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Research Article

A New Method for Ranking Fuzzy Numbers By the Center Of Mass

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Abstract

In this paper, we present the first moment have been defined based on the center of mass from the fuzzy number, then with calculating area between the center of mass, a new ranking method has been proposed. At last, we present some numerical examples to illustrate our proposed method, comparing with distance index ranking method.

Keywords : Ranking fuzzy numbers; Distance index; Area; Normal fuzzy numbers; The first moment; The center of mass.

1 Introduction

Ranking fuzzy numbers is a very important procedure for decision making in a fuzzy environment.For example, the concept of optimum or best choice to come true is completely based on ranking or comparison. Therefore, how to set the rank of fuzzy numbers has been one of the main problems. With the development of fuzzy set theory by Zadeh [11], fuzzy ranking has become a topic that has been studied by many researchers since presented by Jain [6] and Dubois and Prade [4]. More than 30 fuzzy ranking indices have been proposed since 1976. For example, In a paper by Cheng [1], the centroid formulae for fuzzy numbers was provided based on S.Murakami [8], then a centroid-based distance method was suggested for ranking fuzzy numbers. The method utilizes the Euclidean distances from the origin to the centroid point of each fuzzy number to compare and rank the fuzzy numbers.Chu and Tsao [2] found that the distance method could not rank fuzzy numbers correctly if they are negative and therefore suggested using area between the centroid point and origin to rank fuzzy numbers. More recently, Ying-Ming Wang et al [10] found that the centroid formulae for numbers provided by Cheng are incorrect and have led to some misapplication by Chu and Tsao [2].

The rest of the paper is organized as follows. In Section 2, we bring some definitions and preliminaries. In Section 3, we introduce how to caculate the first moment and use the center mass of fuzzy numbers. In Section 4, the proposed method will illustrate by some examples and we will show the advantage of the proposed method. The paper ends with conclusion in Section 5.

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2 Preliminaries and notations

The cocept of fuzzy number is defined in this section we bring some necessary that we will use during the follow [4].

Definition 2.1 *A fuzzy number $\widetilde{A}$ is a triangular fuzzy number if its membership function $f_{\widetilde{A}}$ is*

$$f_{\widetilde{A}}(x) = \begin{cases} (x-a)/(b-a), & a \leq x \leq b, \\ (x-c)/(b-c), & b \leq x \leq c, \\ 0, & otherwise. \end{cases} \quad (2.1)$$

where a, b and c are real numbers. The triangular fuzzy number $\widetilde{A}$ can be denoted by $(a, b, c; 1)$. Since $f^L_{\widetilde{A}}(x) = (x-a)/(b-a)$ and $f^R_{\widetilde{A}}(x) = (x-c)/(b-c)$, the corresponding inverse functions can be expressed as $g^L_{\widetilde{A}}(y) = a + (b-a)y$ and $g^R_{\widetilde{A}}(y) = c + (b-c)y$, $y \in [0, 1]$.

Definition 2.2 *A real fuzzy number $\widetilde{A}$ is a fuzzy subset of the real line R with memebership function $f_{\widetilde{A}}$ which satisfies the following properties [7] :*

1. *$f_{\widetilde{A}}$ is a continuous function from R to a closed interval $[0, w]$, $0 \leq w \leq 1$;*
2. *$f_{\widetilde{A}}$ (x)=0, for all $x \in (-\infty, a]$;*
3. *$f_{\widetilde{A}}$ is strictly increasing on $[a, b]$;*
4. *$f_{\widetilde{A}}$ (x)=1, for all $x \in [b, c]$;*
5. *$f_{\widetilde{A}}$ is strictly decreasing on $[c, d]$;*
6. *$f_{\widetilde{A}}$ (x)=0, for all x$\in [d, \infty)$;*

Where a, b, c and d are real numbers. Unless elsewhere specified, it is assumed that $\widetilde{A}$ is convex, normal and bounded (i.e.$-\infty < a, d < \infty$).

For convenience, the fuzzy number in Definition 2.2 can be denoted by $[a, b, c, d; 1]$, and the membership function $f_{\widetilde{A}}$ of the fuzzy number $\widetilde{A} = [a, b, c, d; 1]$ can be expressed as

$$f_{\widetilde{A}}(x) = \begin{cases} f^L_{\widetilde{A}}(x), & a \leq x \leq b, \\ 1, & b \leq x \leq c, \\ f^R_{\widetilde{A}}(x), & c \leq x \leq d, \\ 0, & \text{otherwise}. \end{cases} \quad (2.2)$$

Where $f^L_{\widetilde{A}} : [a, b] \longrightarrow [0, 1]$ and $f^R_{\widetilde{A}} : [c, d] \longrightarrow [0, 1]$. Since $f^L_{\widetilde{A}} : [a, b] \longrightarrow [0, 1]$ is continuous and strictly increasing, the inverse function of $f^L_{\widetilde{A}}$ exists. Similary, $f^R_{\widetilde{A}} : [c, d] \longrightarrow [0, 1]$ is continuous and strictly decreasing, the inverse functions of $f^R_{\widetilde{A}}$ also exists. The inverse functions $f^L_{\widetilde{A}}$ and $f^R_{\widetilde{A}}$ are denoted by $g^L_{\widetilde{A}}$ and $g^R_{\widetilde{A}}$, respectively. Since $f^L_{\widetilde{A}} : [a, b] \longrightarrow [0, 1]$ is continuous and strictly increasing, $g^L_{\widetilde{A}} : [0, 1] \longrightarrow [a, b]$ is also continuous and strictly increasing. Similary, if $f^R_{\widetilde{A}} : [c, d] \longrightarrow [0, 1]$ is continuous and strictly decreasing, then $g^R_{\widetilde{A}} : [0, 1] \longrightarrow [c, d]$ is continuous and strictly decreasing, $g^L_{\widetilde{A}}$ and $g^R_{\widetilde{A}}$ are continuous on a closed interval [0,1] and they are integrable on [0,1]. That is, both $\int_0^1 g^L_{\widetilde{A}} dy$ and $\int_0^1 g^R_{\widetilde{A}} dy$ exist.

Definition 2.3 *The non-normal trapezoidal fuzzy number $\widetilde{B}$, denoted by $(a, b, c, d; w)$, is a fuzzy number with membership function $f_{\widetilde{B}}$ given by*

$$f_{\widetilde{B}}(x) = \begin{cases} w(x-a)/(b-a), & a \leq x \leq b, \\ w, & b \leq x \leq c, \\ w(x-d)/(c-d), & c \leq x \leq d, \\ 0, & otherwise. \end{cases} \quad (2.3)$$

and $0 < w < 1$.

Since $f^L_{\widetilde{B}}(x) = w(x-a)/(b-a)$ and $f^R_{\widetilde{B}}(x) = w(x-d)/(c-d)$, the inverse functions $f^L_{\widetilde{B}}(x)$ and $f^R_{\widetilde{B}}(x)$ are $g^L_{\widetilde{B}}(y) = a + (b-a)y/w$ and $g^R_{\widetilde{B}}(y) = d + (c-d)y/w$ respectively, where $y \in [0, w]$.
by [1],

Definition 2.4 *Let F be the family of all generalized fuzzy numbers. If K is the set of compact subsets of R^2, A and B are two subsets of R^2 then the Hausdorff metric $H : K \times K \longrightarrow [0, \infty]$ is defined by [11]*

$$H(A, B) = \max\{\sup_{b \in B} d_E(b, A), \sup_{a \in A} d_E(a, B)\}$$

where d_E is the usual Euclidean metric for R^2.

Definition 2.5 *The Centriod Point of a fuzzy number corresponds to $\overline{x}$ on the horizontal axis and $\overline{y}$ on the vertical axis. The centriod point $(\overline{x}, \overline{y})$ for a fuzzy number $\widetilde{B}$ is defined as follows:*

$$\overline{x} = \frac{\int_a^b x f_B^L(x)dx + \int_b^c (x)dx + \int_c^d x f_B^R(x)dx}{\int_a^b f_B^L(x)dx + \int_a^b wdx + \int_c^d f_B^R(x)dx} \tag{2.4}$$

$$\overline{y} = \frac{\int_0^w y(g_B^R(y) + g_B^L(y))dy}{\int_0^w (g_B^R(y) + g_B^L(y))dy}$$

where the denominator $\int_0^w (g_{\tilde{B}}^R(y) + g_{\tilde{B}}^L(y))dy$ in 2.4 represents the area of the trapezoidal, and the numerator $\int_0^w y(g_{\tilde{B}}^R(y) + g_{\tilde{B}}^L(y))dy$ is the weighed average of the area.

Now for ranking of fuzzy numbers that was defined priviously, suppose $S = \{\widetilde{A}_1, \widetilde{A}_2, \cdots, \widetilde{A}_n\}$ is a set of convex fuzzy numbers, and the ranking function R is a mapping from S to the real numbers.The distance index between the centriod point $(\overline{x_0}, \overline{y_0})$ and original point (x_0, y_0) is abtained as follow:

$$R(\widetilde{A}) = \sqrt{(\overline{x_0} - x_0)^2 + (\overline{y_0} - y_0)^2}$$

For any two fuzzy numbers $\widetilde{A}_i, \widetilde{A}_j \in S$, following properties are established:

1. if $R(\widetilde{A}_i) \prec R(\widetilde{A}_j)$ then $\widetilde{A}_i \prec \widetilde{A}_j$;
2. if $R(\widetilde{A}_i) \succ R(\widetilde{A}_j)$ then $\widetilde{A}_i \succ \widetilde{A}_j$;
3. if $R(\widetilde{A}_i) = R(\widetilde{A}_j)$ then $\widetilde{A}_i = \widetilde{A}_j$;

3 Main section

3.1 The First Moment and The Center of Mass(COM) of Fuzzy Number

Consider an area S located in the xy plane (Figure 1) and the element of area dS of coordinates x and y. The first moment about an axis is calculated by integrating over S the distance from the axis times the density. The center of mass is found from the first moments. The first moment of the area S with respect to the x axis, and the first moment of the area S with respect to the y axis are defined, respectively, as [12, 13]

$$\text{Around the x-axis} : M_x = \int\int y\delta(x,y)ds \tag{3.5}$$

$$\text{Around the y-axis} : M_y = \int\int x\delta(x,y)ds$$

The center of mass of an area as S with respect to the x axis is defined as the quantity $\overline{x}$, that satisfies the following equation:

$$\overline{x} = \frac{M_y}{M}, \quad M = \int\int \delta(x,y)dS \tag{3.6}$$

where M is the mass and if $\delta(x,y) = 1$ than M is area of S. In a similar way, we define the center of mass of an area as S with respect to the y axis as follow:

$$\overline{y} = \frac{M_x}{M}, \quad M = \int\int \delta(x,y)dS \tag{3.7}$$

In this article consider $\widetilde{A}$ is a generalized fuzzy number denoted by $(a, b, c, d; w)$ (Figure 1), then we introduce, the center of mass (COM) of the generalized fuzzy number $\widetilde{A}$ is denoted by $(\overline{x}, \overline{y})$ whose value can be obtained by equations 3.5, 3.6, and 3.7. For an area made up of some simple shapes, the first moment of the entire area is the sum of the first moments of each of the individual area around the axis desired. For example, the first moment of the generalized trapezoidal fuzzy numbers in Figure 1, can be obtained as follows,

$$(M_x) = (M_x)_1 + (M_x)_2 + (M_x)_3 \tag{3.8}$$
$$(M_y) = (M_y)_1 + (M_y)_2 + (M_y)_3$$

The first moment and the center of mass

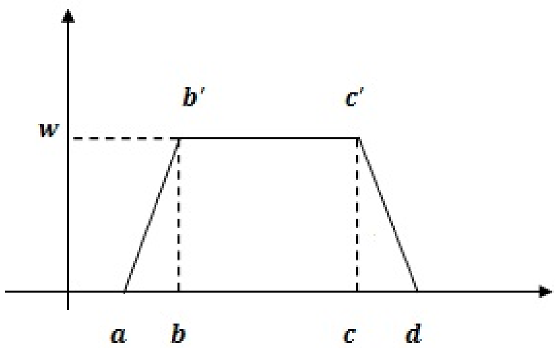

Figure 1: A generalized trapezoidal fuzzy numbers (a,b,c,d;w)

of the generalized trapezoidal fuzzy number $(a, b, c, d; w)$, shown in Figure 1, of the area (1) with respect to x axis, and the first moment of the area (1) with respect to y axis can be calculated, according to equation 3.12, respectively, as

$$(M_x)_1 = \int_0^w \int_{\frac{yb+a(w-y)}{w}}^b y\,dx\,dy = \int_0^w y.\frac{(w-y)(b-a)}{w}dy = \frac{(b-a)w^2}{6} \tag{3.9}$$

$$(M_y)_1 = \int_a^b \int_0^{\frac{w(x-a)}{b-a}} x\,dy\,dx = \int_a^b x.\frac{w(x-a)}{b-a}dx = \frac{w}{b-a}(\frac{b^3}{3} - \frac{ab^2}{2} + \frac{a^3}{6}) \tag{3.10}$$

The first moment of area (2) and (3) with respect to x axis and y axis, can be obtained respectively, as follows,

$$(M_x)_2 = \frac{(c-b)w^2}{2} \tag{3.11}$$
$$(M_x)_3 = \frac{(d-c)w^2}{6}$$
$$(M_y)_2 = \frac{(c^2-b^2)w}{2}$$
$$(M_y)_3 = \frac{w}{c-d}(\frac{c^2 d}{2} - \frac{c^3}{3} - \frac{d^3}{6})$$

So, the (COM) points of generalized trapezoidal fuzzy number $(a, b, c, d; w)$ can be calculated as

$$\overline{x} = \frac{(M_y)_1 + (M_y)_2 + (M_y)_3}{(((c-b)+(d-a)).w)/2} \tag{3.12}$$
$$\overline{y} = \frac{(M_x)_1 + (M_x)_2 + (M_x)_3}{(((c-b)+(d-a)).w)/2}$$

Also, the first moment and the center of mass of the generalized triangular fuzzy number $(a, b, c; w)$, that is shown in Figure 2, can be calculated, according to equations 3.13, 3.14, 3.15 and 3.16, respectively, as follows:

$$M_x = \int_0^w \int_{a+(b-a)y}^{c+(b-c)y} ydxdy = (c-a)(\frac{w^2}{2} - \frac{w^3}{3}) \tag{3.13}$$

$$(M_y)_1 = \int_a^b \int_0^{w\frac{x-a}{b-a}} xdydx = \frac{w}{b-a}[(\frac{b^3}{3} - \frac{ab^2}{2}) - (\frac{a^3}{3} - \frac{a^3}{2})] \tag{3.14}$$

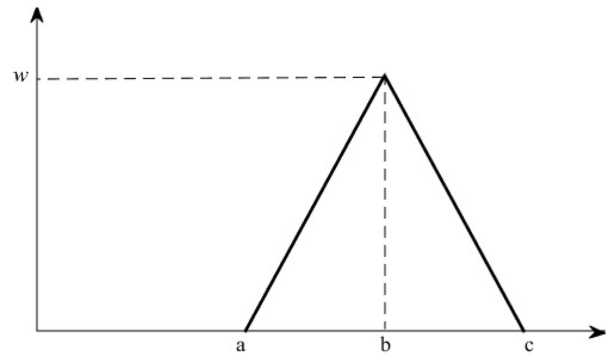

Figure 2: A generalized triangular fuzzy numbers (a,b,c;w)

$$(M_y)_2 = \int_b^c \int_0^{w\frac{x-c}{b-c}} xdydx = \frac{w}{b-c}[(\frac{c^3}{3} - \frac{c^3}{2}) - (\frac{b^3}{3} - \frac{cb^2}{2})] \tag{3.15}$$

By Eq. 3.14, 3.15 and 3.5

$$\overline{x} = \frac{(M_y)_1 + (M_y)_2}{(c-a).w/2} \tag{3.16}$$
$$\overline{y} = \frac{M_x}{(c-a).w/2}$$

3.2 Ranking fuzzy numbers by the center of mass

This section proposes a ranking method with an area between [2] the COM point and original points of a fuzzy number. Consider a fuzzy number $\widetilde{A}$ and its COM point $(\overline{x}, \overline{y})$, where $\overline{x}$ and $\overline{y}$ can be calculated by equations 3.12. The area between the COM points and original points of a fuzzy number. Then, $\widetilde{A}$ is defined as

$$S(\widetilde{A}) = \overline{x} \times \overline{y} \tag{3.17}$$

Here, the area $S(\widetilde{A})$ is used to rank fuzzy numbers. The larger the area $S(\widetilde{A})$, the larger the fuzzy number. Therefore, for any two fuzzy numbers, $\widetilde{A}_i$ and $\widetilde{A}_j$,

1. if $S(\widetilde{A}_i) \prec S(\widetilde{A}_j)$ then $\widetilde{A}_i \prec \widetilde{A}_j$;
2. if $S(\widetilde{A}_i) \succ S(\widetilde{A}_j)$ then $\widetilde{A}_i \succ \widetilde{A}_j$;
3. if $S(\widetilde{A}_i) = S(\widetilde{A}_j)$ then $\widetilde{A}_i = \widetilde{A}_j$;

Table 1: The rank results of distance method and area method

	$\overline{x}_i$	$\overline{y}_i$	$R_i = \sqrt{(\overline{x_0} - x_0)^2 + (\overline{y_0} - y_0)^2}$	$\overline{x}_i$	$\overline{y}_i$	$S_i = \overline{x}_i \times \overline{y}_i$
$\widetilde{A}$	7.714	0.505	7.73	7.714	.286	2.206
$\widetilde{B}$	8	0.3	8.01	18.9	.267	5.046
$\widetilde{C}$	5	0.5	5.03	5.001	0.333	1.665
$\widetilde{D}$	5	0.4	5.02	5	0.373	1.865

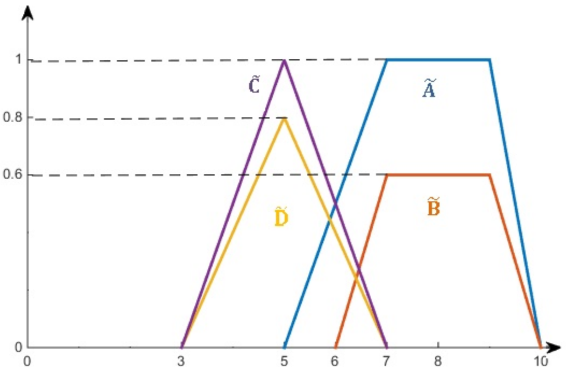

Figure 3: Four fuzzy numbers $\widetilde{A}$, $\widetilde{B}$, $\widetilde{C}$, $\widetilde{D}$

4 Numerical examples

Example 4.1 *In Figure 3, the normal trapezoidal number* $\widetilde{A} = (5, 7, 9, 10; 1)$, *non-normal trapezoidal fuzzy number* $\widetilde{B} = (6, 7, 9, 10; 0.6)$ *and the triangular number* $\widetilde{C} = (3, 5, 7; 1)$ *and the non-normal triangular number* $\widetilde{D} = (3, 5, 7; 0.8)$ *are from [1]*

$$f_{\widetilde{A}} = \begin{cases} \frac{x-5}{2}, & 5 \leq x < 7, \\ 1, & 7 \leq x \leq 9, \\ \frac{10-x}{1}, & 9 < x \leq 10. \end{cases}$$

$$f_{\widetilde{B}} = \begin{cases} 0.6\,\frac{x-6}{1}, & 6 \leq x < 7, \\ 0.6, & 7 \leq x \leq 9, \\ 0.6\,\frac{10-x}{1}, & 9 < x \leq 10. \end{cases}$$

$$f_{\widetilde{C}} = \begin{cases} \frac{x-3}{2}, & 3 \leq x < 5, \\ 1, & x = 5, \\ \frac{7-x}{2}, & 5 < x \leq 7. \end{cases}$$

$$f_{\widetilde{D}} = \begin{cases} 0.8\frac{x-3}{2}, & 3 \leq x < 5, \\ 0.8, & x = 5, \\ 0.8\frac{7-x}{2}, & 5 < x \leq 7. \end{cases}$$

The area between the COM point

$$S_i = \overline{x}_i \times \overline{y}_i \tag{4.18}$$

and the centroid point and $i = \widetilde{A}, \widetilde{B}, \widetilde{C}, \widetilde{D}$.

$$R_i = \sqrt{(\overline{x_0} - x_0)^2 + (\overline{y_0} - y_0)^2}, \tag{4.19}$$

can be calculated by equations (2.4) .
From Table 1, the ordering of fuzzy numbers is $\widetilde{C} \prec \widetilde{D} \prec \widetilde{A} \prec \widetilde{B}$. Also, Table 1 shows the results obtained of ranking method proposed same is by Cheng's method [1]. This is an example to show that is Liou and Wang's method [7], $\widetilde{C} = \widetilde{D}$ for all α,$\alpha \in [0, 1]$.

Example 4.2 *Consider the data used in [3] two triangular fuzzy numbers* $\widetilde{U}_1 = (0.2, 0.5, 0.8; 1)$, $\widetilde{U}_2 = (0.4, 0.5, 0.6; 1)$, *as shown in Figure 4. Through the proposed approach in this paper, the ranking fuzzy numbers by the center of mass values can be obtain as* $S(\widetilde{U}_1) = 0.16665$ *and* $S(\widetilde{U}_2) = 0.16661$.

Then, the ranking order of fuzzy numbers is $\widetilde{U}_1 \succ \widetilde{U}_2$.

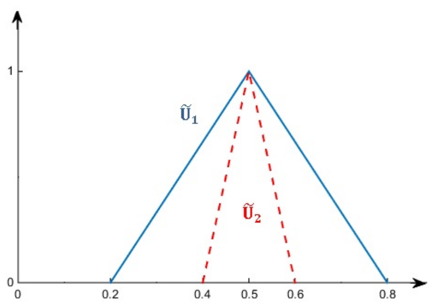

Figure 4: Two fuzzy numbers $\widetilde{U}_1$, $\widetilde{U}_2$

5 Conclusion

Ranking fuzzy numbers is a critical task in a fuzzy decision making process, particularly when ranking a large quantity of fuzzy numbers. Due to lack of information in this respect, an effective, efficient and accurate ranking method becomes essential. In this paper, a new approach presented using the area between the center of mass point for ranking fuzzy numbers. The proposed method could rank any types of fuzzy numbers with different kinds of membership functions. These functions of fuzzy numbers could be triangular, trapezoidal, or of other forms. If can also rank non-normal fuzzy numbers. This method is comprehensive and strong in ranking fuzzy numbers and more importantly, it is a practical and easy-to-use method applicable in the real life problems.

References

[1] C. H. Cheng, A New Approach for Ranking Fuzzy Numbers by Distance Method, *Fuzzy Sets and Systems* 95 (1998) 307-317 .

[2] T. C. Chu, C. T. Tsao, Ranking fuzzy Number with and Area between the centroid point and original point, *Computers and Mathematics with Applications* 43 (2002) 111-117 .

[3] L. H. Chen, H. W. Lu, The preference order of fuzzy numbers, *Computers and mathematics with Applications* 44 (2002) 1455-1465 .

[4] D. Dubois, H. Prade, Operations on fuzzy numbers, *The International Journal of Systems Sciences* 9 (1978) 613-626 .

[5] Y. Deng, Z. Zhenfu, L. Qi, Ranking Fuzzy Numbers with an Area Method using Radius of Gyration, *Computers and Mathematics with Applications* 51 (2006) 1127-1136 .

[6] R. Jain, A procedure for multi-aspect decision making using fuzzy sets, *The International Journal of Systems Sciences* 8 (1978) 11-19.

[7] T. S. Lious, M.J. Wang, Ranking Fuzzy Numbers with Intergral Value, *Fuzzy Sets and Systems* 50 (1992) 18-29.

[8] S. Murakami, H. Maeda, S. Imamura, Fuzzy decision analysis on the development of centralized regional energy control system, *IFAC symp. On fuzzy inform. Know ledge Representation and Decision Anal.* 12 (1983) 363-368 .

[9] Y. J. Wang, H. S. Lee, The revised method of ranking fuzzy numbers with an area between the centroid and original points, *Computers and Mathematics with Applications* 55 (2008) 2033-2042 .

[10] Y. M. Wang, J.B. Yang, D.L. Xu, K.S. Chin, On the Centroids of Fuzzy Numbers, *Fuzzy Sets and Systems* 157 (2006) 919-926 .

[11] F. Hausdroff, *Set Theory* 1 (1962) 109-118 .

[12] G. B. Thomas, R. L.Finny, *Calculus and Analytic Geometry* (1988) 1084-1089 .

[13] R. Resnick, D. Halliday, K. S. Krane, *Physics* (2002) 269-273 .

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