Available online at http://ijim.srbiau.ac.ir/
Int. J. Industrial Mathematics (ISSN 2008-5621)
Vol. 14, No. 2, 2022 Article ID IJIM-1481, 7 pages
DOI: http://dx.doi.org/10.30495/ijim.2022.58349.1481
Research Article

Impulsive Control of Attitude Satellite With Quaternion Parameters

M. R. Niknam *[†], N. Abdi Sobouhi [‡]

Received Date: 2021-01-13 Revised Date: 2021-10-12 Accepted Date: 2022-02-01

Abstract

This article uses impulsive control along with quaternion parameters instead of Euler angles in kinematics equations of satellite. The quaternion parameters are applied to overcome singularity problem in the numerical solution. It is assumed that the satellite is subjected to deterministic external perturbations. At first, the chaotic behavior of system is investigated when there is no control on the system. Then, impulsive control is used to stabilize the satellite attitude around the equilibrium point of origin. Finally, simulation results are given to visualize the effectiveness and feasibility of the proposed method.

Keywords : Impulsive control; Lyapunov exponent; Satellite attitude; Chaotic system; Quaternion.

---

1 Introduction

Satellites are purposely located in orbit around the Earth, other planets, or the Sun. They should put themselves in the right direction relative to the Sun and Earth. Especially they have to maintain, their solar panels toward the Sun and their antennas toward the Earth.

It is important for us to control the position of the satellites due to the gradual deviation of their orientation as well as placing them in the new desired position. Many control have been introduced for this purpose so far. In general, they are classified as active or passive methods. Passive method has been studied in [8]. Some of the active methods include generalized predictive control method [5], sliding-mode approach [1], control method based on Lyapunov [3], nonlinear control based on linear matrix inequality [15], nonlinear H_∞ control [16], finite-time stabilization of satellite quaternion attitude [11], and Robust and optimal attitude control of spacecraft with disturbances [13].

In recent years, impulsive control has been widely used to stabilize chaotic systems [19, 20, 21, 9, 14]. Its necessity and importance lies in that, in some cases, the system cannot be controlled by continuous control.

The main purpose of this paper is to study the possibility of using impulsive control on chaotic attitude of satellite along with quaternion parameters instead of Euler angles in the satellite's kinematics equations. The reason behind using impulsive control is its considerable and compa-

---

*Corresponding author. rezanik82@yahoo.com, Tel:+98(914)4002744.

[†] Department of Mathematics, Khalkhal Branch, Islamic Azad University, Khalkhal, Iran.

[‡] Department of Education, Farhangian University, Tabriz, Iran.

rable speed as for achieving the equilibrium point of system and stablizing system around this point compared with other control methods, such as optimal control of satellite attitude [10, 12]. Also, quaternion parameters are used to overcome singularity problem in the numerical solution. This paper is organized as follows: Section 2, expresses the governing equations of satellite attitude based on quaternion parameters. Section 3, describes chaotic behavior of system using Lyapunov exponents (LEs). Impulsive control of satellite attitude is explained in section 4, and simulation results are shown in section 5. Finally, our concluding remarks are given.

2 Quaternion and motion equations

2.1 Quaternion

The unit quaternion vector provides a non-singular representation of satellite kinematic equations. The four-component quaternion vector is defined as [18]

$$q = iq_1 + jq_2 + kq_3 + q_4, \tag{2.1}$$

where i, j, and k are imaginary numbers satisfying the condition

$$\begin{aligned} i^2 &= j^2 = k^2 = -1, \\ ij &= -ji = k, \\ jk &= -kj = i, \\ ki &= -ik = j. \end{aligned} \tag{2.2}$$

In this definition q_4 is a scalar part, and $Q = [q_1 \;\; q_2 \;\; q_3]^T$ form a vector part. Thus the quaternion $q = [q_1 \;\; q_2 \;\; q_3 \;\; q_4]^T$ may be written as $q = [Q^T \;\; q_4]^T$. The norm of q is defined as

$$|q|= \sqrt{q_1^2 + q_2^2 + q_3^2 + q_4^2}. \tag{2.3}$$

2.2 Motion equations

The mathematical model of a satellite is described by kinetic and kinematic equations of motion.

2.2.1 Kinetic Equations

The relationship between angular velocity and torque in the body frame is expressed by kinetic equations. If we consider the satellite as a rigid object and also the inertia of its body is diagonal and along to the actuators, then kinetic equations can be obtained from a Newton-Euler formula [2]

$$\begin{aligned} I_x\dot{w}_x &= \left[(I_y - I_z)w_y w_z + \tau_x\right], \\ I_y\dot{w}_y &= \left[(I_z - I_x)w_x w_z + \tau_y\right], \\ I_z\dot{w}_z &= \left[(I_x - I_y)w_x w_y + \tau_z\right], \end{aligned} \tag{2.4}$$

where w_x, w_y, w_z are angular velocities around axes of the body, I_x, I_y, I_z are the inertial moments of satellite around its principal axes, and τ_x, τ_y, τ_z are torques around these axes.

2.2.2 Kinematic Equations

By regarding the satellite as an rigid object, the kinematics equations based on quaternion parameters are expressed as follows

$$\begin{aligned} \dot{q}_1 &= \tfrac{1}{2}\left(w_x q_4 - w_y q_3 + w_z q_2\right), \\ \dot{q}_2 &= \tfrac{1}{2}\left(w_x q_3 + w_y q_4 - w_z q_1\right), \\ \dot{q}_3 &= \tfrac{1}{2}\left(w_y q_1 - w_x q_2 + w_z q_4\right), \\ \dot{q}_4 &= -\tfrac{1}{2}\left(w_x q_1 + w_y q_2 + w_z q_3\right). \end{aligned} \tag{2.5}$$

The relationship between attitude and angular velocity is explained by the kinematic equations. In the following, we use the notation SA to refer to the equations (2.4) and (2.5).

3 Analysis of chaos in the SA system

In this section the LEs of the SA system are obtained by assuming initial conditions and constant values given in Table 1, under the perturbing torques [6, 17]

$$\begin{bmatrix} \tau_x \\ \tau_y \\ \tau_z \end{bmatrix} = \begin{bmatrix} -1200 & 0 & (1000)\frac{\sqrt{6}}{2} \\ 0 & 350 & 0 \\ -(1000)\sqrt{6} & 0 & -400 \end{bmatrix} \begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix}. \tag{3.6}$$

The value of each of exponents is depicted in Table 2, and existing positive LEs, indicate that the system is chaotic.

Table 1: Initial conditions and constant values of the SA system.

Attitudes	Values	Constants	Values
Groups	Stroke	Non- Stroke	PercentageCorrect
q_{10}	0.2425	$I_x(kgm^2)$	3000
q_{20}	0.04915	$I_y(kgm^2)$	2000
q_{30}	0.4645	$I_z(kgm^2)$	1000
q_{40}	0.8503		
$w_{x_0}(r/s)$	0.2		
$w_{y_0}(r/s)$	0.1		
$w_{z_0}(r/s)$	0.2		

Table 2: LEs values of the SA system.

LEs	Values	LEs	Values
λ_{q_1}	-7.8762 e-08	λ_{w_x}	+0.13533
λ_{q_2}	-7.8762 e-08	λ_{w_y}	+0.00954
λ_{q_3}	-7.8762 e-08	λ_{w_z}	-0.76986
λ_{q_4}	-7.8762 e-08		

4 Impulsive control of the SA system

Consider the general nonlinear system

$$\dot{x} = f(t, x), \tag{4.7}$$

where $f : R^+ \times R^n \longrightarrow R^n$ is contuniuous, $x \in R^n$ is the state variable. An impulsive control law of system (4.7) is given by a sequence, $\{t_i, u_i(x(t_i))\}$, which has the effect of suddenly changing the state of the system at the instants t_k, where $t_1 < t_2 < ... < t_k < ...,\lim\ t_k \to \infty$ as $k \to \infty$ and $t_1 > t_0$; that is

$$\Delta x\ |_{t_i} = x(t_i^+) - x(t_i) = u_i(x(t_i)), \tag{4.8}$$

where $x(t_i^+) = \lim_{t \longrightarrow t_i^+} x(t)$ and $x(t_i) = \lim_{t \longrightarrow t_i^-} x(t)$. Furthermore, $u_i(x(t_i))$, can be chosen as $B_i x(t_i)$ with B_i being $n \times n$ matrices. Accordingly, the impulsively controlled system can be expressed as follows

$$\begin{cases} \dot{x}(t) = f(t, x), & t \neq t_i \\ \Delta x = B_i x, & t = t_i \quad (i = 1, 2, ...) \\ x(t_0^+) = x_0, & \end{cases} \tag{4.9}$$

which is also called an impulsive differential system [7].

Decomposing the linear and nonlinear parts of the SA system we have

$$\dot{x}(t) = Ax(t) + Nx(t), \tag{4.10}$$

where

$$x = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ w_x \\ w_y \\ w_z \end{bmatrix}, N = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{(I_y - I_z)}{I_x} w_y w_z \\ \frac{(I_z - I_x)}{I_y} w_x w_z \\ \frac{(I_x - I_y)}{I_z} w_x w_y \end{bmatrix}, \tag{4.11}$$

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-1200}{I_x} & 0 & \frac{1000\sqrt{6}}{2I_x} \\ 0 & 0 & 0 & 0 & 0 & \frac{350}{I_y} & 0 \\ 0 & 0 & 0 & 0 & \frac{-1000\sqrt{6}}{I_z} & 0 & \frac{-400}{I_z} \end{bmatrix}. \tag{4.12}$$

The impulsive control system is given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Nx(t), & t \neq t_i \\ u_i(x(t_i)) = x(t_i^+) - x(t_i^-) = B_i x(t_i^-), & \\ x(t_0^+) = x_0, & \end{cases} \tag{4.13}$$

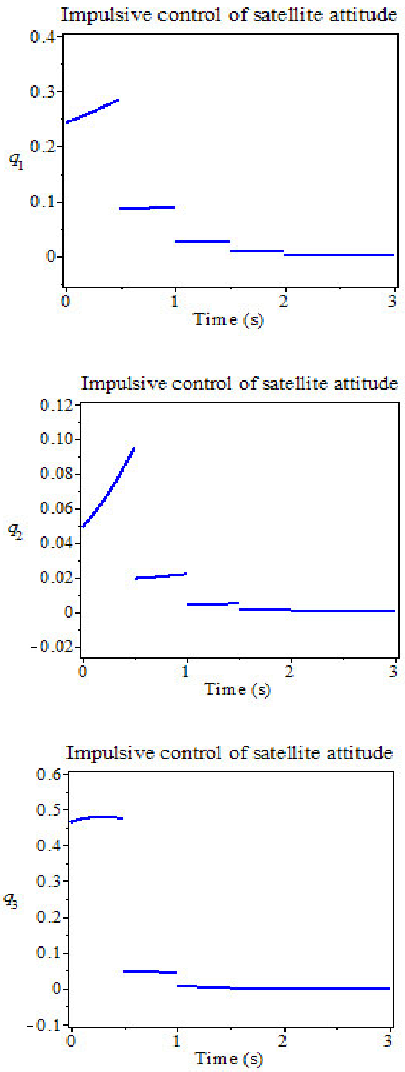

Figure 1: Time series responses corresponding to quaternion parameters in system (4.13) via impulsive control.

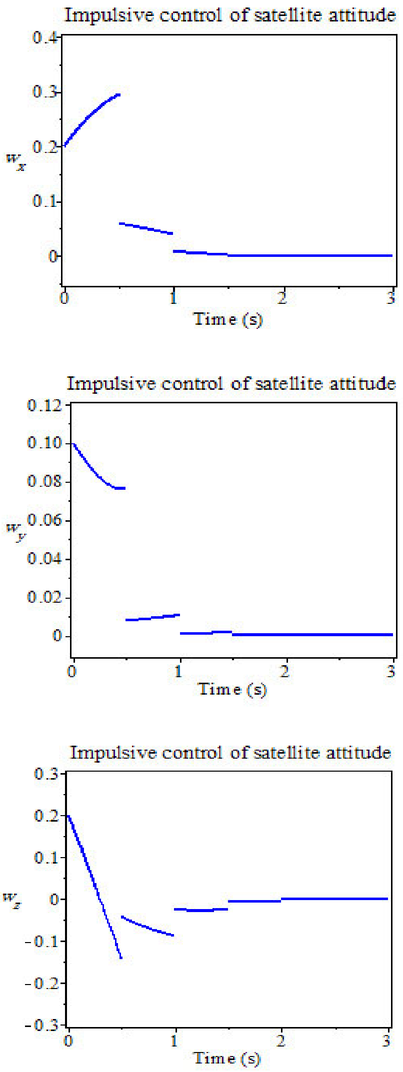

Figure 2: Time series responses corresponding to angular velocities in system (4.13) via impulsive control.

where $t_i, (i = 1, 2, ...)$ denote the instants when impulsive control occur, $u_i(x(t_i)), (i = 1, 2, ...)$ are control functions in the time t_i, and $B_i, (i = 1, 2, ...)$ are $n \times n$ matrices . For convenience, define the following notations

$$\begin{cases} \lambda_m(A) = \frac{1}{2}\lambda_{max}(A + A^T), \\ \beta_i = \lambda_{max}[(I + B_i)^T(I + B_i)], i = 1, 2, ... \end{cases} \tag{4.14}$$

where I is the $n \times n$ identity matrix, and $\lambda_{max}(A)$ is the maximal eigenvalue of matrix A.

Theorem 4.1. *(1) If $2\lambda_m(A) = \lambda < 0$ (λ is a constant) and there is a constant $0 \leq \alpha < -\lambda$, such that*

$$\beta_i \leq e^{\alpha(t_i - t_{i-1})}, \qquad i = 1, 2, ... \tag{4.15}$$

then the trivial solution (4.13) is globally exponentially stable.

(2) If $2\lambda_m(A) = \lambda \geq 0$ (λ is a constant) and there is a constant $\alpha \geq 1$, such that

$$\alpha\beta_i e^{\lambda(t_{i+1} - t_i)} \leq 1, \qquad i = 1, 2, ... \tag{4.16}$$

then $\alpha = 1$ implies that the trivial solution (4.13) is stable; and $\alpha > 1$ implies that the trivial solution (4.13) is globally and asymptotically stable.

Proof. [4]. □

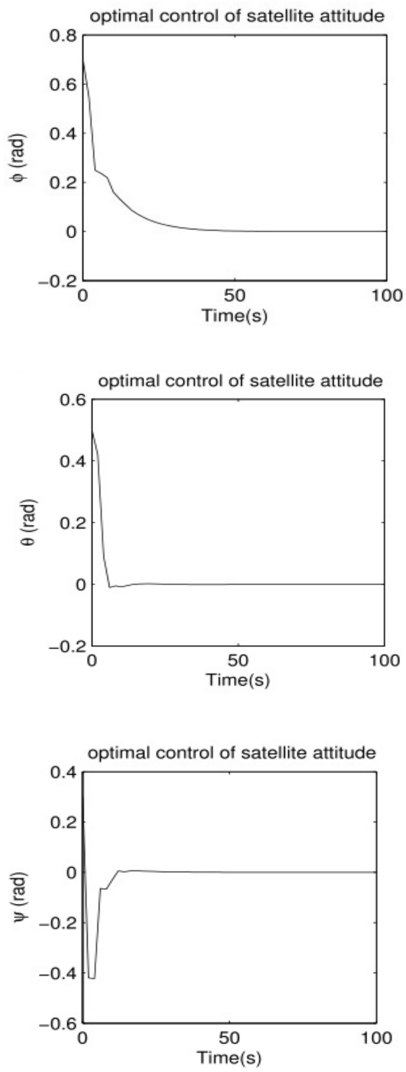

Figure 3: Time series responses corresponding to Euler angles in SA system via optimal control.

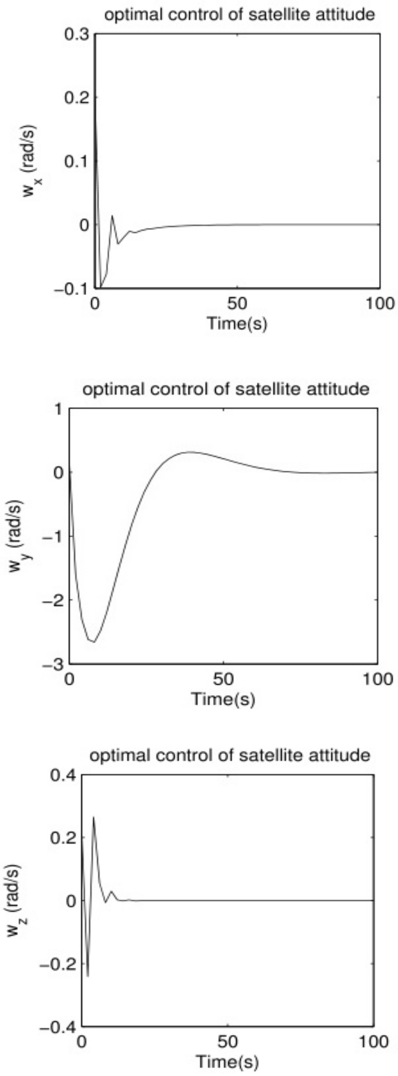

Figure 4: Time series responses corresponding to angular velocities in SA system via optimal control.

For convenience, the gain matrices B_i are often selected as a constant matrix and the impulsive distances $\tau_i = t_i - t_{i-1}, (i = 1, 2, \ldots)$ are set to be a positive constant. Then we have the following corollary.

Corollary 4.1. *Assume* $\tau_i = \tau > 0$, *and matrices* $B_i = B$, *(i=1,2,...)*

(1) If $2\lambda_m(A) = \lambda < 0$ *(*λ *is a constant) and there is* α *constant* $0 \leq \alpha < -\lambda$, *such that* $\beta_i \leq e^{\alpha\tau}$, *then the trivial solution (4.13) is globally and exponentially stable.*

(2) If $2\lambda_m(A) = \lambda \geq 0$ *(*λ *is a constant) and there is* α *constant* $\alpha \geq 1$, *such that,* $\alpha\beta_i e^{\lambda\tau} \leq 1$, *then* $\alpha = 1$ *implies that the trivial solution (4.13) is stable; and* $\alpha > 1$ *implies that the trivial solution (4.13) is globally and asymptotically stable.*

5 Numerical simulation of impulsive control

In this section, in order to demonstrate and verify the performance of the proposed method, some numerical simulations are presented using Maple. Assuming the initial point and the constant values given in Table 1, we control satellite attitude to its equilibrium point $(0, 0, 0, 1, 0, 0, 0)$. considering these assumptions and mentioned subjects we have

$$A+A^T=\begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 & -0.8 & 0 & \frac{-5\sqrt{6}}{6} \\ 0 & \frac{1}{2} & 0 & 0 & 0 & 0.35 & 0 \\ 0 & 0 & \frac{1}{2} & 0 & \frac{-5\sqrt{6}}{6} & 0 & -0.8 \end{bmatrix}. \tag{5.17}$$

The eigenvalues of $A+A^T$ are

$$-2.9267, -0.3547, -0.1764, 0, 0.0854, 0.7047$$

and 1.4176, thus $2\lambda_m(A) = 1.4176 > 0$. If the matrices $B_i, (i = 1, 2, ...)$ are selected as a constant matrix

$$B = diag(b_1, b_2, b_3, b_4, b_5, b_6, b_7)$$

$$= (0, 0, 0, -0.5, -0.7, -0.7, -0.7)$$

then

$$\beta = max\{(1+b_1)^2, ..., (1+b_7)^2\} = 0.25. \tag{5.18}$$

It follows from Corollary 4.1 that impulsive distance is

$$0 \le \tau \le -\frac{ln\alpha + ln(0.25)}{1.4176}. \tag{5.19}$$

Now, taking $\alpha = 1$ obtain

$$0 \le \tau \le 0.9779. \tag{5.20}$$

Figure 1 and Figure 2 illustrate the simulation results of the SA system with $\tau = 0.5$. In these figures, time series responses corresponding to quaternion parameters and angular velocities demonstrate the appropriate performance of the impulsive control with regard to the stabilization and suppression of chaos. These responses are comparable with solutions in Figure 3 and Figure 4 that show the time series corresponding to Euler angles and angular velocities obtained by the optimal control method [12]. In particular the quick stability of angular velocities in impulsive control is considerable compared with ones in the optimal control methods [10, 12].

6 Conclusion

In this paper, the impulsive control along with quaternion parameters were applied on the SA system when its attitude was confused by a disturbed torque. Whilst this method can solve the singularity problem in the numerical solution of system,the simulation results obtained from this method demonstrated its quick stability as for achieving the equilibrim point of system compared with other control methods.

References

[1] J. L. Crassidis, F. L. Markley, Sliding mode control using modified Rodrigues parameters, *J Guidance Control Dyn* 19 (1996) 1381-1383.

[2] K. M. Fauske, Attitude stabilization of an underactuated rigid spacecraft, *Siving thesis, Department of Engineering and Cybernetics, Norwegian University of Technology and Science* 13 (2003) 511-523.

[3] D. Fragopoulos, M. Innocenti, Stability considerations in quaternion attitude control using discontinuous Lyapunov functions, *IEE Proc Control Theory Appl.* 151 (2004) 253-258.

[4] K. Kemih, A. Kemiha, M. Ghanes, Chaotic attitude control of satellite using impulsive control, *Chaos, Solitons & Fractals* 42 (2009) 735-744.

[5] K. Kemih, W. Y. Liu, Constrained generalized predictive control of chaotic Lu system, *ICIC Express Lett, An Int. J. Res. Surv.* 1 (2007) 39-44.

[6] N. Koncar, A. J. Jones, Adaptive real-time neural network attitude control of chaotic satellite motion, *Proc, SPIE 2492, Applications and Science of Artificial Neural Networks*, (1995), `http://dx.doi.org/10.1117/12.205121/`.

[7] V. Lakshmikantham, D. D. Bainov, P. S. Simeonov, Theory of impulsive differen-

tial eqations, *World Scientific, Sangapore,* (1989).

[8] C. Li, M. Guangfu, S. Bin, Passivity-based nonlinear attitude regulation of rigid spacecraft subject to control saturation, *The Sixth World Congress on Intelligent Control and Automation* 2 (2006) 8421-8425.

[9] X. D. Li, X. Y. Yang, J. D. Cao, Event-triggered impulsive control for nonlinear delay systems, *Automatica* 117 (2020) 108-129.

[10] M. R. Niknam, H. Kheiri, N. Abdi Sobouhi, Optimal control of satellite attitude and its stability based on quaternion parameters, *Computational Methods for Differential Equations* 10 (2022) 168-178.

[11] M. R. Niknam, A. Heydari, Finite-time stabilization of satellite quaternion attitude, *Computational Methods for Differential Equations* 3 (2015) 274-283.

[12] M. R. Niknam, H. Kheiri, A. Heydari, Three-axis optimal control of satellite attitude based on Ponteryagin maximum principle, *International Journal of Industrial Mathematics* 8 (2016) 37-44.

[13] Y. Park, Robust and optimal attitude control of spacecraft with disturbances, *International Journal of Systems Science* 46 (2015) 1222-1233.

[14] R. F. Rao, S. M. Zhong, Impulsive control on delayed feedback chaotic financial system with Markovian jumping, *Adv. Diff. Equ.* 20 (2020) 1-18. `http://dx.doi.org/10.1186/s13662-019-2438-0/`

[15] L. L. Show, J. C. Juang, Y. W. Jan, An LMI-based nonlinear attitude control approach, *IEEE Trans Control Syst. Tech.* 11 (2003) 73-87.

[16] L. L. Show, J. C. Juang, Y. W. Jan, C. T. Lin, Quaternion feedback attitude control design: a nonlinear H_∞ approach, *Asian J. Control* 5 (2003), 406-411.

[17] A. P. M. Tsui, A. J. Jones, The control of higher dimensional chaos: comparative results for the chaotic satellite attitude control problem, *Physica* 135 (2000) 41-62.

[18] R. Wisniewski, Satellite attitude control using only electromagnetic actuation, *Ph.D. Thesis, The University of Aalborg, Denmark,* (1996).

[19] X. Zhang, A. Khadra, D. Li, D. Yang, Impulsive stability of chaotic systems represented by T-S model, *Chaos, Solitons & Fractals* 41 (2009) 1863-1869.

[20] Y. Zhang, J. Sun, Robust synchronization of coupled delayed neural networks under general impulsive control, *Chaos, Solitons & Fractals* 41 (2009) 1476-1480.

[21] R. Zhang, Z. Xu, S. X. Yang, X. He, Generalized synchronization via impulsive control, *Chaos, Solitons & Fractals* 38 (2008) 97-105.

Mohammad Reza Niknam has got PhD degree in Applied Mathematics from University of Payame Noor in 2017. Now he is an assistant professor in Azad University. His area of interest includes Dynamical systems, chaotic, Optimization and Control problems.

Nadereh Abdi Sobouhi has got M. Sc. degree in Applied Mathematics from Shahid Bahonar University of Kerman in 2000. She is a faculty member of Farhangian University and her research interests are in the areas of applied mathematics including Optimization, Mathematics education.