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Research Article

A New Hybrid Methodology Based on Data Envelopment Analysis and Neural Network for Optimization of Performance Evaluation

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Abstract

There are numerous models of data envelopment analysis (DEA) for solving the efficiency evaluation a set of homogeneous Decision-Making Units (DMUs) that use similar sources to produce similar outputs. However, the efficiency boundary in these models is very sensitive to outliers and random factors. In this way, researchers have always sought a method that, in addition to having the high flexibility of nonparametric methods, compensates for the weaknesses of this view. The approach suggested by scholars in this regard is the use of a combination of Artificial Neural Network (ANN) and DEA. In this paper, a new method of combining ANN and DEA (ANN-DEA) presented in which the input and output values for a large number of DMUs determined as neural network inputs. It can be seen that the use of the neural network to solve the data envelopment analysis problem does not require solving the model for each DMU, and therefore compared with the conventional method, in the proposed algorithm processing time and memory usage significantly reduced. We have also compared the new model with the existing approach of ANN-DEA. To illustrate the ability of the proposed methodology some case studies are used, including a set of 500 Iranian bank branches. The results indicate a high accuracy and less computational time of the proposed hybrid model and have practical outcomes for decision makers.

Keywords : Data Envelopment Analysis; Artificial Neural Network; Levenberg Marquardt ; Efficiency; Linear Programming.

1 Introduction

With the advent of technology and the complexity and volume of information, senior executives have required themselves to apply scientific methods to determine and increase the productivity of the organization under their jurisdiction. Data Envelopment Analysis (DEA) is linear programming for measuring the relative efficiencies of homogeneous decision-making units (DMUs) without knowing production functions,

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just by utilizing input and output information [1, 2]. The first models in DEA are the CCR and BCC models that the efficiency of each DMU obtained as the maximum of a ratio of weighted outputs to weighted inputs subject to that the similar ratio for all DMUs is less than or equal to one [1, 2]. DEA technique has just been effectively connected in various cases such as broadcasting companies [3], banking institutions [4, 5, 6], R&D organizations [7, 8], health care services [9, 10], manufacturing [11, 12], telecommunication [13], and supply chain management [14, 15, 16, 17]. However, the efficiency boundary in this method is very sensitive to random factors. Therefore, if the data accompanied by statistical errors, the boundary derived from these methods will be diverted [18]. In this way, researchers have always sought a technique that, in addition to having the high flexibility of nonparametric methods, compensates for the weaknesses of this view. The approach suggested in this regard is the use of Artificial Neural Network (ANN). In an evaluation of efficiency, artificial neural networks are a semi-parametric view. Because, as the nonparametric methods, there is no specific assumption about the statistical properties of the data and the form of the boundary production function. However, the boundary of this method, unlike the boundary of non-parametric methods, is not necessarily definitive, and thus the sensitivity of this boundary is less than the nonparametric boundaries. Therefore several scholars have been proposed combination of ANN and DEA. Athanassopoulos and Curram [19] proposed ANN as a tool for measuring efficiency. They evaluated the efficiency of 250 bank branches in Canada and compared the results of ANN with the results of the basic models of data envelopment analysis. Correlation coefficients showed that the highest correlation (87%) found between the two models of CCR and BCC. Also, the correlation between DEA and ANN was at best 68 percent and related to the CCR model. In 1997, London's metro efficiency with time series data was analyzed by Costa and Marcellus [20]which resulted of ANN with the corrected least squares, and the DEA was very similar. Pendharkar and Roger [21] to predict nonlinear functions using the ANN applied the DEA to separate training data. The results showed that the predictive performance of an ANN that trained on the efficient training data subset is higher than the predictive performance of an ANN that trained on the inefficient training data subset. In 2004, Santin et al., [22] used an ANN to simulate a nonlinear production function and compared its results with more common methods, such as DEA and Stochastic Frontier Analysis (SFA) and showed that ANN is more stable compared to the above methods.Delgado [23] evaluated the efficiency of 72 municipality administrations in Spain using basic models of DEA and ANN and presented two approaches for efficiency measurement. Wu et al. [24], with the help of a combination of DEA and ANN, evaluated the performance of 142 large banks in Canada. Comparison of the obtained results with the proposed model and DEA showed a strong correlation between them. Similar to the work of [24], Mostafa [25] evaluated the performance of Arabic banks. In his study, the efficiency of 85 top Arabic banks studied. Comparison of the results of ANN and DEA showed that there are correlations between the rankings of the two methods (94%). Celebi and Bayraktar [26] put forward integration of ANN and DEA for evaluation of suppliers under incomplete information of evaluation criteria which is a common problem in real life situations. Emrouznejad and Shale [27] presented a combination algorithm of DEA and ANN for evaluating the performance of a very large scale datasets. They showed that the conventional DEA methods for such collections require huge amounts of computer processing regarding computer memory and CPU time, and the proposed method facilitates the computation of the efficiency of the large data set. Kwon et al. [28] used a combination of ANN and two-stage data envelopment analysis for assessing the efficiency of the financial banking operations across large U.S. banks. Shabanpour et al. [29]used a combination of neural network and dynamic DEA as two nonparametric models to forecast future efficiency of green suppliers. To this end, firstly, they predicted inputs, outputs, and links of the green suppliers using ANN. Then, the forecasted data derived from ANN are used in dynamic DEA. Recent years we can see a great variety of applying ANNs as an excellent method to assist in estimating efficiency score of DMUs.

Readers can also refer to [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48] for review the more recent researches on ANN-DEA approaches.In this paper, a new method of combining multi-layer perceptron neural networks and data envelopment analysis is presented in which the input and output values for a large number of DMUs are determined as neural network inputs. We have also compared the new model with the existing ANN-DEA. To represent the ability of the new methodology, some case studies are used, including a set of 500 Iranian bank branches. The results indicate that the proposed network has some advantages over the existing ANN-DEA models and has helpful outcomes for decision makers.

The rest of this study is as follows: Section 2 presents some essential concepts regarding DEA, CCR model, ANN, and Multi-layer perceptron network. In Section 3, we study the existing approach of the combination of DEA and ANN, briefly. The new approach of ANN-DEA proposed in Section 4. Section 5 illustrates the proposed model for assessing the efficiency of two case studies. Finally, the paper is concluded.

2 Preliminaries

We start with some fundamental concepts and starter results that we tend to seek advice from later.

2.1 CCR model of Data Envelopment Analysis

Data envelopment analysis (DEA) is a linear programming method for assessing the efficiency and productivity of decision-making units (DMUs) which is originated by Charnes, Cooper, and Rhodes (CCR model). The efficiency of a DMU is established as the ratio of sum weighted output to sum weighted input, subjected to happen between one and zero. Let DMUp is under consideration, then CCR model for the relative efficiency is as follows [1]:

$$\begin{aligned}
&\theta_p^* = \max \frac{\sum_{r=1}^{s} u_r y_{rp}}{\sum_{i=1}^{m} v_i x_{ip}} \\
&s.t. \\
&\qquad \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \leq 1, \ \forall j \\
&\qquad u_r, v_j \geq 0 \qquad \forall r, j.
\end{aligned} \tag{2.1}$$

In this model, each DMU (suppose that we have n DMUs) uses m inputs x_{ij} $(i = 1, 2, \cdots, m)$, to obtains s outputs y_{rj} $(i = 1, 2, \cdots, s)$. Here u_r $(i = 1, 2, \cdots, s)$ and v_i $(i = 1, 2, \cdots, m)$. are the weights of the i th input and r th output. This fractional program is calculated for every DMU to find out its best input and output weights. To simplify the computation, the nonlinear program shown as (1) can be converted to a linear programming (LP) and the model was called the CCR model:

$$\begin{aligned}
&\theta_p^* = \max \sum_{r=1}^{s} u_r y_{rp} \\
&s.t: \\
&\qquad \sum_{i=1}^{m} v_i x_{ip} = 1 \\
&\qquad u_r, y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \quad \forall j \\
&\qquad u_r, v_i \geq 0 \qquad \forall r, j.
\end{aligned} \tag{2.2}$$

We solve the model (2) n -times to work out the efficiency of n DMUs. If $\theta_p^* = 1$, we say that the DMUp is efficient otherwise it is inefficient.

2.2 Artificial neural networks

Artificial Neural Networks (ANN) is an information processing model that seeks to imitate the behavior of biological nervous systems. The neural network can learn from the examples and understand the exact functional relationships among the data, even if the basic relationships are unknown and difficult to describe [49, 50]. Various studies have shown that neural networks have better performance than traditional techniques such as multivariate regression and, unlike regression, works well for large data sets. The ability to learn the relationships of the complex nonlinear input-output, handle subsequent training operations and modify themselves to the data provided is essential features of the neural network. The neural networks are considered as systems of interconnected "neurons", which send messages to each other. The connections within the network can be efficiently balanced dependent on inputs and outputs, making them ideal for supervised learning. The middle layers or hidden layers process the information received from the input layer and place it on the output layer. There are three main steps for solving problems in ANNs: training, validation, and testing. Each ANN learns by getting examples, and the learning is a process that ultimately leads to learning. Network learning occurs when communication weights between layers change so that the difference between predicted and calculated values is little. Having achieved these conditions, learning has been completed. The weights express memory and network knowledge. Validation indicates the ability of the network for presenting a valid response for new inputs that does not belong to the training dataset. Testing also improves the performance of the mentioned network. Figure 3 shows the mean and variance of error in the predicted efficiency values from the training data.

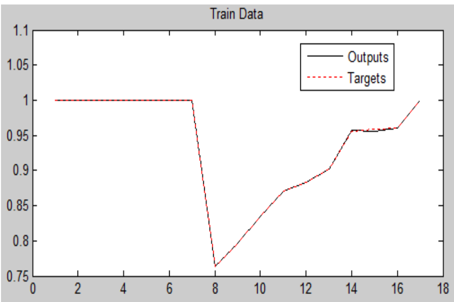

Figure 1: Real and predicted values of efficiency of the training data (hospital data)

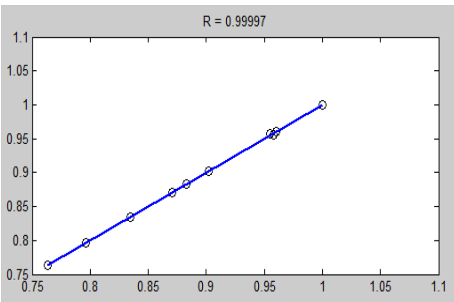

Figure 2: Regression analysis of proposed model for the training data (hospital data)

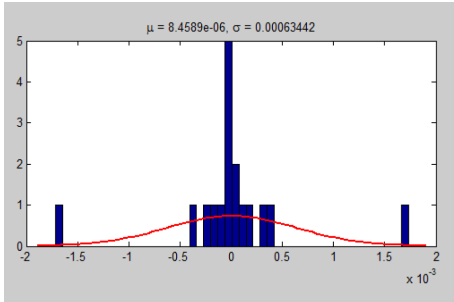

Figure 3: Mean and variance of error in the predicted efficiency of the training data (hospital data)

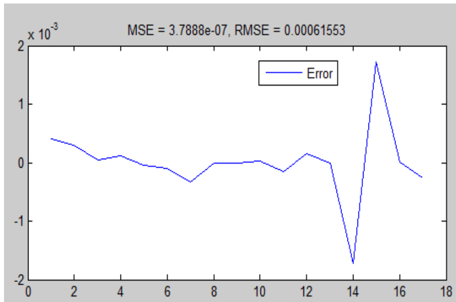

Figure 4: MSE and RMSE in the predicted efficiency of the training data (hospital data)

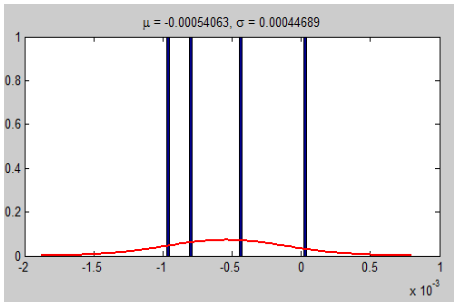

Figure 5: Mean and variance of error in the predicted efficiency of the validation data (hospital data)

ANN are divided into two categories of feed-forward and recurrent networks from the perspective of communications for training a network. One of the most conventional types of training methods for feed-forward networks is Back-Propagation (BP) [51]. Several improvements for BP were developed which much bet-

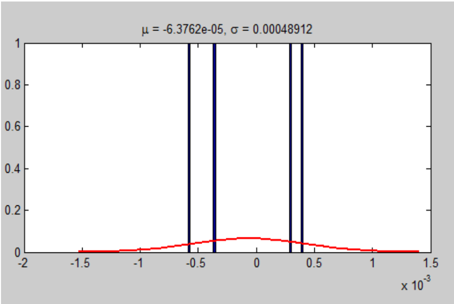

Figure 6: Mean and variance of error in the predicted efficiency of the test data (hospital data)

ter results can be obtained using second-order methods such as LevenbergMarquardt (LM) algorithm. The LM Method was derived by modifying the ordinary least squares norm and is a combination of the Gauss-Newton and Steepest Descent methods[52].

2.3 Multi-layer perceptron network

Networks called multilayer perceptron (MLP) are one of the most common networks in predicting and solving nonlinear problems and are among feed-forward networks. This network contains an input layer, one or more layers of hidden data and an output layer. The training of these networks is based on the BP training algorithm. This algorithm is mainly composed of two main paths. During the network training of MLP using the BP algorithm, the computation from the network inputs to the network outputs is performed first (pathway), and then the calculated error values are propagated to the previous layers (return path). The output is calculated as a layer to the layer, and the output of each layer will be the input of the next layer[53].

3 Existing approach in ANN-DEA

The existing approach of the combination of a multi-layer perceptron artificial neural network and DEA which mentioned in Section 1 for performance evaluation is as follows [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48]:

Step 1. Consider the inputs and outputs of each DMU as the input of ANN.

Step 2. By using any models of DEA, obtain the efficiency of each DMU and consider it as the output of ANN.

Step 3. Normalization:
After pre-processing data, another important issue to be addressed is normalization of data. Here, we use the following formula:

$$\begin{aligned} X^* &= mX - b, \\ m &= \frac{H-L}{\text{Max}(X)-\text{Min}(X)}, \\ b &= \frac{\text{Max}(X)L+\text{Min}(X)H}{\text{Max}(X)-\text{Min}(X)}, \end{aligned} \tag{3.3}$$

where, X^* is the normalized variable and X the main variable. Using this formula, we can use data at any desired interval of $[L, H]$. For normalization, it was preferred that the $L = 0$ and $H = 1$. This is because the difference between the variables can be shown better and also the ANN is better educated with this variables.

Step 4. Divide DMUs into three groups of training data, validation data, and test data.

Step 5. Network training: Here, we use the BP and the LevenbergMarquardt (LM) algorithms.

Step 6. Calculate the efficiency of all DMUs using the trained neural network. To determine the best structure of the neural network to predict the efficiency numbers we use the following indicators:

$$\begin{aligned} \text{MSE} &= \frac{\sum_{i=1}^{n}(O_i-P_i)}{n}, \\ \text{RMSE} &= \sqrt{\text{MSE}}, \end{aligned} \tag{3.4}$$

where, n is the number of data, O_i and P_i are the i-th value of the exact and predicted values, respectively.

It should be noted that in designing of the ANN model, the size of the training, validation, and test data, the number of hidden layers, number of neurons in each layer, the kind of activation (transfer) function, the learning rate and the number of epochs should be determined. There are no systematic methods in determining these. So the best design of the network is achieved using the trial and error approach, i.e., selecting a few alternatives and then running simulations to find out the one with the best results. In this case, Hecht-Nielsen [54] proved that under some conditions in perceptron neural networks with a Single hidden layer and a sigmoid activation function in the hidden layer and also with a linear function in the output layer, it would be possible to approximate all the functions with any degree of approximation. This is known as the universal approximation theorem [55, 56, 57, 58].

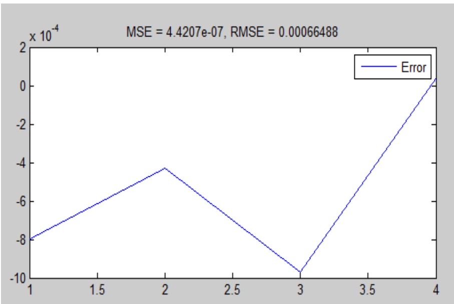

Figure 7: MSE and RMSE in the predicted efficiency of the validation data (hospital data)

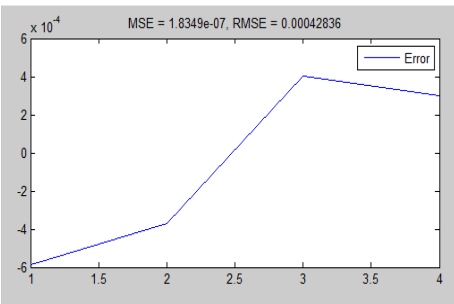

Figure 8: MSE and RMSE in the predicted efficiency of the test data (hospital data)

4 New approach in ANN-DEA

In our model, we are trying to find a more realistic performance for DMUs, so that its exaggeration is less than existing ones, and the DMU's efficiency has less dependency on efficient DMUs. Note that since in the DEA, the efficiency is obtained based on the existing DMUs, it is possible that the DEA models exaggerated levels of per-

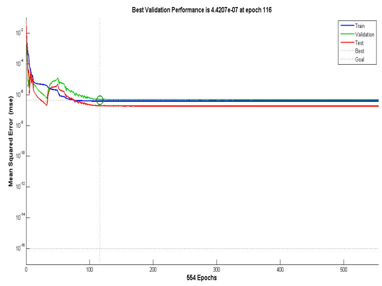

Figure 9: The iteration processes of the proposed ANN-DEA model (hospital data)

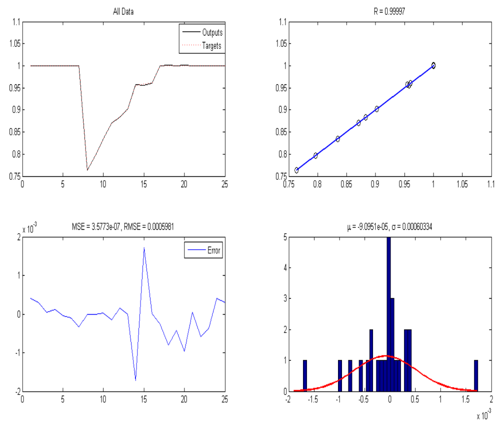

Figure 10: The results of hospital data by the proposed model

formance to evaluate DMUs. The reason for this dilemma is the unsmooth nature of the production frontier. Our proposed solution is to create a set of DMUs using a data production process subject that transmits the performance frontier from the linear piecewise functions to the interpolated smooth curve. The method we propose is to use the linear combination of efficient DMUs to create a set of efficient virtual DMUs. Also by bringing these efficient virtual data, the neural network can also provide a more accurate approximation to the efficiency value. In other words, using ANN, we create a generalized semi- frontier that can show a more realistic value of efficiency. Another advantage of the proposed method is the use of specific data for the training of ANN. In the proposed method, we use a certain number of the most efficient DMUs and the worst DMUs for network training. Using these data for training makes the network more efficiently learn the pattern of performance.

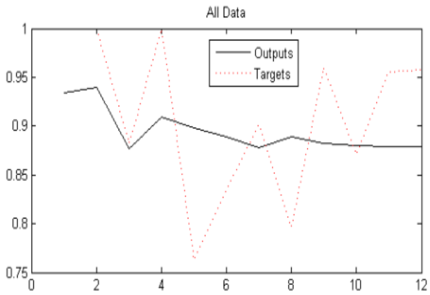

Figure 11: Real and predicted values of efficiency by the existing approach (hospital data)

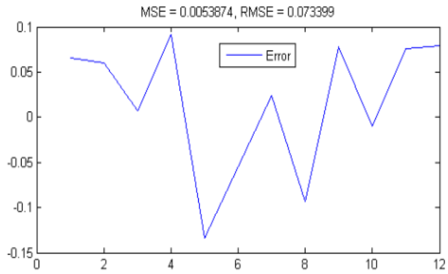

Figure 14: MSE and RMSE of the existing approach for hospital data

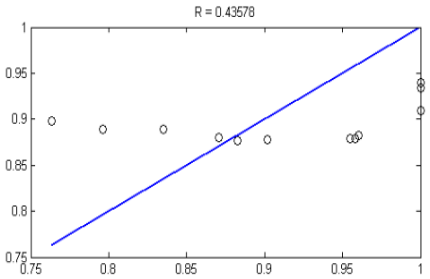

Figure 12: Regression analysis of the existing approach for hospital data

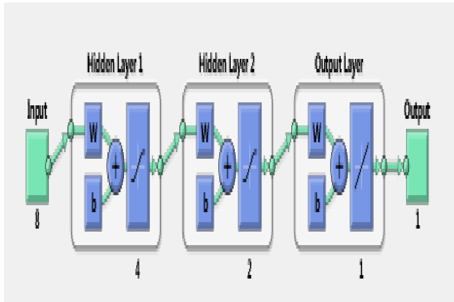

Figure 15: An overview of the structure of ANN for the 500 Iranian bank branches

4.1 Proposed algorithm

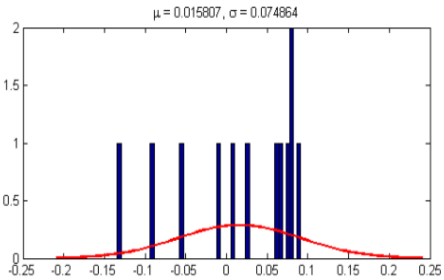

Figure 13: Mean and variance of error in the predicted efficiency by the existing approach (hospital data

Step 1. Consider the inputs and outputs of each DMU and obtain their efficiency by using one of the DEA models.

Step 2. Do the process of producing virtual DMUs:

In this step, we identify the most efficient DMUs specified in step 1. Then, depending on the number of units, we select the total or a number of DMUs, and therefore we create virtual DMUs from their convex combinations. In this way, assuming that the number of DMUs to be chosen is K, first select some arbitrary values for $0 < \lambda < 1$ (suppose the minimum value for λ is 0.1 and the maximum value for that is equal to the kappa. Divide this distance into h equally.), and then we create the virtual DMUs' inputs and outputs as the following Matlab pseudo-code:

Require: $XX = []$, $YY = []$,
for $i = 0.1{:}h{:}\kappa$ **do**
  for $j = 1$ to K **do**
    for $jj = j + 1$ to K **do**
      $Hx = (1 - i)*(:, j) + i*(:, jj)$,
      $XX = [XX, Hx]$,
      $Hy = (1 - i)*(:,j)+i*(:, jj)$,
      $YY = [YY, Hy]$
    end for
  end for
end for

where, inn(:,j), ouu(:,j), XX, and YY are the j-th selected input DMUs, the j-th selected DMU outputs, the obtained virtual inputs, and the obtained virtual outputs, , respectively.

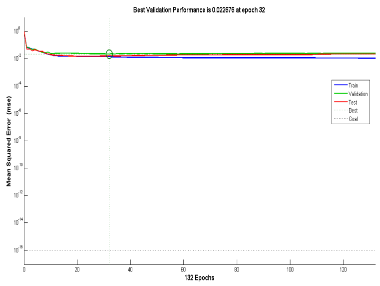

Figure 16: The iteration processes of the existing ANN-DEA model (Banks data)

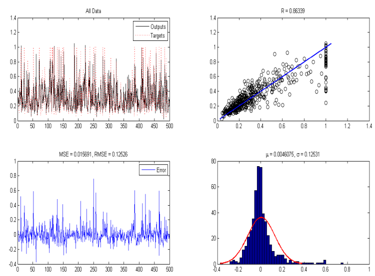

Figure 17: The results of bank data by the existing approach.

Step 3. Construct the virtual DMUs' inputs and outputs and obtain their efficiency by using one of the DEA models.

Step 4. Add the most efficient virtual DMUs from Step 3 to the original DMU set.

Step 5. Normalize the DMUs' information.

Step 6. The input of ANN is compiled from the inputs and outputs of each DMU derived from Step 4 and the output of ANN is the corresponding DMUs' efficiency.

Step 7. Divide DMUs into three groups of training data, validation data, and test data: In this step, training data is between 60% and 75% of the entire DMUs. Select α% of all training data from the most efficient DMUs and $(1-\alpha)$% of the total training data from the worst-performing DMUs (The default $\alpha$is 50)

Step 8. Modify the weights of ANN using

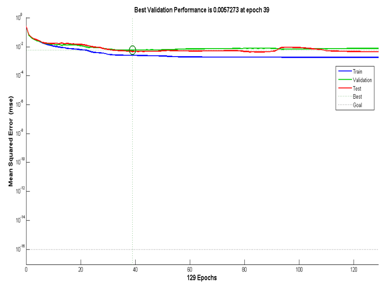

Figure 18: The iteration processes of the proposed ANN-DEA model (Banks data)

training data to achieve the desired result and accuracy.

Step 9. Calculate the efficiency of all DMUs using the trained neural network.

Step 10. Compare the results of the DEA and ANN-DEA models.

5 Empirical illustrations

In this section, we will address a few examples of case studies using the proposed approach and the existing approach.

5.1 Case Study: The Efficiency of Public Hospitals [58]

Table 1 shows the status of 12 hospitals with two inputs (number of Doctors and number of nurses) as well as two outcomes (the number of outpatients and the number of inpatients) where their efficiency is derived from the CCR model.

Now, we investigate the proposed method to solve this problem. The first step in the algorithm is given in Table 1, but for Step 2, i.e., production of virtual DMUs, we set kappa = 0.5 and $h = 0.1$. We run the Step 3 and obtain thirteen virtual DMUs. As a result, we will have 25 DMUs.
Then, we select roughly 70% of the data, namely 17 DMUs for training data, 5 DMUs for validation data and 3 DMUs for testing data. Also, according to Step 7 of the algorithm, we select 17 training data from the best DMUs and 10 DMUs from the worst DMUs. Note that the validation

Table 1: CCR model on Hospital Data

DMU	Inputs		Outputs		DEA
	Doctor	Nurse	Outpatient	Inpatient	Efficiency
A	20	151	100	90	1
B	19	131	150	50	1
C	25	160	160	55	0.883
D	27	168	180	72	1
E	22	158	94	66	0.764
F	55	255	230	90	0.835
G	33	235	220	88	0.902
H	31	206	152	80	0.796
I	30	244	190	100	0.960
J	50	268	250	100	0.871
K	53	306	260	147	0.955
L	38	284	250	120	0.958

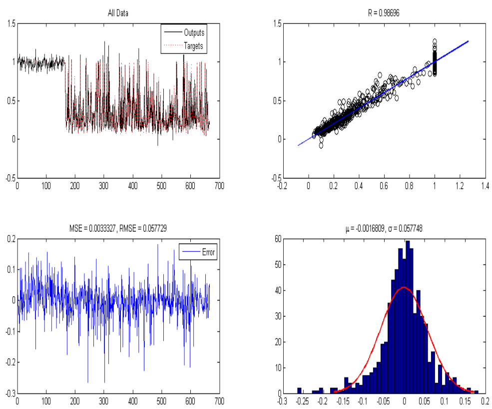

Figure 19: The results of bank data by the new approach.

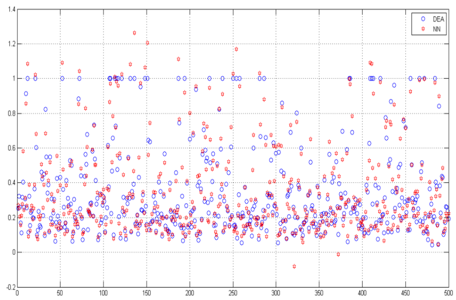

Figure 20: The efficiency scores of 500 banks using DEA and ANN-DEA

and test data chosen randomly. We converged to a configuration consisting of one hidden layer with three neurons that uses the sigmoid transfer function. The output layer has only one neuron with a linear transfer function. Figures 1 and 2 show the actual and predicted values of the efficiency of the training data and the corresponding regression analysis, respectively.

Figure 4 shows the MSE and RMSE for the training data.

Figures 5 and 6 show the mean and variance of error in the predicted efficiency values from the validation and test data, respectively.

Figures 7 and 8 show the MSE and RMSE in the predicted efficiency values from the validation and test data, respectively.

Figure 10 at a glance shows all of the specifications for the entire data.

Figure 9 plots the iteration processes for training, validation, testing, and best validation performance for predicting the efficiency score using proposed ANN-DEA. The iteration process plot shows the value of the performance function (MSE) against the number of epochs. Reference to Figure 9 indicates that the proposed ANN-DEA iteration is stopped when the epoch number reaches 116.

Figure 13 shows the mean and variance of error in the predicted efficiency values by the existing approach for the hospital data, respectively.

Now, if we solve this case study by the existing approach mentioned in Section 3 and with similar conditions, we will have the following results:
Figures 11 and 12 show the actual and predicted values (with the existing approach) of the efficiency of the hospital data and the corresponding regression analysis, respectively.
Figure 14 shows the MSE and RMSE of the hospital data by the existing approach.
As a result, it can be easily seen that the proposed ANN-DEA model is more suitable than the existing approach. Furthermore, since the execution time of the ANN is much lower than the implementation time of DEA models for all DMUs, it can be used as an appropriate tool for calculating the efficiency value.

5.2 Case Study: The Efficiency of 500 Iranian bank branches

To illustrate the practical benefits of the proposed method, a large set of information including 500 Iranian banks branches was collected. Furthermore, for the selection of the most suitable and acceptable items of the banking system, which are commonly used for measuring efficiency in the literature, we consider the following categories:

- Input variables :
  - Personnel: Personnel expenses.
  - Payable interest: interest expense and revenue.
  - Deferred receivables: concerns to installments of deferred receivables and deferred payment credits.
- Output variables :
  - Facility: consists of term loans, cash credit, overdraft, letters of credit, and bank guarantees.
  - Sum of deposits: demand deposits+ short-term investment deposits + long-term investment deposits + foreign currency deposits.
  - Received interest: represents earning assets into investment and interest income.
  - Fee received: fee income + fee-based services
  - Other deposits: Other earning asset+ commercial deposits + retail deposits.

It is worth stressing that all inputs and outputs are measured in terms of Iranian million Rials. Tables 2 and 3 relate a summary of the statistical properties for inputs and outputs.

First, we examine the problem with the existing approach mentioned in Section 3 to evaluate the efficiency of 500 banks branches. In this study, we use MLP with two hidden layers shown in Figure 15. Table 4 summarizes the details of suggested ANN for evaluating the efficiency score with existing approach.
Figure 16 plots the iteration processes for training, validation, testing, and best validation performance for predicting the efficiency score using existing approach of ANN-DEA. The iteration process plot shows the iteration is stopped when the epoch number reaches 32.

Figure 17 at a glance shows all of the specifications for the entire data by using of the existing approach.

Now, we investigate the proposed method to solve this case study. In the first step, we find that the number 33 DMUs it is efficient. For the second step of the proposed model, since the number of DMUs is large, we consider the linear combinations, just for$\lambda = 0.5$. We run Step 3 and obtain 166 virtual DMUs. As a result, we will have 666 DMUs. Then, we select roughly 70% of the data for training data, 15% of the data for validation data and 15% of the data for test data. Moreover, according to Step 7 of the algorithm, we select 170 training data from the best DMUs and 297 DMUs from the worst DMUs. Note that the validation and test data are randomly selected. Then we solve the problem based on Table 4 and Figure 15.

Figure 18 plots the iteration processes for training, validation, testing, and best validation performance for predicting the efficiency score using proposed approach of ANN-DEA. The iteration process plot shows the iteration is stopped when the epoch number reaches 39.

Table 2: Summary statistical information of the inputs

	Personnal	Payable interest	Deferred receivables
Maximum	88.15	513160	1064400
Minimum	2.26	41.603	1.4824
Average	47.00	250375	571621
Standard deviation	24.90	145350	309567
Median	49.02	240513	589287

Table 3: Summary statistical information of the outputs

	Facility	Sum of deposits	Received interest	Fee received	Other deposits
Maximum	8818600	10857000	875880	394980	5216200
Minimum	2057.4	9327.6	0.569	0.056	0.21
Average	4279750	5334375	423961	198079	2577414
Standard deviation	2503537	3106773	250777	115155	1470699
Median	4354128	5201364	423450	203734	2600899

Table 4: Details of trained ANN-DEA for the Banks data

Percentage of training data	70
Percentage of validation data	15
Percentage of test data	15
Number of hidden layers	2
Number of neurons in 1 *st* hidden layer	4
Number of neurons in 1 *st* hidden layer	2
Hidden/ output layer transfer functions	Sigmoid/ Linear
Maximum Epoch	1000
MSE	0.01

Table 5: Comparison of the existing approach and Proposed Method (Banks Data)

Attribute	The existing approach	The existing approach
Average of error	0.0046729	0.0016609
Variance of error	0.12531	0.067748
MSE	0.015691	0.00333327
RMSE	0.12526	0.062729
Regression	0.66339	0.96696

Figure 19 at a glance shows all of the specifications for the entire data by using of the proposed approach. Finally, Table 5 shows the comparison of the two combination methods with different attributes.

In Figure 20, we can see the efficiency of 500 banks branches using the DEA method and the proposed method. Considering the explanation of the proposed method regarding the exaggeration of the efficiency score of some DMUs in the absence of suitable efficient DMUs and also the use of smooth interpolation in the ANN, the efficiency of the proposed method can be more reliable. Furthermore, it can be seen that the efficiency scores of some inefficient DMUs using the ANN is lower than the DEA method. It is also observed that the efficiency scores of some branches are more than one. This results in DEA is not permissible. However, this issue is common in the hybrid approach because the ANN with the statistical characteristics of the data creates a random frontier in accordance with efficient DMUs;

see [24].

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