



Efficiency Evaluation and Ranking DMUs in the Presence of Interval Data with Stochastic Bounds

**Hamid. Sharafi^{a*}, Mohsen. Rostamy-Malkhalifeh^a, Alireza. Salehi^a,
Mohammad.Izadikhah^b**

(a) *Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran.*

(b) *Department of Mathematics, Islamic Azad University, Arak Branch, Arak, Iran.*

Received 08 November 2014, Revised 02 December 2014, Accepted 17 January 2015

Abstract

On account of the existence of uncertainty, DEA occasionally faces the situation of imprecise data, especially when a set of DMUs include missing data, ordinal data, interval data, stochastic data, or fuzzy data. Therefore, how to evaluate the efficiency of a set of DMUs in interval environments is a problem worth studying. In this paper, we discussed the new method for evaluation and ranking interval data with stochastic bounds. The approach is exemplified by numerical examples.

Keywords: Data envelopment analysis (DEA), Decision making unit (DMU), Efficiency, Ranking, Interval data, stochastic bounds.

1. Introduction

Data Envelopment Analysis (DEA), as suggested by Charnes et al [1] is an productive approach for measuring relative efficiency values of a set of similar decision making units (DMUs) that have multiple inputs and multiple outputs. In DEA, the so-called ‘efficient frontier’ or ‘production frontier’

*Corresponding Author: Hamid_sharafi2013@yahoo.com

is construct as the envelope of all DMUs. The set of feasible activities or DMUs is called a production possibility set. The representation of the efficient frontier in DEA is judgmentally influenced by the values of inputs and outputs of observations in the data set. The change in input and output values can reason the change in the structure of the efficient frontier and the relative efficiency values of DMUs. An fascinating problem is how to preserve the relative efficiency value of a considered DMU if the internal technical structure of the considered DMU somewhat changes in a short term.

Under many conditions, exact data are insufficient to model real-life situations. For example, human judgments including preferences are often vague and cannot estimate his preference with an exact numerical data, therefore these data may be have some structures such as bounded data, ordinal data, interval data, fuzzy data and stochastic data. In recent years, in different applications of DEA, inputs and outputs have been observed whose values are unclear R.G. Dyson, et al [2] T. Joro et al [3], T. J Stewart et al [4].

The interval DEA first proposed by Cooper et al [5] and the fuzzy DEA first proposed by Sengupta [6]. Cooper, Park, et al. [7] has growed an interval approach that permits mixtures of imprecise and precise data by transforming the DEA model into an ordinary

linear programming (LP) form. One of the hardships in the interval approach is the evaluation of the lower and upper bounds of the relative efficiencies of the DMUs. despite this difficulty, several researchers have proposed diverse variations of the interval approach (Despotis & Smirlis [8]; Entani, Maeda, & Tanaka [9]; Kao [10]; Kao & Liu [11]; Wang, Greatbanks, & Yang, [12]). Despotis and Smirlis [8] have developed an interval approach for referring to with imprecise data in DEA by convert a non-linear DEA model to an LP equivalent. The upper and lower bounds for the efficiency scores of the DMUS are explained. They use a post-DEA model and the endurance indices to discriminate among the efficient DMUs. They further formulate another post-DEA model to determine input thresholds that turn an inefficient DMU into an efficient one.

Note that we have knowledge of data envelopment analysis methodology has many benefits, such as no requirement for a priori weights or explicit specification of functional relations among the multiple inputs and outputs. However, there is feebleness in conventional DEA models; in fact, original DEA models do not allow stochastic variations in input and output such as data entry errors. As a result, DEA efficiency measurement may be sensitive to such variations. A DMU which is measured as efficient relative to other DMUs, can turn inefficient if such random

variations are considered. Stochastic input and output variations into DEA have been studied by, for example, Cooper, Deng, Huang, and Li [13,14], Land, Lovell, and Thore [15] and Olesen and Petersen [16], Morita and Seiford [17], Khodabakhshi and Asgharian [18], Khodabakhshi [19] and khodabakhshi et al [20].

The current article proceeds as follows: In Section 2, we survey some necessary preliminaries. Therefore, in Section 3, we suggest stochastic models and their deterministic equivalent for efficiency evaluation interval data with stochastic bounds. An example is presented in section 4. The paper terminates with conclusion.

2. Preliminaries

Suppose we have n DMUs, where each $DMU_j (j = 1, \dots, n)$ consumes m inputs $X_j = (x_{1j}, \dots, x_{mj})$ to produce s outputs $Y_j = (y_{1j}, \dots, y_{sj})$. Contrary to the original DEA model, interval DEA model assume that some crisp values for inputs x_{ij} and outputs y_{rj} are not known; it is only known that the input-output values are in certain bounded intervals, i.e. $x_{ij} \in [x_{ij}^L, x_{ij}^U]$ and $y_{rj} \in [y_{rj}^L, y_{rj}^U]$. Where upper and lower bounded of the intervals are given as fixed numbers and it is assumed that $x_{ij} > 0$ and $y_{rj} > 0$. In this case, the efficiency can be an interval. The upper limit of interval efficiency is obtained from the optimistic viewpoint and the lower

limit is obtained from the pessimistic viewpoint. The following model provides such an upper bound for DMU_j : (D.K. Despotis et.al (2002))

$$\begin{aligned} \theta_o^U &= \min \theta \\ s.t. & \\ \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L &\leq \theta x_{io}^L, \quad i = 1, \dots, m \\ \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j y_{rj}^L + \lambda_o y_{ro}^U &\geq y_{ro}^U, \quad r = 1, \dots, s \\ \lambda_j &\geq 0, (j = 1, \dots, n) \end{aligned} \tag{2.1}$$

We denote by θ_o^U the efficiency score attained by DMU_o in (1). The model below provides a lower bound of the efficiency score for DMU_o :

$$\begin{aligned} \theta_o^L &= \min \theta \\ s.t. & \\ \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^L + \lambda_o x_{io}^U &\leq \theta x_{io}^U, \quad i = 1, \dots, m \\ \sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j y_{rj}^U + \lambda_o y_{ro}^L &\geq y_{ro}^L, \quad r = 1, \dots, s \\ \lambda_j &\geq 0, (j = 1, \dots, n) \end{aligned} \tag{2.2}$$

2.1 Minimax regret-based approach in DEA

Here we introduce the minimax regret approach (MRA) developed by Wang et al [12]. The approach has some attractive features and can be used to compare and rank the intervals of scores even if they are equi-

centered but different in widths. The approach is summarized as follows:

Let $A_i = [a_i^L, a_i^U]$, ($i = 1, \dots, n$) be the intervals, and suppose $A_i = [a_i^L, a_i^U]$ is chosen as the best interval, and let $b = \max_{j \neq i} a_j^U$. Obviously, if $a_i^L < b$, the DM might suffer the loss of opportunity or regret and feel regret. The maximum loss of opportunity he/she might suffer is given by :

$$\max(r_i) = b - a_i^L = \max_{j \neq i} a_j^U - a_i^L.$$

If $a_i^L \geq b$, the DM will definitely suffer no loss of opportunity and feel no regret. In this situation, his/her regret is defined to be zero, i.e. $r_i = 0$. Combining the above two situations, we have

$$\max(r_i) = \max\{\max_{j \neq i} (a_j^U) - a_i^L, 0\}.$$

Thus; the minimax regret criterion will choose the interval satisfying the following condition as the best (most desirable) interval:

$$\min_i \max(r_i) = \min_i \{ \max\{ \max_{j \neq i} (a_j^U) - a_i^L, 0 \} \}$$

$$i = 1, \dots, m$$

Based on the analysis above, we consider the following definition for comparing and ranking intervals.

Definition 2.1. Let $A_i = [a_i^L, a_i^U]$, ($i = 1, \dots, m$) be a set of n intervals. The maximum loss of opportunity (also called maximum regret) of each interval A_i is defined as:

$$R(A_i) = \max\{ \max_{j \neq i} (a_j^U) - a_i^L, 0 \}$$

$$i = 1, \dots, m. \quad (2.3)$$

It is evident that the interval with the smallest maximum loss of opportunity is the most desirable interval.

Since the maximum losses of opportunity are relative numbers. So, they can only be used to choose the most desirable interval from among a set of intervals.

3. Stochastic models

In this section, we present stochastic upper bound model and stochastic lower bound model for efficiency score for interval data in the presence of stochastic bounds.

We used by notation in Cooper et al [14] and

$$\text{let } x_{ij} \in [\tilde{x}_{ij}^L, \tilde{x}_{ij}^U], y_{ij} \in [\tilde{y}_{ij}^L, \tilde{y}_{ij}^U],$$

$$\tilde{X}_j^L = (\tilde{x}_{1j}^L, \dots, \tilde{x}_{mj}^L), \tilde{X}_j^U = (\tilde{x}_{1j}^U, \dots, \tilde{x}_{mj}^U),$$

$$\tilde{Y}_j^L = (\tilde{y}_{1j}^L, \dots, \tilde{y}_{sj}^L) \text{ and } \tilde{Y}_j^U = (\tilde{y}_{1j}^U, \dots, \tilde{y}_{sj}^U)$$

represent random input and output vectors, and

$$X_j^L = (x_{1j}^L, \dots, x_{mj}^L), X_j^U = (x_{1j}^U, \dots, x_{mj}^U),$$

$$Y_j^L = (y_{1j}^L, \dots, y_{sj}^L) \text{ and } Y_j^U = (y_{1j}^U, \dots, y_{sj}^U)$$

stand for the corresponding vector of expected values of input and output, and let us consider all input and output components to be jointly normally distributed .

3.1 Stochastic model for upper bound

By using model (2.1), the proposed stochastic upper bound model can be obtained as:

$$\theta_o^U = \min \theta$$

$$s.t. \quad P\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^U + \lambda_o \tilde{x}_{io}^L \leq \theta \tilde{x}_{io}^L\right) \geq 1 - \alpha$$

$$i = 1, \dots, m.$$

$$P\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^L + \lambda_o \tilde{y}_{ro}^U \geq \tilde{y}_{ro}^U\right) \geq 1 - \alpha$$

$$r = 1, \dots, s.$$

$$\lambda \geq 0 \quad (3.1)$$

Where α is a predetermined number between 0 and 1 which specifies the significance level and P means “probability”.

3.1.1 Deterministic equivalent

Now in this part we show how to obtain θ_o^U from deterministic equivalent of the stochastic model (3).

From first constraint in model (3) we have:

$$p\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^U + \lambda_o \tilde{x}_{io}^L \leq \theta \tilde{x}_{io}^L\right) \geq 1 - \alpha,$$

$$i = 1, \dots, m \quad (3.3)$$

Then:

$$p\left(-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^U + \lambda_o \tilde{x}_{io}^L + \theta \tilde{x}_{io}^L \leq 0\right) \leq \alpha,$$

$$i = 1, \dots, m \quad (3.4)$$

Above equation is equivalent:

$$p\left(\frac{(-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^U - \lambda_o \tilde{x}_{io}^L + \theta \tilde{x}_{io}^L) + (\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L - \theta x_{io}^L)}{\omega_i^U}\right) \leq$$

$$\left(\frac{\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L - \theta x_{io}^L}{\omega_i^U}\right) \leq \alpha \quad (3.4)$$

Where:

$$(\omega_i^U)^2 = \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{x}_{ij}^U, \tilde{x}_{ik}^U)$$

$$+ 2(\lambda_o - \theta_o) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{x}_{ij}^U, \tilde{x}_{io}^L)$$

$$+ (\lambda_o - \theta_o)^2 \text{var}(\tilde{x}_{io}^L) \quad (3.5)$$

To obtain the deterministic equivalent of (3.4) and (3.5) we write:

$$Z_i = \frac{-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^U - \lambda_o \tilde{x}_{io}^L + \theta \tilde{x}_{io}^L + (\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L - \theta x_{io}^L)}{\omega_i^U}$$

$$; i = 1, \dots, m \quad (3.6)$$

If we assume the input and output to be normally distributed, then Z_i is also normally distributed with mean zero and unit variance, since Z_i is normally distributed.

The deterministic equivalent of (3.4) is as:

$$\frac{\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L - \theta x_{io}^L}{\omega_i^U} \leq \Phi^{-1}(\alpha) \quad (3.7)$$

Where (3.7), represents the normal cumulative distribution function and $\Phi^{-1}(\alpha)$ is its inverse.

For the second constraint of model (4.1) we have:

$$p\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^L + \lambda_o \tilde{y}_{ro}^U \geq \tilde{y}_{ro}^U\right) \geq 1 - \alpha,$$

$$r = 1, \dots, s \quad (3.8)$$

By the similar manner like relations (3.2) until (3.7) we can obtain the deterministic equivalent of this constraint as follow:

$$\frac{-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^L - \lambda_o \tilde{y}_{ro}^U + \tilde{y}_{ro}^U}{\omega_r^U} \leq \Phi^{-1}(\alpha)$$

$$r = 1, \dots, s \quad (3.9)$$

Where:

$$(\omega_r^U)^2 = \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{y}_{rj}^L, \tilde{y}_{rk}^L)$$

$$+ 2(\lambda_o - 1) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{y}_{rj}^L, \tilde{y}_{ro}^U)$$

$$+ (\lambda_o - 1)^2 \text{var}(\tilde{y}_{ro}^U) \quad (3.10)$$

Hence, by using (3.7) and (3.9), the deterministic equivalent of (3.1) can be represented as:

$$\theta_o^U = \min \theta$$

s.t.

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^U + \lambda_o x_{io}^L - \Phi^{-1}(\alpha) \omega_i^U + s_i^- = \theta x_{io}^L$$

$$i = 1, \dots, m$$

$$-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^L - \lambda_o \tilde{y}_{ro}^U + \tilde{y}_{ro}^U - \Phi^{-1}(\alpha) \omega_r^U + s_r^+ = 0$$

$$r = 1, \dots, s$$

$$s_i^-, s_r^+, \lambda_j \geq 0$$

$$(\omega_i^U)^2 = \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{x}_{ij}^U, \tilde{x}_{ik}^U)$$

$$+ 2(\lambda_o - \theta_o) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{x}_{ij}^U, \tilde{x}_{io}^L)$$

$$+ (\lambda_o - \theta_o)^2 \text{var}(\tilde{x}_{io}^L)$$

$$(\omega_r^U)^2 = \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{y}_{rj}^L, \tilde{y}_{rk}^L)$$

$$+ 2(\lambda_o - 1) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{y}_{rj}^L, \tilde{y}_{ro}^U)$$

$$+ (\lambda_o - 1)^2 \text{var}(\tilde{y}_{ro}^U)$$

3.2 Stochastic model for lower bound

By using the model (2.2), the proposed lower bound model for efficiency score for interval data in the presence of stochastic bounds can be obtained as:

$$\theta_o^L = \min \theta$$

s.t.

$$P\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{x}_{ij}^L + \lambda_o \tilde{x}_{io}^U \leq \theta \tilde{x}_{io}^U\right) \geq 1 - \alpha$$

$$i = 1, \dots, m$$

$$P\left(\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^U + \lambda_o \tilde{y}_{ro}^L \geq \tilde{y}_{ro}^L\right) \geq 1 - \alpha$$

$$r = 1, \dots, s$$

$$\lambda \geq 0 \quad (3.12)$$

Where α is a predetermined number between 0 and 1 which specifies the significance level and P means “probability”.

3.2.1 Deterministic equivalent

By the similar manner like (3.11) we can obtain the deterministic equivalent of lower bound model as follows:

$$\theta_o^L = \min \theta$$

s.t.

$$\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j x_{ij}^L + \lambda_o x_{io}^U - \Phi^{-1}(\alpha) \omega_i^L + s_i^- = \theta x_{io}^U$$

$$i = 1, \dots, m$$

$$-\sum_{\substack{j=1 \\ j \neq o}}^n \lambda_j \tilde{y}_{rj}^U - \lambda_o \tilde{y}_{ro}^L + \tilde{y}_{ro}^L - \Phi^{-1}(\alpha) \omega_r^L + s_r^+ = 0$$

$$r = 1, \dots, s$$

$$s_i^-, s_r^+, \lambda_j \geq 0 \quad (3.11)$$

$$\begin{aligned}
 (\omega_i^L)^2 &= \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{x}_{ij}^L, \tilde{x}_{ik}^L) \\
 &+ 2(\lambda_o - \theta_o) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{x}_{ij}^L, \tilde{x}_{io}^U) \\
 &+ (\lambda_o - \theta_o)^2 \text{var}(\tilde{x}_{io}^U)
 \end{aligned}$$

$$\begin{aligned}
 \bar{y}_r^L &= \frac{1}{15} \sum_{j=1}^{15} y_{rj}^L, \bar{y}_r^U = \frac{1}{15} \sum_{j=1}^{15} y_{rj}^U, \\
 \bar{x}_i^U &= \frac{1}{15} \sum_{j=1}^{15} x_{ij}^U, \bar{x}_i^L = \frac{1}{15} \sum_{j=1}^{15} x_{ij}^L,
 \end{aligned}$$

$$\begin{aligned}
 (\omega_r^L)^2 &= \sum_{j \neq o} \sum_{k \neq o} \lambda_j \lambda_k \text{cov}(\tilde{y}_{rj}^U, \tilde{y}_{rk}^U) \\
 &+ 2(\lambda_o - 1) \sum_{j \neq o} \lambda_j \text{cov}(\tilde{y}_{rj}^U, \tilde{y}_{ro}^L) \\
 &+ (\lambda_o - 1)^2 \text{var}(\tilde{y}_{ro}^L)
 \end{aligned}$$

Table 1. The data set

DMUs	Input 1	Input 2	Output 1	Output 2	Output3
1	[149, 170]	[2860, 3200]	[96,1 01]	[13150, 14530]	[1124, 1247]
2	[168, 195]	[2610, 3218]	[89,9 2]	[8385,8 571]	[956,1 311]
3	[134, 189]	[2690, 3388]	[69,8 1]	[8712,9 571]	[745,1 265]
4	[150, 187]	[3211, 3381]	[81,8 6]	[7422,7 506]	[794,8 24]
5	[151, 158]	[3217, 3505]	[85,9 0]	[7302,7 936]	[828,9 50]
6	[136, 164]	[2750, 3289]	[98,1 09]	[14873, 15231]	[947,1 009]
7	[138, 198]	[3142, 3369]	[89,9 0]	[8124,8 312]	[812,8 61]
8	[129, 172]	[2945, 3187]	[88,9 3]	[7436,8 125]	[712,9 87]
9	[128, 161]	[2820, 3186]	[67,7 6]	[12350, 12860]	[1021, 1425]
10	[133, 155]	[2590, 2872]	[96,1 02]	[8950,1 0950]	[872,8 91]
11	[117, 136]	[2745, 3386]	[87,9 3]	[9842,1 1840]	[912,1 145]
12	[122, 174]	[2974, 3416]	[112, 118]	[8453,9 851]	[1284, 1367]
13	[131, 162]	[2654, 3018]	[80,8 6]	[11520, 12563]	[1009, 1381]
14	[150, 180]	[2987, 3248]	[114, 116]	[8741,1 0256]	[764,9 35]
15	[112, 132]	[2864, 2951]	[97,1 03]	[9463,1 2120]	[1241, 1321]

(3.13)

4. Application

To calculate the results of the models (3.11)and(3.13) we consider two cases, in the first case we have chosen $\alpha = 0.55$ and hence $\Phi^{-1}(\alpha) = 0.12$ and in the second case we have chosen $\alpha = 0.45$ and hence $\Phi^{-1}(\alpha) = -0.12$. We assume that inputs and outputs for all DMUs are independence therefore $\text{cov}(\tilde{x}_{ij}^L, \tilde{x}_{ik}^L) = 0, \text{cov}(\tilde{x}_{ij}^L, \tilde{x}_{io}^U) = 0, \text{cov}(\tilde{y}_{rj}^U, \tilde{y}_{rk}^U) = 0, \text{cov}(\tilde{y}_{rj}^U, \tilde{y}_{ro}^L) = 0$. We will also assume that variance for inputs and outputs can be obtained as:

$$\text{Var}(\tilde{y}_r^L) = \frac{1}{14} \sum_{j=1}^{15} (y_{rj}^L - \bar{y}_r^L)^2, r = 1, 2, 3,$$

$$\text{Var}(\tilde{y}_r^U) = \frac{1}{14} \sum_{j=1}^{15} (y_{rj}^U - \bar{y}_r^U)^2, r = 1, 2, 3,$$

$$\text{Var}(\tilde{x}_i^L) = \frac{1}{14} \sum_{j=1}^{15} (x_{ij}^L - \bar{x}_i^L)^2, i = 1, 2,$$

$$\text{Var}(\tilde{x}_i^U) = \frac{1}{14} \sum_{j=1}^{15} (x_{ij}^U - \bar{x}_i^U)^2, i = 1, 2$$

$$\text{Var}(\tilde{y}_r^L) = \frac{1}{14} \sum_{j=1}^{15} (y_{rj}^L - \bar{y}_r^L)^2, r = 1, 2, 3,$$

$$\text{Var}(\tilde{y}_r^U) = \frac{1}{14} \sum_{j=1}^{15} (y_{rj}^U - \bar{y}_r^U)^2, r = 1, 2, 3,$$

$$Var(\tilde{x}_i^L) = \frac{1}{14} \sum_{j=1}^{15} (x_{ij}^L - \bar{x}_i^L)^2, i = 1, 2,$$

$$Var(\tilde{x}_i^U) = \frac{1}{14} \sum_{j=1}^{15} (x_{ij}^U - \bar{x}_i^U)^2, i = 1, 2$$

$$\bar{y}_r^L = \frac{1}{15} \sum_{j=1}^{15} y_{rj}^L, \bar{y}_r^U = \frac{1}{15} \sum_{j=1}^{15} y_{rj}^U,$$

$$\bar{x}_i^U = \frac{1}{15} \sum_{j=1}^{15} x_{ij}^U, \bar{x}_i^L = \frac{1}{15} \sum_{j=1}^{15} x_{ij}^L,$$

Table 2. The result of 3.11 and 3.12 models with $\alpha = 0.45$

DMUs	θ_o^L	θ_o^U	R_j	Ranking
1	0.8330	1.0000	0.167	6
2	0.7144	1.0000	0.2856	10
3	0.5470	1.0000	0.453	15
4	0.6197	0.8258	0.3803	14
5	0.6265	0.8607	0.3735	13
6	0.9443	1.0000	0.0557	1
7	0.6822	0.9138	0.3178	12
8	0.7133	1.0000	0.2867	11
9	0.7661	1.0000	0.2339	8
10	0.8630	1.0000	0.137	5
11	0.7205	1.0000	0.2795	9
12	0.8942	1.0000	0.1058	4
13	0.7810	1.0000	0.219	7
14	0.9029	1.0000	0.0971	2
15	0.8998	1.0000	0.1002	3

and $x_{ij}^L, x_{ij}^U, y_{rj}^L$ and y_{rj}^U are the expected values of inputs and outputs for DMUj which are presented in Table 1. We will also assume that outputs and inputs for different DMUs are independent. This independence assumption

then implies that $cov(y_{rj}^L, y_{rk}^L) = 0,$
 $cov(y_{rj}^L, y_{rk}^U) = 0, \quad cov(y_{rj}^U, y_{rk}^U) = 0,$
 $cov(x_{ij}^L, x_{ik}^L) = 0, \quad cov(x_{ij}^L, x_{ik}^U) = 0$ and
 $cov(x_{ij}^U, x_{ik}^U) = 0.$

In table 2 and 3 we calculate lower and upper bond of efficiency for each DMUs and Complete ranking by equation (2.3).

Table 3. The result of 3.11 and 3.12 models with $\alpha = 0.55$

DMUs	θ_o^L	θ_o^U	R_j	Ranking
1	0.7927	1.0000	0.2073	6
2	0.6802	1.0000	0.3198	9
3	0.5125	1.0000	0.4875	15
4	0.5883	0.7839	0.4117	14
5	0.5961	0.8184	0.4039	13
6	0.8891	1.0000	0.1109	1
7	0.6498	0.8619	0.3502	12
8	0.6788	0.9524	0.3212	10
9	0.7235	1.0000	0.2765	8
10	0.8225	1.0000	0.1775	5
11	0.6745	1.0000	0.3255	11
12	0.8569	1.0000	0.1431	4
13	0.7378	1.0000	0.2622	7
14	0.8665	1.0000	0.1335	2
15	0.8572	1.0000	0.1428	3

As you see in table 2, DMU2, DMU8 and DMU11 have ranking 10, 11 and 9 respectively, where in table 3 these DMUs ranking 9, 10 and 11 respectively.

5. Conclusion

In real world condition there may exists probabilistic or stochastic data. However, there are very few published studies on performance measurement which concurrently incorporate

stochastic data with Interval data. In the present article we have developed stochastic version of the proposed Ranking method and obtained a deterministic equivalent for the stochastic version. This deterministic equivalent can be converted to a quadratic problem.

References

- [1] A. Charnes, W.W. Cooper and E. Rhodes, Measuring the efficiency of decision making units, *Euro. J. Operat. Res.*, 2 (1978) 429-444.
- [2] R.G. Dyson, E. Thanassoulis, Reducing weight flexibility in data envelopment analysis, *J. Operat. Res. Soc.* 39 (1988) 563–576.
- [3] T. Joro, P. Korhonen, J. Wallenius, Structural comparison of data envelopment analysis and multiple objective linear programming, *Manage. Sci.* 44 (1998) 962–970.
- [4] T. J Stewart, Relationships between data envelopments analysis and multiple criteria decision analysis, *J. Operat. Res. Soc.* 47 (1996) 654–665
- [5] W.W. Cooper, K.S. Park and G. Yu, IDEA and AR-IDEA: models for dealing with imprecise data in DEA, *Management Science* 45 (1999) 597-607.
- [6] Sengupta, J. K. (1992). “A Fuzzy System Approach in Data Envelopment Analysis,” *Computers Math. Applic.* 24, 259–266.
- [7] Cooper, W. W., Park, K. S., & Yu, G. (1999). IDEA and AR-IDEA: Models for dealing with imprecise data in DEA. *Management Science*, 45, 597–607.
- [8] Despotis, D. K., & Smirlis, Y. G. (2002). Data envelopment analysis with imprecise data. *European Journal of Operational Research*, 140, 24–36.
- [9] Entani, T., Maeda, Y., & Tanaka, H. (2002). Dual models of interval DEA and its extension to interval data. *European Journal of Operational Research*, 136, 32–45.
- [10] Kao, C. (2006). Interval efficiency measures in data envelopment analysis with imprecise data. *European Journal of Operational Research*, 174, 1087–1099.
- [11] Kao, C., & Liu, S. T. (2000). Fuzzy efficiency measures in data envelopment analysis. *Fuzzy Sets and Systems*, 113, 427–437.
- [12] Y.M. Wang, R. Greatbanks, J.B. Yang, Interval efficiency assessment using data envelopment analysis, *Fuzzy Set. Syst.* 153 (2005) 347-370.
- [13] Cooper, W. W., H. Deng, Z. Huang and S.X. Li (2002), Chance constrained programming approaches to technical efficiencies and inefficiencies in stochastic data envelopment analysis, *Journal of the Operational Research Society*, 53, (12), 1347–1356.
- [14] Cooper, W. W., H. Deng, Z. Huang and S.X. Li (2004), Chance constrained programming approaches to congestion in stochastic data envelopment analysis,

European Journal of Operational Research, 155, (2), 487–501

[15]Land, K. C., Lovell, C. A. K., & Thore, S. (1988). Chance constrained data envelopment analysis. *Managerial and Decision Economics*, 14, 541–554.

[16]Olesen, O. B., & Petersen, N. C. (1995). Chance constrained efficiency evaluation. *Management Science*, 41, 442–457.

[17] Morita, Hiroshi, & Seiford, L. M. (1999). Characteristics on stochastic DEA efficiency. *Journal of Operational Research Japan*, 42(4), 389–404.

[18]Khodabakhshi, M., & Asgharian, M. (2009). An input relaxation measure of efficiency in stochastic data envelopment analysis. *Applied Mathematical Modelling*, 33, 2010–2023.

[19] Khodabakhshi, M. (2010). Chance constrained additive input relaxation model in stochastic data envelopment analysis. *International Journal of Information and Systems Sciences*, 6(1), 99–112.

[20] Khodabakhshi, M., M. Asgharian and Greg N. Gregoriou (2010), An input-oriented super-efficiency measure in stochastic data envelopment analysis: Evaluating chief executive officers of US public banks and thrifts, *Expert Systems with Applications*, 37, 2092-2097.