Available online at http://ijdea.srbiau.ac.ir
Int. J. Data Envelopment Analysis (ISSN 2345-458X)
Vol. 1, No. 2, Year 2013 Article ID IJDEA-00126, 12 pages
Research Article
International Journal of Data Envelopment Analysis
Science and Research Branch (IAU)

Non radial model of dynamic DEA with the parallel network structure

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Abstract

In this article, Non radial method of dynamic DEA with the parallel network structure is presented and is used for calculation of relative efficiency measures when inputs and outputs do not change equally. In this model, DMU divisions under evaluation have been put together in parallel. But its dynamic structure is assumed in series. Since in real applications there are undesirable inputs and outputs in the proposed model, the assumption of the existence of the intermediate products have been considered. After obtaining period–divisional efficiencies, by considering its weighted arithmetic mean, models are presented for the evaluation of period, divisional and overall efficiency for decision making unit

Keywords: dynamic data envelopment analysis – parallel network – overall efficiency – links and variable carry-overs.

1 Introduction

Data envelopment analysis is a Non parametric method for measuring relative efficiency of decision making units based on multiple inputs and outputs that was invented by Fare and universalized by Charnes et al [2]. One of the drawbacks of this model is the omission of the internal structure of the DMUs. For example, many companies and organizations are comprised of several divisions each one of these division which specific inputs & outputs are linked together and other divisions as well. Also, in real life the activities of such organizations are connected together in several different consecutive. So, for the assessment of the performance of these organizations and companies a model is needed to assess both the period efficiencies and divisional efficiencies and, eventually, the efficiency of overall system.

For the first time in2000, Fare and Grosskopf [5] presented an article under the title of "Network data envelopment analysis" in which the importance of network DEA was emphasized. After that, multiple

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models of DEA with network structure were presented (for further studies one can refer to Costelli et al [1] and Chen [3], Cook et al [4] and Lin et al [7]). Also Tone et al [8], developed network DEA according to the SBM model. In this model links and carry-overs between divisions have specific groupings (good link, fixed link). In addition to the structure of desired DMU division, they paid attention to the connections between which this shows the development of network DEA model towards internal structure of the assessed DMUs with the variable links. Ton et al [9], proposed a combinatory model of two models of developed network DEA [8] and dynamic DEA for SBM model [10]. This combinatory model not only enables us in the assessment of overall efficiencies of desired DMU but also is a good guide for further analysis of the period efficiency and divisional efficiency of DMUs.

In this paper the Non radial method of dynamic DEA with parallel network structure has been presented with the assumption of the existence of various links & connections in the structure of the network and dynamic model. Obtaining overall efficiencies, period efficiencies, divisional efficiencies and period-divisional efficiencies in each period of time and in each part of DMUs' decision making sub-units con be assumed as one of the merits of this method considering the volatile links & connections.

2 Dynamic DEA with parallel network structure

In dynamic DEA with parallel network structure we deal with decision making units n (DMU_j, j=1,…, n). Each DMU is divided to q divisions (p=1,…, q) which are placed parallel together. Therefore overall system inputs are divided among all divisions and overall outputs results from the output of all divisions. In this paper their efficiencies and the desired DMU efficiencies in T time period (t=1,…,T) is examined.

The dynamic structure model consists of internal connections that transport intermediate products of t period to t+1 period. In the first period, we don't have any connection from previous period besides, in the last period of T, we didn't consider any connection for the next period. We grouped these connections into two groups of desirable and undesirable. Desirable carry-overs are treated as outputs (transitional profit, net earned surplus) which we call them as "good" and undesirable carry-overs are treated as inputs (loss carried forward, bad debt, dead stock) which are named "bad" accordingly. So if we consider the number of all dynamic connections in this model as "h", we will have:

(n-good) + (n-bad) =h

Non radial model dynamic DEA with parallel network can be expressed as follows:

$$E_o = Max\ \frac{1}{Tq}\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rop}^t + \sum_{d\in D^p}\beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)\right)$$

$$s.t$$

$$\frac{1}{Tq}\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p}\alpha_d^t z_{dop}^{(t,t+1)bad}\right)\right) = 1 \qquad (1)$$

$$\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rjp}^t + \sum_{d\in D^p}\beta_d^t z_{djp}^{(t,t+1)good} - \sum_{i\in I^p} v_i^t x_{ijp}^t - \sum_{d\in D^p}\alpha_d^t z_{djp}^{(t,t+1)bad} + u_{0p}^t\right)\right) \le 0$$

$$u_r^t \ge 0 \qquad r = 1,\dots, s_p$$

$$v_i^t \ge 0 \qquad i = 1,\dots, m_p$$

$$\beta_d^t \ge 0 \qquad d = 1,\dots, n-good$$

$$\alpha_d^t \ge 0 \qquad d = 1,\dots, n-bad$$

$$u_{0p}^t : free$$

x_{ijp}^t is input resource i to DMU_j for division p in period t.

y_{rjp}^t is output product r from DMU_j for division p in period t.

$z_{djp}^{(t,t+1)good}$ is intermediate products d from DMU_j at division p from period t to period t+1 with treated as output.

$z_{djp}^{(t,t+1)bad}$ is intermediate products d from DMU_j at division p from period t to period t+1 with treated as input.

This model will be able to calculate the overall efficiency of the desired DMU according to sub-unit and dynamic connection after T time period.

3 Calculation of the overall efficiency based on the weighted mean of divisions and periods.

In normal state of DEA, to calculate the efficiency, we divide total weighted outputs to total weighted inputs of the desired DMU. Now that the internal structure DMU is so efficient, to calculate in terms of divisional efficiency & overall efficiency, we use the model of (Zhu et al. 2004) "overall efficiency calculation of decision making unit with network structure by the use of arithmetic mean of the divisional efficiency".

3.1 Period – divisional efficiencies

In this part, by considering the inputs and outputs in one division of the desired DMU during a specific time period, we can evaluate the efficiency for that division in that period. Thus by using the definition of relative efficiency, p division efficiency in t period for the decision making units is defined as follows and will be represented by ρ_{op}^t .

$$\rho^{t}_{op} = Max \frac{\sum_{r\in R^{p}} u^{t}_{r} y^{t}_{rop} + \sum_{d\in D^{p}} \beta^{t}_{d} z^{(t,t+1)good}_{dop} + u^{t}_{0p}}{\sum_{i\in I^{p}} v^{t}_{i} x^{t}_{iop} + \sum_{d\in D^{p}} \alpha^{t}_{d} z^{(t,t+1)bad}_{dop}}$$

$s.t$

$$\frac{\sum_{r\in R^{p}} u^{t}_{r} y^{t}_{rjp} + \sum_{d\in D^{p}} \beta^{t}_{d} z^{(t,t+1)good}_{djp} + u^{t}_{0p}}{\sum_{i\in I^{p}} v^{t}_{i} x^{t}_{ijp} + \sum_{d\in D^{p}} \alpha^{t}_{d} z^{(t,t+1)bad}_{djp}} \le 1 \qquad (2)$$

$u^{t}_{r} \ge 0 \qquad r = 1,\dots, s_{p}$

$v^{t}_{i} \ge 0 \qquad i = 1,\dots, m_{p}$

$\beta^{t}_{d} \ge 0 \qquad d = 1,\dots, n - good$

$\alpha^{t}_{d} \ge 0 \qquad d = 1,\dots, n - \mathrm{bad}$

u^{t}_{0p} : *free*

Linear form of model (2) is as follow:

$$\rho^{t}_{op} = Max \sum_{r\in R^{p}} u^{t}_{r} y^{t}_{rop} + \sum_{d\in D^{p}} \beta^{t}_{d} z^{(t,t+1)good}_{dop} + u^{t}_{0p}$$

$s.t$

$$\sum_{i\in I^{p}} v^{t}_{i} x^{t}_{iop} + \sum_{d\in D^{p}} \alpha^{t}_{d} z^{(t,t+1)bad}_{dop} = 1 \qquad (3)$$

$$\sum_{r\in R^{p}} u^{t}_{r} y^{t}_{rjp} + \sum_{d\in D^{p}} \beta^{t}_{d} z^{(t,t+1)good}_{djp} + u^{t}_{0p} - \sum_{i\in I^{p}} v^{t}_{i} x^{t}_{ijp} - \sum_{d\in D^{p}} \alpha^{t}_{d} z^{(t,t+1)bad}_{djp} \le 0$$

$u^{t}_{r} \ge 0 \qquad r = 1,\dots, s_{p}$

$v^{t}_{i} \ge 0 \qquad i = 1,\dots, m_{p}$

$\beta^{t}_{d} \ge 0 \qquad d = 1,\dots, n - good$

$\alpha^{t}_{d} \ge 0 \qquad d = 1,\dots, n - \mathrm{bad}$

u^{t}_{0p} : *free*

Theorem 1: A) Model (3) is always possible. **B)** $0 < \rho^{t}_{op} \le 1$

Proof: A) if assume $x^{t}_{kop} = \min x^{t}_{iop} \;\; \forall k \ne i = 1,\dots, m_{p}$ and $v^{t}_{k} = \frac{1}{x^{t}_{kop}}$, Also if consider $\forall j \;\; j = 1,\dots, n \quad v^{t}_{k} = 0 \;\; \forall k \ne i = 1,\dots, m_{p}$, $\alpha^{t}_{d} = 0 \;\forall d = 1,\dots, n - bad \quad u^{t}_{r} = 0 \;\; \forall r = 1,\dots, s_{p}$, $\beta^{t}_{d} = 0 \;\; \forall\; \mathrm{d} = 1,\dots, \mathrm{n} - \mathrm{good}$ and $u^{t}_{0p} = 0$, we have:

$$v^{t}_{1} x^{t}_{1op} + \dots + v^{t}_{k} x^{y}_{kop} + \dots + v^{t}_{m_p} x^{t}_{m_p op} + \alpha^{t}_{1} z^{(t,t+1)bad}_{1op} + \dots + \alpha^{t}_{n-bad} z^{(t,t+1)bad}_{n-bad\; op} = 1$$

$$\mathrm{u}^{t}_{1} \mathrm{y}^{t}_{1jp} + \dots + \mathrm{u}^{t}_{s_p} \mathrm{y}^{t}_{s_p jp} + \beta^{t}_{1} z^{(t,t+1)good}_{1jp} + \dots + \beta^{t}_{n-good} z^{(t,t+1)good}_{n-good\; jp} -$$
$$\mathrm{v}^{t}_{1} \mathrm{x}^{t}_{1jp} - \dots - \mathrm{v}^{t}_{k} \mathrm{x}^{t}_{kjp} - \dots - \mathrm{v}^{t}_{m_p} \mathrm{x}^{t}_{m_p jp} - \alpha^{t}_{1} z^{(t,t+1)bad}_{1jp} - \dots - \alpha^{t}_{n-bad} z^{(t,t+1)bad}_{n-bad\; jp} + u^{t}_{0p} \le 0$$

Model (3) is always possible.

B) Due to the previous possible solution and this fact that in each optimum solution at least one of constraints multiplicand (Dual form) is as equality, we have:

$$\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} - \sum_{i=1}^{m} v_i^t x_{iop}^t - \sum_{d=1}^{n-bad} \alpha_d^t z_{dop}^{(t,t+1)bad} + u_{0p}^t = 0$$

$$\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t = \sum_{i=1}^{m} v_i^t x_{iop}^t + \sum_{d=1}^{n-bad} \alpha_d^t z_{dop}^{(t,t+1)bad} = 1$$

$$\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} = 1$$

We know $z_{djp}^{(t,t+1)good} \geq 0$, $y_{rjp}^t \geq 0$, $\beta_d^t \geq 0$, $u_r^t \geq 0$. Then the sum of positive multi term is always positive, so $\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} \geq 0$. We claim $\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} > 0$ because if we suppose contradiction $\sum_{r=1}^{s} u_r^t y_{rop}^t + \sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good} = 0$ then we will have: $\sum_{R=1}^{s} u_r^t y_{rop}^t = -\sum_{d=1}^{n-good} \beta_d^t z_{dop}^{(t,t+1)good}$. This isn't compatible with positive total. $0 < \rho_{op}^t \leq 1$.

Definition 1: if $\rho_{op}^{*t} = 1$, DMU_o is called period-divisional efficient.

By noticing model (2) the period and division efficiency can be defined as convex linear combination.

3.2 Period efficiency

Period efficiency is actually the calculation of overall performance of the desired DMU divisions that can only be evaluated in a specific time period. For this reason it is called period efficiency (the single – period). Calculation of this efficiency is actually the calculation of the desired DMU considering the efficiency of all their divisions. We display it by τ_o^t . This efficiency can be evaluated by the weighted mean of period – divisional efficiency (ρ_{op}^t). Which is defined as follows: $\tau_o^t = \sum_{p=1}^{q} w^p \rho_{op}^t \quad (4)$ Notice that w^p weights shows the share of p division in the efficiency of the desired period for the unit under evaluation and is $w^p = \dfrac{\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}}{\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right)}$. Due to this definition w^p , $\sum_{p=1}^{q} w^p = 1$. Based on equation (4) we are period efficiency model as follows:

$$\tau_o^t = Max \frac{\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rop}^t + \sum_{d\in D^p} \beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)}{\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right)}$$

$s.t:$

$$\frac{\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rjp}^t + \sum_{d\in D^p} \beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t\right)}{\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{ijp}^t + \sum_{d\in D^p} \alpha_d^t z_{djp}^{(t,t+1)bad}\right)} \le 1 \qquad \forall\, t, p, j \qquad (5)$$

$u_r^t \ge 0 \qquad r = 1,\dots, s_p$

$v_i^t \ge 0 \qquad i = 1,\dots, m_p$

$\beta_d^t \ge 0 \qquad d = 1,\dots, n-good$

$\alpha_d^t \ge 0 \qquad d = 1,\dots, n-bad$

$u_{0p}^t : free$

Model (6) is linear model of from (5).

$$\tau_o^t = Max\ \frac{1}{q}\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rop}^t + \sum_{d\in D^p} \beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)$$

$s.t:$

$$\frac{1}{q}\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right) = 1$$

$$\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rjp}^t + \sum_{d\in D^p} \beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t - \sum_{i\in I^p} v_i^t x_{ijp}^t - \sum_{d\in D^p} \alpha_d^t z_{djp}^{(t,t+1)bad}\right) \le 0 \quad \forall\, t, p, j \qquad (6)$$

$u_r^t \ge 0 \qquad r = 1,\dots, s_p$

$v_i^t \ge 0 \qquad i = 1,\dots, m_p$

$\beta_d^t \ge 0 \qquad d = 1,\dots, n-good$

$\alpha_d^t \ge 0 \qquad d = 1,\dots, n-bad$

$u_{0p}^t : free$

Theorem 2: A) Model (6) is always possible. **B)** $0 < \tau_o^t \le 1$

Proof: is similarly to theorem 1 proving.

Definition2: if $\tau_o^{*t} = 1$**,** DMU$_o$ is called period efficient.

Corollary 1: $\tau_o^{*t} = 1$ if and only if $\rho_{op}^{*t} = 1$ at least in one of the divisions.

3.3 Divisional efficiency

One of the benefits of calculating divisional efficiency is that the overall efficiency or inefficiency could be assumed.

Also, if we want to calculate the performance of each one of desired DMU units in a long-time period, we need to calculate divisional efficiency. Calculation this performance is in fact accounted efficiency for each division in a long- time. We show divisional efficiency by δ_{op} and we define as the weighted mean of period-divisional efficiency: $\delta_{op} = \sum_{t=1}^{T} w^t \rho_{op}^t$ (7) ,w^t weight show the share t period in the performance of the desired division for decision making unit and is $w^t = \dfrac{\sum_{i \in I^p} v_i^t x_{iop}^t + \sum_{d \in D^p} \alpha_d^t z_{dop}^{(t,t+1)}}{\sum_{t=1}^{T}\left(\sum_{i \in I^p} v_i^t x_{iop}^t + \sum_{d \in D^p} \alpha_d^t z_{dop}^{(t,t+1)}\right)}$. Due to the definition w^t we resulted $\sum_{t=1}^{T} w^t = 1$.

By considering relation (7), divisional efficiency is defined like following:

$$\delta_{op} = Max \; \frac{\sum_{t=1}^{T}\left(\sum_{r \in R^p} u_r^t y_{rop}^t + \sum_{d \in D^p} \beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)}{\sum_{t=1}^{T}\left(\sum_{i \in I^p} v_i^t x_{iop}^t + \sum_{d \in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right)}$$

$s.t$:

$$\frac{\sum_{t=1}^{T}\left(\sum_{r \in R^p} u_r^t y_{rjp}^t + \sum_{d \in D^p} \beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t\right)}{\sum_{t=1}^{T}\left(\sum_{i \in I^p} v_i^t x_{ijp}^t + \sum_{d \in D^p} \alpha_d^t z_{djp}^{(t,t+1)bad}\right)} \leq 1 \qquad \forall \text{ t, p, j} \qquad (8)$$

$u_r^t \geq 0 \qquad r = 1, \ldots, s_p$

$v_i^t \geq 0 \qquad i = 1, \ldots, m_p$

$\beta_d^t \geq 0 \qquad d = 1, \ldots, n-good$

$\alpha_d^t \geq 0 \qquad d = 1, \ldots, n-bad$

u_{0p}^t : *free*

Model (8) can be changed in to linear model (9).

$$
\begin{aligned}
&\delta_{op} = Max\ \frac{1}{T}\sum_{t=1}^{T}\left(\sum\nolimits_{r\in R^p} u_r^t y_{rop}^t + \sum\nolimits_{d\in D^p} \beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)\\
&s.t:\\
&\frac{1}{T}\sum_{t=1}^{T}\left(\sum\nolimits_{i\in I^p} v_i^t x_{iop}^t + \sum\nolimits_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right) = 1\\
&\sum_{t=1}^{T}\left(\sum\nolimits_{r\in R^p} u_r^t y_{rjp}^t + \sum\nolimits_{d\in D^p} \beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t - \sum\nolimits_{i\in I^p} v_i^t x_{ijp}^t - \sum\nolimits_{d\in D^p} \alpha_d^t z_{djp}^{(t,t+1)bad}\right) \le 0 \quad \forall\ t, p, j \qquad (9)\\
&u_r^t \ge 0 \qquad r = 1,\dots, s_p\\
&v_i^t \ge 0 \qquad i = 1,\dots, m_p\\
&\beta_d^t \ge 0 \qquad d = 1,\dots, n-good\\
&\alpha_d^t \ge 0 \qquad d = 1,\dots, n-bad\\
&u_{0p}^t :\ free
\end{aligned}
$$

Theorem 3: A) this model is always possible. B) $0 < \delta_{op} \le 1$.

Proof: proving is similar to theorem 1.

Definition3: if $\delta^*_{op} = 1$ then DMU_o is called divisional efficient.

Corollary 2: $\delta^*_{op} = 1$ if and only if $\rho_{op}^{*t} = 1$ at least in one of the period.

3.4 Overall efficiency

By the use of (2),(5)and(8) models, the overall performance of decision making unit can be written as convex linear combination of parts and periods efficiency and period- divisional efficiency as model (10). $E_o = \sum_{t=1}^{T}\sum_{p=1}^{q} w_p^t \rho_{op}^t \qquad (10)$. In this model ($w_p^t$) represents the share of p part of t period in the performance of the unit under evaluation which results from the following equation:

$$w_p^t = \frac{\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}}{\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p} \alpha_d^t z_{dop}^{(t,t+1)bad}\right)\right)}$$

. According to the definition: $\sum_{t=1}^{T}\sum_{p=1}^{q} w_p^t = 1$

According to what was said, the proposed model for accounting the overall efficiency of the unit under evaluation is as follows:

$$E_o = Max \frac{\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rop}^t + \sum_{d\in D^p}\beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)\right)}{\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p}\alpha_d^t z_{dop}^{(t,t+1)bad}\right)\right)}$$

$s.t:$

$$\frac{\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rjp}^t + \sum_{d\in D^p}\beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t\right)\right)}{\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{ijp}^t + \sum_{d\in D^p}\alpha_d^t z_{djp}^{(t,t+1)bad}\right)\right)} \le 1 \quad \forall\, t, p, j \qquad (11)$$

$u_r^t \ge 0 \quad r = 1,\dots, s_p$

$v_i^t \ge 0 \quad i = 1,\dots, m_p$

$\beta_d^t \ge 0 \quad d = 1,\dots, n-good$

$\alpha_d^t \ge 0 \quad d = 1,\dots, n-bad$

$u_{0p}^t : free$

Model (11) can be changed in to model (12).

$$E_o = Max \frac{1}{Tq}\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rop}^t + \sum_{d\in D^p}\beta_d^t z_{dop}^{(t,t+1)good} + u_{0p}^t\right)\right)$$

$s.t:$

$$\frac{1}{Tq}\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{i\in I^p} v_i^t x_{iop}^t + \sum_{d\in D^p}\alpha_d^t z_{dop}^{(t,t+1)bad}\right)\right) = 1$$

$$\sum_{t=1}^{T}\left(\sum_{p=1}^{q}\left(\sum_{r\in R^p} u_r^t y_{rjp}^t + \sum_{d\in D^p}\beta_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t - \sum_{i\in I^p} v_i^t x_{ijp}^t - \sum_{d\in D^p}\alpha_d^t z_{djp}^{(t,t+1)bad}\right)\right) \le 0 \quad \forall\, t, p, j \qquad (12)$$

$u_r^t \ge 0 \quad r = 1,\dots, s_p$

$v_i^t \ge 0 \quad i = 1,\dots, m_p$

$\beta_d^t \ge 0 \quad d = 1,\dots, n-good$

$\alpha_d^t \ge 0 \quad d = 1,\dots, n-bad$

$u_{0p}^t : free$

Theorem 4: A) This model is always possible. B) $0 < E_o^* \le 1$.

Proof: is similar to previous.

Definition4: if $E_o^* = 1$ then DMU_o is called overall efficient.

Corollary3: $E_o^* = 1$ if and only if $\rho_{op}^t = 1$ at least in one of the period and division.

Theorem 5: Overall efficiency is unique.

Proof: suppose $(u^*, v^*, \alpha^*, \beta^*)$ is the optimum solution of model (12). Suppose posterior there exists another possible solution as $(\tilde{u}, \tilde{v}, \tilde{\alpha}, \tilde{\beta})$ such that $\mathrm{E}_o(u^*, v^*, \alpha^*, \beta^*) = E_o(\tilde{u}, \tilde{v}, \tilde{\alpha}, \tilde{\beta})$. However

$$\sum_{i\in I^p} v_i^{*t} x_{iop}^t + \sum_{d\in D^p} \alpha_d^{*t} z_{dop}^{(t,t+1)bad} = \sum_{i\in I^p} \tilde{v}_i^t x_{iop}^t + \sum_{d\in D^p} \tilde{\alpha}_d^t z_{dop}^{(t,t+1)bad} = 1 \Rightarrow$$

$$\sum_{i\in I^p} (v_i^{*t} - \tilde{v}_i^t) x_{iop}^t + \sum_{d\in D^p} (\alpha_d^{*t} - \tilde{\alpha}_d^t) z_{dop}^{(t,t+1)bad} = 1$$

$$\sum_{r\in R^p} u_r^{*t} y_{rjp}^t + \sum_{d\in D^p} \beta_d^{*t} z_{djp}^{(t,t+1)good} + u_{0p}^t - \sum_{i\in I^p} v_i^{*t} x_{ijp}^t - \sum_{d\in D^p} \alpha_d^{*t} z_{djp}^{(t,t+1)bad} = \sum_{r\in R^p} \tilde{u}_r^t y_{rjp}^t + \sum_{d\in D^p} \tilde{\beta}_d^t z_{djp}^{(t,t+1)good} + u_{0p}^t - \sum_{i\in I^p} \tilde{v}_i^t x_{ijp}^t - \sum_{d\in D^p} \tilde{\alpha}_d^t z_{djp}^{(t,t+1)bad} \le 0$$

$$\Rightarrow \sum_{r\in R^p} (u_r^{*t} - \tilde{u}_r^t) y_{rjp}^t + \sum_{d\in D^p} (\beta_d^{*t} - \tilde{\beta}_d^t) z_{djp}^{(t,t+1)good} - \sum_{i\in I^p} (v_i^{*t} - \tilde{v}_i^t) x_{ijp}^t - \sum_{d\in D^p} (\alpha_d^{*t} - \tilde{\alpha}_d^t) z_{djp}^{(t,t+1)bad} \le 0$$

Since all coefficients must be positive then we have:

$$u_r^{*t} - \tilde{u}_r^t \ge 0 \Rightarrow u_r^{*t} \ge \tilde{u}_r^t \quad \forall r \qquad , v_i^{*t} - \tilde{v}_i^t \ge 0 \Rightarrow v_i^{*t} \ge \tilde{v}_i^t \quad \forall i$$

$$\beta_d^{*t} - \tilde{\beta}_d^t \ge 0 \Rightarrow \beta_d^{*t} \ge \tilde{\beta}_d^t \quad \forall d \qquad , \alpha_d^{*t} - \tilde{\alpha}_d^t \ge 0 \Rightarrow \alpha_d^{*t} \ge \tilde{\alpha}_d^t \quad \forall d \qquad (*)$$

And because $y_{rjp}^t, z_{djp}^{(t,t+1)good}, x_{ijp}^t, z_{djp}^{(t,t+1)bad}$ are constant for both of the solution, then according to (*) $\mathrm{E}_o(u^*, v^*, \alpha^*, \beta^*) \ge E_o(\tilde{u}, \tilde{v}, \tilde{\alpha}, \tilde{\beta})$ This is contradiction by $\mathrm{E}_o(u^*, v^*, \alpha^*, \beta^*) = E_o(\tilde{u}, \tilde{v}, \tilde{\alpha}, \tilde{\beta})$.

4 A numerical example

We applied this model to a dataset gathered from an insurance company in of exists in Taiwan. (For further studies you may refer to [6]).This company has five evaluation unite each one consists of two parts with an input, an output, a good intermediate product and a bad intermediate product. The performance of the company has been evaluated in two time periods. The data are given in table 1.

Table1

Inputs & outputs and intermediate products data.

DMU_j		$X_j.t_1$	$X_j.t_2$	Z_j.good	Z_j.bad	$Y_j.t_1$	$Y_j.t_2$
1	Division1	1567746	950432	11473162	546337	1043778	264098
	Division2	1453797	1085019	7695461	342489	3144484	371984
2	Division1	1962448	672414	7222378	643178	1486014	18259
	Division2	757515	547997	3631484	995620	692731	163927
3	Division1	6699063	353161	37392862	1753794	7851229	39252
	Division2	1396002	988888	7396396	465509	1401200	332283
4	Division1	601320	594259	3174851	371863	248709	177331
	Division2	145442	53518	316829	131920	355624	26537
5	Division1	15993	10502	52063	14574	82141	4181
	Division2	2627707	668363	9747908	952326	1713598	415058

According to the table1 and using the proposed models for calculating the ρ_p^t , δ_p and τ^t , E , the performance of this insurance company according to parts and each of the periods is calculated and its value are given in tables (2) and (3).

Table (2) consists of the performance values of each division of DMU_j in each time period and also the performance of each division in overall time period.

Table2

Period–divisional efficiency - divisional efficiency

DMU_j		ρ_p^1	ρ_p^1	δ_p
1	Division1	1.0000	1.0000	1.0000
	Division2	1.0000	1.0000	1.0000
2	Division1	0.6464	0.8620	0.9120
	Division2	0.9649	1.0000	1.0000
3	Division1	1.0000	0.3645	1.0000
	Division2	1.0000	1.0000	1.0000
4	Division1	0.7321	1.0000	1.0000
	Division2	1.0000	1.0000	1.0000
5	Division1	1.0000	1.0000	1.0000
	Division2	1.0000	1.0000	1.0000

Table (3) is also consists of DMU's under evaluation values. This performance is calculated by the efficiency of each division in each period. Also, each DMU's overall efficiency value is given in this table.

Table3

Period efficiency - Overall efficiency

DMU_j	τ^1	τ^2	**E**
1	1.0000	1.0000	1.0000
2	0.9670	1.0000	1.0000
3	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000

5 Conclusion

In normal state in DEA, calculating the performance value, the sum of weight outputs is divided to the sum of weighted inputs of the desired DMU. By using the above model at first we calculated the efficiency of each part of the desired DMU in a time period and then according to the weighted mean of all parts, we evaluated the desired DMU efficiency in different time periods and ultimately in the overall period. The difference of this method from the conventional method was in efficiency calculation that performance and nonperformance of on unite was achieved with respect to efficiency and inefficiency of its divisions. But in the usual method, if a unit was inefficiency, we looked for the causes of the desired unit in its sub-units.

Another feature of the presented model is that the same thing can be done for another organization that hasn't any similarity to the surveyed organization by using the parallel network dynamic DEA during the time period and then determined the growth of this organization during the time, eventually compared these two heterogeneous units according to performance growth over the various years. Because simple models of DEA, the basic requirement to compare the decision making units together was their homogeneity. It may also be valuable to investigate the Malmquist index under the Non radial model of dynamic DEA with the parallel network structure model.

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