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Evaluation of research projects based on a mix model with fuzzy DEA-RA

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Abstract
In this paper, first the DEA-RA (integration of DEA with Ratio Analysis) output and input oriented is proposed. Then, by using the fully fuzzy problem [1] and the problem that only the parameters are fuzzy, the DEA-RA joint model [2] has been proposed. Therefore, the lexicographic approach and the α-cut approach have been used for these two types of the problems and evaluation of research projects. DEA-RA models behave similarly to DEA models. In this study, the input and output parameters are not ratio, but if the input to output ratios and vice versa are defined, DEA-RA models can also be a criterion for calculating the efficiency and evaluation of units.

Keywords: DEA, DEA-RA, Lexicographic approach, α-cut approach.

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1. Introduction

Today, DEA is a useful tool for evaluating the performance of decision-making units that managers can use it as one of the important factors in decision making. The envelopment and multiplier CCR models are modeled based on the idea of Charnes, Cooper and Rhodes in 1978 [3], which are the criteria for calculating efficiency in constant returns to scale technology. Then BCC models were developed by Banker et.al. in 1984. In 2001 [4], the non-radial SBM model was proposed by Tone and its relationship with the envelopment and multiplier DEA models was studied. The Classic DEA models were formed based on the principles, but with the need to use DEA in the application and practical studies, more careful attention paid to the definitions of efficiency and details of the principles of the subject were highly considered by researchers. In 2005, Emrouznejad and Amin [5] discussed the challenge of DEA with ratio data. This matter led to detailed studies in this field. In 2005, Emrouznejad and Amin [5] discussed classical DEA models with ratio data, which some inputs or outputs are inherently ratio data but the numerator and denominator of the ratio are known. In 2015 and 2017, Olesen and Petersen et al. [6] discussed the DEA with ratio data that is not specified in the numerator and denominator of the ratio and some input or output parameters are inherently ratio. But Despic et al. in 2007 [7]and Liu et al. in 2011 [8] proposed the DEA-RA models, which are a combination of DEA and Ratio Analysis. In these models, the input and output parameters are not Ratio. But input to output ratios (vice versa) need to be defined. The similar behaviors of the DEA and DEA-RA models are very significant. In this regard, Wu and Liang 2005 [9] and Zha and Liang 2014 [10] show relations DEA and Ratio Analysis. Therefore, it is very important to pay attention to these references.

In general, the present article has the following objectives:

A. Modeling of the DEA-RA model in the nature of input and output with fuzzy data

B. Calculating the efficiency of research projects based on input to output ratios and vice versa

C. The relationship between fuzzy lexicographic and α-cut approaches in DEA-RA models

Finally, the structure of this article is presented as follows:

In the second section, the basic concepts of fuzzy, fuzzy linear programming and DEA and DEA-RA models are briefly expressed. In the third section, first the DEA-R joint model is solved with fuzzy data in two ways, lexicographic and α-cut approaches. In the fourth section, research projects of Fars province are evaluated based on the proposed models. At the end, the conclusion is presented.

2. Basic concepts

In this section, we present the linear programming fuzzy andbasic concepts of DEA-R.

2-1 Definitions and theories of fuzzy sets

In this section, we review some basic definitions of fuzzy sets theory.

Definition 2.1.1[1] A fuzzy set$\tilde{a}$, defined on a universal set of real numbers, is said to be fuzzy if its membership function has the following properties:

i. $\tilde{a}$Is convex, i.e. $\forall x, y \in \Re, \forall \lambda \in [0,1], \mu_{\tilde{a}}(\lambda x + (1-\lambda)y) \geq \min\{\mu_{\tilde{a}}(x), \mu_{\tilde{a}}(y)\}$,

ii. $\tilde{a}$Is normal, i.e. $\exists \bar{x} \in \Re; \mu_{\tilde{a}}(\bar{x}) = 1$,

iii. $\mu_{\tilde{a}}$Is piecewise continuous.

Definition 2.1.2[11] If $\tilde{A} = (a, b, c)$ is a triangular fuzzy number, then the membership function of this fuzzy number is as follows:

$$\mu_A(x) = \begin{cases} \dfrac{x-a}{c-a} & a \le x \le c \\ \dfrac{b-x}{b-c} & b \le x \le c \\ 0 & \text{otherwise} \end{cases}$$

Definition 2.1.3[12] A fuzzy subset of $\tilde{A}$ in which the membership degree is at least as large as $\alpha > 0$ is called α-cut of $\tilde{A}$ and denoted by $\tilde{A}_\alpha$.

The set of all these triangular fuzzy numbers is denoted by TF($\mathfrak{R}$). A triangular fuzzy number $(a_1, a_2, a_3) = \tilde{a}$ can be reduced to the real number "a" if $a_3 = a_2 = a_1 = a$ Conversely, a real number "a" can be considered as a triangular fuzzy number $(a, a, a) = (a_1, a_2, a_3) = \tilde{a}$.

Definition 2.1.4[1] The triangular fuzzy number $(a_1, a_2, a_3) = \tilde{a}$ is called non-negative (respectively positive) if and only if $a_1 \le 0$(respectively $a_1 < 0$). The sets of non-negative and positive triangular fuzzy numbers are denoted by $TF(\mathfrak{R})^+$ and $TF(\mathfrak{R})^{++}$, respectively.

Definition2.1.5[1] Two fuzzy triangle numbers $(a_1, a_2, a_3) = \tilde{a}$ and $(b_1, b_2, b_3) = \tilde{b}$ are equal if and only if $a_1 = b_1$, $a_2 = b_2$ and $a_3 = b_3$.

Definition 2.1.6[1] Suppose $(a_1, a_2, a_3) = \tilde{a}$ and $(b_1, b_2, b_3) = \tilde{b}$ are two non-negative triangular fuzzy numbers and $k \in \mathfrak{R}$. Then the arithmetic operations on $\tilde{a}$ and $\tilde{b}$ are defined as follows:

$k\tilde{a} = (ka_1, ka_2, ka_3)$ if $k \ge 0$,

$k\tilde{a} = (ka_3, ka_2, ka_1)$ if $k \le 0$

$\tilde{a} \oplus \tilde{b} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$,

$\tilde{a} \otimes \tilde{b} = (a_1b_1, a_2b_2, a_3b_3)$.

$$. \frac{\tilde{a}}{\tilde{b}} = \left(\frac{a_1}{b_3}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

The standard form of a fully fuzzified linear programming problem is considered as model (1).

$$\begin{aligned} &\min \tilde{C}^T\tilde{X} \\ &\text{S.t.} \tilde{A}\tilde{X} = \tilde{b} \qquad (1) \end{aligned}$$

Where

$\tilde{C}^T = [\tilde{c}_j]_{1\times n}, \tilde{X} = [\tilde{x}_j]_{n\times 1}, \tilde{A} = [\tilde{a}_{ij}]_{m\times n}, \tilde{b} = [\tilde{b}_i]_{m\times 1}, \tilde{a}_{ij}, \tilde{c}_j, \tilde{b}_i \in TF(\mathfrak{R}), \tilde{x}_j \in TF(\mathfrak{R})^+$

$i = 1, \dots, m$ and $j = 1, \dots, n$.

The optimal solution of model (1) is $\tilde{X}^* = ((X^*)^L, (X^*)^C, (X^*)^U)$, if it applies to the following 3 conditions:

i. $\tilde{X}^* = [\tilde{x}_j^*]_{n\times 1}$ where $\tilde{x}_j^* \in TF(\mathfrak{R})^+ j = 1, \dots, n$,
ii. $\tilde{A}\tilde{X}^* = \tilde{b}$,
iii. For each $\tilde{X} = ((X)^L, (X)^C, (X)^U)$ of the set $\tilde{S} = \left\{\tilde{X} \middle| \tilde{A}\tilde{X} = \tilde{b}, \tilde{X} = [\tilde{x}_j]_{n\times 1}, \tilde{x}_j \in TF(\mathfrak{R})^+\right\}$, the relationship $\tilde{C}^T\tilde{X} \ge \tilde{C}^T\tilde{X}^*$ is established.

2-2 Basic Concepts of DEA-RA

In this section, we will firstly present the relationship between DEA and DEA-R models in constant returns to scale technology based on the ideas of Despic et al.; then we will briefly show the production possibility set based on ratio-based data presented by Liu et al.

Suppose that n DMUs receiving m inputs $X_j = (x_{1j}, \dots, x_{mj})$, produce s outputs $Y_j = (y_{1j}, \dots, y_{sj})$. Generally, the inputs matrix is of dimensions $m \times n$ and the outputs matrix of dimensions $s \times n$.

In data envelopment analysis, production possibility set is constructed based on the following axioms:

A) Axiom of inclusion of observation, which states all DMUs are always observable in the production possibility set [13].

B) Axiom of free disposability, which states that production, is always possible with more input and less output.

C) Axiom of convexity, which states that convex combinations of any two members are always in the set.

D) Axiom of minimum extrapolation, meaning the smallest constructed set which satisfies the mentioned axioms.

Although, the ray unboundedness condition for constant returns to scale technology will be considered. Introducing DEA models without explicit inputs and studying the conditions of free disposability and convexity for the Asset, Liu et al. showed that DEA models with ratio-based data and DEA models without explicit inputs are equivalents.

$$As = \left\{ (F) \middle| \begin{array}{c} F \leq \sum_{j=1}^{n} \lambda_j \left(\frac{Y_j}{X_j}\right), \\ \sum_{j=1}^{n} \lambda_j = 1, \lambda_j \geq 0, j = 1, \dots, n \end{array} \right\}$$

As is a convex, closed and bounded set. Liu et al. proposed the following model for evaluation of DMU_o, which data consists of output to input ratios [12].

$$\begin{aligned} Max \quad & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{y_{ro}}{x_{io}}\right) \\ s.t. \quad & \sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{y_{rj}}{x_{ij}}\right) \leq 1 \\ & \text{J=1,…,n} \\ & w_{ir} \geq 0, \quad i = 1, \dots, m, \quad r = 1, \dots, s. \end{aligned}$$

Theorem 1. The model (2) is feasible and bounded.

Proof: Suppose $\bar{w}_{ir} = 0,\ i = 1, \dots, m,\quad r = 1, \dots, s$, so we have $\sum_{i=1}^{m}\sum_{r=1}^{s} \bar{w}_{ir}\left(\frac{y_{rj}}{x_{ij}}\right) = 0\,, \quad j = 1, \dots, n.$

Therefore the model (2) is feasible. On the other hand, because we have it in the constraints of model (2) $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{y_{rj}}{x_{ij}}\right) \leq 1,\ j = 1, \dots, n$ and $o \in \{1, \dots, n\}$, then $\sum_{i=1}^{m}\sum_{r=1}^{s} w_{ir}\left(\frac{y_{ro}}{x_{io}}\right) \leq 1$. So the model (2) is bounded.

$$\begin{aligned} & Max \quad z = \beta - \theta \\ & \text{S.t.} \quad (3) \\ & \sum_j \lambda_j \frac{y_{rj}}{x_{ij}} \geq \beta \frac{y_{ro}}{x_{io}} && ;\forall r, i \\ & \sum_j \mu_j \frac{x_{ij}}{y_{rj}} \leq \theta \frac{x_{io}}{y_{ro}} && ;\forall r, i \\ & \sum_j \lambda_j = 1 && ;\forall j \\ & \sum_j \mu_j = 1 && ;\forall j \\ & \lambda_j \geq 0 && ;\mu_j \geq 0 \end{aligned}$$

Definition 2.2.1 DMU_oin model (3) is efficient in DEA-R whenever$\theta^* = 1$and$\beta^* = 1$.

The dual of the above model by considering the variables u and v is obtained as follows:

$$\begin{aligned} & \text{Max } z = \sum_i\sum_r u_{ir}\frac{y_{ro}}{x_{io}} - \sum_i\sum_r v_{ir}\frac{x_{io}}{y_{ro}}; \\ & \text{S.t.} \quad (4) \\ & \sum_i\sum_r u_{ir}\frac{y_{rj}}{x_{ij}} \leq 1 && ;\ \forall j \\ & \sum_i\sum_r v_{ir}\frac{x_{ij}}{y_{rj}} \geq 1 && ;\ \forall j \\ & u_{ir} \geq 0;\ \ v_{ir} \geq 0; \end{aligned}$$

Theorem 2. The model (4) is feasible.

Proof: Suppose$\bar{u}_{ir} = 0,\quad i = 1, \dots, m,\ \ r = 1, \dots, s,$ so we have $\sum_{i=1}^{m}\sum_{r=1}^{s} u_{ir}\left(\frac{y_{rj}}{x_{ij}}\right) = 0\,,\ \ j = 1, \dots, n$.Also, let $\bar{v}_{ir} = M$ and $M = \frac{1}{\min\left\{\sum_i\sum_r\frac{x_{ij}}{y_{rj}}, j=1,\dots,n\right\}}$. Therefore we have$\sum_i\sum_r \bar{v}_{ir}\frac{x_{ij}}{y_{rj}} \geq 1\,, \forall j$.

Definition 2.2.2 The DMU_o in Model (4) is called efficient in DEA-R whenever$\sum_i\sum_r v_{ir}^*\frac{x_{io}}{y_{ro}} = 1, \sum_i\sum_r u_{ir}^*\frac{y_{ro}}{x_{io}} = 1$.

Model (4) is a linear programming problem which is the dual of model (3). According to the duality theorem, the optimal solution of both models is equal. Both models are proposed based on input-output ratios i.e. $\frac{X_j}{Y_j}$ and as well as output-input ratios i.e. $\frac{Y_j}{X_j}$. It is obvious that, zero values of the $\frac{X_j}{Y_j}$ function are the criteria for the efficiency of decision-making units.

3. The DEA-RA joint model in fuzzy mode

In this section, first, the DEA-RA joint model in constant returns to scale technology in fully fuzzy mode is proposed and solved by the lexicographic approach. Then, the DEA-RA fuzzy model is solved with fuzzy data based on the alpha-cut, optimistic and pessimistic view approaches, and the efficiency interval is also calculated for each decision-making unit.

3-1 fully fuzzified DEA-RA model

Model (3) in fully fuzzified mode is proposed to evaluate decision making units that are completely fuzzy as follows:

$$Max\ z = \tilde{\beta} - \tilde{\theta}$$

S.t. (5)

$$\sum_j \tilde{\lambda}_j \otimes \frac{\widetilde{y_{rj}}}{\widetilde{x_{ij}}} \geq \tilde{\beta} \otimes \frac{\widetilde{y_{ro}}}{\widetilde{x_{io}}} \qquad ;\forall r, i$$

$$\sum_j \tilde{\mu}_j \otimes \frac{\widetilde{x_{ij}}}{\widetilde{y_{rj}}} \leq \tilde{\theta} \otimes \frac{\widetilde{x_{io}}}{\widetilde{y_{ro}}} \qquad ;\forall r, i$$

$$\sum_j \tilde{\lambda}_j = 1 \qquad ;\forall j$$

$$\sum_j \tilde{\mu}_j = 1 \qquad ;\forall j$$

$$\tilde{\lambda}_j \in TF(\Re)^+ \qquad ;\forall j$$

$$\tilde{\mu}_j \in TF(\Re)^+ \qquad ;\forall j$$

Model (5) is a fuzzy programming problem that by considering variables $\tilde{\mu}_j = \tilde{e}_o$, $\tilde{\lambda}_j = \tilde{e}_o$, $\tilde{\theta} = \tilde{1}$ and $\tilde{\beta} = \tilde{1}$ a feasible solution can be obtained for the model.

Where the set of non-negative triangular fuzzy numbers is represented by $TF(\Re)^+$. In this step, fuzzy numbers are placed with their triangular triad, that is, we place,

$\widetilde{x_{ij}} = (x_{ij,1}, x_{ij,2}, x_{ij,3})$,

$\widetilde{x_{io}} = (x_{io,1}, x_{io,2}, x_{io,3})$,

$\widetilde{y_{rj}} = (y_{rj,1}, y_{rj,2}, y_{rj,3})$,

$\widetilde{y_{ro}} = (y_{ro,1}, y_{ro,2}, y_{ro,3})$,

$\tilde{\mu}_j = (\mu_{j,1}, \mu_{j,2}, \mu_{j,3})$,

$\tilde{\lambda}_j = (\lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3})$,

$\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$ and $\tilde{\beta} = (\beta_1, \beta_2, \beta_3)$.

$$Max\ z = (\beta_1, \beta_2, \beta_3) - (\theta_1, \theta_2, \theta_3)$$

S.t. (6)

$$\sum_j (\lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3}) \otimes \frac{(y_{rj,1}, y_{rj,2}, y_{rj,3})}{(x_{ij,1}, x_{ij,2}, x_{ij,3})}$$

$$\geq (\beta_1, \beta_2, \beta_3) \otimes \frac{(y_{ro,1}, y_{ro,2}, y_{ro,3})}{(x_{io,1}, x_{io,2}, x_{io,3})} \qquad ;\forall r, i$$

$$\sum_j (\mu_{j,1}, \mu_{j,2}, \mu_{j,3}) \otimes \frac{(x_{ij,1}, x_{ij,2}, x_{ij,3})}{(y_{rj,1}, y_{rj,2}, y_{rj,3})}$$

$$\leq (\theta_1, \theta_2, \theta_3) \otimes \frac{(x_{io,1}, x_{io,2}, x_{io,3})}{(y_{ro,1}, y_{ro,2}, y_{ro,3})} \qquad ;\forall r, i$$

$$\sum_j (\lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3}) = 1 \qquad ;\forall j$$

$$\sum_j (\mu_{j,1}, \mu_{j,2}, \mu_{j,3}) = 1 \qquad ;\forall j$$

$$(\lambda_{j,1}, \lambda_{j,2}, \lambda_{j,3}) \in TF(\Re)^+ \qquad ;\forall j$$

$$(\mu_{j,1}, \mu_{j,2}, \mu_{j,3}) \in TF(\Re)^+ \qquad ;\forall j$$

Now apply the arithmetic fuzzy operation:

$$Max\ z = (\beta_1 - \theta_3, \beta_2 - \theta_2, \beta_3 - \theta_1)$$

S.t. (7)

$$\sum_j (\lambda_{j,1}\frac{y_{rj,1}}{x_{ij,3}},\lambda_{j,2}\frac{y_{rj,2}}{x_{ij,2}},\lambda_{j,3}\frac{y_{rj,3}}{x_{ij,1}})$$
$$\geq (\beta_1\frac{y_{ro,1}}{x_{io,3}},\beta_2\frac{y_{ro,2}}{x_{io,2}},\beta_3\frac{y_{ro,3}}{x_{io,1}}) \quad ;\forall r,i$$
$$\sum_j (\mu_{j,1}\frac{x_{ij,1}}{y_{rj,3}},\mu_{j,2}\frac{x_{ij,2}}{y_{rj,2}},\mu_{j,3}\frac{x_{ij,3}}{y_{rj,1}})$$
$$\leq (\theta_1\frac{x_{io,1}}{y_{ro,3}},\theta_2\frac{x_{io,2}}{y_{ro,2}},\theta_3\frac{x_{io,3}}{y_{ro,1}}) \quad ;\forall r,i$$
$$\sum_j (\lambda_{j,1},\lambda_{j,2},\lambda_{j,3}) = 1 \quad ;\forall j$$
$$\sum_j (\mu_{j,1},\mu_{j,2},\mu_{j,3}) = 1 \quad ;\forall j$$
$$(\lambda_{j,1},\lambda_{j,2},\lambda_{j,3}) \in TF(\Re)^+ \quad ;\forall j$$
$$(\mu_{j,1},\mu_{j,2},\mu_{j,3}) \in TF(\Re)^+ \quad ;\forall j$$
$$\lambda_{j,1} \geq 0; \lambda_{j,2}-\lambda_{j,1} \geq 0; \lambda_{j,3}-\lambda_{j,2} \geq 0 \quad ;\forall j$$
$$\mu_{j,1} \geq 0; \mu_{j,2}-\mu_{j,1} \geq 0; \mu_{j,3}-\mu_{j,2} \geq 0 \quad ;\forall j$$
$$\theta_2-\theta_1 \geq 0; \theta_3-\theta_2 \geq 0;$$
$$\beta_2-\beta_1 \geq 0; \beta_3-\beta_2 \geq 0;$$

Rewrite the constraints:

$$\sum_j (\lambda_{j,1}\frac{y_{rj,1}}{x_{ij,3}},\lambda_{j,2}\frac{y_{rj,2}}{x_{ij,2}},\lambda_{j,3}\frac{y_{rj,3}}{x_{ij,1}})$$
$$\geq \left(\beta_1\frac{y_{ro,1}}{x_{io,3}},\beta_2\frac{y_{ro,2}}{x_{io,2}},\beta_3\frac{y_{ro,3}}{x_{io,1}}\right) \quad ;\forall r,i \qquad (8)$$
$$\sum_j (\mu_{j,1}\frac{x_{ij,1}}{y_{rj,3}},\mu_{j,2}\frac{x_{ij,2}}{y_{rj,2}},\mu_{j,3}\frac{x_{ij,3}}{y_{rj,1}})$$
$$\leq (\theta_1\frac{x_{io,1}}{y_{ro,3}},\theta_2\frac{x_{io,2}}{y_{ro,2}},\theta_3\frac{x_{io,3}}{y_{ro,1}}) \quad ;\forall r,i$$
$$\sum_j (\lambda_{j,1},\lambda_{j,2},\lambda_{j,3}) = 1 \quad ;\forall j$$
$$\sum_j (\mu_{j,1},\mu_{j,2},\mu_{j,3}) = 1 \quad ;\forall j$$
$$(\lambda_{j,1},\lambda_{j,2},\lambda_{j,3}) \in TF(\Re)^+ \quad ;\forall j$$
$$(\mu_{j,1},\mu_{j,2},\mu_{j,3}) \in TF(\Re)^+ \quad ;\forall j$$
$$\lambda_{j,1} \geq 0; \lambda_{j,2}-\lambda_{j,1} \geq 0; \lambda_{j,3}-\lambda_{j,2} \geq 0 \quad ;\forall j$$
$$\mu_{j,1} \geq 0; \mu_{j,2}-\mu_{j,1} \geq 0; \mu_{j,3}-\mu_{j,2} \geq 0 \quad ;\forall j$$
$$\theta_2-\theta_1 \geq 0; \theta_3-\theta_2 \geq 0;$$
$$\beta_2-\beta_1 \geq 0; \beta_3-\beta_2 \geq 0;$$

Convert LP model to MOLP model:

$$\text{Max}z = \beta_1-\theta_3$$
$$\text{Max}z = \beta_2-\theta_2$$
$$\text{Max}z = \beta_3-\theta_1$$
$$\text{S.t.} \qquad (9)$$
$$\theta_2-\theta_1 \geq 0; \theta_3-\theta_2 \geq 0;$$
$$\beta_2-\beta_1 \geq 0; \beta_3-\beta_2 \geq 0;$$

Remaining Constraints in (8).

By using the Lexicographic approach, solve the MOLP model in order, that is, first solve $Maxz = \beta_3-\theta_1$ as the first objective function, in the second step solve$Maxz = \beta_2-\theta_2$, and finally solve$Maxz = \beta_1-\theta_3$, thus:

$$\text{Max z} = \beta_3-\theta_1$$
$$\text{S.t.} \qquad (10)$$
$$\theta_2-\theta_1 \geq 0; \theta_3-\theta_2 \geq 0;$$
$$\beta_2-\beta_1 \geq 0; \beta_3-\beta_2 \geq 0;$$

Remaining Constraints in (8).

The optimal solution of previous model, (10), is $\beta_3^*-\theta_1^*$, which is used as follows in the next step:

$$\text{Max z} = \beta_2-\theta_2$$
$$\text{S.t.} \qquad (11)$$
$$\beta_3-\theta_1 = \beta_3^*-\theta_1^*;$$
$$\theta_2-\theta_1 \geq 0; \theta_3-\theta_2 \geq 0;$$
$$\beta_2-\beta_1 \geq 0; \beta_3-\beta_2 \geq 0;$$

Remaining Constraints in (8).

The optimal solution of above model, (11), is the phrase $\beta_2^*-\theta_2^*$ that put in the next model:

$\text{Max } z = \beta_1 - \theta_3$

S.t.(12)

$\beta_3 - \theta_1 = \beta_3^* - \theta_1^*$;

$\beta_2 - \theta_2 = \beta_2^* - \theta_2^*$;

$\theta_2 - \theta_1 \geq 0; \theta_3 - \theta_2 \geq 0;$

$\beta_2 - \beta_1 \geq 0; \beta_3 - \beta_2 \geq 0;$

Remaining Constraints in (8).

Finally, the optimal solution is $(\beta_1^* - \theta_3^*, \beta_2^* - \theta_2^*, \beta_3^* - \theta_1^*)$.

3-2 DEA-RA model with fuzzy parameters

In this section, the DEA-RA fuzzy model in constant returns to scale technology in both optimistic and pessimistic view approaches is proposed for evaluating DMU_o with fuzzy data.

$$Max z = \sum_i \sum_r u_{ir} \frac{\widetilde{y_{ro}}}{\widetilde{x_{io}}} - \sum_i \sum_r v_{ir} \frac{\widetilde{x_{io}}}{\widetilde{y_{ro}}};$$

$S.t.$ (13)

$$\sum_i \sum_r u_{ir} \frac{\widetilde{y_{rj}}}{\widetilde{x_{ij}}} \leq 1 \quad ; \forall j$$

$$\sum_i \sum_r v_{ir} \frac{\widetilde{x_{ij}}}{\widetilde{y_{rj}}} \geq 1 \quad ; \forall j$$

$v_{ir} \geq 0;$

$u_{ir} \geq 0;$

This approach includes two types, optimistic and pessimistic view types, which are discussed below. In the first case, that is, the optimistic view approach, we take the unit under evaluation in the best case, that is, the minimum input and maximum output, and we put the other units in the worst case that is, the maximum input and minimum output. Therefore, we let the inputs of the unit under evaluation in the lowest value, i.e.x_{io}^l, and its outputs in the highest value, i.e. y_{ro}^u, and for other units, we put the inputs in their highest value, i.e. x_{ij}^u, and the output in their lowest value, i.e. y_{rj}^l.

Optimistic view approach:

$$\text{Max } z = \sum_i \sum_r u_{ir} \frac{y_{ro}^u}{x_{io}^l} - \sum_i \sum_r v_{ir} \frac{x_{io}^l}{y_{ro}^u};$$

S.t (14)

$$\sum_i \sum_r u_{ir} \frac{y_{rj}^l}{x_{ij}^u} \leq 1 \quad ; \forall j$$

$$\sum_i \sum_r v_{ir} \frac{x_{ij}^u}{y_{rj}^l} \geq 1 \quad ; \forall j$$

$v_{ir} \geq 0;$

$v_{ir} \geq 0;$

Model (14) is a linear programming problem. In the second case, optimistic view approach, we take the unit under evaluation in the worst case, i.e. maximum input and minimum output, and the rest of the units in the best case, i.e. minimum input and maximum output. Therefore, we put the inputs of the unit under evaluation in the most value, i.e. x_{io}^u, and its outputs in the lowest value, i.e. y_{ro}^l, and for the rest of the units, we let the inputs in their lowest value, i.e. x_{ij}^l, and the output in their highest value, i.e.y_{rj}^u. Thus:

Pessimistic view approach:

$$\text{Max } z = \sum_i \sum_r u_{ir} \frac{y_{ro}^l}{x_{io}^u} - \sum_i \sum_r v_{ir} \frac{x_{io}^u}{y_{ro}^l};$$

S.t. (15)

$$\sum_i \sum_r u_{ir} \frac{y_{rj}^u}{x_{ij}^l} \leq 1 \quad ; \forall j$$

$$\sum_i \sum_r v_{ir} \frac{x_{ij}^l}{y_{rj}^u} \geq 1 \quad ; \forall j$$

$v_{ir} \geq 0;$

$v_{ir} \geq 0;$

Model (15) is a linear programming problem that in the pessimistic view puts the unit under evaluation in the worst case, i.e. inputs at the most value and outputs at the lowest value, while other decision-making units are in the best case, i.e. less input and more outputs. Obviously, the efficiency interval for model (15) can be an interval for the efficiency in worst case to evaluateDMU_o.

For Optimistic view:

$$E_1^* = \sum_i \sum_r v_{ir} \frac{x_{io}^l}{y_{ro}^u}, E_2^* = \sum_i \sum_r u_{ir} \frac{y_{ro}^u}{x_{io}^l} \qquad (16)$$

For Pessimistic view:

$$E_1^* = \sum_i \sum_r v_{ir} \frac{x_{io}^u}{y_{ro}^l}, E_2^* = \sum_i \sum_r u_{ir} \frac{y_{ro}^l}{x_{io}^u} \qquad (17)$$

4. Applied study

One of the indicators of development in any country is the rate of science production and the progress of industry in that country. The best way to achieve economic and social progress is to establish a connection between educational centers and industrial centers, while the university has special importance. University research officials play an important role in selecting projects that can be done at the university. Projects must be selected that, as a result of doing them, the production of science and industrialization will take place. Therefore, choosing the right project from the proposed projects is very important.

The purpose of reviewing the proposed research projects is to make a preliminary judgment on the project with available the information about the project.

A practical study has been selected from a university unit in Fars-Iran province in 2018. To apply Model (3), research projects proposed by faculty members will be considered as a DMU.

21 research projects have been proposed by the faculty members of a university unit in Fars-Iran province in 2018.

To select the appropriate research project, many indicators can be considered. Including: the scientific conditions of the executive committee of each project (the degree of education, the number of scientific articles, the extent of their relationship with industry, the environment, etc.), estimating the cost and profit of doing that project, the time required to do that project etc. In the case of the 21 mentioned projects, 3 indicators were available as input and 2 indicators as output which based on them the implementation of the proposed projects is prioritized. The mentioned indicators are as follows:

X_1: The number of people-hours required to do the proposal project on a daily basis

X_2: Hours required, having the equipment and laboratories of the university on a daily basis

X_3: Estimated cost for each proposal project in dollars

Y_1: Amount of sponsor allowance per project in dollars

Y_2: The profit from doing the project in dollars

Since the collected information is not accurate, the data are defined as a fuzzy. Therefore, have been tried to be expressed the results in fuzzy. According to the results, the implementation of the proposed projects will be prioritized and, if necessary, a suitable pattern for to correct will be introduced for each project. The available data from these projects are shown in Table 1.

Table 1 shows the input and output data of 21 research projects. Although the models are radial, but the X_3 input plays an important role, in addition the Y_2 output is also very important. Therefore, three input and two output indicators have an important role in determining the efficiency of research projects.

Table 1. Input and output data of research projects

DMU	X_1	X_2	X_3	Y_1	Y_2
1	(150, 178.5, 180)	(85, 87, 89)	(2320.7, 2320.7, 2320.7)	(19.01, 30.52, 46.66)	(2744.06, 2754.71, 2766.53)
2	(78, 78.83, 79)	(89, 89, 89)	(7660.22, 7660.22, 7660.22)	(19.01, 35.13, 53.57)	(12825.22, 12986.78, 13174.27)
3	(102, 102, 102)	(107, 107.66, 111)	(4320, 4320, 4320)	(51.84, 72.58, 98.5)	(12752.64, 13073.46, 13713.41)
4	(91, 92.33, 94)	(93, 94.5, 96)	(2592, 2592, 2592)	(17.28, 52.41, 95.04)	(18696.96, 18698.1, 18698.69)
5	(89, 90.66, 92)	(83, 83, 83)	(2903.04, 2903.04, 2903.04)	(15.55, 46.93, 74.3)	(16403.9, 16598.87, 16849.73)
6	(85, 87, 90)	(92, 92, 92)	(2592, 2592, 2592)	(19.01, 29.95, 46.66)	(2823.55, 2848.88, 2870.21)
7	(89, 89.5, 91)	(85, 85, 85)	(1947.46, 1947.46, 1947.46)	(25.92, 38.59, 51.84)	(12617.86, 12807.07, 13032.58)
8	(50, 51, 52)	(81, 81.5, 82)	(2808, 2808, 2808)	(34.56, 51.84, 57.75)	(2073.6,3456, 4320)
9	(79, 81, 83)	(81, 81, 81)	(5832, 5832, 5832)	(19.01, 43.48, 57.02)	(967.68, 1033.91, 1085.18)
10	(102, 103, 105)	(97, 97, 97)	(6480, 6480, 6480)	(12.1, 21.31, 32.83)	(13022.21, 13938.32, 15123.46)

11	(96, 97.33, 100)	(90, 91, 92)	(5724.86, 5724.86, 5724.86)	(81.22, 160.98, 222.91)	(24395.9, 25154.77, 25909.63)
12	(106, 108.5, 112)	(84, 88.33, 92)	(2764.8, 2764.8, 2764.8)	(74.3, 100.5, 167.62)	(17635.97, 18131.32, 18619.2)
13	(107, 108.83, 111)	(95, 95, 95)	(2980.8, 2980.8, 2980.8)	(15.55, 45.5, 62.21)	(11418.62, 11573.56, 11790.14)
14	(94, 97.16, 101)	(78, 78, 78)	(3317.76, 3317.76, 3317.76)	(139.97, 308.45, 418.18)	(20729.09, 20908.51, 21187.01)
15	(196, 198.83, 200)	(186, 186.5, 187)	(6912, 6912, 6912)	(51.84, 63.64, 77.76)	(2258.5, 2309.18, 2332.8)
16	(75, 78.66, 81)	(88, 88.83, 90)	(4432.32, 4432.32, 4432.32)	(17.28, 27.06, 38.02)	(14489.28, 14624.63, 10145.09)
17	(82, 85, 88)	(92, 93.16, 94)	(4838.4, 4838.4, 4838.4)	(48.38, 61.62, 74.3)	(1088.64, 1116.29, 1140.48)
18	(77, 80.16, 82)	(92, 92.83, 94)	(2816.64, 2816.64, 2816.64)	(10.37, 20.44, 27.65)	(17706.82, 17714.87, 17722.37)
19	(84, 88.5, 90)	(104, 104, 104)	(5875.2, 5875.2, 5875.2)	(25.92, 34.56, 48.38)	(8190.72, 8408.72, 8740.22)
20	(94, 101.83, 108)	(91, 91.66, 92)	(2253.31, 2253.31, 2253.31)	(22.46, 35.7, 43.2)	(8199.36, 8431.78, 8900.93)
21	(97, 101, 103)	(95, 95.16, 96)	(7267.97, 7267.97, 7267.97)	(22.46, 27.92, 36.29)	(2783.81, 2802.23, 2827.01)

Table 2. The lexicographic results of the model (9)

DMU	Lexicographic DEA-R	Lexicographic α	Lexicographic β
DMU1	5.10883	(0.1852,0.1863,0.1863,0.1942)	(5.1502,5.5237,5.5 237,5.4801)
DMU2	0.89880	(0.6258, 0.6374, 0.6374, 0.6655)	(1.5027, 1.5688, 1.5688, 1.5979)
DMU3	1.24923	(0.5635, 0.5573, 0.5573, 0.5672)	(1.8012 , 1.8340, 1.8340, 1.8033)
DMU4	0.00000	(1.0000, 1.0000, 1.0000, 1.0000)	(1.0000, 1.0000, 1.0000, 1.0000)
DMU5	0.28762	(0.8699, 0.8673, 0.8673, 0.8695)	(1.1564, 1.1585, 1.1585, 1.1532)
DMU6	5.66454	(0.1717, 0.1721, 0.1721, 0.1743)	(5.7381, 6.0029, 6.0029, 5.9286)
DMU7	0.15372	(0.9277, 0.9217, 0.9217, 0.9780)	(1.0809, 1.0942, 1.0942, 1.0780)
DMU8	2.11769	(0.3574, 0.3202, 0.3202, 0.4796)	(2.0852, 3.1232, 3.1232, 2.8804)
DMU9	5.85646	(0.1622, 0.1691, 0.1691, 0.1653)	(6.0508, 5.9142, 5.9142, 6.1636)
DMU10	1.39770	(0.5494, 0.5236, 0.5236, 0.5084)	(1.9671, 1.9099, 1.9099, 1.8203)
DMU11	0.00000	(1.0000, 1.0000, 1.0000, 1.0000)	(1.0000, 1.0000, 1.0000, 1.0000)
DMU12	0.04833	(1.0000, 0.9884, 0.9884, 1.0000)	(1.0401, 1.0607, 1.0607, 1.0255)
DMU13	1.22769	(0.5695, 0.5691, 0.5691, 0.5654)	(1.8045, 1.7902, 1.7902, 1.7690)
DMU14	0.00000	(1.0000, 1.0000, 1.0000, 1.0000)	(1.0000, 1.0000, 1.0000, 1.0000)
DMU15	7.20142	(0.0893, 0.1008, 0.1008, 0.1870)	(5.3466, 9.9186, 9.9186, 1.1E+1)
DMU16	0.82736	(0.5029, 0.7304, 0.7304, 0.7427)	(1.3517, 1.3749, 1.3749, 1.9903)
DMU17	3.08681	(0.2037, 0.2284, 0.2284, 0.3967)	(2.5208, 4.3792, 4.3792, 4.9098)
DMU18	0.00000	(1.0000, 1.0000, 1.0000, 1.0000)	(1.0000, 1.0000, 1.0000, 1.0000)
DMU19	2.30489	(0.3855, 0.3676, 0.3676, 0.3730)	(2.6806, 2.7201, 2.7201, 2.5939)
DMU20	1.33715	(0.5476, 0.5476, 0.5476, 0.5594)	(1.8913, 1.8950, 1.8950, 1.8263)
DMU21	7.12500	(0.1195, 0.1200, 0.1200, 0.1573)	(6.3554, 8.5794, 8.5794, 8.6139)

In table 2, using the lexicographic approach, theα variable corresponds to the efficiency obtained from decreasing the inputs and the β variable corresponds to increasing the outputs. In the fuzzy case, if α and β are one in the form of fuzzy numbers, it indicates that the decision maker unit is an efficient. Considering the third and fourth columns of Table 2, research projects 4, 11 and 18 are efficient. The advantage of the proposed model is that it is based on the nature of input and output.

Table 3. The results from Pessimistic and Optimistic view approaches of models 15 and 14

DMU	Pessimistic DEA-R	$[E_1, E_2]$	Optimistic DEA-R	$[E_1, E_2]$
DMU1	-5.22	[5.399,0.182]	-4.96	[5.150,0.194]
DMU2	-0.97	[1.598,0.626]	-0.84	[1.503,0.665]
DMU3	-1.22	[1.775,0.555]	-1.21	[1.763,0.555]
DMU4	0.00	[1.000,1.000]	0.00	[1.000,1.000]
DMU5	-0.28	[1.150,0.867]	-0.29	[1.150,0.865]
DMU6	-5.65	[5.823,0.169]	-5.56	[5.736,0.174]
DMU7	-0.15	[1.078,0.928]	-0.10	[1.023,0.925]
DMU8	-2.45	[2.798,0.347]	-1.61	[2.085,0.480]
DMU9	-6.00	[6.164,0.162]	-5.89	[6.051,0.165]
DMU10	-1.27	[1.820,0.549]	-1.46	[1.967,0.508]
DMU11	0.00	[1.000,1.000]	0.00	[1.000,1.000]
DMU12	-0.02	[1.000,0.975]	-0.04	[1.000,0.961]
DMU13	-1.19	[1.756,0.565]	-1.21	[1.769,0.554]
DMU14	0.00	[1.000,1.000]	0.00	[1.000,1.000]
DMU15	-11.11	[1.E+1,0.089]	-5.16	[5.347,0.187]
DMU16	-1.49	[1.988,0.502]	-0.61	[1.347,0.740]
DMU17	-4.71	[4.910,0.204]	-2.12	[2.521,0.397]
DMU18	0.00	[1.000,1.000]	0.00	[1.000,1.000]
DMU19	-2.21	[2.594,0.386]	-2.31	[2.681,0.373]
DMU20	-1.28	[1.826,0.548]	-1.26	[1.788,0.529]
DMU21	-8.25	[8.368,0.116]	-6.20	[6.355,0.157]

Table 3 shows that based on the fuzzy data Table 1 as well as the use of optimistic and pessimistic view approaches, the efficiency intervals are obtained for each of the 14 and 15 models. Hence, it is observed that research projects 4, 11 and 18 are efficient. Obviously, the model considers the behavior of inputs and outputs radially.

5. Conclusion

Classic DEA models and their manipulated models cannot be a useful tool for evaluating the performance of organizations. Similarly, DEA-RA models, which are a combination of DEA and Ratio-Analysis, behave similarly. On the other hand, DEA-RA models are also related to DEA models. And some problems such as using the non-Archimedean number ε and dividing the weighted sum of the outputs to the weighted sum of the inputs $\left(\frac{UY}{\mathrm{VX}}\right)$ are not included. In fact, one of that's reasons is related to the definition of efficiency as the weighted sum of input to output ratios or vice versa$\left(W\frac{Y}{X}\right)$. In the present paper, input-output oriented DEA-RA models with fuzzy data have been used to evaluate research projects in Fars-Iran province. Fully fuzzified problem is proposed in DEA-RA models in joint nature and as well as problem with fuzzy parameters. For future research, evaluation of units with non-radial DEA-RA models is recommended.

References

[1] Hatami-Marbini A, Ebrahimnejad A, Lozano S. Fuzzy Efficiency Measures in Data Envelopment Analysis Using Lexicographic Multiobjective Approach. Computers & Industrial engineering.2017;1–9.

[2] Mozaffari M. R, Gerami J, Jablonsky J. Relationship between DEA models without explicit inputs and DEA-R models. Central European Journal of Operations Research. 2014; 22:1–12.

[3] Charnes A, Cooper W. W, Rhodes E. Measuring the efficiency of decision making units. European Journal of Operational Research. 1984; 2:429-444.

[4] Banker RD, Charnes A, Cooper W.W. Some models estimating technical and scale inefficiencies in data envelopment analysis. Management Science. 1984; 30:1078-1092.

[5] Emrouznejad A, Amin G.R. DEA models for ratio data: convexity consideration. Applied Mathematical Modeling. 2009; 1:486–498.

[6] Olesen O.B, Petersen N.C. Identification and use of efficient faces and facets in DEA. Journal of Productivity Analysis. 2003; 20:323–360.

[7] Despic O, Despic M, Paradi J.C. DEA-R: Ratio-based comparative efficiency model, its mathematical relation to DEA and its use in applications. Journal of Productivity Analysis. 2007; 28:33–44.

[8] Sharp J A, Liu W B and Meng W. A modified slacks-based measure model for data envelopment analysis with 'natural' negative outputs and inputs. Journal of Operational Research Society.2006;57, 1–6. 2006.

[9] Wu D., Liang L., Huang Z.M., Li S. Aggregated ratio analysis in DEA. International Journal of Information Technology and Decision Making. 2005;4(3):369-384.

[10] Zha Y., Liang L. Aggregated ratio analysis in DEA. International Journal of Information Technology and Decision Making. 2014; 13:1-13. DOI: 10.1142/S0219622014500114.

[11] Rashidi K., Cullinane K. A comparison of fuzzy DEA and fuzzy TOPSIS in sustainable supplier selection: implications for sourcing strategy. Expert Syst. 2019; 121:266–281.

[12] Zimmerman H.J. Fuzzy set theory and it applications. Kluwer, Massachusetts. Academic Publishers. 1996.

[13] Seiford L M and Zhu J. Modeling undesirable factors in efficiency evaluation, European journal of operational research. 2002;142, 16–20.

[14] Farrell M.J. The measurement of productivity efficiency. Journal of The Royal Statistical Society Series A: General. 1957;120(3): 253–281.

[15] Foroughi A.A, Jafari Y. A modified method for constructing efficient solutions structure of MOLP.Applied Mathematical Modeling. 2009;33: 2403–2410.

[16] Mozaffari M.R, Kamyab P, Jablonsky J, Gerami J. Cost and revenue efficiency in DEA-R models. Computers & Industrial Engineering. 2014;78:188–194.

[17] Silva PortelaM, Thanassoulis E and Simpson G (2004). Negative data in DEA: a directional distance approach applied to bank branches, Journal of the Operational Research Society, 55(10), 1111–1121.
[18] Smith P (1990). Data Envelopment Analysis Applied to Financial Statements," Omega, 18(2), 131-138.
[19] Ueda T and Hoshiai Y (1997). Application of component analysis for parsimonious summarization of DEA inputs and/or outputs, Journals of operational research society Japan, 40: 466–478.
[20] Wei C.K, Chen L.C, Li R.K, Tsai C.H. A study of developing an input- oriented ratio-based comparative efficiency model. Expert Systems with Applications. 2011; 38:2473–2477.
[21] Wei C.K, Chen L.C, Li R.K, Tsai C.H. Exploration of efficiency underestimation of CCR model: Based on medical sectors with DEA-R model. Expert Systems with Applications. 2011; 38:3155–3160.
[22] Wei C.K, Chen L.C, Li R.K, Tsai C.H. Using the DEA-R model in the hospital industry to study the pseudo-inefficiency problem. Expert Systems with Applications. 2011; 38:2172–2176.
[23] Wei Q.L, Yan.H, Hao.G. Characteristics and structures of weak efficient surfaces of production possibility sets. Journal of Mathematical Analysis and Applications. 2007; 327:1055–1074.
[24] Wei. Q.L., G. Yu. Analyzing the properties of K-cone in generalized data envelopment analysis model. J. Econometrics. 1997; 80:63–84.