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Research Article

Performance Measurement Techniques In the Presence of Undesirable Products

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Abstract

Performance measurement in the presence of undesirable outputs is an important subject in production processes that has attracted considerable attention among researchers. In production analysis, there is a group of papers in which researchers have proposed to model undesirable outputs. These works can be divided in to two groups. The first group develops production models in which undesirable outputs are considered as inputs and in the second one undesirable outputs are modeled as outputs instead of inputs. This paper provides a comparison between these two groups and we axiomatically show that the production technology with the weak disposability assumption is the complete and correct technology. To facilitate comparison numerical examples are used to analysis the results.

Keywords : Data Envelopment Analysis; Activity analysis; Undesirable outputs; Efficiency; Production processes.

1 Introduction

Data Envelopment Analysis (DEA) is a powerful technique to evaluate the relative efficiency of homogeneous decision making units (DMUs) with multiple incommensurate inputs and outputs. This non-parametric efficiency analysis technique is increasingly applied for measuring the level of efficiency of observed DMUs. DEA provides a mathematical programming method for estimating best practice production frontiers and evaluating the relative efficiency of operational units. Given the input/output data, DEA evaluates the performance of the DMUs in the technology set $T = \{(x, y) : x\ can\ produce\ y\}$. T is extrapolated from the observed data (x^k, y^k) : $k = 1, ..., K$ in which $(x^k = x_1^k, , x_2^k, ..., x_N^k)$ and $(y^k = y_1^k, y_2^k, ..., y_N^k)$ are respectively the input and output vectors of DMU_k. On the input side, the Farrell efficiency of DMU_O is measured by $\theta_o = Min\{\theta : (\theta x^o, y^o) \in T\}$ and similarly, on the output side, this efficiency index is measured by $\phi_o = Max\{\phi : (x^o, \phi y^o) \in T\}$ The directional distance function approach that combines both the input and output sides is formulated as $\rho^* = Max\{\rho : (x^o - \rho d_x, y^o + \rho d_y)\}$. DMU_o is said to be efficient if and only if $\rho^* = 0$.

Classic DEA models assume that all inputs and outputs are desirable and in the models formu-

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lated for efficiency analysis we normally want to increase the good outputs and to decrease the inputs. However in production processes there may exist both desirable and undesirable outputs and clearly we need to reduce undesirable outputs. Traditional DEA models such as CCR model of Charnes, Cooper and Rhodes [3] and BCC model of Banker, Charnes and Cooper [2] cannot deal with the undesirable outputs. In this sense, modeling undesirable outputs in production process is an important subject in the field of production theory that has attracted considerable attention among researchers.

There is a group of papers in which DEA researchers have proposed to model undesirable outputs. (See for instances, Chung et al. [4], Scheel [16], Seiford and Zhu [17], Kuosmanen [12].) These studies on modeling undesirable outputs or environmental outputs (not environmental factors that reflect the environment that the DMUs do business in it) have been divided into two groups: The first group of approaches develops techniques in which environmental outputs are considered as inputs instead of outputs. However, in the second group, researches proposed approaches in which undesirable outputs are modeled as outputs. In what follows, we briefly survey some of these studies.

Hailu and Veeman [7] have treated the undesirable outputs as inputs and a classic DEA model is used to evaluate the relative efficiency of the DMUs in the presence of desirable and undesirable outputs. Jung et al. [10] and Kumar-Mandal and Madheswaran [11] have made a performance analysis of overall efficiency in the oil and cement industries, respectively. As another DEA-based work, Lu et al. [13] and Jin et al. [9] have used CO2 emission as an undesirable output in their studies. Wu et al. [20] have also used this output in an investigation of cost performance of CO2 reduction. See also Wang et al. [19] that have proposed a meta-frontier DEA analysis of energy efficiency.

Many additional and theoretical articles in the field have adapted the models to treat with undesirable outputs. See for instances Podinovski and Kuosmanen [14], Rashidi K. and Farzipoor Saen [15], Hu and Liu [8] and Zanella et al. [21].

In this paper we will compare the technology set and the results of the different approaches to deal with undesirable outputs. Three different approaches have been studied and collated in this paper. The first approach that we will study, considers the bad outputs as inputs and we will show that this leads to incorrect performance measurement. The second approach takes the weak disposability assumption of Shephard [18] in to account and using this assumption, undesirable outputs are considered as outputs instead of inputs. Finally, in the third approach, the inverse of undesirable outputs are considered as desirable outputs and then classic DEA models have been used to performance analysis. Our results show that the correct and complete technology is given in the case that the weak disposability axiom is used to model undesirable outputs. The paper is organized as follows. The next section illustrates the DEA model in which bad outputs are modeled as inputs. Next, we will introduce the weak disposable technology of Färe and Grosskopf [6] and Kuosmanen [12]. The inverse of bad outputs is considered as good outputs in section 5. A simple example and a real application are given in sections 6 and 7, respectively. Conclusion will end the paper.

2 Bad outputs as inputs

At the first glance, it seems to be rational to treat with undesirable outputs as inputs. Hailu and Veeman [7] have developed a production model in which undesirable outputs are considered as inputs. To include undesirable outputs in the production technology, they introduced a nonorthodox monotonicity condition on their technology. This condition permits us to model undesirable outputs as inputs. They claimed that their treatment to undesirable outputs in the construction of the inner bound differs from the approaches adopted by Ball et al. [1] and Färe et al. [5], who used strict equality constraints for the undesirable outputs. Suppose there are K firms and the production process in firm k uses N inputs $x_n^k : n = 1, ..., N$ to produce M desirable outputs $v_m^k : m = 1, ..., M$ and J undesirable outputs $w_j^k : j = 1, ..., J$. The mathematical formulation of the approach proposed by Hailu and Veeman [7] is as follows:

$$
\begin{aligned}
TE_o = Min \quad & \theta \\
s.t. \quad & \sum_{k=1}^{K} \lambda^k w_j^k \le w_j^o; \quad j = 1, ..., J, \\
& \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o; \quad m = 1, ..., M, \\
& \sum_{k=1}^{K} \lambda^k x_n^k \le \theta x_n^o; \quad n = 1, ..., N, \\
& \sum_{k=1}^{K} \lambda^k = 1; \\
& \lambda^k \ge 0; \qquad k = 1, ..., K.
\end{aligned} \tag{2.1}
$$

in which the super-script "o" shows the firm under evaluation. If we use the directional distance function with $(d_x, d_v, d_w) = (x_o, v_o, w_o)$, model (2.1) is re-formulated as follows:

$$
\begin{aligned}
TE_o = Max \quad & \theta \\
s.t. \quad & \sum_{k=1}^{K} \lambda^k w_j^k \le w_j^o - \theta w_j^o; \ j = 1, ..., J, \\
& \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m^o + \theta v_m^o; \ m = 1, ..., M, \\
& \sum_{k=1}^{K} \lambda^k x_n^k \le x_n^o - \theta x_n^o; \ n = 1, ..., N, \\
& \sum_{k=1}^{K} \lambda^k = 1; \\
& \lambda^k \ge 0; \qquad k = 1, ..., K.
\end{aligned} \tag{2.2}
$$

As the model shows, Hailu and Veeman [7] have used the monotonicity condition in their methodology and Färe and Grosskopf [6] have correctly stated that in the presence of bad outputs, this condition is inconsistent with physical laws. We will show this in detail in section 6.

3 Bad outputs as outputs

As we stated in the preceding section, considering bad outputs as inputs leads to incorrect performance analysis. To show this, consider the monotonicity condition when x and w are the inputs:

Monotonicity assumption: If $(x, v, w) \in T$ and $v \ge v'$, $w \le w'$ and $x \le x'$ then, we must have $(x', v', w') \in T$.

Based on this assumption any vector (x, v, w') with $w' \ge w$ belongs the technology set T and this means that a fixed amount of resources can produce an unbounded amount of bad outputs; and this is not rational. In fact, the monotonicity condition in the presence of undesirable outputs violates the boundedness of production technology for each input vector x .

Färe and Grosskopf [6] have used the weak disposability assumption of Shepherd [18] to define the weakly disposable technology. Consider the following output set:

$$p(x) = \{(v, w) : x \ can \ produce \ (v, w)\}$$

Weak disposability (Shepherd [18]): Outputs are weakly disposable if $(v, w) \in p(x)$ and $0 \le \theta \le 1$ then $(\theta v, \theta w) \in p(x)$.

If we have two outputs, good and bad, weak disposability means that a $\theta\%$ reduction in bad output is possible if it accompanied by a $\theta\%$ reduction in good output with constant input.

Taking the weak disposability assumption of Shepherd [18] in to consideration and under variable return to scale, Färe and Grosskopf [6] proposed the following production set:

$$
\begin{aligned}
T_F = \{(v, w, x) : & \\
& \theta \sum_{k=1}^{K} \lambda^k v_m^k \ge v_m; \quad m = 1, ..., M, \\
& \theta \sum_{k=1}^{K} \lambda^k w_j^k = w_j; \quad j = 1, ..., J, \\
& \sum_{k=1}^{K} \lambda^k x_n^k \le x_n; \quad n = 1, ..., N, \\
& \sum_{k=1}^{K} \lambda^k = 1; \\
& \lambda^k \ge 0; \qquad k = 1, ..., K, \\
& 0 \le \theta \le 1\}.
\end{aligned} \tag{3.3}
$$

This technology is under variable return to scale and satisfies weak disposability assumption. The single abatement factor θ allows for the simultaneous reduction of desirable and undesirable outputs. Instead of single abatements factor, Kousmanen [12] has used nonuniform abatements factor across firms and defined a linear technology set. In the next section, we introduce the technology set of Kuosmanen [12].

4 Technology of Kuosmanen

As we stated before, Färe and Grosskopf [6] have used a single abatement factor θ to each firm k.

Kuosmanen [12] proposed a simple formulation of weak disposability that allows for non-uniform abatement factors, and with a numerical example he demonstrated that the technology set proposed by Färe and Grosskopf [6] with θ as a single abatement factor, does not show all feasible points and it contradicted the convexity which is an underlying assumption of the model. Kuosmanen [12] considered free disposability of desirable outputs, weak disposability of undesirable outputs, convexity and variable returns to scale assumptions. Then, under these assumptions, he revised the production technology of Färe and Grosskopf [6] as follows:

$$
\begin{aligned}
T_K = \{(v,w,x) : & \\
& \sum_{k=1}^{K} \theta^k \lambda^k v_m^k \geq v_m; \quad m = 1,...,M \\
& \sum_{k=1}^{K} \theta^k \lambda^k w_j^k = w_j; \quad j = 1,...,J \\
& \sum_{k=1}^{K} \lambda^k x_n^k \leq x_n; \quad n = 1,...,N \\
& \sum_{k=1}^{K} \lambda^k = 1; \\
& \lambda^k \geq 0; \qquad k = 1,...,K \\
& 0 \leq \theta^k \leq 1 \qquad k = 1,...,K\}.
\end{aligned} \tag{4.4}
$$

The technology set (3.3) is a special case of (4.4) when $\theta^1 = \theta^2 = ... = \theta^k$. Clearly, T_k is not linear but, it can be restated in an equivalent linear form as follows:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \sum_{k=1}^{K} \lambda^k w_j^k = \theta w_j^o; \quad j = 1,...,J, \\
& \quad \sum_{k=1}^{K} \lambda^k v_m^k \geq v_m^o; \quad m = 1,...,M, \\
& \quad \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \leq x_n^o; \quad n = 1,...,N, \\
& \quad \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1; \\
& \quad \lambda^k, \mu^k \geq 0; \quad k = 1,...,K.
\end{aligned} \tag{4.5}
$$

Again, if we use the directional distance function, model (4.5) can be written as follows:

$$
\begin{aligned}
& Max \quad \theta \\
& s.t. \sum_{k=1}^{K} \lambda^k w_j^k = w_j^o - \theta w_j^o; \quad j = 1,...,J, \\
& \quad \sum_{k=1}^{K} \lambda^k v_m^k \geq v_m^o + \theta v_m^o; \quad m = 1,...,M, \\
& \quad \sum_{k=1}^{K} (\lambda^k + \mu^k) x_n^k \leq x_n^o - \theta x_n^o; \quad n = 1,...,N, \\
& \quad \sum_{k=1}^{K} (\lambda^k + \mu^k) = 1; \\
& \quad \lambda^k, \mu^k \geq 0; \qquad k = 1,...,K.
\end{aligned} \tag{4.6}
$$

Model (4.6) is now a linear programming model and in this model, bad outputs are modeled as outputs.

5 The inverse of bad outputs as good outputs

In this section undesirable outputs are considered as outputs, but the inverse of these outputs are considered as good outputs. In a rational sight, to decrease the bad outputs w , we increase the inverse $\frac{1}{w}$ we assume that is not zero and we will relax this assumption later.) If we consider the inverse of undesirable outputs as desirable outputs, under variable returns to scale assumption, the performance measurement model is as follows:

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \sum_{k=1}^{K} \lambda^k \left(\frac{1}{w_j^k}\right) \geq \frac{1}{w_j^o}; \quad j = 1,...,J, \\
& \quad \sum_{k=1}^{K} \lambda^k v_m^k \geq v_m^o; \quad m = 1,...,M, \\
& \quad \sum_{k=1}^{K} \lambda^k x_n^k \leq \theta x_n^o; \quad n = 1,...,N, \\
& \quad \sum_{k=1}^{K} \lambda^k = 1; \\
& \quad \lambda^k \geq 0; \quad k = 1,...,K.
\end{aligned} \tag{5.7}
$$

Using the directional distance function with $(d_x, d_v, d_w) = (x_o, v_o, w_o)$ model (5.7) can be reformulated as follows:

$$
\begin{aligned}
& Max \quad \theta \\
& s.t. \sum_{k=1}^{K} \lambda^k (\tfrac{1}{w_j^k}) \geq \tfrac{1}{w_j^o} + \theta(\tfrac{1}{w_j^o}); j = 1, ..., J, \\
& \sum_{k=1}^{K} \lambda^k v_m^k \geq v_m^o + \theta v_m^o; \quad m = 1, ..., M, \\
& \sum_{k=1}^{K} \lambda^k x_n^k \leq x_n^o - \theta x_n^o; \quad n = 1, ..., N, \\
& \sum_{k=1}^{K} \lambda^k = 1; \\
& \lambda^k \geq 0; \quad k = 1, ..., K.
\end{aligned} \tag{5.8}
$$

Needless to say that the first constraint in (5.8) is absolutely different from the corresponding constraint in (2.1) that considers the undesirable outputs w as inputs.

6 A simple example

Now, we illustrate the above mentioned three different technologies by a simple example consisting of three DMUs A, B and C with one desirable output, one undesirable output and one input. The data are summarized in Table 1. Fig. 1 illustrates the three output sets graphically in $v - w$ space. (Note that input value to all three units is one.)

In all three methods, $DMUs$ A and B are efficient and C is inefficient. The results of the three different methods are listed in the last three columns of Table 1. Consider the region $P(1) = \{(v, w) : 0 \leq v \leq 1, 0 \leq w \leq 1, v \leq 2w\}$. $P(1)$ shows all points (v, w) that can be produced by the input one. This region is tetragon OSQR in figure (c) that belongs to the output set, but this region does not belong to the output sets in figures 1(a) and 1(b).

Now, consider the region $P'(1) = \{(v, w) : 0 \leq v \leq 1, 6 \leq w\}$. Clearly, this region does not belong to the output set (c), but it does belong to the set (a). This means that the points $(x, v, w) = (1, 1, w)$ belong to the technology set (a) for any large positive w . This is clearly inconsistent with physical laws. These show the limitations of the inverse method and the method proposed by Hailu and Veeman [7] that considers the bad outputs as inputs. We conclude this section an important finding that the correct and complete technology set is given in Kuosmanen [12] that takes the weak disposability assumption in to consideration.

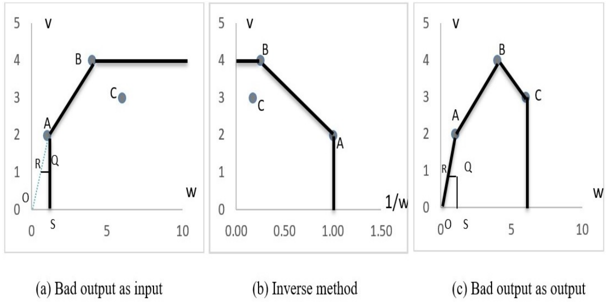

Figure 1: Three different technologies

7 Numerical example

After formulating the different performance measurement models, we apply the models to a real case example consisting 92 power plants. The plants uses three inputs (Capital (x_1), Coal (x_2) and natural gas (x_3)) to produce the single desirable output Net electricity (v_1). Two undesirable outputs (Sulphor dioxide (w_1) and Nitrogen dioxide (w_2)) are produced along with the good output. Table 2 shows the summary statistics of the data.

Three different approaches (Kuosmanen [12], Hailu and Veeman [7] and the Inverse method) have been used to this data set with five different directions $\{(v, w, x) = (v_o, 0, 0), (0, w_o, 0), (0, 0, x_o), (v_o, w_o, 0), (v_o, w_o, x_o)\}$. The statistical descriptive of the results are summarized in Table 3.

In all directions, the number of efficient units in two methods Hailu and Veeman [7] and Inverse method are respectively 7 and 5 firms. However, in Kuosmanen [12] the number of efficient firms is more than two others methods. For example, the number of efficient firms in direction $(v, w, x) = (v_o, w_o, x_o)$ is 20 firms. It seems that the discrimination power of the inverse method is relatively better than the other two approaches, but, what is important is that the correct and complete technology is given in Kuosmanen [12] and the other two approaches do not provide the correct technology and hence the performance evaluation in these two approaches is not reliable.

Table 1: The data set for the simple example

DMU	x	v	w	Efficiency	Efficiency Bad output as input	Efficiency Bad output as output	Efficiency Inverse method
A	1	2	1	1	1	1	1
B	1	4	4	0.25	1	1	1
C	1	3	6	0.1667	0.75	0.4176	0.75

Table 2: The summery statistics of data

	Capital	Coal	Gas	Net electricity	Sulphor dioxide	Nitrogen dioxide
Mean	240000014.7	188.27	4.71044E+13	4686524843	40745.19	17494.02
STDEV	146352514.9	112.75	3.99822E+13	4065294367	48244.78	16190.11
Domain of variations	710675620	496	1.73796E+14	18045453000	251051.40	72101.05

Table 3: The results of different approaches

Direction	Results	Hailu-Veeman	Inverse	Kuosmanen
d(v,w,x)	Number efficient firms	7	5	20
	Mean	0.1231	0.1287	0.0913
	STDEV	0.0828	0.0717	0.0741
	Domain of Variations	0.3585	0.2810	0.3112
d(v,w,0)	Number efficient firms	7	5	18
	Mean	0.2003	0.3112	0.1677
	STDEV	0.1414	0.1960	0.1402
	Domain of Variations	0.5712	0.7818	0.5712
d(v,0,0)	Number efficient firms	7	5	20
	Mean	0.3025	0.3228	0.2162
	STDEV	0.2345	0.2074	0.1902
	Domain of Variations	1.1176	0.9247	0.9036
d(0,w,0)	Number efficient firms	7	5	12
	Mean	0.3658	0.8050	0.3477
	STDEV	0.2092	0.4071	0.2210
	Domain of Variations	0.7447	0.490	0.7447
d(0,0,x)	Number efficient firms	7	5	20
	Mean	0.2323	0.2210	0.1522
	STDEV	0.1278	0.1122	0.1098
	Domain of Variations	0.5278	0.4388	0.3631

The maximum and minimum inefficiency means occur in two different approaches and directions. The maximum inefficiency occurs in the inverse method in the direction $(v, w, x) = (0, w_o, 0)$, but the minimum inefficiency means occur in the direction $(v, w, x) = (v_o, w_o, x_o)$ in Kuosmanen [12].

8 Conclusions

This paper has focused on modeling undesirable outputs in production processes in the context of DEA. Three different approaches have been investigated here. These approaches have been divided in to two groups: in the first group undesirable outputs are modeled as inputs and in the second one they have been considered as outputs.

It has been shown that considering bad outputs as inputs is not rational and it violates the physical law. Moreover, we have shown that the inverse method that considers the inverse of bad outputs as good inputs cannot correctly extrapolate the production technology. Next, we have rationally shown that the approach proposed by Kuosmanen [12] that takes the weak disposability assumption in to consideration constructs the complete and correct production technology.

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