



Satisfaction Function in Present Undesirable Factors

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Abstract

Data Envelopment Analysis (DEA) is an efficient method to perform evaluation of units. In DEA we try to evaluate units with undesirable factors in input & outputs by satisfaction function, testing some models. On the other hand benefiting this concept, we can identify non-efficient units. Also we can recognize why these units are inefficient and calculate the reason of their inefficiency and how they turn efficient. In DEA we cannot know why some units are non-efficient and how these units can be efficient, but by this paper we can do this work.

Keywords: Decision Making Units, Efficiency, Game Theory, Undesirable Factor, Satisfaction Functions

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1. Introduction

One efficient method for units performance evaluation is DEA. Data envelopment Analysis was first introduced by Charnes, Cooper and Rhods in 1978 [1]. They represented CCR model for efficiency evaluation of units under observation. The CCR model is a non-parametric method for efficiency evaluation, that the efficiency border is made by comparison of units condition under observation. Farel proposed the concept of non-parametric models of efficiency evaluation in 1957[2], but before the CCR model, this concept was not spread. After that, this model attracted many researchers 'attention and a lot of studies were conducted in this case.

Banker, Charnes and cooper used the concept of efficiency to scale in design of BCC model. The efficiency border of BCC model is created on the basis of efficiency to variable scale. In all of DEA models, for each unit under study a number is obtained as efficiency, , the units are divided into two groups of efficient and non-efficient based on it . The non- efficient units has the efficiency lower than one and the efficient units has efficiency equal one.

In data envelopment analysis by using some models we try to evaluate units, this method by using some models formulated based on existing cases, recognizes the efficient and non-efficient units. Practically, there are always cases that undesirable factors exist; this job is done by help of unit's comparison. But by this work we just explain about efficiency or non-

efficiency units.

One of very fundamental models that exist with undesirable factors, is one paper by Fär in2002 [5], but that is non-linear. After that Seford proposed another model in 1989 [4].Then Countless articles were written about undesirable factors. But none of these articles didn't explain about non-efficient units and how those can be efficient. In most of models whose DEA exists, when one efficient unit is verified, it doesn't discuss about how this unit becomes efficient. On the other hand in practice undesirable factors exist, unfortunately in most of DEA models this subject is ignoring ignored. Therefore in this paper by using satisfaction function we try to evaluate units that have undesirable factors in inputs and outputs. Then by this concept, the non-efficient units are verified and we can know cause of non-efficiency and how we can change them to efficient units. For this subject in all desirable and undesirable inputs and outputs we carry out sensivity analysis and the by using some parameters we do this change.

2. Suggestion model

In this section the u unit is evaluated and the dominated units are shown by v . Also the symbol b is for introduction of undesirable inputs and outputs and the symbol g is for introduction of desirable inputs and outputs. Also in this section the symbol O is for output and I is for input.

Suppose d is existing decision maker. In all cases u is under evaluation. Also the vectors of

inputs and outputs are shown with O and I . But practically it is possible that all desirable inputs and outputs do not exist and some undesirable factors might exist. Therefore the vector of inputs and outputs besides undesirable factors is considered follow:

$$I = \begin{pmatrix} I_u^g \\ I_u^b \end{pmatrix}, O = \begin{pmatrix} O_u^g \\ O_u^b \end{pmatrix}$$

Where I_u^g and O_u^g respectively are vectors of desirable factors and I_u^b and O_u^b respectively are vectors of undesirable factors. But the inputs and outputs may not be in the same unit, then for this we use one method namely Linearization method. By this calculation we have

$$\begin{aligned} O_u^g &= \left[\frac{O_u^{1g}}{\max O_u^g}, \frac{O_u^{2g}}{\max O_u^g}, \dots, \frac{O_u^{ng}}{\max O_u^g} \right] \\ O_u^b &= \left[\frac{O_u^{1b}}{\max O_u^b}, \frac{O_u^{2b}}{\max O_u^b}, \dots, \frac{O_u^{nb}}{\max O_u^b} \right] \quad (1) \\ I_u^g &= \left[\frac{I_u^{1g}}{\max I_u^g}, \frac{I_u^{2g}}{\max I_u^g}, \dots, \frac{I_u^{ng}}{\max I_u^g} \right] \\ I_u^b &= \left[\frac{I_u^{1b}}{\max I_u^b}, \frac{I_u^{2b}}{\max I_u^b}, \dots, \frac{I_u^{nb}}{\max I_u^b} \right] \end{aligned}$$

By that definition and for evaluation units, we need one criterion. For the definition of this criterion we follow this:

$$\begin{aligned} g_s^g(u) &= W^s O_u^g, g_s^b(u) = W^s O_u^b \quad (2) \\ g_r^g(u) &= W^r I_u^g, g_r^b(u) = W^r I_u^b \end{aligned}$$

Notice in the above definitions W is weight of each input and output factor and the weight vector of input and output follows this definition:

$$W^r = [w_1^r, w_2^r, \dots, w_n^r], W^s = [w_1^s, w_2^s, \dots, w_n^s]$$

And the above phrases w_i^s is Weight of acceptance and w_i^r is Weight of rejection when is show follow:

$$w_j^s = \frac{\sum_{k=1}^d \rho_{kj}}{\sum_{j=1}^n \sum_{k=1}^d \rho_{kj}}, w_i^r = \frac{\sum_{k=1}^d \sigma_{ki}}{\sum_{i=1}^m \sum_{k=1}^d \sigma_{ki}} \quad (3)$$

As inputs must be always reduced then the weight of rejection is w_i^r and the output always must be increased then the weight of acceptance is w_j^s .

Since the Weight of selection is w_j^s and the Weight of rejection is w_i^r then satisfaction function is defined as follows:

$$\begin{aligned} P_s^b(u) &= \frac{g_s^b(u)}{\sum_{g_s} g_s^b(u)}, P_s^g(u) = \frac{g_s^g(u)}{\sum_{g_s} g_s^g(u)} \quad (4) \\ P_r^g(u) &= \frac{g_r^g(u)}{\sum_{g_r} g_r^g(u)}, P_r^b(u) = \frac{g_r^b(u)}{\sum_{g_r} g_r^b(u)} \end{aligned}$$

Where the satisfaction function for 4 cases include desirable and desirable factors are written.

The goal of this paper is to identify efficient and non-efficient units and explain about how non-efficient units can become efficient and what they are efficient. Therefore first we have some definition.

The satisficing set: We define efficient set that uses its sources to produce efficient output. The satisficing shows by Σ and defined as follows:

$$\Sigma = \left\{ P_s^g(u) \geq p_r^g(u) \ \& \ p_s^b(u) \leq p_s^b(u), \right\}_{u = 1, 2, \dots, n} \quad (5)$$

The set B(u): the set $B(u)$ is set of units are strictly better than u and defined as follows:

$$B^g(u) = B_s^g(u) \cup B_r^g(u)$$

$$B^b(u) = B_s^b(u) \cup B_r^b(u)$$

Where we have:

$$B_s^g(u) = \{v \in U | p_r^g(v) < p_r^g(u), p_s^g(v) \geq p_s^g(u)\}$$

$$B_s^b(u) = \{v \in U | p_r^b(v) > p_r^b(u), p_s^b(v) \leq p_s^b(u)\}$$

$$B_r^g(u) = \{v \in U | p_r^g(v) \leq p_r^g(u), p_s^g(v) > p_s^g(u)\}$$

$$B_r^b(u) = \{v \in U | p_r^b(v) \geq p_r^b(u), p_s^b(v) < p_s^b(u)\}$$

The satisficing equilibrium: For satisficing unit u another satisficing units exist that are better than u , It is clear that in this case any decision maker prefer the second unit. So the set is interesting that includes satisficing unit and no unit is better than them. This satisficing set is namely \mathcal{E} . In another definition the satisficing equilibrium are units that use reference to produce outputs in the best way and this satisficing equilibrium is defined as follows:

$$\mathcal{E} = \{B^g(u) = \emptyset \wedge B^b(u) = \emptyset, u = 1, \dots, n\} \quad (6)$$

S is the set of units when they are good enough and use their own resource good enough.

$$S = \sum \cap \mathcal{E}$$

$$O_u^{gi} \geq O_v^{gi}, O_u^{bi} \leq O_v^{bi}, I_u^{gj} \leq I_v^{gj}, I_u^{bj} \geq I_v^{bj} \quad (7)$$

Definition: The unit $u \in U$ dominated $v \in U$ if and only if following inequality for each item with at least one strict inequality for each output i and each input j established.

$$\begin{cases} O_u^{gi} \geq O_v^{gi} \\ I_u^{gj} \leq I_v^{gj} \end{cases}$$

$$\begin{cases} O_u^{bi} \leq O_v^{bi} \\ I_u^{bj} \geq I_v^{bj} \end{cases}$$

Theorem: Suppose u, v in U then we have:

$$u \geq v \Rightarrow u \in B(v), v \notin \mathcal{E}$$

Notice that one non-dominated units cannot be

introduced for satisficing equilibrium. Of course the units in S have no problem, but units in \sum must have Sensitivity analysis on their needs so that we can bring into S . For this work Sensitivity parameters $0 \leq \delta_u^{gi}, 0 \leq \delta_u^{bi}, 0 \leq \gamma_u^{gi}, 0 \leq \gamma_u^{bi}$ introduce and we must suppose these parameters in order to $O_u^{gi}, O_u^{bi}, I_u^{bi}, I_u^{gi}$ and Consider the following:

$$\begin{aligned} O_u^{gi} + \delta_u^{gi}, O_u^{bi} - \delta_u^{bi} \\ I_u^{gi} - \gamma_u^{gi}, I_u^{bi} + \gamma_u^{bi} \end{aligned}$$

Notice relationship above means to improve the units in \sum , the desirable output must increase in size of δ_u^{gi} but undesirable output must decrease in size of δ_u^{bi} . Because always when undesirable factors exist, the aim is decrease of undesirable output. Similarly we must decrease desirable inputs I_u^{gi} in size of γ_u^{gi} and undesirable input I_u^{bi} increase in size of γ_u^{bi} .

Consider the following conditions:

$$0 < I_u^b(i) + \gamma_u^{bi} \leq 1, 0 < O_u^b(i) - \delta_u^{bi} \leq 1$$

$$0 < I_u^g(i) - \gamma_u^{gi} \leq 1, 0 < I_u^b(i) + \gamma_u^{bi} \leq 1$$

In the above case, we have:

$$P_s^g(u) \geq P_r^g(u)$$

$$P_s^b(u) \leq P_r^b(u)$$

These parameters can be achieved through problem solving below:

$$\text{Min } \delta_u^g, \delta_u^b, \gamma_u^g, \gamma_u^b$$

$$C_o(\delta_u^g) \geq C_i(\gamma_u^g)$$

$$C_o(\delta_u^b) \leq C_i(\gamma_u^b)$$

$$\text{S. t. } \varepsilon_i \leq I_u^b(i) + \gamma_u^{bi} \leq 1$$

$$\varepsilon_o \leq O_u^b(i) - \delta_u^{bi} \leq 1$$

$$\varepsilon_i \leq I_u^g(i) - \gamma_u^{gi} \leq 1$$

$$\varepsilon_o \leq I_u^b(i) + \gamma_u^{bi} \leq 1$$

These are defined as follows:

$$C_o(\delta_u^g) = \frac{W^s(O_u^g + \delta_u^g)}{\sum_{v \in U, v \neq u} W^s O_v^g + W^s(O_u^g + \delta_u^g)}$$

$$C_I(\gamma_u^g) = \frac{W^r(I_u^g - \gamma_u^g)}{\sum_{v \in U, v \neq u} W^r I_v^g + W^r(I_u^g - \gamma_u^g)}$$

$$C_o(\delta_u^b) = \frac{W^r(O_u^b - \delta_u^b)}{\sum_{v \in U, v \neq u} W^r O_v^b + W^r(O_u^b + \delta_u^b)}$$

$$C_I(\gamma_u^b) = \frac{W^r(I_u^b - \gamma_u^b)}{\sum_{v \in U, v \neq u} W^r I_v^b + W^r(I_u^b - \gamma_u^b)}$$

With the above definitions, we have:

$$\frac{\delta_u^g(i)}{O_u^g(i)} = \frac{\delta_u^b(i)}{O_u^b(i)}, \quad \frac{\gamma_u^g(i)}{I_u^g(i)} = \frac{\gamma_u^b(i)}{I_u^b(i)}$$

The above items are some Amounts when unit u must increase in desirable output and decrease in undesirable output in order to be efficient when the other units are unchanged.

Conclusion

The advantage of the proposed method in this paper is by this paper in addition to verify efficient and non-efficient units; we can know why some units are non-efficient and how these units can be efficient. This job is conducted by using variables in sensitivity analysis. So proposed method in this paper is an applied method for evaluation units under review.

References

[1] Tchangani, A. P., (2006).A satisficing

game theory approach for group evaluation of production unit Decision support system.42, 778– 788.

[2] Charnes, A. (1978). W.W.Cooper, E. Rhodes, Measuring the efficiency of decision making units, Eur. Oper. Res, 2(6) , 429-444.

[3] Charnes, A. (1985). W.W.Cooper, B. Golany, I.Seiford, J. Stutz, Foundations of data envelopment analysis for pareto-koopmans efficient empirical production functions, J. Econometrics, 30 (1) , 91-107.

[4] Fär, R., Grosskopf, S., Lovell, C.A.K., Pasurka, C., 1989.Multilateral productivity comparisons when some outputs are undesirable: a nonparametric approach. The Review of Economics and Statistics 71, 90–98.

[5] Seiford, L.M. Zhu, J. (2002). Modeling undesirable factors in efficiency evaluation, Eur.J.Oper. Res, 142 (1) , 16-20.

[6] H,Yang .M.G, Pollitt. (2009). Incorporating both undesirable outputs and uncontrollable variable into DEA: the performance of Chinese coal-fired power plants, Eur. J. Oper. Res, 197 (3), 1095-1105.

[7] Banker, R.D. Charnes, A. (1984). W.W.Cooper.some models for estimating technical and scale inefficiencies in data envelopment analysis. Manage, 30 (9) , 1078-1092.