overline{q}_{12}+\underline{q}_{22}+4\overline{q}_{22}+2\overline{p}_{12})$$
$$=60+63i$$

$$(3\overline{p}_{21}+2\overline{q}_{11}-3\overline{q}_{21}+\underline{q}_{11})+\mathrm{i}\ (3\overline{q}_{21}+3\overline{p}_{21}-2\overline{p}_{11}-\underline{p}_{11})$$
$$=-10+33i$$

$$(3\overline{p}_{22}-3\overline{q}_{22}+2\overline{q}_{12}+\underline{q}_{12})+\mathrm{i}\ (3\overline{q}_{22}-2\overline{p}_{12}+3\overline{p}_{22}-\underline{p}_{12})$$
$$=30+23i$$

$$(-4\overline{p}_{11}+\underline{p}_{21}-4\overline{p}_{21}-2\overline{q}_{11})+\mathrm{i}\ (-4\overline{q}_{11}+\underline{q}_{21}-4\overline{q}_{21}-2\overline{p}_{11})$$
$$=-46-38i$$

$$(-4\overline{p}_{12}-3\overline{p}_{22}-2\overline{q}_{12})$$
$$+i\ (-4\overline{q}_{12}+\underline{q}_{22}-4\overline{q}_{22}-2\overline{p}_{12})$$
$$=-88-67i$$

$$(-3\overline{p}_{21}-7\overline{q}_{21}-2\overline{q}_{11}+\underline{q}_{11})$$
$$+i\ (-2\overline{p}_{11}-7\overline{p}_{21}-3\overline{q}_{12}+\underline{p}_{11})$$
$$=-60-97i$$

$$(-3\overline{p}_{22}-2\overline{q}_{12}+\underline{q}_{12}-7\overline{q}_{22})$$
$$+i\ (-2\overline{p}_{12}-7\overline{p}_{22}-3\overline{q}_{22}+\underline{p}_{12})$$
$$=-86-107i$$

Finally, we have:

$$\begin{pmatrix} [x_{11}] & [x_{12}] \\ [x_{21}] & [x_{22}] \end{pmatrix}$$
$$=\begin{pmatrix} [\underline{p}_{11},\overline{p}_{11}]+\mathrm{i}\,[\underline{q}_{11},\overline{q}_{11}] & [\underline{p}_{12},\overline{p}_{12}]+\mathrm{i}\,[\underline{q}_{12},\overline{q}_{12}] \\ [\underline{p}_{21},\overline{p}_{21}]+\mathrm{i}\,[\underline{q}_{21},\overline{q}_{21}] & [\underline{p}_{22},\overline{p}_{22}]+\mathrm{i}\,[\underline{q}_{22},\overline{q}_{22}] \end{pmatrix}$$

$$=\begin{pmatrix} [1,2]+\mathrm{i}\,[0,3] & [8,10]+\mathrm{i}\,[1,7] \\ [4,9]+\mathrm{i}\,[6,7] & [10,11]+\mathrm{i}\,[5,6] \end{pmatrix}$$

Example 4.2 *Consider the following system*

$$\begin{pmatrix} [-1,0]+\mathrm{i}\,[2,3] & [1,7]+\mathrm{i}\,[-4,-2] \\ [-4,-2]+\mathrm{i}\,[3,5] & \mathrm{i}\,[1,5] \end{pmatrix}$$

$$\begin{pmatrix} [x_{11}] & [x_{12}] \\ [x_{21}] & [x_{22}] \end{pmatrix}$$

$$= \begin{pmatrix} [-15,8]+\mathrm{i}\,[-20,4] & [-29,11]+\mathrm{i}\,[-26,11] \\ [2,28]+\mathrm{i}\,[-15,5] & [11,47]+\mathrm{i}\,[-26,8] \end{pmatrix}$$

We put:

$$\mathrm{A} = \begin{pmatrix} [-1,0] & [1,7] \\ [-4,-2] & [0,0] \end{pmatrix},$$

$$B = \begin{pmatrix} [2,3] & [-4,-2] \\ [3,5] & [1,5] \end{pmatrix},$$

$$\mathrm{P} = \begin{pmatrix} [\underline{p}_{11},\overline{p}_{11}] & [\underline{p}_{12},\overline{p}_{12}] \\ [\underline{p}_{21},\overline{p}_{21}] & [\underline{p}_{22},\overline{p}_{22} \end{pmatrix},$$

$$Q = \begin{pmatrix} [\underline{q}_{11},\overline{q}_{11}] & [\underline{q}_{12},\overline{q}_{12}] \\ [\underline{q}_{21},\overline{q}_{21}] & [\underline{q}_{22},\overline{q}_{22} \end{pmatrix},$$

$$\mathrm{U} = \begin{pmatrix} [-15,8] & [-29,11] \\ [2,28] & [11,47] \end{pmatrix},$$

$$V = \begin{pmatrix} [-20,4] & [-26,11] \\ [[-15,5] & [-26,8] \end{pmatrix}$$

Using theorem 3.1, we have:

$$(7\underline{p}_{21}-\underline{p}_{11}+\overline{p}_{21}-3\underline{q}_{11}-2\overline{q}_{21}+4\underline{q}_{21})$$
$$+i\ (7\underline{q}_{21}-\underline{q}_{11}+\overline{q}_{21}+3\underline{p}_{11}-2\overline{p}_{21}+2\overline{p}_{11}-4\underline{p}_{21})$$
$$=-7-16\ i$$

$$(7\underline{p}_{22}-\underline{p}_{12}+\overline{p}_{22}-3\underline{q}_{12}+2\overline{q}_{22}-2\overline{q}_{12}+4\underline{q}_{22})$$
$$+i\ (7\underline{q}_{22}-\underline{q}_{12}+\overline{q}_{22}+3\underline{p}_{12}-2\overline{p}_{22}+2\overline{p}_{12}-4\underline{p}_{22}$$

$$(-2\overline{p}_{11}-4\underline{p}_{11}-5\underline{q}_{11}-5\underline{q}_{21}-3\overline{q}_{11}-\overline{q}_{21})$$
$$+i\ (-2\overline{q}_{11}-4\underline{q}_{11}+5\underline{p}_{11}+5\underline{p}_{21}+3\overline{p}_{11}+\overline{p}_{21})$$
$$=30-10\ i$$

$$(-2\overline{p}_{12}-4\underline{p}_{12}-5\underline{q}_{12}-5\underline{q}_{22}-3\overline{q}_{11}-\overline{q}_{21})$$
$$+i\ (-2\overline{q}_{12}-4\underline{q}_{12}+5\underline{p}_{12}+5\underline{p}_{22}+3\overline{p}_{12}+\overline{p}_{22})$$
$$=58-18\ i$$

$$(7\underline{p}_{21}+\underline{p}_{11}-\overline{p}_{21}+3\underline{q}_{11}-2\overline{q}_{21}-2\overline{q}_{11}+4\underline{q}_{21})$$
$$+i(7\underline{q}_{21}+\underline{q}_{11}-\overline{q}_{21}+3\underline{p}_{11}-2\overline{p}_{21}-2\overline{p}_{11}+4\underline{p}_{21})$$
$$=-23-24\ i$$

$$(7\underline{p}_{22}+\underline{p}_{12}-\overline{p}_{22}+3\underline{q}_{12}-2\overline{q}_{22}-2\overline{q}_{12}+4\underline{q}_{22})$$
$$+i\ (7\underline{q}_{22}+\underline{q}_{12}-\overline{q}_{22}+3\underline{p}_{12}-2\overline{p}_{22}-2\overline{p}_{12}+4\underline{p}_{22})$$
$$=-40-37\ i$$

$$(-2\overline{p}_{11}+4\underline{p}_{11}+5\underline{q}_{11}+5\underline{q}_{21}-3\overline{q}_{11}-\overline{q}_{21})+$$
$$i\ (-2\overline{q}_{11}+4\underline{q}_{11}+5\underline{p}_{11}+5\underline{p}_{12}-3\overline{p}_{11}-\overline{p}_{21})$$
$$=-26-20\ i$$

$$(-2\overline{p}_{12}+4\underline{p}_{12}+5\underline{q}_{12}+5\underline{q}_{22}-3\overline{q}_{12}-\overline{q}_{22})+$$
$$i(-2\overline{q}_{22}+4\underline{q}_{22}+5\underline{p}_{12}+5\underline{p}_{22}-3\overline{p}_{12}-\overline{p}_{22})$$
$$=-36-34\ i$$

Finally, we obtain:

$$\begin{pmatrix} [x_{11}] & [x_{12}] \\ [x_{21}] & [x_{22}] \end{pmatrix} =$$

$$\begin{pmatrix} [-2,-1]+\mathrm{i}\,[-2,0] & [-3,-2]+\mathrm{i}\,[-4,-2] \\ [-1,0]+\mathrm{i}\,[-2,0] & [-3,-2]+\mathrm{i}\,[-3,-1] \end{pmatrix}$$

5 Conclusion

In the paper, interval complex matrix system is solved. With the help of two addition and subtraction systems and using the features of interval complex numbers and matrix properties, we find the positive and negative solutions as well as the solutions including zero for the system. The proposed method is simple and doesn't need complex computations. In fact, the solution of interval matrix is achieved by solving a real crisp system. Finally, useful numerical examples are presented and the applicability of the method and the simplicity of the system solution is verified by the proposed method. In the future, we will solve the complex matrix fuzzy system by α-cut.

References

[1] S. Abolmasoumi, M. Alavi, A method for calculating interval linear system, *Journal of mathematics and computer science* 8 (2014) 193-204.

[2] E. Siahlooei, S. A. ShahzadehFazeli, Two Iterative Methods for Solving Linear Interval Systems, *Applied Computational Intelligence and Soft Computing* 11 (2018) 1256-1265.

[3] S. M. Rump, On the solution of interval linear systems, *Computing* 47 (1992) 337-353.

[4] T. Allahviranloo, F. Babakordi, Algebraic solution of fuzzy linear system as: $AX + BX = Y$, *Soft Computing* 21 (2017) 7463-7472.

[5] F. Khalili Goodarzi, N. A. Taghinezhad, S. H. Nasseri, A new fuzzy approach to solve a novel model of open shop scheduling problem, *University Politehnica of Bucharesr Scientific Bulletin-series A-Applied Mathematics and Physics* 76 (2014) 199-210.

[6] S. H. Nasseri, N. A.Taghi-Nezhad, A. Ebrahimnejad, A novel method for ranking fuzzy quantities using center of in circle and its application to a petroleum distribution center evaluation problem, *International Journal of Industrial and Systems Engineering* 27 (2017) 457-484.

[7] F. Taleshian, J. Fathali, N. A. Taghi-Nezhad, Fuzzy majority algorithms for the 1-median and 2-median problems on a fuzzy tree, *Fuzzy Information and Engineering* 10 (2018) 225-248.

[8] F. Babakordi , M. Adabitabar Firozja, Solving Fully Fuzzy Dual Matrix System With Optimization Problem, *Int. J. Industrial Mathematics* 12 (2020) 11-22.

[9] G. Alefeld, J. Herzberger, Introduction to interval computations, *Academic Press, New York,* (1983).

[10] M. S. petkovic, L. D. Petkovic, Complex interval arithmetic and itsapplications, *Wiley-VCH, Berlin*, (1998).

[11] Y. Candau, T. Raissi, N. Ramdani, L. Ibos, Complex interval arithmetic using polar form, *Reliable Comput.* 12 (2006) 1-20.

[12] D. M. Gay, Solving Interval Linear Equations, *SIAM Journal on Numerical Analysis* 19 (1982) 858-870.

[13] M. Hladik, Solution sets of complex linear interval systems of equations, *Reliab Computing* 14 (2010) 78-87.

[14] D. Behera, S. Chakraverty, Solving fuzzy complex system of linear equations, *Information Sciences* 277 (2014) 154-162.

[15] D. Behera, S. Chakraverty, Erratum to "Solving fuzzy complex system of linear equations", *Information Sciences* 369 (2016) 788-790.

[16] M. Ghanbari, A discussion on "Solving fuzzy complex system of linear equations", *Information Sciences* 402 (2017) 165-169.

[17] M. Chehlabi, Solving fuzzy dual complex linear systems, *Journal of Applied Mathematics and Computing* 60 (2019) 87-112.

[18] M. Ghanbari, T. Allahviranloo, E. Haghi, Estimation of algebraic solution by limiting the solution set of an interval linear system, *Soft computing* 5 (2012) 98-11.

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