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Ranking of Non-Extreme Efficient units based on multi ideal DMUs in PPS

A.R. Salehi, F. Hosseinzadeh Lotfi*, M.Rostamy-Malkhlifeh
Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran

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Abstract
Data envelopment analysis (DEA) is a body of research methodologies to evaluate overall efficiencies, identify the sources, and estimate the amounts of inefficiencies in inputs and outputs. In DEA, the best performers are called DEA efficient and the efficiency score of a DEA efficient unit is denoted by a unity. The standard DEA method assigns an efficiency score less than one to inefficient decision-making units (DMUs), from which a ranking can be derived. However, efficient DMUs all have an efficiency of one, so that for these units no ranking can be given. Since in evaluating by traditional DEA models many DMUs are classified as efficient, a large number of methods for fully ranking both efficient and inefficient DMUs have been proposed. In the last decade, ranking DEA efficient units has become the interests of many DEA researchers and a variety of models (called super-efficiency models) were developed to rank DEA efficient units. Super efficiency data envelopment analysis model can be used in ranking the performance of efficient DMUs. While the models developed in the past are interesting and meaningful, they have the disadvantages of being infeasible or instable occasionally. But the main problem of super-efficient models is lack of differentiation between non- extreme efficient DMUs, so these models cannot rank these DMUs. In this paper, we propose a new method for Ranking Non-Extreme Efficient Decision making units in Data Envelopment Analysis based on benchmark. One of the main advantages of our approach is that, this method doesn't apply any new models, rather this model applies a combination of the well-known models for ranking DMUs. Therefore, understanding our proposed method is easy for readers. One numerical example is examined to illustrate the potential applications of the proposed method.

Keywords: Ranking, Non-extreme efficient, Super–efficiency, DEA, benchma

[1] Corresponding Author: farhad@hosseinzadeh.ir

1. Introduction

Data envelopment analysis (DEA) is a methodology for assessment the performances of a group of decision-making units (DMUs) that utilize multiple inputs to produce multiple outputs. DEA, originally presented by Charnes et al. [1], is a well-known mathematical programming tools for evaluating the relative efficiency of a set of comparable processing decision-making units (DMUs). DEA successfully divides the units into two categories: efficient DMUs and inefficient DMUs. Unlike the inefficient DMUs, the efficient ones cannot be ranked based on their efficiencies because they all get the efficiency score equal to one.

However, it is not reasonable to claim that the efficient DMUs have the same performance in actual practice. Now, the question arises how to rank the efficient DMUs? To address this question, different methods have been developed to achieve complete ranking of these DMUs. So, one of the interesting research subjects is to discriminate between efficient DMUs. Therefore, the researchers proposed some methods to discriminant the efficient units. This concept has named ranking efficient units in DEA. There are many ranking methods and each of them has some advantages and drawbacks to rank efficient units. To review ranking methods, see also Adler et al .[2] and Hosseinzadeh et al .[3].

In the following, we summarized some of well-known methods for ranking DMUs.

Charnes et al .[4], counted the number of times that an efficient DMU is as the benchmark unit for other DMUs, and used it to rank the units. As regards, the reference set of a DMU is not found easily, therefore, their model is not an applicable method. Charnes et al .[5], proposed another method to find the benchmark DMUs. They changed the outputs of units and then they evaluated how the efficiency score of DMUs changed. However, they didn't distinguish how they can do it.

Super-efficient approach is another method pioneered by Anderson and Peterson [6] (AP model). In their method, the corresponding column to the DMU under evaluation is omitted from the technological matrix. Later, Mehrabian et al .(MAJ) [7] have modified AP model. In some circumstances, the mentioned models may be infeasible and in particular AP model may be unstable because the extreme sensitivity may be occurring by small variations in data, when some units have the small values in some inputs. Saati et al. [8], have modified MAJ model and solved its infeasibility and then Jahanshahloo et al .[9] have changed the type of data normalization in order to receive a much better result. In order to remove the drawbacks of AP and MAJ models, Salehi et al [10] proposed a method for ranking all DMUs by using strong and weak supporting hyperplanes. They evaluated the rail freight and the passenger transportation in some Asian countries. some authors have used specific norms. For instance, Jahanshahloo et al [11], have practiced norm for ranking efficient units. Mirzaei and Salehi [12] extended a new model to calculate ranking efficient DMUs in the fuzzy environment. Amirteimoori et al .[13], have experienced norm to find the gap between evaluated efficient units and the new PPS. Gradient line and ellipsoid norms have been used by Jahanshahloo et al .[14], in order to rank efficient units. Tone [15] and [16], has used SBM model in this way.

Sexton et al. [17] proposed the cross-efficiency method. In cross-efficiency evaluation, each DMU is self and peer evaluated. Specifically, each unit determines a set of weights in the traditional DEA model, resulting in n sets of weights. Then, each DMU is evaluated by the n sets of weights obtaining n efficiency scores. The cross-efficiency of each unit is the average of the *n* efficiency scores. Although, cross-efficiency evaluation has been extensively applied in various cases, but there exists factor that possibly reduces the usefulness of the cross-efficiency evaluation method. This factor is that the cross-efficiency scores may not be unique due to the presence of alternative optimal weights. As a result, it is suggested that secondary goals are introduced in cross-efficiency evaluation. For more studies about secondary goal models, see Doyle and Green [18], Liang et al. [19], Wang and Chin [20], Dotoli et al. [21], Wu et al. [22], Jahanshahloo et al. [23] and Wu et al. [24].

Although secondary goal models were suggested to solve the problem of the cross-efficiency evaluation, but, the existing secondary goal models in the literature have some drawbacks. Note, none of the secondary goal models in the literature guarantees that the optimal weights are unique. Hence, the problem of existing alternative optimal solutions does not solve completely. This is the main drawbacks of secondary goal models. On the other hand, most of the existing secondary goal models in literature solve *n(n-1)* model to obtain the rank of units. Therefore, if *n* be a large number, then the number of models that should be solved is very large, so the computational complexity is very high and this is another drawbacks of secondary goal models.

Finally, it should be noted that there are some techniques and strategies in DEA which they effect on ranking. For example, Thompson et al [25], used the assurance regions. In their technique, the number of efficient DMUs may be decrease. But it isn't a suitable method because finding suitable weights isn't easy. Adler et al .[26] proposed another method to difference between DMUs. In their model, they decrease the number of inputs and outputs by component analysis. Therefore, the number of efficient DMUs is decreased. But in general, this model cannot be used for a complete ranking.

All ranking methods evaluate units from a particular perspective and each of them has advantages and drawbacks compared to others. Therefore, none of the methods has superiority over the others. We are looking for a method that can use all the benefits of ranking methods by integrating them, as far as possible, and also this approach provides a new method for ranking decision making units. Note, super efficiency method cannot rank the non-extreme efficient DMUs in this paper we develop a methodology for ranking non-extreme efficient DMUs by representation of non-extreme efficient DMUs with extreme efficient DMUs and determination of weight based on self-super efficiency measure. We propose this method to complete the methods which rank all units under evaluation, but our focus is on the ranking of non-extreme efficient units.

2. Preliminaries and basic ranking methods

In this section, we describe some of the main ranking methods and their advantages and drawbacks. Also, due to the fact that the proposed algorithm use Topsis method to rank all units, therefore, Topsis method is also mentioned at the end of this section.

Assume that there are n DMUs, $DMU_j, j=1,...,n,$ and each DMU consumes m inputs to produce s outputs. The i^{th} input and r^{th} output for DMU_j are denoted by x_{ij} and y_{rj}, respectively, for $i=1,...,m$ and $r=1,...,s.$ Charnes et al. [1] proposed CCR model, in which the efficiency score of each unit is evaluated as the ratio of the weighted sum of the outputs to the weighted sum of the inputs. However, the original formulation of the

CCR model is a non-linear problem. It can be transformed into its input-oriented linear programming format as in the following model by using the Charnes–Cooper transformation (Charnes & Cooper [1]).

$$
\begin{aligned}
\max\ & z=\sum_{r=1}^{s}u_r y_{ro} \\
s.t.\ & \sum_{i=1}^{m}v_i x_{io}=1, \\
& \sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}\geq 0\ ,\ j=1,\dots,n \\
& u_r\geq\varepsilon\quad ,\quad r=1,\dots,s \\
& v_i\geq\varepsilon\quad ,\quad i=1,\dots,m.
\end{aligned}
\qquad (1)
$$

2.1 AP model

Super efficiency models introduced in DEA technique is based upon the idea of leave one out and assessing

this unit trough the remanding units. In this subsection we are going to review AP, MAJ AND Super SBM ranking models in data envelopment analysis. To describe the DEA efficiency measurement, let there are n DMUs and the performance of each DMU is characterized by a production process of m inputs $(x_{ij}\ i=1,\dots,m)$ to yields s outputs $(y_{rj}\ r=1,\dots,s)$. Andersen and Petersen [6] developed a new procedure for ranking efficient units. The methodology enables an extreme efficient DMUo to achieve an efficiency score greater than or equal to one by removing the O-th constraint in the primal formulation, they omitted the efficient DMU from the PPS, and ran CCR model [1] for other units to rank them. The mathematical formulation of model (2) is as follows:

$$
\begin{aligned}
\max\ & z=\sum_{r=1}^{s}u_r y_{ro} \\
s.t.\ & \sum_{i=1}^{m}v_i x_{io}=1, \\
& \sum_{i=1}^{m}v_i x_{ij}-\sum_{r=1}^{s}u_r y_{rj}\geq 0,\ j=1,\dots,n\ ,\ j\neq o \\
& u_r\geq\varepsilon\quad ,\quad r=1,\dots,s \\
& v_i\geq\varepsilon\quad ,\quad i=1,\dots,m.
\end{aligned}
\qquad (2)
$$

The dual formulation of the super-efficient model, as seen in model (3), computes the distance between the Pareto frontier, evaluated without DMUo, and the unit itself i.e. for$\{j=1,\dots,n, j\neq o\}$.

$$
\begin{aligned}
\min\ & \theta \\
s.t.\ & \sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j x_{ij}\leq\theta x_{ip}\ ,\ i=1,\dots,m \\
& \sum_{\substack{j=1\\ j\neq p}}^{n}\lambda_j y_{rj}\geq y_{rp}\quad ,\ r=1,\dots,s \\
& \lambda_j\geq 0\quad ,\quad j=1,\dots,n.
\end{aligned}
\qquad (3)
$$

However, The AP method has the following problems:

- First, Andersen and Petersen refer to the DEA objective function value as a rank score for all units, despite the fact that each unit is evaluated according to different weights. This value in fact explains the proportion of the maximum efficiency score that each unit O attained with its chosen weights in relation to a virtual unit closest to it on the frontier. Furthermore, if we assume that the weights reflect prices, then each unit has different prices for the same set of inputs and outputs within the same organization.
- Second, the super-efficient methodology can give ‘‘specialized’’ DMUs an excessively high ranking. To avoid this problem, Sueyoshi [27] suggest a method to avoid this problem.

- The third problem lies with an infeasibility issue, which if it occurs, means that the super-efficient technique cannot provide a complete ranking of all DMUs. Mehrabian et al. [7] suggested a modification to the dual formulation in order to ensure feasibility; we will refer to it later . Notice that, the AP model is feasible when we use this model in output oriented form.
- Fourth, In some cases, small changes in the data may change a lot θ^*, of course, this The problem does not occur in output oriented form.
- Fifth, AP model does not have any suggestion for ranking non-extreme efficient units I fact, super efficiency method cannot rank the non-extreme efficient DMUs.

Remark 1: For efficient units $\theta^* \geq 1$ and for inefficient units $0 < \theta^* < 1$ .also, the optimal objective value of AP model (model (3)) is greater than one or the model is infeasible for extreme efficient units and equal one for non-extreme efficient units, We'll use this to identify extreme and non-extreme units.

2.2. MAJ model

To solve the important difficulties of AP models, Mehrabian et al. [7], proposed another model (model (4)) for ranking efficient units. Their proposed model is:

$$
\begin{aligned}
\min \quad & 1+w \\
s.t. \quad & \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j x_{ij} \leq x_{ip} + w, \quad i=1,\dots,m, \\
& \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j y_{rj} \geq y_{rp} \quad , \quad r=1,\dots,s, \qquad (4) \\
& \lambda_j \geq 0 \quad , \quad j=1,\dots,n.
\end{aligned}
$$

Despite these drawbacks, possibly because of the simplicity of the concept, many published papers have used this approach.

Theorem 1. The necessary and sufficient conditions for feasibility of MAJ model is that in evaluating of DMUp, or $y_{rp} = 0$, $r = 1, \dots, s$ or there exists DMUj, $j \neq p$such that $y_{rj} \neq 0$.

Proof. See [7].

2.3 Super-efficiency evaluated by SBM

In this section, we discuss the super-efficiency issue by using the slacks-based measure (SBM) of

efficiency. Tone [15] presented super efficiency of SBM model. This model has the advantages of non-radial models, and it is always feasible and stable, the mathematical formulation of model (5) is as follows:

$$
\begin{aligned}
\min \quad & \frac{\sum_{i=1}^{m} \frac{\bar{x}_i}{x_{ip}}}{\sum_{r=1}^{s} \frac{\bar{y}_r}{y_{rp}}} \\
s.t. \quad & \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j x_{ij} \leq \bar{x}_i \quad , \quad i=1,\dots,m, \\
& \sum_{\substack{j=1\\ j\neq p}}^{n} \lambda_j y_{rj} \geq \bar{y}_r \quad , \quad r=1,\dots,s, \qquad (5) \\
& \bar{x}_i \geq x_{ip} \quad , \quad i=1,\dots,m, \\
& 0 \leq \bar{y}_r \leq y_{rp} \, , \quad r=1,\dots,s \quad , \\
& \lambda_j \geq 0 \quad , \quad j \neq p.
\end{aligned}
$$

The fractional program [Super-SBM] can be transformed into a linear programming problem (see [15]).

2.3. Cross-efficiency method

Sexton et al. [17] proposed the cross-efficiency method. In cross-efficiency evaluation, each DMU is self and peer evaluated. Specifically, each unit determines a set of weights in the traditional DEA model, resulting in *n* sets of weights. Then, each DMU is evaluated by the n sets of weights obtaining n efficiency scores. The cross-efficiency of each unit is the average of the n efficiency scores.

In this method, by solving model (1) n times for each DMU, we can obtain the optimal solution $\{v_{1o}^*, ..., v_{mo}^*, u_{1o}^*, ..., u_{so}^*\}$ for each $DMU_o (o = 1, ..., n)$. Then, the cross efficiency score of DMU_j corresponding to DMU_o can be calculated as the following Eq. (6).

$$E_{oj} = \frac{\sum_{r=1}^{s} u_{ro}^* y_{rj}}{\sum_{i=1}^{m} v_{io}^* x_{ij}}, \quad (o, j = 1, ..., n) \qquad (6)$$

Then, the cross-efficiency score of DMU_j can be calculated as the average of $E_{oj} (o = 1, ..., n)$ in Eq. (7).

$$E_j = \frac{1}{n} \sum_{o=1}^{n} E_{oj} \qquad (7)$$

There are three principal advantages of cross-efficiency evaluation:

(i) This approach provides a unique ordering of the DMUs (Doyle and Green [28]).

(ii) It eliminates unrealistic weight schemes without incorporating weight restrictions (Anderson et al. [29]).

(iii) Cross-efficiency method distinguishes good and poor performers among the units (Boussofiane et al. [30]).

Because of these advantages, cross-efficiency evaluation has been extensively applied in various cases (See Sexton et al. [17], Liang et al. [19], and so on for more details).

A factor that possibly reduces the usefulness of the cross-efficiency evaluation method is that the cross-efficiency scores may not be unique due to the presence of alternative optimal weights. As a result, it is suggested that secondary goals are introduced in cross-efficiency evaluation. Doyle and Green [18] proposed two linear programming problems which are known as the aggressive formulation and benevolent formulation for cross-efficiency evaluation. The aggressive formulation aims to minimize the cross-efficiencies and the benevolent formulation aims to maximize the cross-efficiencies of other DMUs.

Liang et al. [19] extended Doyle and Green's models by incorporating alternative secondary objective functions based on deviations to its ideal point 1. However, Wang and Chin [20] pointed out that the ideal points in the model of Liang et al. [19] are not realizable for the inefficient units. They improved the models by changing the target efficiency from the ideal point 1 to the CCR efficiency. It could be found that, the traditional benevolent and aggressive models only consider the desirable targets (1 or the original efficiency scores) as the referenced efficiencies for all units. However, Dotoli et al. [21] pointed out that the undesirable targets are also important indicators that the DMUs need to consider. Wu et al. [22], Jahanshahloo et al. [23] incorporated a symmetric technique into DEA cross-efficiency evaluation and gave secondary goal models which can choose symmetric weights for units. Wu et al. [24] incorporated a target identification model to get reachable targets for all DMUs. They proposed several secondary goal models for weights selection considering both desirable and undesirable cross-efficiency targets for all DMUs. Although secondary goal models were suggested to solve the problem of the cross-efficiency evaluation, but, the existing secondary goal models in the literature have some drawbacks. Note, none of the secondary goal models in the literature guarantees that the optimal weights are unique. Hence, the problem of existing alternative optimal solutions does not solve completely. This is the main drawbacks of secondary goal models. On the other hand, most of the existing secondary goal models in literature solve *n(n-1)* model to obtain the rank of units. Therefore, if *n* be a large number, then the number of models that should be solved is very large, so the computational complexity is very high and this is another drawbacks of secondary goal models.

2.4. TOPSIS procedure

TOPSIS method is one of the best techniques for MCDM developed by Hwang and Yoon in [31], is a simple ranking method in conception and application. The standard TOPSIS method attempts to choose alternatives that simultaneously have the shortest distance from the positive ideal solution and the farthest distance from the negative-ideal solution. The positive ideal solution maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution maximizes the cost criteria and minimizes the benefit criteria. TOPSIS makes full use of attribute information, provides a cardinal ranking of alternatives, and does not require attribute preferences to be independent (Chen and Hwang [32]). To apply this technique, attribute values must be numeric, monotonically increasing or decreasing, and have commensurable units. After forming an initial decision matrix, the procedure starts by normalizing the decision matrix. This is followed by building the weighted normalized decision matrix in Step 2, determining the positive and negative ideal solutions in Step 3, and calculating the separation measures for each alternative in Step 4. The procedure ends by computing the relative closeness coefficient. The set of alternatives (or candidates) can be ranked according to the descending order of the closeness coefficient. It is necessary to pass following steps in order to solve a problem by the technique:

Step 1. Calculate the normalized decision matrix. The normalized value n_{ij} is calculated as:

$$n_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} {x_{ij}}^2}} \quad i = 1,...,m \ , j = 1,...,n$$

Step 2. Calculate the weighted normalized decision matrix. The weighted normalized value is calculated as:

$$v_{ij} = w_j . n_{ij} \qquad i = 1,..,m \quad j = 1,...n$$

Where w_j is the weight of the *jth* attribute or criterion, and $\sum_{j=1}^{m} w_j = 1$.

Step 3. Determine the positive ideal and negative ideal solution

$$A^+ = \{V_1^+, ..., V_n^+\} = \{(\max v_{ij} \,|\, i \in I), (\min v_{ij} \,|\, i \in J)\}$$

$$A^- = \{V_1^-, ..., V_n^-\} = \{(\min v_{ij} \,|\, i \in I), (\min v_{ij} \,|\, i \in J)\}$$

Step 4. Calculate the separation measures, using the *n*-dimensional Euclidean distance. The Separation of each alternative from the positive ideal solution is given as:

$$d_i^+ = \{\sum_{j=1}^{n} (v_{ij} - v_j^+)^2\}^{\frac{1}{2}} \qquad i = 1,..,m$$

Similarly, the separation from the negative ideal solution is given as:

$$d_i^- = \{\sum_{j=1}^{n} (v_{ij} - v_j^-)^2\}^{\frac{1}{2}} \qquad i = 1,..,m$$

Step 5. Calculate the relative closeness to the ideal solution, where the relative closeness of the Alternative A_i with respect to A^+ is defined as:

$$c_i = \frac{d_i^-}{d_i^+ + d_i^-} \qquad i = 1,...,m$$

Since $d_i^- \geq 0$ and $d_i^+ \geq 0$ clearly, $C_i \in [0,1]$.

Step 6. Rank the preference order. For ranking alternatives using this index, we can rank alternatives in decreasing order.

3. Proposed method

In this section, our method for ranking non-extreme efficient units will be described in details. One of the main advantages of our approach is that, this method doesn't apply any new models, rather this model applies a combination of the well-known models for ranking DMUs. Therefore, understanding our proposed method is easy for readers. First, we describe the observations that motivated us to develop a new method for ranking DMUs. And then, we describe our proposed approach in details. One of the most important techniques for ranking the units is to use Ideal and Anti-Ideal DMUs. The issue has attracted attentions of several scholars. For example, Wang and Ying Luo [33] proposed a method for ranking all units by defining the ideal and anti-ideal DMUs. They defined the Ideal (IDMU) and Anti-Ideal (ADMU) DMUs as follows:

Definition 1. An IDMU is a virtual DMU, which can use the least inputs to generate the most outputs. While an ADMU is a DMU, which consumes the most inputs only to produce the least outputs.

Note that a virtual IDMU may not exist in practical production activity at least at current technical level, while a virtual ADMU may exist in practical production activity because the waste of resources is always allowed in the theory of production possibility set (PPS). Given that the ideal unit is not often in the PPS, therefore, it is a virtual unit that is outside the set of observations. The ideal DMU belongs to the PPS only if there exists one decision making unit that dominates all DMUs. As we know, this is practically impossible. Therefore, it can be said that the evaluation of decision-making units by comparing them with a unit that is not in practical production activity cannot be a reasonable criterion for ranking DMUs, because the efficiency of this virtual unit has not been calculated according to the potential and the current facilities of the practical production activity. Hence, all decision making units can object why their efficiency scores were not measured according to the current technical level in practical production activity. Therefore, to overcome this problem, it seems that one technique is to look for ideal units that belongs to the practical production activity. The important question that arises here is how this technique defines the ideal unit belonging to the PPS. On the other hand, this technique should be as good as possible from the point of view of computational complexity compared to the existing techniques. We propose a new method for ranking DMUs that will be further described in detail in this section. Our method applies extreme efficient DMUs, provided by AP model, as ideal units instead of using a virtual DMU. At first, we describe our method in details and then summarize it in an algorithm. Regarding Remark 1, the super efficiency of the extreme efficient units is greater than 1, therefore, these units have a better performance than other units according to the potential and the current facilities of the practical production activity. So, we use the information derived from AP model without the need to solve a new model.

Extreme efficient DMUs are actually members of the reference set and we know benchmarks are in fact a combination of members of the reference set .In order to reach the benchmark, we need to model the members of the reference set.

In the business world, companies use benchmarking as a point of reference as well. They use benchmark reports as a way to compare themselves to others in the industry. Benchmarking is the practice of a business comparing key metrics of their operations to other similar companies. Companies use benchmarking as a way to help become more competitive. By looking at how other companies are doing, they can identify areas where they are underperforming. Companies are also able to identify ways that can improve their own operations without having to recreate the wheel. They are able to accelerate the process of change because they have models from other companies in their industry to help guide their changes, the purpose of benchmarking is to help the management of a decision making unit (DMU) to improve performance and productivity.

As it's noted in this section and in regard to the importance of the member of reference set, we can define them as on ideal among observed units. In this paper, introducing the extreme efficient units which are benchmark among observed units and regarding to their high importance, we propose them as ideal. Furthermore, the ranking of non-extreme efficient units which it can't done using super efficiency method, we rank them via benchmarks and MCDM techniques. So, we evaluate decision making units using one of super efficiency methods, this operation helps us to reorganization of non-extreme efficient units in addition of ranking extreme efficient units. As you know, in evaluation of decision making units by supper efficiency models, it assumes the value 1 for non-extreme efficient units, so it can't differentiate between these units. In this paper, we propose to calculate the distance of each of non-extreme efficient units from all of the extreme efficient units (using norm 2).

Now assume that $\overline{DMU_e}, e = 1,..,l$ are extreme efficient units and $\overline{\overline{DMU_n}}, n = 1, ..., q$ are non-extreme efficient units, we should consider if q=1 there is no need to ranking non extreme efficient DMU. Now we calculate the distance of each of non-extreme efficient units from all of the extreme efficient units, the results show in following table:

Table 1. Distance of each of non-extreme efficient units from all of the extreme efficient units

	$\overline{DMU}_1$	$\overline{DMU}_2$	$\overline{DMU}_3$	$\cdots$	$\overline{DMU}_l$
$\overline{\overline{DMU}}_1$	d_{11}	d_{12}	d_{13}	$\cdots$	d_{1l}
$\overline{\overline{DMU}}_2$	d_{21}	d_{22}	d_{23}	$\cdots$	d_{2l}
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\overline{\overline{DMU}}q$	d_{q1}	d_{q2}	d_{q3}	$\cdots$	d_{ql}

d_{ne} is the distance of n-th non extreme efficient unit from e-th extreme efficient unit Which is defined as follows:

$$d_{ne} = \left\| \begin{pmatrix} \overline{\overline{x}}_{in} \\ \overline{\overline{y}}_{rn} \end{pmatrix} - \begin{pmatrix} \overline{x}_{ie} \\ \overline{y}_{re} \end{pmatrix} \right\|$$
$$i = 1,\dots,m \quad , \quad r = 1,\dots,s \quad , \quad n = 1,\dots,q \quad , \quad e = 1,\dots,l \qquad (8)$$

In Table 1, the first row shows the effect of the first extreme efficient unit on other non -extreme efficient unit, the smaller the sum of these values means that this unit has a greater effect on all non -extreme efficient. Considering that our purpose is ranking of non-extreme efficient units using extreme efficient units, so we deal with Multi criteria decision making (MCDM) problem, which in alternatives are non-extreme efficient units and criteria are extreme efficient units. There are some different techniques to solve MCDM problems. In this paper we propose TOPSIS method which is summarized in section 2 because the TOPSIS method is also of great popularity in multi-criteria decision-making. To do the TOPSIS method, you must have both the weight of the criteria and the decision matrix data. To obtain the weight of the criteria, experts can be used or we can calculate the weight of the criteria using the AHP method. The decision matrix data is real and quantitative, such as the amount of profit, cost, price, weight, and ... Having a decision matrix for analysis of the methodology, but if the criteria are qualitative and we cannot obtain the actual value of each option relative to any criterion, it is better to use the TOPSIS questionnaire. In this questionnaire, the rate of the score of each option relative to each criterion is obtained in the form of a Likert spectrum or any other contractual spectrum. Given that the data of the decision-making matrix are judgments, it is better to distribute more than one questionnaire to the target statistical society and to integrate comments all respondents will draw up a final decision matrix to reach a consensus on qualitative and judicious standards. The number of criteria and options in the TOPSIS method is not limited and you can choose a large number according to your issue. So in general, in the TOPSIS method:

- It can be done with a small number of criteria or options.
- Possessing positive and negative criteria can be done.
- With qualitative and quantitative measures, it is possible to do.
- Options are ranked in the TOPSIS method.
- There must be criteria and options.
- Topsis questionnaires can be distributed to a large number in the statistical community.
- If there is actual data for the decision matrix, the use of the TOPSIS method is very appropriate.

Remark 2: Since the members of decision matrix are kind of distance, the positive and negative ideal should identify carefully, given that Positive ideal is a virtual alternative with best score, so minimum distance should be considered and it’s true for negative ideal reversely.

Remark 3: one of the most steps of TOPSIS method is determination of weights of criteria. We do this based on 2 different methods

1. Entropy method [24]. The result is showed in table 5.
2. Considering that non-extreme efficient unit which is closer to extreme efficient unit, has better performance, so we propose to assume the super efficiency value of extreme efficient unit as its weight

Now, we provide an algorithm to explain the steps of our method to rank all units, as follows:

Step 1: Solve model (3) to evaluate decision making units.

Step 2: Determine the inefficient units, the extreme and non-extreme efficient units by Remark 1.

Step 3: Consider the extreme efficient DMUs as ideal units.

Step 4: Determine the distance of the non-extreme efficient units from all of the extreme efficient DMUs by applying Eq (8).

Step 5: Make decision matrix as described in Table 1.

Step 6: Apply TOPSIS method to rank DMUs.

Note that, the rank of inefficient units and extreme efficient units is specified by solving model (3) in Step 1, and the other steps of our algorithm determine the rank of non-extreme efficient units.

4. An application example

The real data set, documented in table 2, contains 23 banks with four inputs and four outputs. The inputs and outputs in this study are presented in Table 3. This data is related to a performance appraisal project of one of Iran's commercial banks conducted by the team of authors. Below, each of the input and output factors is defined and a brief indication of how to calculate and measure their unit.

Table 2. Contains 23 banks with four inputs and four outputs

DMUs	Payable interest	Personnel	Non-performing loans	number of branch	The total sum of four main deposits	Loans granted	Received interest	Fee
1	4707.86	175.8	60801	31	1033890	42954	611224	31671.6
2	32641.32	477.94	264991	52	5398005	966040	5090776	108826.2
3	24603.99	511.76	238510	53	5795565	871880	4839322	131011.6
4	19435.03	457.39	187639	51	4641078	853001.7	4321139	109026.5
5	9097.12	348.65	85897	48	2332104	815245	3284772	65056.46
6	34766.12	276.55	402614	36	4313779	539228	7878616	231066.5
7	41239.42	408.88	105778	46	6136069	298420	5115135	29197.01
8	2978.41	459.78	321776	49	4923925	1802130	4887652	123469.1
9	4902.54	254.34	110543	46	1097316	122046	1127011	12581.5
10	2278.13	142.75	30084	34	555997	22165	168786	3672.26
11	23642.26	736.26	58238	141	3736368	190077	1353879	23249.96
12	8394.97	529.64	64750	98	1437663	60187	929473	20853.48
13	411.48	28.16	2059	6	125767	11638	66532	5208.43
14	36923.89	320.66	303668.7	39	4921209	458958.7	6957456	163776.6
15	2698.51	175.95	26732	45	524945	21484	277671	5134.55
16	3490.86	181.79	22065	48	568498	86932	495530	5618.5
17	1551.69	681.88	186281	144	2866310	245966	2055363	34231.45
18	1862.82	132.14	24805	33	415291	22353	339450	8397.85
19	7887.42	46.98	27059	14	245523	7189	269819	3189.24
20	1658.79	415.75	111632	89	1898925	115275	1222240	24371.74
21	19811.22	368.72	92524	47	3600092	642970	3894893	53103.31
22	1658.79	101.68	21245	28	467922	22728	446906	7491.01
23	2297.71	146.92	24579	34	513104	36438	502190	8826.21

Table 3. The inputs and outputs

ID	variables	type of variables
1	Payable interest	input
2	Personnel	input
3	Non-performing loans	input
4	number of branch	input
5	The total sum of four main deposits	output
6	Loans granted	output
7	Received interest	output
8	Fee	output

I1: Payable interest : Interest payable is the amount of interest on its debt and capital leases that a company owes to its lenders and lease providers as of the balance sheet date. This amount can be a crucial part of a financial statement analysis, if the amount of interest payable is greater than the normal amount - it indicates that a business is defaulting on its debt obligations. Interest payable can include both billed and accrued interest, though (if material) accrued interest may appear in a separate "accrued interest liability" account on the balance sheet. In the case of capital leases, a company may have to infer the amount of interest payable, based on a deconstruction of the underlying capital lease. Interest is considered to be payable irrespective of the status of the underlying debt as short-term or long-term debt. Short-term debt is payable within one year, and long-term debt is payable in more than one year. Interest payable is a liability, and is usually found within the current liabilities section of the balance sheet. In this paper, the scale of this input is in millions of Rials. Figure 1 shows the rate of the first input of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the first input value of each decision-making unit is displayed.

I2: Personnel : This factor is the Bank staff rating, which is derived from the combination of staff numbers, work experience, age, specialization, executive posts and salaries. Each factor normalized first, and then the weighted sum is calculated. Figure 2 shows the rate of the second input of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the second input value of each decision-making unit is displayed.

I3: Non-performing loans: When banks lend out money, they do so with the hope that their borrowers will make their payments as scheduled. But that doesn't always happen. Sometimes borrowers run out of money or fall into situations where they can't repay their debt, and that's how non-performing loans become a problem for so many banks. A non-performing loan, or NPL, is one that is in or close to default. This typically happens when principal and interest payments on the loan are overdue by 90 days or more. Non-performing loans are generally considered bad debt because the chances of them getting paid back are minimal. The more non-performing loans a bank has on its books, the more its stock price is likely to be affected. In this paper, the scale of this input is in millions of Rials. Figure 3 shows the rate of the third input of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the third input value of each decision-making unit is displayed.

I4: number of branch: This input indicator represents the number of branches in the sub-category of the bank in a particular geographic area of study. Figure 4 shows the rate of the fourth input of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the fourth input value of each decision-making unit is displayed.

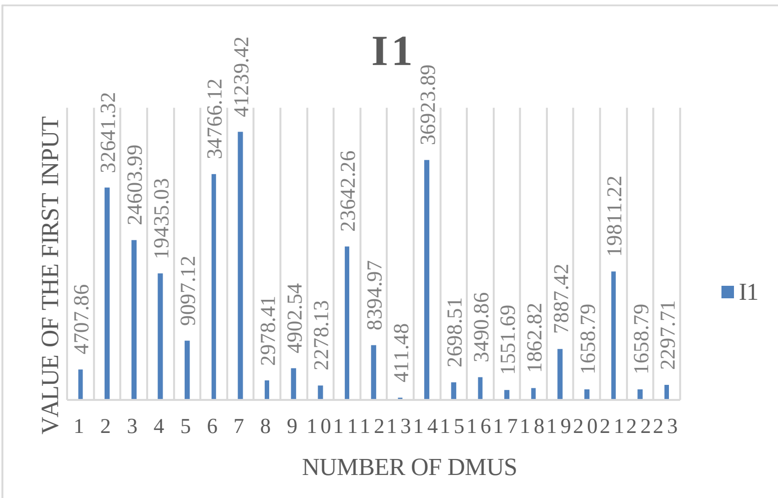

Figure 1. shows the rate of the first input of each decision-making unit

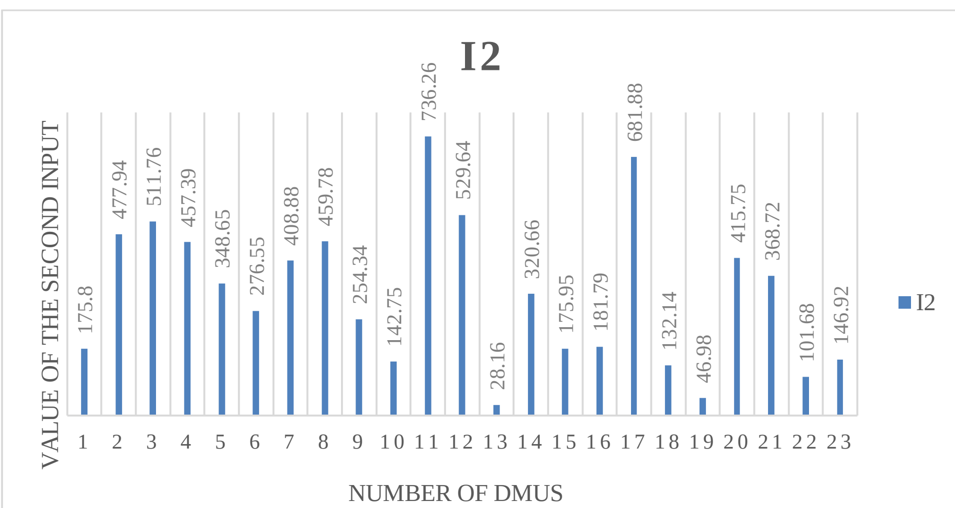

Figure 2. Shows the rate of the second input of each decision-making unit

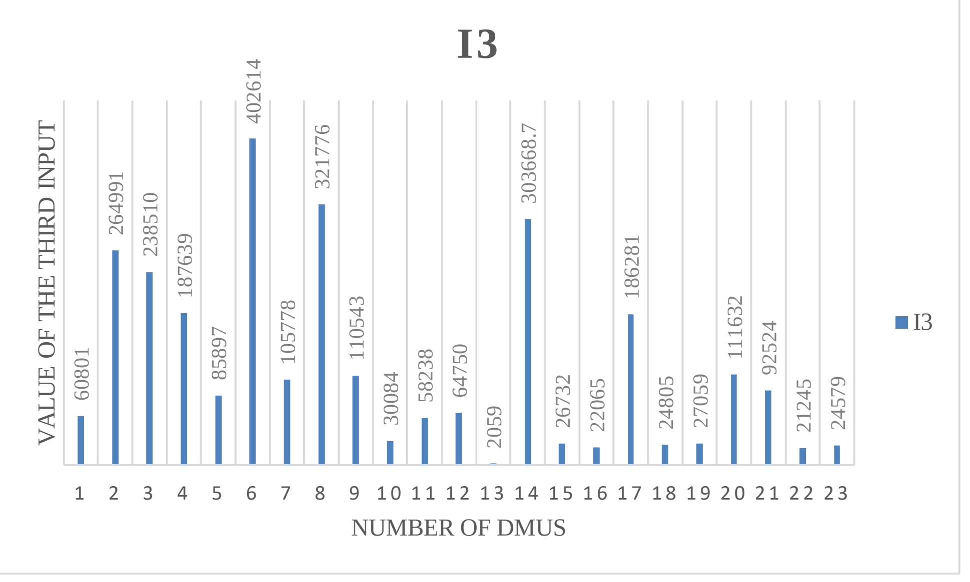

Figure 3. Shows the rate of the third input of each decision-making unit

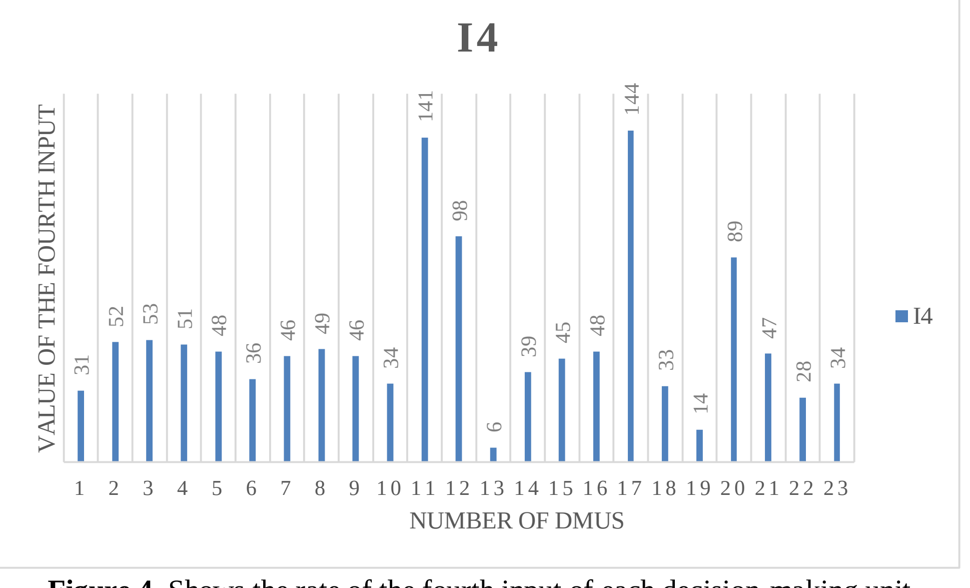

Figure 4. Shows the rate of the fourth input of each decision-making unit

O1: The total sum of four main deposits : This indicator is derived from the sum of the four main accounts described briefly. In this article, the unit of measurement is Million Rials.

- **Current account:** A current account is a personal bank account, which you can take money out of at any time using your chequebook or cash card.
- **Savings account:** A savings account is a bank account with a limited number of transactions per month and which pays a higher interest rate than a checking account.

- **Investment account :**a bank account in which money is saved long-term to accrue interest
- **Deposit account:** A deposit account is a type of bank account where the money in it earns interest.

Figure 5 shows the rate of the first output of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the first output value of each decision-making unit is displayed.

O2: Loans granted : the total amount of loans a bank paid to legal or legal persons. Figure 6 shows the rate of the second output of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the second output value of each decision-making unit is displayed.

O3: Received interest : The amount of benefit each bank receives from Customer for a loan or an allowance. Figure 7 shows the rate of the third output of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the third output value of each decision-making unit is displayed.

O4: Fee: The term bank charge covers all charges and fees made by a bank to their customers. In common parlance, the term often relates to charges in respect of personal current accounts or checking account. These charges may take many forms, including:

- Monthly charges for the provision of an account
- Charges for specific transactions (other than overdraft limit excesses)
- Interest in respect of overdrafts (whether authorised or unauthorised by the bank)
- Charges for exceeding authorised overdraft limits, or making payments (or attempting to make payments) where no authorised overdraft exists

Figure 8 shows the rate of the fourth output of each decision-making unit. In the horizontal axis, the number of units and the vertical axis, the fourth output value of each decision-making unit is displayed.

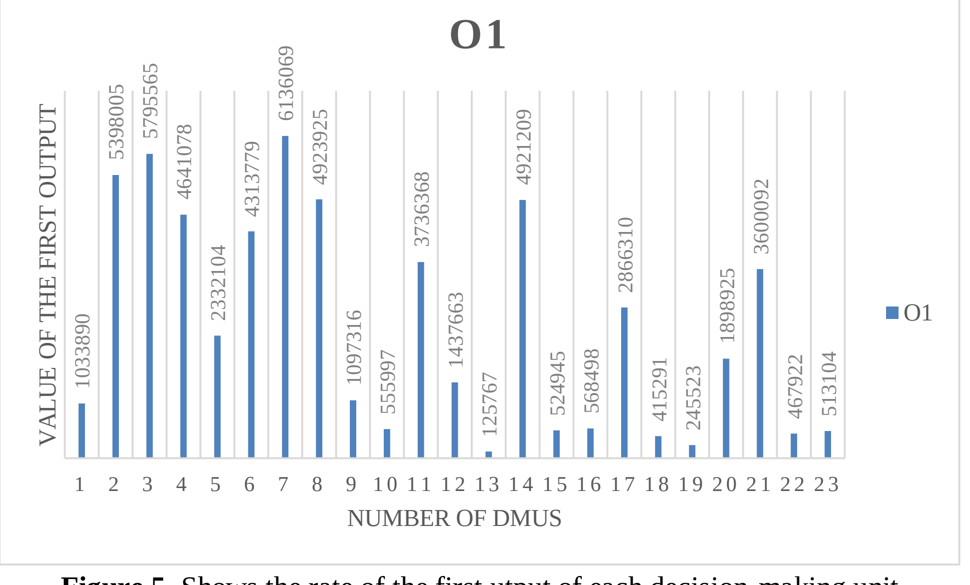

Figure 5. Shows the rate of the first utput of each decision-making unit

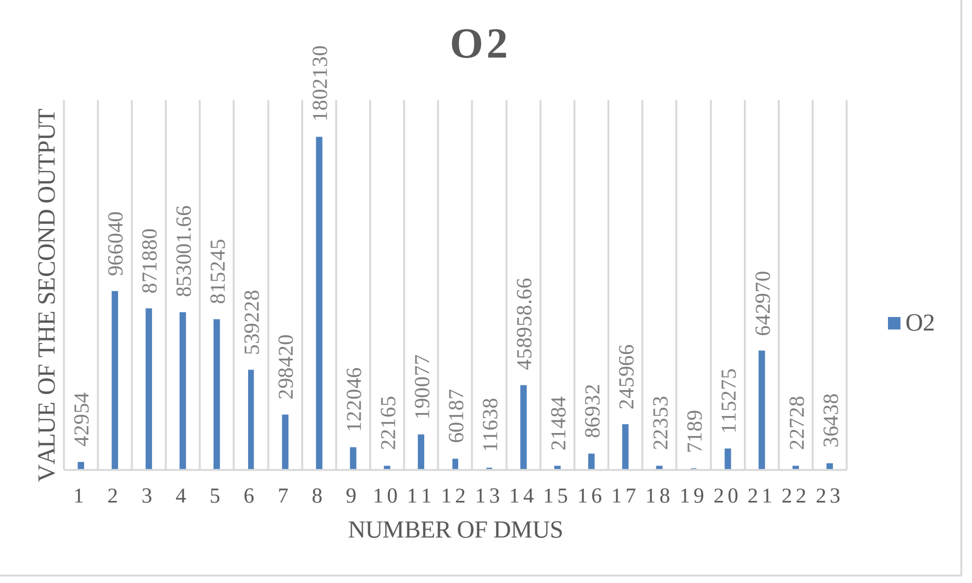

Figure 6. Shows the rate of the second output of each decision-making unit

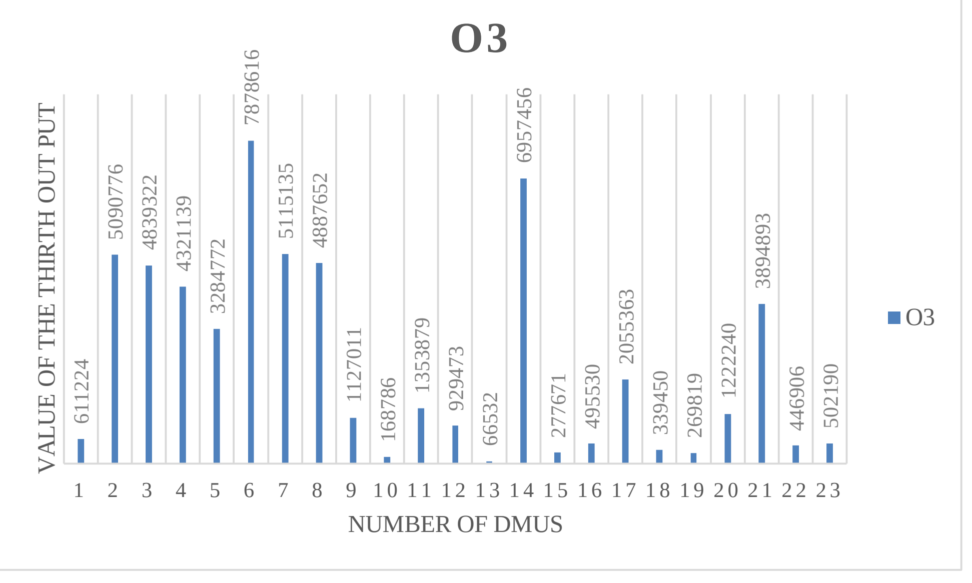

Figure 7. Shows the rate of the third output of each decision-making unit

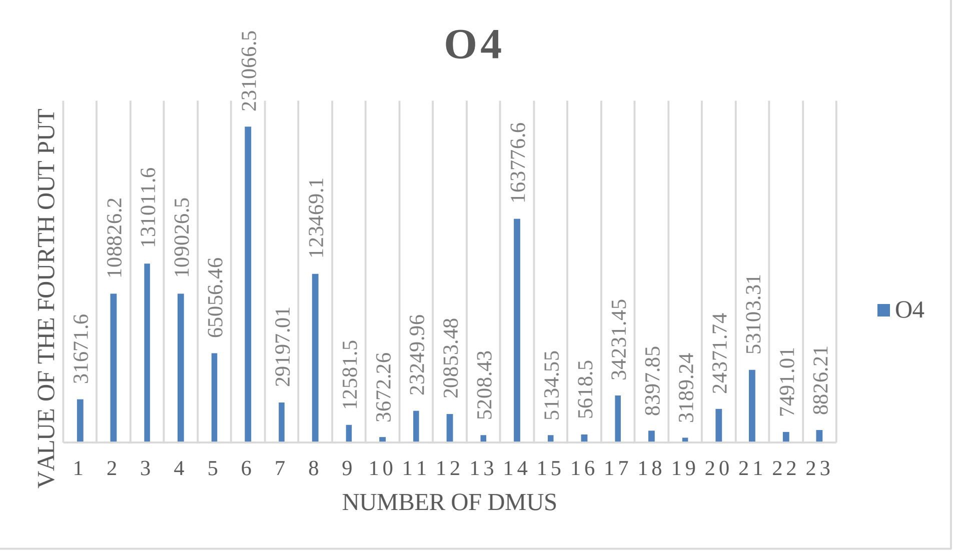

Figure 8. Shows the rate of the fourth output of each decision-making unit

Table 4. General information

	Payable interest	Personnel	Non-performing loans	number of branch	The total sum of four main deposits	Loans granted	Received interest	Fee
Min data	411.48	28.16	2059	6	125767	7189	66532	3189.24
Max data	41239.42	736.26	402614	144	6136069	1802130	7878616	231066.5
Average	12562.62391	320.8857	120620.5	52.69565	2502145	358926.3	2440688	52566.57
Variance	171950998.1	36497.36	1.25E+10	1132.994	4.14E+12	1.96E+11	5.59E+12	3.69E+09
Standard deviation	13113.00874	191.0428	111869.3	33.65998	2033985	443085.3	2364081	60762.93

The minimum values and the maximum mean variance and the deviations to the criteria for inputs and outputs are summarized in Table 4.

In evaluating, the performance of 23 banks studied by classical methods of data envelopment analysis, such as Method CCR, the results of shows in the second column of Table 5. As described in Section 2, the units under evaluation whose efficiency is 1 is efficient and decision-making units whose efficiency is less than one are inefficient. It is noticeable that decision units 3,4,5,6,7,8,11,13,14,17,20,21, 23 efficient units and decision-maker units 1,2,9,10,12,15, 16,18,19 are inefficient.

First, for each DMU, we calculate super efficiency value by AP model. The results are shown in table 5, which CCR efficiency values, super efficiency values and finally the ranking of each unit are in second, third and 4th columns, respectively. Therefore, unit 8 has best ranking and unit 19 has worst ranking. Units 4,14,20,21,23 are non-extreme efficient that super efficiency models cannot rank them, so they are considered with same ranking. In order to formation of decision making matrix, we calculate the distance of each non-extreme efficient unit from all of the extreme efficient units using equation 8, the results show in following table 6.

Now according to step1in TOPSSIS method, we calculate the normalized decision matrix the results show in following table 7.

Finally, by application of TOPSIS method, the rankings of non-extreme efficient units are displayed in table 8 The complete ranking values are in 5th column of table 5.

Table 5. the efficiency score and ranking by the AP model (*: extreme efficient DMU)

	CCR-EFF	AP-EFF	ranking	Complete ranking with proposed method	Cross efficiency method	Ranking with cross efficiency	extreme efficient DMU
DMU_1	0.740709	0.74	11	16	0.517826	15	
DMU_2	0.913777	0.91	10	15	0.631739	10	
DMU_3	1	1.07	7	7	0.775652	7	*
DMU_4	1	1.00	9	13	0.792174	6	
DMU_5	1	1.76	3	3	0.84087	3	*
DMU_6	1	1.63	5	5	0.628696	12	*
DMU_7	1	1.65	4	4	0.820435	4	*
DMU_8	1	6.14	1	1	0.906522	1	*
DMU_9	0.493645	0.49	17	22	0.382609	22	
DMU_{10}	0.633904	0.63	12	17	0.405652	19	
DMU_{11}	1	1.06	8	8	0.582174	14	*
DMU_{12}	0.546805	0.54	16	21	0.401304	20	
DMU_{13}	1	3.3	2	2	0.858261	2	*
DMU_{14}	1	1.00	9	14	0.673913	8	
DMU_{15}	0.550199	0.55	15	20	0.389565	21	
DMU_{16}	0.614473	0.61	13	18	0.45087	17	
DMU_{17}	1	1.1	6	6	0.629565	11	*
DMU_{18}	0.576577	0.57	14	19	0.434783	18	
DMU_{19}	0.433583	0.43	18	23	0.188696	23	
DMU_{20}	1	1.00	6	6	0.633043	9	*
DMU_{21}	1	1.00	9	9	0.816957	5	
DMU_{22}	1	1.00	9	12	0.594783	13	
DMU_{23}	1	1.00	9	11	0.499565	16	

Table6: the distance of each non-extreme efficient unit from all of the extreme efficient units

	DMU_3	DMU_5	DMU_6	DMU_7	DMU_8	DMU_{11}	DMU_{13}
DMU_4	3800431.157	5023460.178	4253257.811	3212443.182	2581578	2630585.361	3598010
DMU_{14}	6846161.29	8302908.383	7238253.669	6426983.121	829173.4	972906.9517	231658.1
DMU_{20}	6890157.442	8324839.256	7278334.907	6473543.712	863970.1	1019676.332	225368.7
DMU_{21}	1266810.386	3594797.164	1785087.949	1149024.634	4828091	4734640.632	5832716
DMU_{22}	2329584.453	1112777.386	2225554.772	2468058.094	6985051	6979184.279	8015053
DMU_{23}	2406626.437	4064260.131	2835505.207	2034587.787	3768105	3716369.124	4788946

Table7: The normalized decision matrix

	DMU_3	DMU_5	DMU_6	DMU_7	DMU_8	DMU_{11}	DMU_{13}
DMU_2	0.344	0.36	0.35	0.31	0.26	0.27	0.31
DMU_4	0.62	0.59	0.61	0.62	0.08	0.10	0.01
DMU_{12}	0.62	0.59	0.61	0.631	0.08	0.10	0.01
DMU_{16}	0.11	0.25	0.15	0.11	0.49	0.48	0.50
DMU_{18}	0.211247638	0.079860773	0.188338438	0.240761766	0.718931	0.720447873	0.691761
DMU_{19}	0.218233835	0.291680044	0.239955731	0.198476263	0.387829	0.383633692	0.413323

Table 8. The weights of extreme efficient units

	$W_{DMU\,3}$	$W_{DMU\,5}$	$W_{DMU\,6}$	$W_{DMU\,7}$	$W_{DMU\,8}$	$W_{DMU\,11}$	$W_{DMU\,13}$
super efficiency value	1.07	1.76	1.63	1.65	6.14	1.06	3.33

Table 8. the rankings of non-extreme efficient units

DMU21	0.59
DMU20	0.56
DMU23	0.52
DMU22	0.48
DMU4	0.411
DMU14	0.39

6. Conclusion.

In many cases, it is necessary to give a full ranking of the DMUs. One of the ranking methods of extreme efficient units is based on super efficiency. In this kind of methods, after elimination of units from observation set, the distance of unit from production possibility set is a measure for ranking of it. As this distance increases, the corresponding unit has a better ranking. This indicates whatever the distance of extreme efficient unit's other efficient units in production possibility set are more, they have better ranking. Thus, we can't use this method for ranking of extreme efficient units, unless the measures change. In this paper we propose a new ranking system for all DMUs by using benchmarking. Therefore, this method is able to rank all extreme and non-extreme efficient DMUs, the calculation complicity of this propose model is low. as an advantage, alternative optimal solution affords no problem in results of proposed model. In addition, this method uses the results of super efficiency models and doesn't need to renew a model. It seems that ranking by this approach is more precise than other methods.

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