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Malmquist Productivity Index Based on Time Effect

M. Vaez-Ghasemi*

Department of Mathematics, Islamic Azad University, Rasht branch, Iran

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Abstract

In recent years, measuring and analyzing productivity changes is the main focus of various researches who study performance of organizations. All through widespread application of Malmquist Productivity Index, different types of data should be considered thoroughly, otherwise any defective study of the related data and deciding factors may yield poor results. Practical Malmquist Productivity Index (PMPI) models, presented in this research, are fundamentally capable of measuring the productivity of units in a competitive atmosphere, along with the hidden economic indexes such as time value of money, amortization and promoted skills of employees. Also these models would provide the productivity comparison over different periods of time. Moreover, these models are reliable as well as tangible for superior managers and it is noteworthy that they would offer significantly favorable conditions, lack of which may cause the unit under evaluation to face a great deal of regression.

Keywords: *Malmquist Productivity Index; Time value of Money; Amortization; Data Envelopment Analysis; Productivity*

* Email: mohsen.vaez@iaurasht.ac.ir

1. Introduction

Nowadays Malmquist Productivity Index (MPI) used in applicational issues and innovative topics have been developed in accordance with the application of the Malmquist Productivity Index. An important issue about MPI is that with which it is possible to measure the total factor productivity (TFP) of a Decision Making Unit (DMU). As stated in literature, one of the major sources of economic development is productivity growth. Thus having an in-depth interpretation of the factors affected productivity can be helpful for senior managers for better decision making. In 1982 Caves D.W., Christensen L.R. and Diewert W.E. [4] (CCD), introduced Malmquist Productivity Index in production analysis. Nowadays application which uses the Malmquist Productivity Index is being used widely. In recent years among researchers who are studying the performance entities, the measurement and analysis of productivity changes have attracted a specific attention. To address a few we summarize some of these works as follows.

Kontodimopoulos and Niakas [10], using nationally representative panel data, examined total factor productivity of dialysis facilities in Greece over a 12-year period. As they stated, they used data envelopment analysis (DEA) to compute Malmquist productivity indices, which were decomposed into technical efficiency change and technological change. Allowing for constant returns to scale technology (CRS) the DEA models are considered to be input-oriented. Armagan et al. [1] in their study, accepted NUTS (The Nomenclature of Territorial Units for Statistics) regions in Turkey as a decision making unit and the efficiency values of these regions, changes in the total factor productivity and technology were calculated for the 10-year period covering 1994_ 2003. They used methods of data envelopment analysis and Malmquist Productivity Index to measure the crop production of NUTS1 regions in Turkey. It was revealed that in 10 year period covered by this study, there was a decrease in the technical efficiency and total factor productivity in the regions, except for the locomotive regions. The authors notified that this decrease which was reflected all over the country was due to the fact that the real price level remained the same. Oliveira et al. [12] provided a paper studying the evolution of productivity. The analysis, for a period of 10 years, from 1995 until 2004, was based on the estimation of the Malmquist index and its components. As previous applications of data envelopment analysis and its subsequent Malmquist indices to efficiency and productivity measurements have been criticized for not providing statistical inferences regarding the significance of observed results, in his research, Odeck [11], combined DEA and a Malmquist index with a bootstrap method in order to provide succinct statistical inferences that determine the performance of grain producers in Eastern Norway. In their paper, Chang et al. [6], used DEA to calculate Malmquist indexes of productivity and efficiency changes. They notified that this index is used because it can distinguish between changes in technical efficiency and changes in the performance efficiencies for each organization.

Hosseinzadeh Lotfi et al.[8] in their paper discussed about shortcomings of data envelopment analysis (DEA) in efficiency assessment in successive periods. They introduced a combined model to evaluate efficiency in successive years. Moreover, Hosseinzadeh Lotfi et al.[9] consider the factors influenced by time value of money and introduced a modified MPI for better analyzing the situations in such conditions. In an application this subject mooted and also demonstrated. In classic models of MPI, each of the factors which are involved in the period under examination has been considered without any variations. But this is not the case in real world applications, thus in this paper input- output data are divided into four distinct groups. First; those on which time value of money have influences, second, those of invariable possessions that can be influenced by effective factors such as amortization and third, those of human resource which, through years, gain experience and finally the data which are not included in any of the above-mentioned groups and not influenced by time value of money, amortization and cannot gain experience. Practical Malmquist Productivity Index (PMPI) models, which are presented in this paper, are fundamentally capable of calculating the productivity of units in this competitive world, along with the hidden economics indexes such as time value of money, amortization and the increase in skills of employees, while comparing productivity over different periods. The provided application in this paper aims to calculate MPI of commercial banks where there exist all of the aforementioned types of indexes. The acquired results demonstrate the validity of the provided models.

The current article proceeds as follows: In the next section, Malmquist Productivity Index is briefly reviewed. Then, in Section 3, the proposed method, Modified Malmquist Productivity Index (PMPI) is discussed. An illustrative example is documented in section 4 in which main findings are highlighted and section 5 with conclusions and recommendations concludes the paper.

2. Background

2. 1. Malmquist Productivity Index

As stated in literature for studying the performance entities, it is necessary to measure and analyze productivity changes. DEA methodology can be used in Malmquist Productivity Index in order to measure the productivity changes over time. From DEA models, which can be formulated as linear programming (LP) problems, production function frontier can be estimated, Charnes et al. [5]. In two different time points, (time t and $t+1$), this frontier can shift from frontier t to frontier $t+1$. The main idea behind DEA methodology is to develop the 'best practice frontier' through 'efficient units' usually called' best practice units'. Therefore, DMUs located onto this frontier are mentioned as efficient otherwise inefficient. The degree of technical efficient DMUs is one and the degree of inefficiency of DMUs is calculated through a comparison process to this frontier. In this process Euclidian distance of input-output ratio of each DMU calculated from the frontier of production function. The Malmquist DEA approach calculates an efficiency measure for two successive years, while allowing the best frontier to shift from time t and $t+1$.

Consider DMU_l as a unit from a set of n units to be assessed. Let $x_l \in R^{m+}$ and $y_l \in R^{s+}$ be semipositive input and output vectors of DMU_l. One way of characterization of production technology is production possibility set T, which is defined as:

$$T = \{(x, y)| \quad x \geq \sum_{j=1}^{n} \lambda_j x_j, \quad y \leq \sum_{j=1}^{n} \lambda_j y_j, \quad \lambda_j \geq 0, \quad j = 1, \dots, n\}$$

As mentioned in literature Malmquist Productivity Index can be calculated via several functions, in this paper using DEA methodology distance function will be calculated as follows:

$$D(X_l, Y_l) = Min\{\theta: (\theta X_l, Y_l) \in T\}$$

The resultant distance function can be computed by solving linear programming problems. Consider an input-oriented CCR model as follows:

$$D^f(x_l^k, y_l^k) = \min \qquad \theta$$

$$s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij}^{f} \leq \theta x_{il}^{k}, \quad i = 1,\dots,m,$$

$$\sum_{j=1}^{n} \lambda_j y_{rj}^{f} \geq y_{rl}^{k}, \quad r = 1,\dots,s, \qquad (2.1)$$
$$\lambda_j \geq 0, \quad j = 1,\dots,n.$$

Considering this notification, four LP problems can be defined. In the this model l is the unit under assessment and each of k and f vary between time t and $t+1$. As an instance for assessing DMU_l consider k=t and f=t+1, $D^{t+1}(x_l^t, y_l^t)$, this means that DMU_l is considered in time t while technology is considered in time $t+1$.

In regards of this subject, Caves D.W., Christensen L.R. and Diewert W.E. [4] have introduced the Malmquis Productivity Index as follows in which the results obtained from the mentioned models are being used.

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \left(\frac{D^t(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t) D^{t+1}(x_l^t, y_l^t)}\right)^{\frac{1}{2}} \qquad (2.2)$$

This index measures the productivity of unit l at the production (x_l^{t+l}, y_l^{t+l}) relative to (x_l^t, y_l^t) in which x_l^t and y_l^t are the input and output vectors for unit l, used in period t. Also, x_l^{t+1} and y_l^{t+1} are the input and output vectors for unit l, used in period t+1.

$$M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = \frac{D^{t+1}(x_l^{t+1}, y_l^{t+1})}{D^t(x_l^t, y_l^t)} \left[\frac{D^t(x_l^{t+1}, y_l^{t+1}) D^t(x_l^t, y_l^t)}{D^{t+1}(x_l^{t+1}, y_l^{t+1}) D^{t+1}(x_l^t, y_l^t)}\right]^{\frac{1}{2}} \qquad (2.3)$$

The interpretation of this equation is that $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) > 1$ indicates an improvement in total productivity, $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) < 1$ indicates a decline, and $M(x_l^{t+1}, y_l^{t+1}, x_l^t, y_l^t) = 1$ shows an unchanged productivity growth, Caves D.W., Christensen L.R. and Diewert W.E.[4], Chen Y., [6].

2. 2. Fuzzy background and Metric for fuzzy numbers

This section gives a brief review of essential notions of fuzzy set theory which will be used throughout this paper. Below, we give definitions and notations taken from Bezdek (1993), Zimmermann [15], Dubois and Prade [7] and Zadeh [16].

DEFINITION 1. Consider X to be the universal set. If $\tilde{A}$ is a set of ordered pairs

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) | x \in X\},$$

then $\tilde{A}$ is called a fuzzy set in X where $\mu_{\tilde{A}}(x)$ whoes the membership value of x in $\tilde{A}$.

DEFINITION 2. If the following properties hold, then a convex fuzzy set $\tilde{A}$ on $\mathfrak{R}$ is considered to be a fuzzy number: (a) Its membership function is piecewise continuous. (b) There exist only one x_0 that $\mu_A(x_0) = 1$.

DEFINITION 3. Let $\tilde{E} = (a_1, a_2, a_3, a_4)$ and $\tilde{E} = (b_1, b_2, b_3, b_4)$ be two fuzzy numbers and p=1 with $s(\alpha) = 1$

$$d(\tilde{A},\tilde{B}) =: d_{TMF}^{(1)}(\tilde{A},\tilde{B},1) = \int_0^1 D_{TMI}^{(1)}([A_l{}^\alpha, A_u{}^\alpha],[B_l{}^\alpha, B_u{}^\alpha])d\alpha = \int_0^1 (\int_0^1 |(1-x)A_u{}^\alpha + xA_l{}^\alpha - ((1-x)B_u{}^\alpha + xB_l{}^\alpha)|dx)d\alpha \quad (2.4)$$

Also, the distance from $\tilde{A}$ to the origin is defined as follows:

$$d(\tilde{A}) =: d_{TMF}^{(1)}(\tilde{A},1) = \int_0^1 D_{TMI}^{(1)}([A_l{}^\alpha, A_u{}^\alpha])d\alpha = \int_0^1 (\int_0^1 |(1-x)A_u{}^\alpha + xA_l{}^\alpha|dx)d\alpha. \quad (2.5)$$

DEFINITION 4. The ranking method for two positive fuzzy numbers is as follows:

$$\tilde{\mathbf{A}} \lesssim \tilde{\mathbf{B}} \iff \mathbf{d}_{\mathbf{TMF}}^{(1)}(\tilde{\mathbf{A}}) \le \mathbf{d}_{\mathbf{TMF}}^{(1)}(\tilde{\mathbf{B}}) \quad (2.6)$$

$$\tilde{\mathbf{A}} \cong \tilde{\mathbf{B}} \iff \mathbf{d}_{\mathbf{TMF}}^{(1)}(\tilde{\mathbf{A}}) = \mathbf{d}_{\mathbf{TMF}}^{(1)}(\tilde{\mathbf{B}}) \quad (2.7)$$

$$\tilde{\mathrm{A}} \lesssim \tilde{\mathrm{B}} \iff \mathrm{d}_{\mathrm{TMF}}^{(1)}(\tilde{\mathrm{A}}) \ge \mathrm{d}_{\mathrm{TMF}}^{(1)}(\tilde{\mathrm{B}}) \quad (2.8)$$

3. Main subject

Amortization plays an important role in economic comparisons after "taxes". In the following some formulas relate to engineering economic has been used as stated in Riggs [14], Blank and Tarquin [3] and Oskunejad [13]

This notion has different definitions. As an instance the following definition can be mentioned.

1- Deduction in value of an asset: this deduction consists of difference in value of an asset in two various times, and may be caused due to any reasons.

2- Subtraction of salvage value from the distribution of expenses or initial value of an asset, during beneficial life of related asset. Amortization of an asset can be achieved due to different reasons such as technological development, machinery erosion, variation in general regulation related to machinery or building, variation in the quantity and type of service which is under consideration. Thus, in economic comparison as well as productivity comparison of the organizations "book value" of an asset should be taken into consideration. It worth mentioning that book value of an asset (book value in each time) is the difference of value or initial expenses of that asset and aggregated amortization amounts up to that time. While MPI is being calculated, in different periods, book value of the asset in that period should be calculated as well. "Straight line method" is the simplest and most common method for the calculation of amortization. In this method the annual amortization is fixed and it can be obtained according to the following equation existing in literature: $D = \frac{P-SV}{n}$ in which "D" is the quantity of annual amortization. "P" is the initial expenses of the asset.

"SV" is the salvage value of the asset and "n" is the years of amortization.

As stated in the above mentioned references, due to this fact that every year assets are amortized to a certain degree, the book value after "m" years can be calculated as follows: $BV_m = P - mD, m = 1,\dots,n.$ In computing amortization of the equipment and invariable possessions, straight line method is used. Also, for buildings "declining balance method" is utilized in which the annual erosion has been reduced at fixed and monotone rate. It should be noted that the quantity of annual amortization is being computed from the multiplication of fixed rate to the book value of previous year. $D = BV_{m-1}(d)$ in which "d" is the fixed rate. In this paper asset value in year t is computed from the "accumulated depreciation" in year t-1. The data which are of money kind and influenced by the time value of money are of special importance but they are not widely considered and incorporated in evaluations. One of the situations where these data ought to be exactly considered is the process of comparing different periods, time value of money and profit rate should be taken into account. As an instance, if we have 50 units of money in yeart, and we want to compare the value of this amount to year$t + 1$, it should be multiplied to the önce paid factor". In general this means $F = P(1 + e)^n$ in which "F" indicates value of money in future, "P" shows value of money in present time, "e" is the profit rate and n shows the number of periods, considering time t and$t + 1$, $n = 1$ Economic comparison of performance of organizations, human resource is one of the important indicators which is not much paid attention to, from the view point of variation in skills and dexterity or years of service. This is evident that 20 employees, in year$t$, are not equal to those in year$t + 1$. it is evident that while comparing year t and $t + 1$, in year t these employees have gained much more experience. Also, they might pass educational and mastery courses. As a result, productivity of employees, in this period, is different from that of previous year. Moreover, replacement of employees is, in turn, of great importance, that is, when an employee after being retired, with years of experience, is being replaced with a rookie this replacement is certainly not equivalent.

Therefore in this paper for considering human resource two major factors, specialties and skills, have been taken into account for evaluating scores of the personnel.

According to what has been discussed previously, we present four LP problems for evaluating Practical Malmquist Productivity Index (PMPI) as following. Under constant returns to scale, the LP for $D^t(x_l^t, y_l^t)$, with m inputs and s outputs, is as follows:

$$\begin{aligned} &\bar{D}^t(x_l^t, y_l^t) = \min \quad \theta \\ s.t.\quad &\sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{il}^t, \quad i = 1,\dots,m, \\ &\textstyle\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rl}^t, \quad r = 1,\dots,s, \\ &\lambda_j \ge 0, \quad j = 1,\dots,n \end{aligned} \qquad (3.9)$$

Similarly, the other three LP problems are as follows:

$$\begin{aligned} &\bar{D}^{t+1}(x_l^t, y_l^t) = \min \quad \theta \\ s.t.\quad &\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{il}^t, \quad i \in I_1 \\ &\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{rl}^t, \quad r \in R_1, \\ &\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta (1+e)^1 x_{il}^t, \quad i \in I_2, \\ &\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge (1+e)^1 y_{rl}^t, \quad r \in R_2, \\ &\sum_{j=1}^{n} \lambda_j (x_{ij}^{t+1} - D) \le \theta x_{il}^t, \quad i \in I_3, \\ &\sum_{j=1}^{n} \lambda_j (y_{rj}^{t+1} - D) \ge y_{rl}^t, \quad r \in R_3, \\ &\lambda_j \ge 0, \quad j = 1,\dots,n. \end{aligned} \qquad (3.10)$$

Where I_1 and R_1 show the subsets of inputs and outputs respectively for which Time Value of Money is non-impressive and I_2 and R_2 show the subsets of inputs and outputs respectively for which Time Value of Money is influential. Moreover, I_3 and R_3 show the subsets of inputs and outputs respectively for which amortization is effective. It also should be mentioned that $I = \{1,\dots,m\}$, $R = \{1,\dots,s\}$ and $I = I_1 \cup I_2 \cup I_3$, $R = R_1 \cup R_2 \cup R_3$.

$$\begin{aligned} &\bar{D}^{t+1}(x_l^{t+1}, y_l^{t+1}) = \min \quad \theta \\ s.t.\quad &\sum_{j=1}^{n} \lambda_j x_{ij}^{t+1} \le \theta x_{il}^{t+1}, \quad i = 1,\dots,m, \\ &\sum_{j=1}^{n} \lambda_j y_{rj}^{t+1} \ge y_{rlo}^{t+1}, \quad r = 1,\dots,s, \\ &\lambda_j \ge 0, \quad j = 1,\dots,n. \end{aligned} \qquad (3.11)$$

In model (3.10) subsets of inputs and outputs are the same as what has been discussed previously. It is noteworthy that in models (3.9) and (3.11) Time Value of Money is not included. Time value of money does not influence the procedure since two similar periods are being compared with each other and Time Value of Money is fixed in a period. Moreover, according to the aforesaid formula$(1+e)^n$, when n is equal to zero, one is multiplied to the input and output parameters. But, in models (3.10) and

(3.12), which are considered in various periods, Time Value of Money, for the indexes under the influence of it, is calculated by “single payment compound" factor. The Modified Malmquist Productivity Index is calculated like the preceding classic analysis through the following formula:

$$
\begin{aligned}
&\bar{D}^t(x_l^{t+1}, y_l^{t+1}) = \min \quad \theta \\
&s.t. \quad \sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta x_{il}^{t+1}, \quad i \in I_1, \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge y_{rl}^{t+1}, \quad r \in R_1, \\
&\sum_{j=1}^{n} \lambda_j (1+e)^1 x_{ij}^t \le \theta x_{il}^{t+1}, \quad i \in I_2, \\
&\sum_{j=1}^{n} \lambda_j (1+e)^1 y_{rj}^t \ge y_{rl}^{t+1}, \quad r \in R_2, \\
&\sum_{j=1}^{n} \lambda_j x_{ij}^t \le \theta (x_{il}^{t+1} - D), \quad i \in I_3, \\
&\sum_{j=1}^{n} \lambda_j y_{rj}^t \ge (y_{rl}^{t+1} - D), \quad r \in R_3, \\
&\lambda_j \ge 0, \quad j = 1, \dots, n.
\end{aligned} \tag{3.12}
$$

In model (3.10) subsets of inputs and outputs are the same as what has been discussed previously. It is noteworthy that in models (3.9) and (3.11) Time Value of Money is not included. Time value of money does not influence the procedure since two similar periods are being compared with each other and Time Value of Money is fixed in a period. Moreover, according to the aforesaid formula $(1+e)^n$, when n is equal to zero, one is multiplied to the input and output parameters. But, in models (3.10) and (3.12), which are considered in various periods, Time Value of Money, for the indexes under the influence of it, is calculated by “single payment compound" factor. The Modified Malmquist Productivity Index is calculated like the preceding classic analysis through the following formula:

$$
\bar{M}(x_i^{t+1}, y_i^{t+1}, x_i^t, y_i^t) = \frac{\bar{D}^{t+1}(x_i^{t+1}, y_i^{t+1})}{\bar{D}^t(x_i^t, y_i^t)} \left[\frac{\bar{D}^t(x_i^{t+1}, y_i^{t+1}) \bar{D}^{t+1}(x_i^{t+1}, y_i \quad)}{\bar{D}^t(x_i^t, y_i^t) \bar{D}^{t+1}(x_i^t, y_i^t)}\right]^{\frac{1}{2}} \tag{3.13}
$$

Considering the aforesaid discussion, in regards of (3.13) it can be concluded that,

$\bar{M}(x_i^{t+1}, y_i^{t+1}, x_i^t, y_i^t) > 1$ Indicates productivity gain,

$\bar{M}(x_i^{t+1}, y_i^{t+1}, x_i^t, y_i^t) < 1$ Indicates productivity loss, and

$\bar{M}(x_i^{t+1}, y_i^{t+1}, x_i^t, y_i^t) = 1$

Means no changes in productivity from time t to t + 1.

4. Application

In classic models of MPI, each of the factors which are involved in the period under examination has been considered without any variations. As it was discussed in previous sections in this model (PMPI) input- output data are divided into four distinct groups.

The first group: these data are of money kind and influenced by the time value of money. in the process of comparing different periods, time value of money and profit rate should be taken into consideration. As an instance, if we have 50 units of money in year t, and we want to compare corresponding *value* to year $t+1$, it should be multiplied to the önce paid factor". In general this means $F = P(1+e)^n$ in which F indicates value of money in future, P shows value of money in present time, e is the profit rate and n shows number of periods. The second group: these data are of invariable possessions and can be influenced by effective factors such as amortization. As it was discussed above these conditions should be considered in evaluation, since machineries, as far as they are brand-new will work with higher productivity. Moreover buildings and possessions will gradually lose their efficiency. Therefore, amortization of each possession should be evaluated and on grounds of accumulated amortization subtracted from the book value of possessions. The third group: these data are of human resource and through years it gains experience. Human resource through years can be replaced by more experienced ones. In addition, they can be trained and become more specialized. Consequently, the work will prosper and gain more productivity. These data should be measured due to their specialties and skills, and during years these data have been calculated. Using different techniques such as fuzzy techniques these quantity data can be converted into the quality ones. The fourth group: these data are not included in any of the above- mentioned groups and not influenced by time value of money, amortization and gaining experience. This project is aimed at calculating MPI of commercial banks, includes all four types of indexes. The input-output indexes are gathered in Table 1 in which the groups are listed in consistent with the above-mentioned arrangement.

It should be noted that for sake of simplicity on basis of the fuzzy technique which is briefly reviewed in previous sections, quality scores of specialties and skills have been converted into the quality data, thus this index, is also listed in Table 1. the related information is gathered in four periods from six banks, these periods are from 2007 till 2010. The input-output data, according to the defined indexes, are listed in Tables 2 and 3.

Table1. Description

Input		Output	
Index	**Group**	**Index**	**Group**
Quantity of possessions (I1)	Group 4	Profit marginal (O1)	Group 4
profit(I2)	Group 1	Received profit(O2)	Group 1
Personnel expenses (I3)	Group 1	Total revenue (O3)	Group 1
Invariable possessions (I4)	Group 2	Rate of customer satisfaction(O4)	Group 4
Human resource (I5)	Group 3	Rate of customer attraction(O5)	Group 4
Delayed assets (I6)	Group 1	Total number of online branches(O6)	Group 4
Special electronic equipment(I7)	Group 4		

Table2. Inputs

DMU	I1	I2	I3	I4	I5	I6	I7
2007							
1	82.42%	3733535	1546117	3055092	12,884	1404816	0
2	91.63%	1531782	761666	1536048	6,347	1282345	0
3	84.79%	2713555	1012123	1771364	8,434	2776737	311
4	91.38%	1322229	562000	1171941	4,683	469883	1365
5	85.67%	1580745	612876	1301664	5,107	557990	0
6	88.22%	322760	209150	537390	1,743	339579	25
2008							
1	84.52%	4746010	2045491	3123452	17,046	1473635	734
2	91.23%	2202181	988163	1614630	8,235	1895285	0
3	86.64%	3450791	1267093	2125277	10,559	4280025	532
4	91.52%	1737239	719412	1265707	5,995	568787	1475
5	95.60%	2039642	987139	1547251	8,226	1072620	0
6	88.69%	329968	317444	802856	2,645	398223	290
2009							
1	83.79%	6131088	2622188	3201571	21,852	2885802	832
2	92.22%	3380231	1240252	1904399	10,335	3154069	430
3	78.72%	4582403	1903395	2718657	15,862	8401853	1130
4	94.02%	2241437	990467	1318934	8,254	1631729	1493
5	95.98%	2891489	1258469	1670867	10,487	1463479	0
6	79.01%	732181	491521	908892	4,096	756591	301
2010							
1	87.23%	8301843	3624698	3196189	30,206	4183187	1125
2	96.72%	4416677	1688724	2142701	14,073	3612992	1192
3	82.25%	5216403	2223659	3102316	18,530	13278872	1344
4	93.33%	3350167	1124923	1296421	9,374	1874730	1489
5	97.13%	3951486	1516034	1951767	12,634	1905901	420
6	85.27%	1256218	651419	941905	5,428	1050272	333

Table3. Output

DMU	O1	O2	O3	O4	O5	O6
2007						
1	2.13%	5456846	6242343	3.25%	25.85%	1208
2	3.90%	2986501	3281831	3.21%	24.04%	0
3	3.74%	4774258	5165554	3.41%	23.97%	311
4	2.71%	2109188	2380064	3.12%	27.28%	183
5	2.85%	2583767	2862649	3.43%	36.10%	0
6	5.64%	772256	860719	3.74%	73.49%	25
2008						
1	1.55%	6279449	7943232	3.25%	26.77%	1262
2	2.87%	3568242	4026108	3.21%	49.76%	0
3	2.74%	5209039	5711620	3.41%	40.62%	535
4	3.05%	2815229	3148523	3.12%	32.71%	503
5	2.65%	3456352	3802313	3.43%	32.65%	0
6	6.01%	1025285	1184574	3.74%	46.37%	290
2009						
1	2.51%	9522348	11869855	3.25%	24.33%	1278
2	2.96%	5549420	6509109	3.21%	38.80%	426
3	4.17%	8656018	9870337	3.41%	18.77%	1113
4	3.43%	3985100	4600389	3.12%	22.60%	754
5	2.61%	4916408	5567726	3.43%	24.50%	0
6	5.61%	1691760	1902707	3.74%	38.48%	301
2010						
1	1.50%	11133284	15660622	3.25%	23.96%	1325
2	2.37%	7275909	8507807	3.21%	20.00%	1183
3	3.54%	10133005	11504037	3.41%	21.00%	1222
4	2.65%	5286830	6512891	3.12%	29.30%	904
5	2.36%	6342139	7387085	3.43%	24.20%	395
6	3.13%	2005444	2323583	3.74%	34.00%	333

In accordance with the aforesaid issues and models about PMPI, the results of PMPI are as follows. Considering the results, one important point is that, while time conditions are considered, is that each bank has negative performance and each witnessed a regress in all the periods.

This is due to the fact that time value of money and inflation, caused by poor governmental policies, are not observed.

It should be mentioned that these units must do something to overcome the problem and improve their policies and plans.

The acquired results of PMPI are compared with those of MPI and are presented in Tables 4, 5 and 6.

Table4. Results (2007-2008)

DMU	PMPI	PMPI Status	MPI	MPI Status	Differences
DMU1	0.44	Regress	0.463	Regress	EQUABLE
DMU2	0.456	Regress	0.816	Regress	EQUABLE
DMU3	0.454	Regress	1.056	Progress	CHANGE
DMU4	0.491	Regress	1.126	Progress	CHANGE
DMU5	0.411	Regress	0.731	Regress	EQUABLE
DMU6	0.744	Regress	0.707	Regress	EQUABLE

Table5. Results (2008-2009)

DMU	PMPI	PMPI Status	MPI	MPI Status	Differences
DMU1	0.867	Regress	0.902	Regress	EQUABLE
DMU2	0.807	Regress	0.696	Regress	EQUABLE
DMU3	0.934	Regress	0.526	Regress	EQUABLE
DMU4	0.979	Regress	0.839	Regress	EQUABLE
DMU5	0.879	Regress	0.859	Regress	EQUABLE
DMU6	0.813	Regress	0.655	Regress	EQUABLE

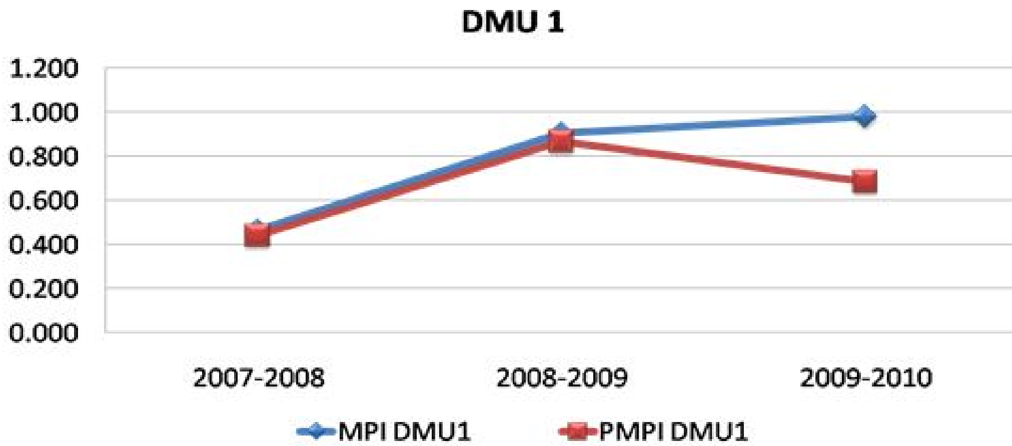
Table6. Results (2009-2010)

DMU	PMPI	PMPI Status	MPI	MPIStatus	Differences
DMU1	0.683	Regress	0.979	Regress	EQUABLE
DMU2	0.69	Regress	0.974	Regress	EQUABLE
DMU3	0.622	Regress	0.922	Regress	EQUABLE
DMU4	0.714	Regress	1.068	Progress	CHANGE
DMU5	0.631	Regress	1.392	Progress	CHANGE
DMU6	0.634	Regress	0.588	Regress	EQUABLE

In the above table, periods and the compared results are presented separately. In the second period of evaluation, 2008-2009, not only the results of classic MPI shows a regress but also each of the units in PMPI witnessed a regress. on the other hand, some of the units, according to the monetary contraction and reduction in consuming resources, such as DMU_3, show a less regression in PMPI than in classic MPI, MPI=0.52, PMPI=0.93. It is worth mentioning that this situation is influenced by the performance of previous periods.

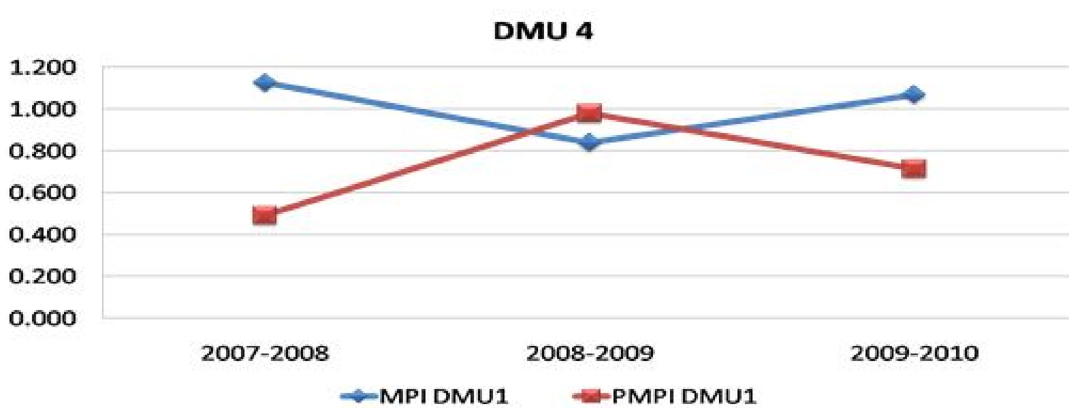
Also, DMU_4, which witnessed a progress in two periods in classic evaluation, in PMPI, while time value of money is considered, has entirely regressed. Diagram of these variations are depicted in figure 1.

Figure1. Evaluation of DMU 4.



This case is somehow different from that of DMU_1 and shows that the performance of this unit is in consistent with time value of money in the first period. Moreover, its route till 2009 is more satisfactory. But due to the world variation in 2010 this bank cannot take advantage of its resources to the full. Considering classic models this unit has a better status while it is being compared to those of previous periods.

Figure2. Evaluation of DMU 1.



5. Conclusion

Practical Malmquist Productivity Index (PMPI) models, presented in this research, are fundamentally capable of measuring the productivity of units in a competitive atmosphere, along with the hidden economic indexes such as time value of money, amortization and promoted skills of employees. Also these models would provide the productivity comparison over different periods of time. Moreover, these models are reliable as well as tangible for superior managers and it is noteworthy that they would offer significantly favorable conditions, lack of which may cause the unit under evaluation to face a great deal of regression. For further study relevant to DEA, more attention should be paid to the variations in personnel during the periods such as turn-over of human resource and developed skills and specialties, which can be incorporated to the model through mathematical expressions, unlike the procedure that has been carried out in this paper in which first the quality data are converted to quantity data and then they are used in the models.

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