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Alternative mixed integer linear programming model for finding the most efficient decision making unit in data envelopment analysis

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Abstract
Finding the Most Efficient Decision Making Unit (DMU) provides more information about efficient DMUs in data envelopment analysis (DEA). Hence, in recent years, many mixed integer linear programming (MILP) models based on a common set of weights have been proposed to determine the most efficient DMU. This paper introduces another MILP model to find the most efficient DMU. In this model, we use a numerical parameter to increase the discrimination power of the proposed model. To illustrate the various potential applications of the proposed model, we compare the performance of our model with the other three models using two real numerical examples.

Keywords: Data envelopment analysis, Most Efficient DMU, Mixed integer linear programming.

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1- Introduction

Data envelopment analysis (DEA) is a mathematical approach that introduced by Charnes et al [1]. to assess the relative efficiency of a homogeneous group of decision-making units (DMUs). DEA successfully divides them into two categories; efficient DMUs and inefficient DMUs. One of the important issues discussed in DEA literature is ranking efficient units since the efficient units obtained in the efficiency score of one cannot be compared with each other on the basis of this criterion any more. Therefore, it seems necessary to provide models for further discrimination. Hence, many different approaches were proposed to rank the efficient DMUs. Some include ranking with multivariate statistics in DEA context, including linear discriminant analysis (Torgersen et al. [2]), discriminant analysis of ratios (Sinuany-Stern et al. [3]), super efficiency ranking methods (Andersen and Petersen [4]), benchmark ranking methods (Sueyoshi [5]), cross efficiency ranking methods (Dyson et al. [6]).

In some cases, the decision-maker must select only one DMU throughout a set of considered DMUs. There have been several studies to extend some integrated DEA models for determining a single efficient DMU, namely the most efficient unit. Karsak and Ahiska [7] proposed an integrated multi-criteria decision making (MCDM) DEA model in order to evaluate the most efficient DMU in advanced manufacturing technology (AMT). Amin et al. [8] introduced a counterexample to show that the model does not always converge and then improved it. Amin and Toloo [9] formulated a new mixed integer linear programming (MILP) model to find the most efficient unit with common set of weights (CSW). Toloo and Nalchigar [10] extended this model into variable returns to scale (VRS) situation for selecting the most BCC-efficient DMU. Amin [11,12] explained some drawbacks of previous MILP models and introduced a new mixed integer non-linear programming (MINLP) model to modify these flaws. It was mathematically proved that these models can determine the best efficient unit; however, the suggested models were nonlinear in nature and consequently hard to solve. The problem of finding the most association rule by considering multiple criteria, which is an important task in data mining, was addressed in Toloo et al [13]. Besides the novel application, they also designed an algorithm for prioritizing association rules. However, Toloo and Nalchigar [14] pointed out some drawbacks in the proposed algorithm of Toloo et al. [13] and improved it. Foroughi [15] proposed a new integrated maximin MILP model that finds the most efficient unit by maximizing the minimum possible distance between a selected unit and the next ranked unit. It was shown that the suggested approach can also be extended to rank all extreme efficient DMUs. Wang and Jiang [16] clarified that Foroughi's model is very complicated and involves many redundant constraints and proposed a new approach to identify the most efficient DMU. Toloo [17] formulated an MILP model for finding the most efficient unit without explicit input and utilized it to determine the best efficient professional tennis player. By excluding the non-Archimedean epsilon, Toloo [18] proposed an approach which finds the most efficient DMU with fewer computations. Toloo [19] addressed the problem of selecting and full ranking suppliers with imprecise data which is an important issue in supply chain management. Toloo [20] formulated a new minimax MILP model for finding the most efficient DMU with the CSWs. Lam [21] introduced a new MILP model for finding the most efficient DMU in DEA. Compared to existing models, the Lam proposed model features an objective that is more intuitively allied to finding the most efficient DMU.

This paper provides a new mixed integer programming (MIP) as a good alternative model for finding the most efficient DMU in DEA. In order to increase the discriminatory power of the proposed model, we use a scalar parameter.

The rest of the paper is classified as follows: Section 2 briefly reviews existing models for finding the most efficient DMU. Section 3 proposes the alternative MILP model to determine the most efficient DMU. Section 4 examines two numerical examples to show the potential applications of the proposed alternative MILP models and their effectiveness in finding the most efficient DMU. The paper concludes in Section 5.

2. Preliminaries

Throughout this paper, we assume that there are n independent and homogenous units, $DMU_j\,(j=1,2,...,n)$, which consume m various inputs, $x_{ij}\,(i=1,2,...,m)$, to produce s different outputs, $y_{rj}\,(r=1,2,...,s)$. The relative efficiency score of a unit is defined as the ratio of total weighted outputs to the total weighted inputs. Let $v_i\,(i=1,2,...,m)$ and $u_r\,(r=1,2,...,s)$ be the weights of ith input and rth output, respectively.

Mathematically, the efficiency score of $DMU_j\,(j=1,2,...,n)$ can be calculated as[22]:

$$e_j=\frac{\sum_{r=1}^{s}u_r y_{rj}}{\sum_{i=1}^{m}v_i x_{ij}},\ j=1,2,...,n$$

Sueyoshi et al.[23] proposed the following linear programming model for obtaining optimal weights and estimating the best relative efficiency score of DMU_p, the DMU under evaluation:

$$\begin{aligned} e_p^* &= Max\sum_{j=1}^{n} y_{ip} \\ s.t.\ & \sum_{r=1}^{s}u_r y_{rj}-\sum_{i=1}^{m}v_i x_{ij}\le 0,\ j=1,2,...,n, \\ & \sum_{i=1}^{m}v_i x_{ip}=1, \qquad (1) \\ & u_r \ge \frac{1}{(m+s)\max_j\{y_{rj}\}},\ r=1,2,...,s, \\ & v_i \ge \frac{1}{(m+s)\max_j\{x_{ij}\}},\ i=1,2,...,m, \end{aligned}$$

The DMU_p is efficient if and only if $e_p^*=1$, otherwise is inefficient. Let v_i^* and u_r^* be the optimal weights of ith input and rth output, respectively. If the DMU_p is efficient, then by definition $\sum_{r=1}^{s}u_r^* y_{rp}-\sum_{i=1}^{m}v_i^* x_{ip}=0$; otherwise there exists at least one other index $j\in\{1,2,...,n\}$ such that $\sum_{r=1}^{s}u_r^* y_{rj}-\sum_{i=1}^{m}v_i^* x_{ij}=0$. The subset of such j is called the reference set or the peer group to the DMU_p. Indeed, this collection of efficient DMUs forces them DMU_p to be inefficient.

Definition 1. If there is a common set of optimal weights, $(\mathbf{u}^*,\mathbf{v}^*)>0$, such that $\mathbf{u}^*\mathbf{y}_p-\mathbf{v}^*\mathbf{x}_p=0$ and moreover $\mathbf{u}^*\mathbf{y}_j-\mathbf{v}^*\mathbf{x}_j<0, j\ne p$, then DMU_p is called the most (best) efficient unit[20].

Wang and Jiang proposed the following model for finding the most efficient DMU under constant returns to scale [16].

$$
\begin{aligned}
Min \sum_{i=1}^{m} v_i \left(\sum_{j=}^{n} x_{ij} \right) - \sum_{i=1}^{m} u_r \left(\sum_{j=}^{n} y_{ij} \right) & \\
s.t. \ \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \le I_j, \ j = 1,2,...,n, & \\
\sum_{j=1}^{n} I_j = 1, & \quad (2) \\
u_r \ge l_r^u, \ r = 1,2,...,s, & \\
v_i \ge l_i^v, \ i = 1,2,...,m, & \\
I_j \in \{0,1\}, \quad j = 1,2,....,n, &
\end{aligned}
$$

Where $l_r^u = ((m+s)\max_j\{y_{rj}\})^{-1}$ and $l_i^v = ((m+s)\max_j\{x_{ij}\})^{-1}$ lower bounds borrowed from model(1). The objective of the model (2) is to maximize the overall efficiency of all of the DMUs. In this model, DMU_p is determined as the best efficient unit if and only if $I_p^* = 0$.

Toloo [18], proposed the following model to identify the most efficient DMU as follows:

$$
\begin{aligned}
Min \quad & d_{\max} \\
s.t. \quad & d_{\max} - d_j \ge 0, \ j = 1,2,...,n \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j = 0, \quad j = 1,2,...,n, \\
& \sum_{i=1}^{n} \theta_j = 1, \qquad (3) \\
& d_j \le M\theta_j, \ j = 1,2,...,n, \\
& \theta_j \le Nd_j, \ j = 1,2,...,n, \\
& \theta_j \in \{0,1\}, \ j = 1,2,...,n, \\
& u_r \ge 1, \quad r = 1,2,...,s, \\
& v_i \ge 1, \quad i = 1,2,...,m,
\end{aligned}
$$

Where M and N are large positive numbers. In this model, DMU_p is determined as the best efficient unit if and only if $\theta_p^* = 0$. The Objective function in model(3) minimizes the maximum inefficiencies of all of the DMUs except the most efficient one selected by the model. Toloo [20], proposed the following minimax model as an alternative MILP for identifying the most efficient unit under constant returns to scale (CRS).

$$
\begin{aligned}
Min \quad & d_{\max} \\
s.t. \quad & d_{\max} - d_j + \beta_j \ge 0, \ j = 1,2,...,n, \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + d_j - \beta_j = 0, \ j = 1,2,...,n, \\
& \sum_{i=1}^{n} d_j = 1, \qquad (4) \\
& d_j \in \{0,1\}, \ j = 1,2,...,n, \\
& u_r \ge l_r^u, \quad r = 1,2,...,s, \\
& v_i \ge l_i^v, \quad i = 1,2,...,m, \\
& \beta_j \le 1, \ j = 1,2,...,n \\
& d_{\max} \ free
\end{aligned}
$$

Toloo have been proved that model (4) is always feasible and the optimal objective value of model (4) is bounded. In model (4), the most efficient DMU has highest efficiency score that can be greater than 1, whereas those of the other DMUs are bounded by 1.

3. The proposed model

Assume that all inputs and outputs are strictly positive. We propose the following model (5) for determining the most efficient DMU:

$$
\begin{aligned}
Min \quad & \sum_{j=1}^{n} s_j \\
s.t. \quad & \\
& \sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} + s_j + \varepsilon\delta_j = 0, \ j = 1,2,...,n, \\
& \sum_{j=1}^{n} \delta_j = n-1, \\
& 0 \le s_j \le M\delta_j, \ j = 1,2,...,n, \qquad (5) \\
& \delta_j \in \{0,1\}, \ j = 1,2,...,n, \\
& u_r \ge l_r^u, \ r = 1,2,...,s, \\
& v_i \ge l_i^v, \ i = 1,2,...,m,
\end{aligned}
$$

Where M is a large positive number and ε is a small positive number.

Suppose that in optimal solution of model (5), if $\delta_p^* = 0$ then $s_p^* = 0$, so DMU_p has an efficiency score equal to one that is selected as the most efficient DMU, and the other DMUs have efficiency scores less than one. In this model, we can increase the value of ε by a reasonable amount to ensure that several DMUs are not placed on the same hyperplane. Thus the discriminatory power of the proposed model increases. The objective function in model (5) maximizes the overall efficiency of all of the DMUs. So, Model (5) uses a more relevant objective when choosing the most efficient DMU than the objectives applied in the other MILP models. Model (5) also uses lower bounds from slack-adjusted DEA models [28], as the lower bound for all of the input and output weights. The following theorems show some of the properties of the proposed model.

Theorem 1. Model (4) always has a feasible solution.

Proof. Let $(\mathbf{u}^o, \mathbf{v}^o)$ be the feasible solution of model (1), Note that sueyoshi et al [23] proved that such solution exits. Suppose that

$$F_j(\mathbf{u},\mathbf{v}) = \sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} \left(j = 1, 2, ..., n \right),$$

therefore $F_j(\mathbf{u}^o, \mathbf{v}^o) \le 0 \left(j = 1, 2, ..., n \right)$. Since $F_j(u, v)$ is an affine function then there exists a neighborhood, $N_\delta(\mathbf{u}^o, \mathbf{v}^o)$, such that

$$\forall \left(\bar{\mathbf{u}}, \bar{\mathbf{v}} \right) \in N_\delta(\mathbf{u}^o, \mathbf{v}^o) \cap$$

$$\left\{ (\mathbf{u},\mathbf{v}) \mid u_r \ge l_r^u,\ r = 1, 2, ..., s, v_i \ge l_i^v,\ i = 1, 2, ..., m, \right\}$$

$$\Rightarrow F_j(\bar{\mathbf{u}}, \bar{\mathbf{v}}) < 0 \left(j = 1, 2, ..., n \right)$$

So model (5) always has feasible solution.

Theorem 2. The optimal objective value of model (4) is bounded.

Proof. Let $(\mathbf{u}^0, \mathbf{v}^0, \mathbf{s}^0, \boldsymbol{\delta}^0)$ be any arbitrarily feasible solution to model (5). From constrains of this model $0 \le s_j^0 (j = 1, 2, ..., n)$ which means the objective function value of any feasible solution is bounded from below. This fact that model (5) is a minimization problem completes the proof.

4. Numerical examples

In all the numerical examples discussed below, models (2), (3), (4), and (5) are applied to determine the most efficient DMU. The datasets utilized in this section are used from previous studies in the DEA literature, as noted in each case.

Example 1. Facility layout design (FLD) in manufacturing systems.

Ertay et al [24] explored a dataset containing 19 FLDs. Table 1 indicates the data set containing 19 FLDs with two inputs and four outputs:

x_1= material handling cost,

x_2 = adjacency score,

y_1 = shape ratio,

y_2 = flexibility,

y_3 = quality.

y_4 = hand-carry utility

The following optimal solution is obtained by solving model (5) with $\varepsilon = 0.001$:

v_1^*=0.000040786351894, v_2^*=0.000009577443206,

u_1^*=0.258238999819774, u_2^*=1.947040498442445,

u_3^*=1.970055161544624, $u_4^* =$ 0.015429861507027,

$\delta_{12}^* = 0, \delta_j^* = 1 (j \ne 12),$

The following optimal solution is obtained by solving model (5) with $\varepsilon = 0.0001$:

v_1^*=0.000041161694003, v_2^*=0.000009577443206

u_1^*=0.262154449519854, $u_2^* =$ 1.947040498446610

u_3^*=1.970055161548814, $u_4^* =$ 0.015650183295409

$\delta_{12}^* = 0, \delta_j^* = 1 (j \ne 12),$

Table 2 summarizes different efficiency scores obtained by models (1), (2), (3), (4) and (5), respectively. The largest efficiency scores achieved by the different integrated MILP models are highlighted in bold. Model (5) reveals that FLD12 is the most efficient DMU, while the results of model (2) and model (4) tell that FLD10 is the most efficient DMU and the result of model (3) discloses FLD14 is the most efficient DMU

Table 1: Data set for 19 FLDs .

DMUs	x_1	x_2	y_1	y_2	y_3	y_4
FLD1	20309.56	6405	0.4697	0.0113	0.041	30.89
FLD2	20411.22	5393	0.438	0.0337	0.0484	31.34
FLD3	20280.28	5294	0.4392	0.0308	0.0653	30.26
FLD4	20053.2	4450	0.3776	0.0245	0.0638	28.03
FLD5	19998.75	4370	0.3526	0.0856	0.0484	25.43
FLD6	20193.68	4393	0.3674	0.0717	0.0361	29.11
FLD7	19779.73	2862	0.2854	0.0245	0.0846	25.29
FLD8	19831	5473	0.4398	0.0113	0.0125	24.8
FLD9	19608.43	5161	0.2868	0.0674	0.0724	24.45
FLD10	20038.1	6078	0.6624	0.0856	0.0653	26.45
FLD11	20330.68	4516	0.3437	0.0856	0.0638	29.46
FLD12	20155.09	3702	0.3526	0.0856	0.0846	28.07
FLD13	19641.86	5726	0.269	0.0337	0.0361	24.58
FLD14	20575.67	4639	0.3441	0.0856	0.0638	32.2
FLD15	20687.5	5646	0.4326	0.0337	0.0452	33.21
FLD16	20779.75	5507	0.3312	0.0856	0.0653	33.6
FLD17	19853.38	3912	0.2847	0.0245	0.0638	31.29
FLD18	19853.38	5974	0.4398	0.0337	0.0179	25.12
FLD19	20355	17402	0.4421	0.0856	0.0217	30.02

Table 2: Efficiency of the FLDs by different models

DMUs	CCR	Model(2)	Model(3)	Model(4)	Model(5) $\varepsilon=0.001$	Model(5) $\varepsilon=0.0001$
FLD1	0.985	0.9649	0.6085	0.7351	0.7876	0.7905
FLD2	0.988	0.9715	0.7373	0.805	0.8569	0.8592
FLD3	0.997	1	0.715	0.8446	0.8759	0.8779
FLD4	0.949	0.8945	0.6587	0.7744	0.8174	0.8192
FLD5	1	0.9253	0.9425	0.8708	0.8693	0.8698
FLD6	0.973	0.9108	0.9173	0.8253	0.8718	0.8733
FLD7	1	0.7908	0.6341	0.7679	0.8131	0.8139
FLD8	0.857	0.8682	0.5488	0.6083	0.6303	0.6332
FLD9	0.889	0.8345	0.7954	0.8334	0.854	0.8543
FLD10	1	**1.4403**	0.9999	**1.1503**	0.9989	0.9999
FLD11	0.998	0.9402	0.9768	0.9229	0.9578	0.9584
FLD12	1	1	0.9998	1	**1**	**1**
FLD13	0.776	0.6757	0.5997	0.6063	0.684	0.6856
FLD14	1	0.941	**1**	0.9312	0.9937	0.9945
FLD15	1	0.9513	0.7451	0.7924	0.8674	0.8699
FLD16	1	0.914	0.9739	0.9158	0.9989	0.9999
FLD17	1	0.7693	0.6911	0.7358	0.8613	0.8632
FLD18	0.852	0.9137	0.6551	0.686	0.6944	0.6968
FLD19	1	0.9238	0.6774	0.7463	0.7893	0.7916

The meanings of these four methods are different and it is understandable that different designs may lead to different selection. To be more specific, model (4) , similar to the proposed approach in model (2), allows the efficiency score of one unit to be larger than one and the efficiency scores of the other DMUs are all less than or equal to one and selects it as the most efficient. Meanwhile, in the model (3) and model (5) the efficiency scores of all DMUs are less than or equal to one. As shown in Table 2, while the epsilon value tends to be zero, efficiency score of FLD10 tends to 1, thus the power of discrimination in the proposed model declines. So we can use this feature to increase the discrimination power of the model (5).

Example 2. Banking industry [25].

Table 3 provides a real data set of twenty bank branches of one of the largest private bank in Iran. Three inputs and three outputs are recorded for each branch:

x_1 = employees, x_2 = assets, x_3 = costs,

y_1 = number of translations, y_2 = deposits,

y_3 = loans.

Table 3: inputs and outputs of 20 branches

DMUs	X_1	X_2	X_3	Y_1	Y_2	Y_3
1	11	1753	10020	5214	72149	57537
2	17	2604	11440	5343	89781	51114
3	7	1155	8427	5145	42654	52485
4	12	1899	11816	3249	97812	67298
5	14	2215	12426	6706	77031	43487
6	14	2357	9907	6259	75923	41442
7	9	1370	10365	3652	47763	43262
8	5	829	5283	3913	45732	14237
9	6	985	11061	3566	55222	41062
10	6	1023	5856	4559	53323	37418
11	8	1311	8745	4441	69734	57883
12	9	1536	7326	5031	49153	47139
13	8	1367	8326	5053	92365	55543
14	7	1193	6525	4762	64235	22347
15	9	1359	8158	6876	89104	45717
16	7	1111	11135	4307	42012	73925
17	7	1182	6920	5331	69360	27246
18	7	1069	5864	4004	51438	26531
19	6	992	5039	2342	39948	20223
20	7	1180	8378	4238	154284	43928

Table 4: Efficiency of bank branches by different models.

DMUs	**Model(1)**	**Model(2)**	**Model(3)**	**Model(4)**	**Model(5)** $\varepsilon = 0.0001$ $\varepsilon = 0.001$
1	0.86	0.736	0.5817	0.7434	0.7165
2	0.7	0.5336	0.5483	0.5424	0.5666
3	**1**	0.9067	0.9325		0.8291
4	0.85	0.6641	0.5867	0.618	0.5991
5	0.64	0.6008	0.4576	0.6222	0.6091
6	0.75	0.5975	0.5537	0.6231	0.6459
7	0.65	0.5997	0.3953	0.5982	0.542
8	**1**	0.8084	0.5519	0.8244	0.7974
9	0.88	0.7356	0.3886	0.6888	0.5789
10	**1**	0.9821	0.7191	1	0.9594
11	0.99	0.8907	0.6451	0.8732	0.8121
12	0.98	0.7683	0.6129	0.8053	0.7902
13	**1**	0.993	0.7832	0.9585	0.9236
14	0.89	0.7952	0.6248	0.8026	0.802
15	**1**	1	0.769	**1.0236**	0.9998
16	**1**	0.8981	0.4647	0.8837	0.7276
17	0.99	0.8841	0.658	0.8923	0.8748
18	0.81	0.7615	0.619	0.7848	0.78
19	0.66	0.596	0.534	0.5907	0.5976
20	1	**1.1521**	**1**	1	**1**

We apply model (5) on the given data set with $\varepsilon = 0.0001$ and $\varepsilon = 0.001$ to obtain the following optimal solution:
v_1^*=0.009803921568627,
v_2^*=0.000064004096262,
$v_3^* = 0.000031831465462$,
u_1^*=0.000034245474977,
u_2^*=0.000001080258916,
u_3^*= 0.000002254537256,
$\delta_{20}^* = 0, \delta_j^* = 1 (j \neq 20)$,

Table 4 demonstrates different efficiency scores obtained by models (1), (2), (3), (4) and (5) respectively. Results in Table 4 shows that 7 out of 20 branches are efficient. Models (2), (3) and (5) find DMU20 as the most efficient branch, while DMU15 is selected as the most efficient branch by model (4). The efficiency score of these DMUs are highlighted in bold in Table 4.

5. Conclusion

The identification of the most efficient DMU is sometimes the main concern of decision makers. Toward this end, numerous integrated DEA models have been formulated and these models usually find a common set of optimal weights. The main advantage of such integrated models is that the evaluation will be carried out by solving only one optimization problem. In this paper, we have proposed an alternative mixed integer linear programming model based on common weight set for identifying the most efficient DMU. In this model, a parameter, ε, is used to increase discrimination power of proposed model. The results of the numerical examples presented herein clearly demonstrate the various potential applications of the proposed approach.

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