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A New Approach to Solve Fully Fuzzy Linear Programming with Trapezoidal Numbers Using Conversion Functions

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Abstract

Recently, fuzzy linear programming problems have been considered by many. In the literature of fuzzy linear programming several models are offered and therefore some various methods have been suggested to solve these problems. One of the most important of these problems that recently has been considered; are Fully Fuzzy Linear Programming (FFLP), which all coefficients and variables of the problem are the same kind of fuzzy numbers. One of most common of them is the model in which all fuzzy parameters are discussed by triangle numbers. In this paper, we first define a fully fuzzy linear programming with trapezoidal numbers and then suggest a new method based on reducing the original problem to the problem with triangle number. Specially, a conversion function for converting two trapezoidal and triangular numbers to each other is offered. Finally, the mentioned method is illustrated by a numerical example.

Keywords: **C**onversion function, Fully fuzzy linear programming, Multi-objective linear programming, Trapezoidal and triangular fuzzy numbers.

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1. Introduction

Until now, most of the problems which have been studied in the field of fuzzy linear programming are the problems that some of their variables and parameters are fuzzy. Such as the value of right side (sources), target functions or decision variables. One of the most important of these problems that recently, has been considered, is Fully Fuzzy Linear Programming (FFLP), which in it all of problem's coefficients and variables, are the same kind of trapezoidal fuzzy numbers. Some kinds of the models and also methods for solving these problems is given in [1, 2, 3, 6]. Usually, a conversion function which is defined from the set of all trapezoidal numbers to the set of all triangular numbers suggest to solve these problems. Then using the nearest concept of estimation of triangular fuzzy number, the main problem converts to two auxiliary problems (max, min), that with solving these two problems, a response vector consist of symmetric triangular numbers will be attained. The response of maximization auxiliary problem, considered as center and the response of minimization auxiliary problem as the fringe of response. Finally, using a conversion function, we define a trapezoidal fuzzy number for each triangular fuzzy number that is determined by the process of solving the problem.

2. Preliminaries

In this section, some necessary backgrounds and notions of fuzzy set theory are reviewed [2].

2.1. Basic definitions and concepts

Definition 2.1. A fuzzy set $\widetilde{A}$ is called a fuzzy number if it satisfies the following conditions:

1. $\widetilde{A}$ is normal, that is, ($hgt(\widetilde{A}) = 1$).
2. $\widetilde{A}$ is convex.
3. There is exactly one $\bar{x} \in R$ with $\mu_{\widetilde{A}}(\bar{x}) = 1$, that is, $core(\widetilde{A}) = \bar{x}$.
4. The membership function $\mu_{\widetilde{A}}(x)$, $x \in R$, is at least piecewise continuous.

The value $\bar{x} = core(\widetilde{A})$ which shows the maximum degree of membership $\mu_{\widetilde{A}}(\bar{x}) = 1$ is called the modal value of the fuzzy number $\widetilde{A}$, in notational accordance with the value that occurs most frequently in data samples. The modal value may also be referred to as peak value, center value, or mean value, where the last two expression are preferably used for symmetric fuzzy numbers.

The parametric form of fuzzy will be

showed as $\widetilde{A} = (\underline{A}(r), \overline{A}(r))$, so that $\underline{A}(r)$ and $\overline{A}(r)$ satisfy the below conditions:

1. Function $\overline{A}(r)$ from left, constantly is a descending consistency.
2. Function $\underline{A}(r)$ from right, constantly is an ascending consistency.
3. $\underline{A}(r) \le \overline{A}(r), \qquad 0 \le r \le 1.$
4. $\underline{A}(r) = \overline{A}(r) = 0, \qquad r \le 0, \; r \ge 1.$

In literature of the theory of fuzzy sets, triangular fuzzy numbers and trapezoidal fuzzy numbers have many applications (see in [4, 5]). Hence, in this article we also consider them and briefly define here.

Definition 2.2. We show each triangular fuzzy number $\hat{a}$ with $\hat{a} = (\hat{a}^m, \hat{a}^\alpha, \hat{a}^\beta)$, where $(\hat{a}^\alpha, \hat{a}^\beta)$ is consisted of the support of triangular fuzzy number **(Fig 1)** $\hat{a}$ and $\hat{a}^m$ is its core. In this case, we can rewrite the triangular fuzzy number $\hat{a}$ as $\hat{a} = (\hat{a}^m, \hat{\alpha}, \hat{\beta})$, where $\hat{\alpha} = \hat{a}^m - \hat{a}^\alpha$, $\hat{\beta} = \hat{a}^\beta - \hat{a}^m$. And its figure is like this [4] :

We show the set of all triangular fuzzy numbers in an abbreviated form of $F_t(R)$.

Definition 2.3. We show each trapezoidal fuzzy number $\widetilde{a}$ with $\widetilde{a} = (\widetilde{a}^L, \widetilde{a}^U, \widetilde{a}^\alpha, \widetilde{a}^\beta)$ where $\left[\widetilde{a}^L, \widetilde{a}^U\right]$ is the core of trapezoidal fuzzy number **(Fig 2)** $\widetilde{a}$ and $(\widetilde{a}^\alpha, \widetilde{a}^\beta)$ is the support of fuzzy number $\widetilde{a}$. Thus, we may rewrite the trapezoidal fuzzy number $\widetilde{a}$ as $\widetilde{a} = (\widetilde{a}^L, \widetilde{a}^U, \widetilde{\alpha}, \widetilde{\beta})$, where $\widetilde{\alpha} = \widetilde{a}^L - \widetilde{a}^\alpha$, $\widetilde{\beta} = \widetilde{a}^\beta - \widetilde{a}^U$. And its figure is like this [4] :

We represent the set of all trapezoidal fuzzy numbers in an abbreviated form $F(R)$.

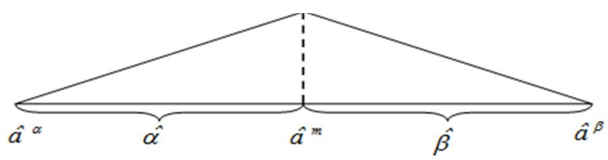

Fig 1. Triangular fuzzy number

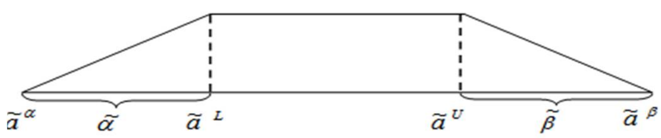

Fig 2. Trapezoidal fuzzy number

Remark 2.4. For each trapezoidal fuzzy number $\tilde{x}$, we say $\tilde{x} \geq \tilde{0}$, if there are $h \geq 0$ and $a \geq 0$ as $\tilde{x} \geq [-a, a, h, h]$ and $[-a, a, h, h]$ will be shown by $\tilde{0}$. It is worth mentioning that $\tilde{0}$ is equal with [0, 0, 0, 0]. We can easily see that, if $\tilde{x} \geq \tilde{y}$ then $(\tilde{x} - \tilde{y}) \geq \tilde{0}$. If $\tilde{x} = \tilde{0}$, then we say that $\tilde{x}$ is a trapezoidal fuzzy number equal with 0. Similarly define as triangular zero.

3. Fully fuzzy linear programming problem

A Fully Fuzzy Trapezoidal Linear Programming (FFTLP) problem is defined as follows:

$$\begin{aligned} \max \quad & \tilde{\boldsymbol{c}}\tilde{\boldsymbol{x}} \\ s.t. \quad & \tilde{\boldsymbol{A}}\tilde{\boldsymbol{x}} \cong \tilde{\boldsymbol{b}} \\ & \tilde{\boldsymbol{x}} \geq \tilde{0} \end{aligned} \qquad (1)$$

where all elements of $\tilde{b}, \tilde{A}, \tilde{x}$ and $\tilde{c}$ are belong to $F(R)$.

Note that the problem of fully fuzzy triangular linear programming defines such as below [2]:

$$\begin{aligned} \max \quad & \hat{\boldsymbol{c}}\hat{\boldsymbol{x}} \\ s.t. \quad & \hat{\boldsymbol{A}}\hat{\boldsymbol{x}} \cong \hat{\boldsymbol{b}} \\ & \hat{\boldsymbol{x}} \geq \hat{0} \end{aligned} \qquad (2)$$

where all elements of $\hat{b}, \hat{A}, \hat{x}$ and $\hat{c}$ are belong to $F_t(R)$.

3.1. Conversion function from trapezoidal fuzzy number to triangular

In this part, by introducing a function *T*, we intend to transform each trapezoidal fuzzy number

to a triangular fuzzy number; it means that: T: $F(R) \rightarrow F_t(R)$. In this case, if $\tilde{a} \in F(R)$ then $T(\tilde{a}) = \hat{a} \in F_t(R)$.

$$\tilde{\boldsymbol{a}} = (\tilde{\boldsymbol{a}}^L, \tilde{\boldsymbol{a}}^U, \tilde{\boldsymbol{\alpha}}, \tilde{\boldsymbol{\beta}}) \xrightarrow{T} \hat{\boldsymbol{a}} = (\hat{\boldsymbol{a}}^m, \hat{\boldsymbol{\alpha}}, \hat{\boldsymbol{\beta}}) \qquad (3)$$

where

$\hat{\alpha} = \hat{a}^m - (\tilde{a}^L - \tilde{\alpha}), \hat{\beta} = (\tilde{a}^U + \tilde{\beta}) - \hat{a}^m$ and

$$\hat{a}^m = \frac{\tilde{a}^L + \tilde{a}^U}{2}.$$

The following figure explores the mentioned concept. **(Fig 3)**

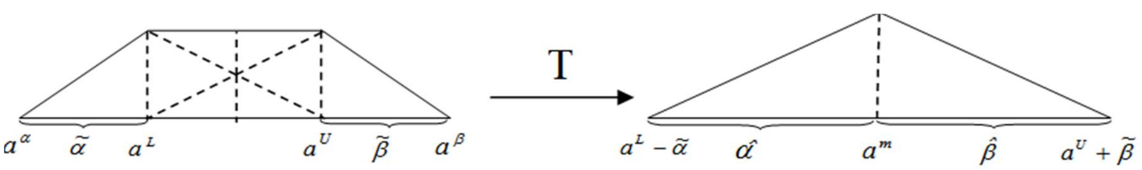

Fig 3. A transformation of the trapezoidal fuzzy number to the triangular fuzzy number

3.2.Nearest symmetric triangular defuzzification (N.s.t)

Here, we state the concept of the nearest symmetric triangular defuzzification which is given in [2].

Definition 3.1. If we suppose that t, is a symmetric triangular number, its parametric form will be as $t[x_0,\delta]$, that centrality in x_0 and 2δ borders, such as below:

$t[x_0,\delta] = (x_0 - \delta + r(\delta), x_0 + \delta - R(\delta)) = (x_0,\delta)$

where $x_0, \delta \in R,\ \ 0 \le r \le 1$.

Note that we show the set of all the symmetric triangular fuzzy numbers with $\hat{S}.T$.

Definition 3.2. If we suppose that t, is an asymmetric triangular fuzzy number, its parametric form is such as below:

$\hat{t} = (c_{\hat{t}}, w_{\hat{t}}^L, w_{\hat{t}}^R),$

$C_{\hat{t}} = core(\hat{t}) = \bar{t}(1) = \underline{t}(1),$

$w_{\hat{t}}^L = C_{\hat{t}} - \underline{t}(0) \ge 0, \quad w_{\hat{t}}^R = \bar{t}(0) - c_{\hat{t}} \ge 0.$

Those are respectively, the center and the left and right fringe of asymmetric triangle fuzzy number, as:

$\underline{t}(r) = c_{\hat{t}} - w_{\hat{t}}^L + w_{\hat{t}}^L r,$

$\bar{t} = c_{\hat{t}} + w_{\hat{t}}^R + w_{\hat{t}}^R r,$

$c_{\hat{t}} w_{\hat{t}}^R, w_{\hat{t}}^L \in R, \quad 0 \le r \le 1.$

The symmetric triangular fuzzy number is a special form of asymmetric triangular fuzzy number,

where $w_s^R = w_s^L = \delta$.

We show the set of all asymmetric triangular fuzzy numbers with $A.\hat{S}.T$.

Definition 3.3. The dominant relations on the set of asymmetric triangular fuzzy numbers will be defined such as below:

$\hat{t} = (c_1, w_1^L, w_1^R)$

$\hat{u} = (c_2, w_2^L, w_2^R) \in A\hat{S}T, \quad K \in R.$

1. $\hat{t} = \hat{u} \quad \leftrightarrow \quad c_1 = c_2,$
$w_1^L = w_2^L,\ w_1^R = w_2^R.$
2. $\hat{t} + \hat{u} = (c_1 + c_2, w_1^L + w_2^L, w_1^R + w_2^R).$
3. $K\hat{t} = \begin{cases} (kc_1, kw_1^L, Kw_1^R), & K \ge 0, \\ (Kc_1, -Kw_1^L, -Kw_I^R), & K < 0. \end{cases}$

Definition 3.4. If $\hat{t} = (\underline{t}(r), \bar{t}(r))$ and $\hat{u} = (\underline{u}(r), \bar{u}(r))$, then $\hat{h} = (\underline{h}(r), \bar{h}(r))$ where

$$\bar{h} = \max\{\underline{t}(r)\underline{u}(r), \bar{t}(r)\bar{u}(r), \bar{t}(r)\underline{u}(r), \underline{t}(r)\bar{u}(r)\}$$

$$\underline{h} = \min\{\underline{t}(r)\underline{u}(r), \bar{t}(r)\bar{u}(r), \bar{t}(r)\underline{u}(r), \underline{t}(r)\bar{u}(r)\}$$

For example; the multiplication of two asymmetric triangular fuzzy numbers is such as below:

$$\hat{t}\hat{u} = (c\, c_{\hat{u}} + c_{\hat{t}} w^L (r-1) + w_{\hat{t}}^L (r-1) c_{\hat{u}} + w_{\hat{t}}^L w_{\hat{u}}^L (r-1)^2,$$
$$c_{\hat{t}} c_{\hat{u}} + c_{\hat{t}} w_{\hat{u}}^R (1-r) + w_{\hat{t}}^R (1-r) C_{\hat{u}} + w_{\hat{t}}^R w_{\hat{u}}^R (1-r)^2)$$

Definition 3.5 Suppose that, $\hat{u}$ is a triangular fuzzy number in the parametric form as $(\underline{u}(r), \overline{u}(r))$.

In this case, for gaining a symmetric triangular fuzzy number, which is close to ˆu, the following function must be minimized:

$$D^2(\hat{u}, s[x,\delta]) = \int_0^1 (\underline{u}(r) - \underline{s[x_0,\delta]}(r))^2 dr + \int_0^1 (\overline{u}(r) - \overline{s[x_0,\delta]}(r))^2 dr . \quad (4)$$

If $s[x_0,\delta]$ be the minimizer of $D(\hat{u}, s[x,\delta])$, then $s[x_0,\delta]$ is a non-fuzzy (crisp) from $\hat{u}$ to the center of x_0 and fringe of δ . Therefore, we have for the minimize of $D(\hat{u}, s[x,\delta])$:

$$\frac{\partial D(\hat{u}, s[x,\delta])}{\partial \delta} = 0$$

$$\frac{\partial D(\hat{u}, s[x,\delta])}{\partial x_0} = 0 \quad (5)$$

So, from solving the previous equation, we have:

$$\delta = \frac{3}{2}\int_0^1 (\overline{u}(r) - \underline{u}(r))(1-r)dr \quad (6)$$

$$x_0 = \frac{1}{2}\int_0^1 (\overline{u}(r) - \underline{u}(r))dr \quad (7)$$

The nearest crisp number is asymmetric triangular to $\hat{u}$ to the center of x_0 and tolerance of δ . As you know, in section (3), a function was introduced that the conversion possibility of a trapezoidal fuzzy number to a triangular fuzzy number has been prepared. Here, first we show that how each problem of triangular fully fuzzy linear programming (as it has verified in (2)) turned to two problems of linear programming. Then, we show that, using function (3) we can turn the problem (1) to the problem (2). Now by using the method of nearest triangular estimation, we solve the obtained auxiliary problem, and then gain to an estimation from of the solution of the main problem using this response. Therefore, we consider the following problem which is consisted of symmetric triangular fuzzy numbers similar to the problem which is discussed in [3].

$$\begin{array}{ll} \max & (C_{\hat{c}}, \delta_{\hat{c}})(C_{\hat{x}}, \delta_{\hat{x}}) \\ s.t. & (C_{\hat{A}}, \delta_{\hat{A}})(C_{\hat{x}}, \delta_{\hat{x}}) = (c_{\hat{b}}, \delta_{\hat{b}}) \\ & (c_{\hat{x}}, \delta_{\hat{x}}) \\ & (c_{\hat{x}}, \delta_{\hat{x}}) \in \hat{N}.s.t \end{array} \quad (8)$$

We can see that the previous problem is a problem of Multi Objective Linear Programming (MOLP) that for maximizing of the objective functions in problem (8), we offer the maximization of problem for center and as minimizer problem for limits.

$$\begin{cases} \max & F_0(X) \\ \hat{x} \in s \end{cases} \quad (9)$$

$$\begin{cases} \min & F_1(x) \\ \hat{x} \in s \\ C_{\hat{c}\hat{x}} = a^* \end{cases} \quad (10)$$

where

$$s = \{\hat{x} \mid \hat{A}\hat{x} \approx \hat{b}, c_{\hat{x}} - w_{\hat{x}}^L \geq 0, \hat{x} \in \hat{N}.s.t\}.$$

That the core (center) response has ordinal priority rather than limits or tolerance response that a^* is an optimum value of target function (9). The final condition in (10) for this reason is considered that the optimum response (10) be set up in (9).

$$\max \quad C_{\hat{c}}C_{\hat{x}} + \frac{1}{4}C_{\hat{c}}w_{\hat{x}}^R + \frac{1}{4}C_{\hat{x}}w_{\hat{c}}^R + \frac{1}{6}w_{\hat{c}}^R w_{\hat{x}}^R - \frac{1}{4}C_{\hat{c}}w_{\hat{x}}^L - \frac{1}{4}C_{\hat{x}}w_{\hat{c}}^L + \frac{1}{6}w_{\hat{c}}^L w_{\hat{x}}^L$$

$$s.t. \quad C_{\hat{A}}C_{\hat{x}} + \frac{1}{4}C_{\hat{A}}w_{\hat{x}}^R + \frac{1}{4}C_{\hat{x}}w_{\hat{A}}^R + \frac{1}{6}w_{\hat{A}}^R w_{\hat{x}}^R - \frac{1}{4}C_{\hat{A}}w_{\hat{x}}^L - \frac{1}{4}C_{\hat{x}}w_{\hat{A}}^L + \frac{1}{6}w_{\hat{A}}^L w_{\hat{x}}^L = C_{\hat{b}} + \frac{1}{4}w_{\hat{b}}^R - \frac{1}{4}w_{\hat{b}}^L \qquad (11)$$

$$\frac{1}{2}C_{\hat{A}}w_{\hat{x}}^R + \frac{1}{2}C_{\hat{x}}w_{\hat{A}}^R + \frac{3}{8}w_{\hat{A}}^R w_{\hat{x}}^R + \frac{1}{2}C_{\hat{A}}w_{\hat{x}}^L + \frac{1}{2}C_{\hat{x}}w_{\hat{A}}^L - \frac{3}{8}w_{\hat{A}}^l w_{\hat{x}}^L = \frac{1}{2}w_{\hat{b}}^R - \frac{1}{2}w_{\hat{b}}^L$$

$$C_{\hat{x}} - w_{\hat{A}}^L \ge 0$$

$$w_{\hat{x}}^R \ge 0, \quad w_{\hat{x}}^L \ge 0.$$

In this case, the problem of (9) and (10) will be rewrite such as below:

$$\min \quad \frac{1}{2}c_{\hat{c}}w_{\hat{x}}^R + \frac{1}{2}C_{\hat{x}}w_{\hat{c}}^R + \frac{3}{8}w_{\hat{c}}^R w_{\hat{x}}^R + \frac{1}{2}C_{\hat{c}}w_{\hat{x}}^L + \frac{1}{2}C_{\hat{x}}w_{\hat{c}}^L - \frac{3}{8}w_{\hat{c}}^L w_{\hat{x}}^L$$

$$s.t. \quad C_{\hat{A}}C_{\hat{x}} + \frac{1}{4}C_{\hat{A}}w_{\hat{x}}^R + \frac{1}{4}C_{\hat{x}}w_{\hat{A}}^R + \frac{1}{6}w_{\hat{A}}^R w_{\hat{x}}^R - \frac{1}{4}C_{\hat{A}}w_{\hat{x}}^L - \frac{1}{4}C_{\hat{x}}w_{\hat{A}}^L + \frac{1}{6}w_{\hat{A}}^L w_{\hat{x}}^L$$

$$w_{\hat{x}}^L \ge 0.$$

$$= C_{\hat{b}} + \frac{1}{4}w_{\hat{b}}^R - \frac{1}{4}w_{\hat{b}}^L \qquad (12)$$

$$\frac{1}{2}C_{\hat{A}}w_{\hat{x}}^R + \frac{1}{2}C_{\hat{x}}w_{\hat{A}}^R + \frac{3}{8}w_{\hat{A}}^R w_{\hat{x}}^R + \frac{1}{2}C_{\hat{A}}w_{\hat{x}}^L - \frac{1}{2}C_{\hat{x}}w_{\hat{A}}^L - \frac{3}{8}w_{\hat{A}}^L w_{\hat{x}}^L = \frac{1}{2}w_{\hat{b}}^R - \frac{1}{2}w_{\hat{b}}^L$$

$$C_{\hat{x}} - w_{\hat{A}}^L \ge 0,$$

$$C_{\hat{c}\hat{x}} = a^*,$$

$$w_{\hat{x}}^R \ge 0,$$

As, a^* is the optimum value of objective function (11).

If problem (11) had a unique response $(C_{\hat{x}}, \delta_{\hat{x}})$, then one part of optimum answer (8) has attained.

If problem (11) had an alternative answer, $(C_{\hat{x}}, \delta_{\hat{x}})$ is the optimum answer of (8), also if the optimum answer be problem (12).

3.3. The definition of conversion function of triangular to trapezoidal

Here, by introducing of function T', we intend to convert every triangular fuzzy number to trapezoidal fuzzy number, it means that: $T' : F_t(R) \to F(R)$

$$R_i = \frac{b_i^U + b_i^L}{\hat{\beta} - \hat{\alpha}}, \qquad \gamma = \max\{R_i\}, \ 0 \le i \le n,$$

$$\begin{aligned} \tilde{x}_i^L &= \hat{x}_i^m - \gamma_i \\ \tilde{x}_i^U &= \hat{x}_i^m + \gamma_i \\ \tilde{x}_i^\alpha &= \hat{x}_i^\alpha - \gamma_i \\ \tilde{x}_i^\beta &= \hat{x}_i^\beta - \gamma_i \end{aligned} \qquad (13)$$

4. The nominal analyzing of problem

Example 4.1. A steel mill, manufacture iron beams in shape of "I" in four sizes; small, medium, large, very large. The rough length of manufactured beams by machine, approximately in every 50 to 70 minutes, is like this: (Table 1)

And also, the sale's profit of each one is roughly 13, 10, 15 and 11 dollars respectively. Furthermore, machines, roughly in 1 hour, produce approximately 500 and 600 foot of various sizes of beams respectively. The objective of scheduling the machines is maximization the profits. According to the viewpoint of the manager of the factory the fuzzy numbers with the following form:

$1\tilde{3}$=(11, 15, 3, 0) $1\tilde{0}$=(9, 11, 1, 2) $1\tilde{5}$=(13, 17, 2, 1) $1\tilde{1}$=(10, 12, 0, 1)

$\tilde{8}$=(7, 9, 1, 0) $1\tilde{2}$ (11, 13, 1, 1) $\tilde{7}$=(6, 8,1,0) $1\tilde{6}$=(14, 18, 4, 2)

$1\tilde{4}$=(12, 16, 0, 2) $1\tilde{8}$=(17, 19, 3, 0) $\widetilde{600}$=(550, 650, 100, 190) $\widetilde{500}$=(460, 540, 90, 130)

$$\tilde{C}=\begin{bmatrix}(11,15,3,0)\\(9,11,1,2)\\(13,17,2,1)\\(10,12,0,1)\end{bmatrix},$$

$$\tilde{A}=\begin{bmatrix}(7,9,1,0)&(11,13,1,1)&(13,17,3,0)&(14,16,3,1)\\(6,8,1,0)&(17,19,3,0)&(14,18,4,2)&(12,16,0,2)\end{bmatrix},$$

$$\tilde{b}=\begin{bmatrix}(460,540,90,130)\\(550,650,100,190)\end{bmatrix}.$$

The above problem can be formulated as follows:

$$\begin{aligned}&\max \quad (11,15,3,0)\tilde{x}_1+(9,11,1,2)\tilde{x}_2\\&+(13,17,2,1)\tilde{x}_3+(10,12,0,1)\tilde{x}_4\\&s.t. \quad (7,9,1,0)\tilde{x}_1+(11,13,1,1)\tilde{x}_2\\&+(13,17,3,0)\tilde{x}_3+(14,16,3,1)\tilde{x}_4\\&\cong(460,540,90,130)\\&\quad (6,8,1,0)\tilde{x}_1+(17,19,3,0)\tilde{x}_2+\\&(14,18,4,2)\tilde{x}_3+(12,16,0,2)\tilde{x}_4\\&\cong(550,650,100,190)\\&\quad \tilde{x}\geq\tilde{0}\end{aligned}$$

That under the function (3), turned to below data:

$$\hat{C}=\begin{bmatrix}(13,5,2)\\(10,2,3)\\(15,4,3)\\(11,1,2)\end{bmatrix},$$

$$\hat{A}=\begin{bmatrix}(8,2,1)&(12,2,2)&(15,5,2)&(15,4,2)\\(7,2,1)&(14,4,1)&(16,6,4)&(14,1,4)\end{bmatrix},$$

$$\hat{b}=\begin{bmatrix}(500,130,170)\\(600,150,240)\end{bmatrix}.$$

Table 1

Very large	Large	Medium	Small	product—machine
$\tilde{15}$	$\tilde{15}$	$\tilde{12}$	$\tilde{8}$	A
$\tilde{14}$	$\tilde{16}$	$\tilde{18}$	$\tilde{7}$	B

Then the related fully fuzzy linear programming problems with triangular numbers is:

$$\max \quad (13,5,2)\hat{x}_1 + (10,2,3)\hat{x}_2 + (15,4,3)\hat{x}_3 + (11,1,2)\hat{x}_4$$

$$s.t. \quad (8,2,1)\hat{x}_1 + (12,2,2)\hat{x}_2 + (15,5,2)\hat{x}_3 + (15,4,2)\hat{x}_4 \cong (500,130,170)$$

$$(7,2,1)\hat{x}_1 + (18,4,1)\hat{x}_2 + (16,6,4)\hat{x}_3 + (14,1,4)\hat{x}_4 \cong (600,150,240)$$

$$\hat{x} \geq \hat{0}$$

Now, by solving the problems (9) and (10) we have the following results:

The optimum answer of maximization problem is equal to $a_{\max} = 606.187$.

The optimum answer of minimization problem is equal to $a_{\min} = 141.484$.

The answer of auxiliary problem is equal with $\hat{a} = (606, 141, 141)$ and

$$\hat{x} = \begin{bmatrix} (28.78,18.84,9.02 \\ (26.01,0,0) \\ (0,0,0) \\ (0,0,0) \end{bmatrix}$$

That is a symmetric triangular fuzzy number. Now, using the function (13), we convert the solution to trapezoidal fuzzy number. Thus we obtain the solution as follows:

$$\tilde{x} = \begin{bmatrix} (28.7,31.8,15.7,5.9 \\ (26.01,26.01,0,0) \\ (0,0,0,0) \\ (0,0,0,0) \end{bmatrix}$$

5. Conclusion

In the common strategy for solving fully fuzzy linear programming usually, the authors had been used the classification functions to solve the problem of fuzzy linear programming. But, in this paper we use a new method without using any classification function and just by applying the most primarily fuzzy concepts to find the solution of the fuzzy programming problem. As well as the introduced functions in this paper, we can choose an appropriate fuzzy data for solving a fully fuzzy linear programming problem. We may use the mentioned approach in this paper for solving the models which is appeared in the real world when it consist of the trapezoidal fuzzy numbers in all parameters.

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